



**NET/JRF (DEC-2011)**

- Q2. Given that the ground state energy of the hydrogen atom is  $-13.6 \text{ eV}$ , the ground state energy of positronium (which is a bound state of an electron and a positron) is  
 (a)  $+6.8 \text{ eV}$                       (b)  $-6.8 \text{ eV}$                       (c)  $-13.6 \text{ eV}$                       (d)  $-27.2 \text{ eV}$

Ans: (b)

Solution: The energy expression for Positronium atom is  $E_n = -\frac{13.6}{2n^2} (\text{eV})$

$$\text{For } n = 1, E_1 = \frac{-13.6}{2} (\text{eV}) = -6.8 \text{ eV}, \quad \therefore E_1 = -6.8 \text{ eV}$$

- Q3. A laser operating at  $500 \text{ nm}$  is used to excite a molecule. If the Stokes line is observed at  $770 \text{ cm}^{-1}$ , the approximate positions of the Stokes and the anti-Stokes lines are  
 (a)  $481.5 \text{ nm}$  and  $520 \text{ nm}$                       (b)  $481.5 \text{ nm}$  and  $500 \text{ nm}$   
 (c)  $500 \text{ nm}$  and  $520 \text{ nm}$                       (d)  $500 \text{ nm}$  and  $600 \text{ nm}$

Ans:

Solution: Given  $\lambda_0 = 500 \text{ nm} = 5 \times 10^{-5} \text{ cm}$ ,  $\bar{\nu}_{\text{stoke}} = 770 \text{ cm}^{-1} \therefore \bar{\nu}_0 = 20,000 \text{ cm}^{-1}$

$$\text{Raman shift } \Delta\bar{\nu} = \bar{\nu}_0 - \bar{\nu}_{\text{stoke}} = 19230 \text{ cm}^{-1}$$

$$\text{Wave number of anti-stokes line is } \bar{\nu}_{\text{anti-stoke}} = \Delta\bar{\nu} + \bar{\nu}_0 = 39,230 \text{ cm}^{-1}$$

$$\text{In wavelength term } \lambda_{\text{anti-stoke}} = 2.549 \times 10^{-7} = 254.9 \text{ nm} \text{ and } \lambda_{\text{stoke}} = 12987 \text{ nm}$$

- Q4. If the hyperfine interaction in an atom is given by  $H = a\vec{S}_e \cdot \vec{S}_p$  where  $\vec{S}_e$  and  $\vec{S}_p$  denote the electron and proton spins, respectively, the splitting between the  $^3S_1$  and  $^1S_0$  state is  
 (a)  $a\hbar^2 / \sqrt{2}$                       (b)  $a\hbar^2$                       (c)  $a\hbar^2 / 2$                       (d)  $2a\hbar^2$

Ans: (b)

Solution: Total spin is  $\vec{S} = \vec{S}_e + \vec{S}_p \Rightarrow S^2 = S_e^2 + S_p^2 + 2\vec{S}_e \cdot \vec{S}_p \Rightarrow \vec{S}_e \cdot \vec{S}_p = \frac{1}{2} [S^2 - S_e^2 - S_p^2]$

$$\Rightarrow H = a\vec{S}_e \cdot \vec{S}_p = \frac{a}{2} [S^2 - S_e^2 - S_p^2] \text{ where } S_e^2 = S_p^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$$

$$\Rightarrow H = \frac{a}{2} \left( S^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2 \right) = \frac{a}{2} \left[ S^2 - \frac{3}{2}\hbar^2 \right]$$

$$\text{For } ^3S_1 : S = 1 \Rightarrow S^2 = S(S+1)\hbar^2 = 2\hbar^2, \text{ For } ^1S_0 : S = 0 \Rightarrow S^2 = S(S+1)\hbar^2 = 0\hbar^2$$

$$\therefore H_1 = \frac{a}{2} \left[ 2 - \frac{3}{2} \right] \hbar^2 = \frac{a}{4} \hbar^2 \text{ for } ^3S_1 \text{ and } H_2 = \frac{a}{2} \left[ 0 - \frac{3}{2} \right] \hbar^2 = -\frac{3}{4} a\hbar^2 \text{ for } ^1S_0$$

$\therefore$  The splitting between  $^3S_1$  and  $^1S_0$  is  $\Delta H = H_1 - H_2 = \left(\frac{1}{4} + \frac{3}{4}\right) a\hbar^2 = a\hbar^2$

Q5. The ratio of intensities of the  $D_1$  and  $D_2$  lines of sodium at high temperature is

- (a) 1:1                      (b) 2:3                      (c) 1:3                      (d) 1:2

Ans: (d)

Solution: The electronic transition for  $D_2$  and  $D_1$  line is

$$D_2: ^2P_{3/2} \rightarrow ^2S_{1/2}, D_1: ^2P_{1/2} \rightarrow ^2S_{1/2} \therefore \frac{I(D_2)}{I(D_1)} = \frac{2J_2 + 1}{2J_1 + 1} = \frac{2 \times \frac{3}{2} + 1}{2 \times \frac{1}{2} + 1} = \frac{4}{2} = \frac{2}{1}$$

Q6. An atom of mass  $M$  can be excited to a state of mass  $(M + \Delta)$  by photon capture. The frequency of a photon which can cause this transition is

- (a)  $\frac{\Delta c^2}{2h}$                       (b)  $\frac{\Delta c^2}{h}$                       (c)  $\frac{\Delta^2 c^2}{2Mh}$                       (d)  $\frac{\Delta c^2}{2Mh} (\Delta + 2M)$

Ans: (d)

Solution: The conservation law of energy and momentum give

$$Mc^2 + h\nu = \left[ (M + \Delta)^2 c^4 + p^2 c^2 \right]^{1/2} \text{ and } \frac{h\nu}{c} = p$$

$$M^2 c^4 + h^2 \nu^2 + 2Mc^2 h\nu = M^2 c^4 + \Delta^2 c^4 + 2M\Delta c^4 + h^2 \nu^2 \Rightarrow 2Mc^2 h\nu = \Delta^2 c^4 + 2M\Delta c^4$$

$$\Rightarrow 2Mc^2 h\nu = 2M\Delta c^4 \left( 1 + \frac{\Delta}{2M} \right) \Rightarrow \nu = \frac{\Delta c^2}{h} \left( 1 + \frac{\Delta}{2M} \right) \Rightarrow \nu = \frac{\Delta c^2}{2Mh} (\Delta + 2M).$$

### NET/JRF (JUNE-2012)

Q7. The first absorption spectrum of  $^{12}\text{C}^{16}\text{O}$  is at  $3.842 \text{ cm}^{-1}$  while that of  $^{13}\text{C}^{16}\text{O}$  is at  $3.673 \text{ cm}^{-1}$ . The ratio of their moments of inertia is

- (a) 1.851                      (b) 1.286                      (c) 1.046                      (d) 1.038

Ans: (c)

Solution: For  $^{12}\text{C}^{16}\text{O}$ :  $2B_1 = 3.842 \text{ cm}^{-1} \Rightarrow B_1 = 1.921 \text{ cm}^{-1}$

For  $^{13}\text{C}^{16}\text{O}$ :  $2B_2 = 3.673 \text{ cm}^{-1} \Rightarrow B_2 = 1.8365 \text{ cm}^{-1}$

Where,  $B = \frac{h}{8\pi^2 IC}$   $\therefore \frac{I_2}{I_1} = \frac{B_1}{B_2} = \frac{1.921}{1.8365} = 1.046$

Q8. The spin-orbit interaction in an atom is given by  $H = a\vec{L} \cdot \vec{S}$ , where  $\vec{L}$  and  $\vec{S}$  denote the orbital and spin angular momenta, respectively, of the electron. The splitting between the levels  ${}^2P_{3/2}$  and  ${}^2P_{1/2}$  is

- (a)  $\frac{3}{2}a\hbar^2$                       (b)  $\frac{1}{2}a\hbar^2$                       (c)  $3a\hbar^2$                       (d)  $\frac{5}{2}a\hbar^2$

Ans. : (a)

Solution: Given  $H = a\vec{L} \cdot \vec{S}$  where  $\vec{J} = \vec{L} + \vec{S}$ .

$$\therefore J^2 = L^2 + S^2 + 2\vec{L} \cdot \vec{S} \Rightarrow \vec{L} \cdot \vec{S} = \frac{1}{2}[J^2 - L^2 - S^2] \therefore H = \frac{a}{2}[J^2 - L^2 - S^2]$$

For  ${}^2P_{3/2}$  :  $S = \frac{1}{2}$  which gives  $S^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$

$L = 1$  which gives  $L^2 = L(L+1)\hbar^2 = 2\hbar^2$

$J = \frac{3}{2}$  which gives  $J^2 = J(J+1)\hbar^2 = \frac{15}{4}\hbar^2 \therefore H_1 = \frac{a}{2}\left[\frac{15}{4} - 2 - \frac{3}{4}\right]\hbar^2 = \frac{a}{2}\hbar^2$

For  ${}^2P_{1/2}$  :  $S = \frac{1}{2}$  which gives  $S^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$

$L = 1$  which gives  $L^2 = L(L+1)\hbar^2 = 2\hbar^2$

$J = \frac{1}{2}$  which gives  $J^2 = J(J+1)\hbar^2 = \frac{3}{4}\hbar^2 \therefore H_2 = \frac{a}{2}\left[\frac{3}{4} - 2 - \frac{3}{4}\right]\hbar^2 = -a\hbar^2$

$\therefore \Delta H = H_1 - H_2 = \frac{a}{2}\hbar^2 + a\hbar^2 = \frac{3}{2}a\hbar^2$

Q9. The spectral line corresponding to an atomic transition from  $J = 1$  to  $J = 0$  states splits in a magnetic field of 0.1 Tesla into three components separated by  $1.6 \times 10^{-3} \text{ \AA}$ . If the zero field spectral line corresponds to  $1849 \text{ \AA}$ , what is the g-factor corresponding to the  $J = 1$

state? (You may use  $\frac{hc}{\mu_0} \approx 2 \times 10^4 \text{ cm}$ )

- (a) 2                      (b) 3/2                      (c) 1                      (d) 1/2

Ans: (c)

Solution: The Zeeman splitting is  $\Delta E = (gM_J)(\mu_B B) = g\mu_B B$  for  $M_J = +1$

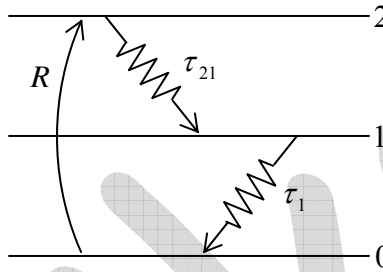
Given, Zeeman splitting separations,  $\Delta\lambda = \frac{\lambda^2}{c} \Delta\nu = 1.6 \times 10^{-3} \text{ \AA}$

$$\Rightarrow \Delta \nu = \frac{c}{\lambda^2} \Delta \lambda = \frac{3 \times 10^8}{(1849 \times 10^{-10})^2} \times 1.6 \times 10^{-1} = 0.1404 \times 10^{10}$$

$$\therefore g = \frac{\Delta E}{\mu_B B} = \frac{6.625 \times 10^{-34} \times 0.1404 \times 10^{10}}{9.27 \times 10^{-24} \times 0.1} = 1.00$$

### NET/JRF (DEC-2012)

Q10. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate  $R$  is  $10^{20}$  atoms  $\text{cm}^{-3} \text{s}^{-1}$  and the decay routes are as shown with  $\tau_{21} = 20$  ns and  $\tau_1 = 1 \mu\text{s}$ , the equilibrium populations of states 2 and 1 are, respectively,

- (a)  $10^{14} \text{ cm}^{-3}$  and  $2 \times 10^{12} \text{ cm}^{-3}$       (b)  $2 \times 10^{12} \text{ cm}^{-3}$  and  $10^{14} \text{ cm}^{-3}$ .  
 (c)  $2 \times 10^{12} \text{ cm}^{-3}$  and  $2 \times 10^6 \text{ cm}^{-3}$       (d) zero and  $10^{20} \text{ cm}^{-3}$

Ans: (b)

Solution:  $\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}}$  and  $\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1}$ .

Under equilibrium condition  $\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$

$$\Rightarrow N_2 = \tau_{21} R = 10^{20} \times 20 \times 10^{-9} = 2 \times 10^{12} \text{ cm}^{-3}$$

$$\Rightarrow N_1 = \frac{\tau_1 N_2}{\tau_{21}} = \frac{10^{-6} \times 2 \times 10^{12} \text{ cm}^{-3}}{20 \times 10^{-9}} = 10^{14} \text{ cm}^{-3}$$

Q11. Consider a hydrogen atom undergoing a  $2P \rightarrow 1S$  transition. The lifetime  $t_{sp}$  of the  $2P$  state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium  $n_0 = 1$ , the ratio of Einstein coefficients for stimulated and spontaneous emission  $B_{21}(\omega)/A_{21}(\omega)$  is given by

- (a)  $0.683 \times 10^{12} \text{ m}^3 \text{J}^{-1} \text{s}^{-1}$       (b)  $0.146 \times 10^{-12} \text{ Jsm}^{-3}$ .  
 (c)  $6.83 \times 10^{12} \text{ m}^3 \text{J}^{-1} \text{s}^{-1}$       (d)  $1.463 \times 10^{-12} \text{ Jsm}^{-3}$ .

Ans: (a)

Solution:  $n_0 = 1$ ,  $\Delta E = 10.2 \text{ eV}$  and  $\frac{B_{21}}{A_{21}} = \frac{\pi^2 c^3}{\hbar \omega^3 n_0^3} = \frac{\hbar^2 \pi^2 c^3}{(\Delta E)^3 n_0^3} = 0.67 \times 10^{12}$ .

Q12. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities  $R_1 = 1$  and  $R_2 = 0.98$ . The mirrors are separated by a distance  $d = 20 \text{ cm}$  and the medium in between has a refractive index  $n_0 = 1$  and absorption coefficient  $\alpha = 0$ . The values of the separation between the modes  $\delta\nu$  and the width  $\Delta\nu_p$  of each mode of the laser cavity are:

- (a)  $\delta\nu = 75 \text{ kHz}$ ,  $\Delta\nu_p = 24 \text{ kHz}$                       (b)  $\delta\nu = 100 \text{ kHz}$ ,  $\Delta\nu_p = 100 \text{ kHz}$   
 (c)  $\delta\nu = 750 \text{ MHz}$ ,  $\Delta\nu_p = 2.4 \text{ MHz}$                       (d)  $\delta\nu = 2.4 \text{ MHz}$ ,  $\Delta\nu_p = 750 \text{ MHz}$

Ans: (c)

Solution: Mode separation  $\delta\nu = \frac{c}{2dn_0} = 750 \text{ MHz}$

where  $c = 3 \times 10^8 \text{ m/sec}$ ,  $d = 20 \times 10^{-2} \text{ m}$  and  $n_0 = 1$ .

Width of each mode  $\delta\nu_p = \frac{1}{2\pi t_c}$  where  $t_c = \frac{2n_0 d}{c \ln\left(\frac{1}{R_1 R_2 e^{-2\alpha d}}\right)}$ .

**Note:** In this question, there is no need to calculate  $\delta\nu_p$ . Since in the given options there is only one option with  $\delta\nu = 750 \text{ MHz}$  and i.e., option (c). You can calculate  $\delta\nu_p$  without calculator so use your common sense.

### NET/JRF (JUNE-2013)

Q13. A muon ( $\mu^-$ ) from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be

- (a)  $5.62 \text{ \AA}$                       (b)  $6.67 \text{ \AA}$                       (c)  $3.75 \text{ \AA}$                       (d)  $13.3 \text{ \AA}$

Ans: (b)

Solution: In case of muonic atom, the reduce mass is  $m' = \frac{m_\mu m_p}{m_\mu + m_p} = 180 m_e$

$$E'_n = \left(\frac{m'}{m_e}\right) \frac{E_1}{n^2} = 180 \frac{E_1}{n^2} \quad \text{where, } E_1 = -13.6 \text{ eV}$$

For ground state of muonic atom  $n=1$ ,  $E'_1 = 180E_1$

For first excited state of muonic atom  $n=2$ ,  $E'_2 = 45E_1$

The longest wavelength of the photon corresponds to the transition between first and ground state of muonic atom.

The energy difference between first excited and ground state is

$$\Delta E = E'_2 - E'_1 = -135E_1 = 1836 \text{ eV} = 2938 \times 10^{-19} \text{ J}$$

In term of wavelength

$$\Delta E = hv = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2938 \times 10^{-19}} = 6.67 \times 10^{-10} \text{ m} = 6.67 \text{ \AA}$$

Q14. Consider the hydrogen-deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is  $938 \text{ MeV}/c^2$ , then the energy difference  $\Delta E$  between the two lowest rotational states is approximately

- (a)  $10^{-1} \text{ eV}$                       (b)  $10^{-2} \text{ eV}$                       (c)  $2 \times 10^{-2} \text{ eV}$                       (d)  $10^{-3} \text{ eV}$

Ans: (b)

Solution: Rotational energy expression  $E = \frac{h^2}{8\pi^2 I} J(J+1) = AJ(J+1)$

Difference between two lowest energy levels is  $\Delta E = 2A$

where  $A = \frac{h^2}{8\pi^2 I} = \frac{\hbar^2}{2I}$  and  $I = \mu r^2$

Here,  $\mu = \frac{M_H M_D}{M_H + M_D} = \frac{M_H \times 2M_H}{M_H + 2M_H} = \frac{2}{3} M_H$ ,                       $\mu = \frac{2}{3} \times 938 \text{ MeV}/c^2$

$$\hbar = 1.01 \times 10^{-34} \text{ (J-sec)} = 1.01 \times 10^{-34} \times \frac{1}{1.6} \times 10^{19} \text{ (eV-sec)} = 6.3 \times 10^{-16} \text{ (eV-sec)}$$

$$\therefore \Delta E = \frac{\hbar^2}{I} = \frac{(6.3 \times 10^{-16} \text{ eV-sec})^2}{\frac{2}{3} \times 938 \times 10^6 \text{ eV}/c^2 \times (0.08 \times 10^{-9} \text{ m})^2} = 9.2 \times 10^{-3} \text{ eV} \cong 10^{-2} \text{ eV}$$

Q15. The electronic energy levels in a hydrogen atom are given by  $E_n = -13.6/n^2 \text{ eV}$ . If a selective excitation to the  $n=100$  level is to be made using a laser, the maximum allowed frequency line-width of the laser is

- (a) 6.5 MHz                      (b) 6.5 GHz                      (c) 6.5 Hz                      (d) 6.5 kHz





- Q19. The spectroscopic symbol for the ground state of  ${}_{13}\text{Al}$  is  ${}^2P_{1/2}$ . Under the action of a strong magnetic field (when  $L-S$  coupling can be neglected) the ground state energy level will split into
- (a) 3 levels                      (b) 4 levels                      (c) 5 levels                      (d) 6 levels

Ans: (c)

Solution: In extremely strong magnetic field coupling between L-S breaks down.  $J$  is no longer a valid quantum number.

The ground state energy level will have  $(2S+1)(2L+1)=6$  states, but 2 states are degenerate. Therefore, total energy levels will be 5.

### NET/JRF (JUNE-2014)

- Q20. A spectral line due to a transition from an electronic state  $p$  to an  $s$  state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is
- (a) 10                      (b) 3                      (c) 6                      (d) 9

Ans: (a)

Solution: For  $p$  state:  $l = 1, s = 1/2 : j = 1/2 \text{ \& } 3/2$ .

This gives two spectral terms  ${}^2P_{3/2}$  &  ${}^2P_{1/2}$

For  $s$  state:  $l = 0, s = 1/2 : j = 1/2$  : This gives spectral terms  ${}^2S_{1/2}$

The terms  ${}^2P_{3/2}$  and  ${}^2S_{1/2}$  corresponding to  $J = 3/2$  &  $J = 1/2$  will break into  $2J+1$  Zeeman levels, which is 4 and 2 respectively.

$$\text{For } {}^2P_{3/2} \quad M_j = -3/2 \quad -1/2 \quad +1/2 \quad +3/2$$

$$\text{For } {}^2S_{1/2} \quad M_j = -1/2 \quad +1/2$$

The selection rule is  $\Delta M_j = 0, \pm 1$  ( $M_j = 0 \rightarrow M_j = 0$  if  $\Delta J = 0$ )

$\Delta M_j = 0$  gives  $\pi$  component,  $\Delta M_j = \pm 1$  gives  $\sigma$  component

Number of  $\pi$  component = 2, Numbers of  $\sigma^+$  components = 2

Number of  $\sigma^-$  components = 2

The terms  ${}^2P_{1/2}$  and  ${}^2S_{1/2}$  corresponding to  $J = 1/2$  &  $J = 1/2$  will break into  $2J+1$  Zeeman levels, which is 2 & 2 respectively.

$$\text{For } {}^2P_{1/2} \quad M_j = -1/2 \quad +1/2, \text{ For } {}^2S_{1/2} \quad M_j = -1/2 \quad +1/2$$

The selection rule is  $\Delta M_J = 0, \pm 1$  ( $M_J = 0 \rightarrow M_J = 0$  If  $\Delta J = 0$ )

$\Delta M_J = 0$  gives  $\pi$  component,  $\Delta M_J = \pm 1$  gives  $\sigma$  component

Number of  $\pi$  component = 2, Numbers of  $\sigma^+$  components = 1

Number of  $\sigma^-$  components = 1

Thus, total number of Zeeman component = 10

Q21. A double slit interference experiment uses a laser emitting light of two adjacent frequencies  $\nu_1$  and  $\nu_2$  ( $\nu_1 < \nu_2$ ). The minimum path difference between the interfering beams for which the interference pattern disappears is

- (a)  $\frac{c}{\nu_2 + \nu_1}$       (b)  $\frac{c}{\nu_2 - \nu_1}$       (c)  $\frac{c}{2(\nu_2 - \nu_1)}$       (d)  $\frac{c}{2(\nu_2 + \nu_1)}$

Ans: (c)

Solution: The condition of maximum intensity for interfering laser beam is:  $d \sin \theta = n\lambda$

The condition of dark intensity for interfering laser beam is:  $d \sin \theta = \left(n + \frac{1}{2}\right)\lambda$

For interference pattern to vanish, the minimum path difference should be  $\lambda/2$

The spectral bandwidth of laser is defined as,  $\Delta \nu = \frac{c\Delta\lambda}{\lambda^2} \Rightarrow \frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda}$

For two closely spaced line of wavelength  $\lambda_1$  and  $\lambda_2$

$$\frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda} = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} = \frac{1}{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}} = \frac{c}{\frac{c}{\lambda_2} - \frac{c}{\lambda_1}} = \frac{c}{\nu_2 - \nu_1}$$

Since, for interference pattern to vanish for two closely spaced line of wavelength  $\lambda_1$  and

$\lambda_2$ , the minimum path difference should be  $= \frac{\lambda}{2} = \frac{c}{2\Delta\nu} = \frac{c}{2(\nu_2 - \nu_1)}$

Q22. How much does the total angular momentum quantum number  $J$  change in the transition of  $Cr(3d^6)$  atom as it ionize to  $Cr^{2+}(3d^4)$ ?

- (a) Increases by 2      (b) Decreases by 2      (c) Decreases by 4      (d) Does not change

Ans: (c)

Solution: In  $Cr(3d^6)$  state

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$

↑	↓	↑	↑	↑	↑
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In this configuration,  $S = 4 \times \frac{1}{2} = 2$  and  $L = 2$

This is the case of more than half filled subshell, thus state with highest  $J$  value will have the lowest energy.

The highest  $J$  - value is  $J = L + S = 4$

Thus, the ground state spectral term for this configuration is  ${}^5D_4$ .

Now in  $Cr^{2+}(3d^4)$

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$



In this configuration,  $S = 4 \times \frac{1}{2} = 2$  and  $L = 2$

Since this is the case of less than half filled subshell, thus, state with lowest  $J$  value will have the lowest energy.

The lowest  $J$  - value is  $J = L - S = 2 - 2 = 0$ .

Thus the ground state spectral term for this configuration is the  $J$  - value decreases from  $J = 4$  to  $J = 0$ .

Thus correct answer is option (c).

### NET/JRF (DEC-2014)

- Q23. An atomic transition  ${}^1P \rightarrow {}^1S$  in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton  $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$ , and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately
- (a) 0.01 nm      (b) 0.1 nm      (c) 1.0 nm      (d) 10 nm

Ans: (a)

Solution: This is the case of Normal Zeeman effect. The Zeeman separation in terms of frequency,  $\Delta\nu = \frac{\mu_B B}{h}$ , where  $\mu_B$  is Bohr magneton

$$\text{In terms of wavelength, } \Delta\lambda = \frac{\lambda^2}{c} \Delta\nu = \frac{\lambda^2}{c} \times \frac{\mu_B B}{h}$$

$$\Delta\lambda = \frac{(250 \times 10^{-9} \text{ m})^2 \times (9.27 \times 10^{-24} \text{ J/T})(1\text{T})}{(3 \times 10^8 \text{ m/s})(6.625 \times 10^{-34} \text{ Js})} \cong 3 \times 10^{-12} \text{ m} \cong 0.003 \text{ nm}$$

None of the answer is matching correctly. But best suitable answer is option (a)

- Q24. If the leading anharmonic correction to the energy of  $n^{\text{th}}$  vibrational level of a diatomic molecule is  $-x_e \left( n + \frac{1}{2} \right)^2 \hbar \omega$  with  $x_e = 0.001$ , the total number of energy levels possible is approximately
- (a) 500                      (b) 1000                      (c) 250                      (d) 750

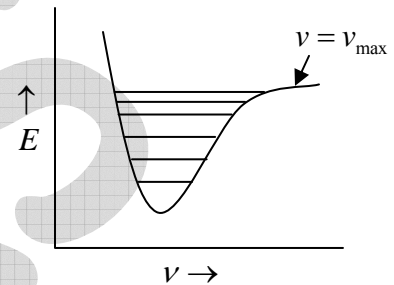
Ans: (a)

Solution: The energy of anharmonic oscillator is  $E_v = \left( v + \frac{1}{2} \right) \hbar \omega - x_e \left( v + \frac{1}{2} \right)^2 \hbar \omega$

where  $v = 0, 1, 2, \dots, v_{\text{max}}$  is vibrational quantum number

$$\text{Now, } \left. \frac{dE_v}{dv} \right|_{v=v_{\text{max}}} = 0 \Rightarrow \hbar \omega - 2x_e \left( v_{\text{max}} + \frac{1}{2} \right) \hbar \omega = 0$$

$$\Rightarrow 1 = 2x_e \left( v_{\text{max}} + \frac{1}{2} \right) \Rightarrow v_{\text{max}} = \frac{1}{2x_e} - \frac{1}{2} \cong \frac{1}{2x_e} = \frac{1}{2 \times 0.001} = 500$$



- Q25. The effective spin-spin interaction between the electron spin  $\vec{S}_e$  and the proton spin  $\vec{S}_p$  in the ground state of the Hydrogen atom is given by  $H' = a\vec{S}_e \cdot \vec{S}_p$ . As a result of this interaction, the energy levels split by an amount
- (a)  $\frac{1}{2} a\hbar^2$                       (b)  $2a\hbar^2$                       (c)  $a\hbar^2$                       (d)  $\frac{3}{2} a\hbar^2$

Ans: (c)

Solution: The Hamiltonian is given as  $H = a\vec{S}_e \cdot \vec{S}_p$  where  $S_e$  and  $S_p$  are electron and proton spin.

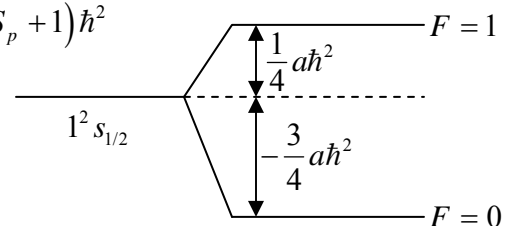
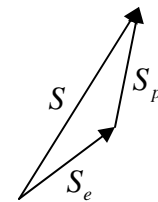
The coupling between  $S_e$  and  $S_p$  gives net resultant spin angular momentum

$$\vec{S} = \vec{S}_e + \vec{S}_p, \quad S^2 = S_e^2 + S_p^2 + 2\vec{S}_e \cdot \vec{S}_p \Rightarrow \vec{S}_e \cdot \vec{S}_p = \frac{1}{2} (S^2 - S_e^2 - S_p^2)$$

$$\therefore H = \frac{a}{2} (S^2 - S_e^2 - S_p^2)$$

where  $S^2 = S(S+1)\hbar^2$ ,  $S_e^2 = S_e(S_e+1)\hbar^2$ ,  $S_p^2 = S_p(S_p+1)\hbar^2$

Since  $S_e = \frac{1}{2}$  and  $S_p = \frac{1}{2} \Rightarrow S = 0, 1$



For  $S = 0$  (singlet state),  $H_1 = \frac{a}{2} \left( 0 - \frac{3}{4} - \frac{3}{4} \right) \hbar^2 = -\frac{3}{4} a \hbar^2$

For  $S = 1$  (Triplet state),  $H_2 = \frac{a}{2} \left( 2 - \frac{3}{4} \hbar^2 - \frac{3}{4} \hbar^2 \right) = \frac{1}{4} a \hbar^2$

$\therefore \Delta H = H_2 - H_1 = \frac{1}{4} a \hbar^2 + \frac{3}{4} a \hbar^2 \Rightarrow \Delta H = a \hbar^2$

### NET/JRF (JUNE-2015)

Q26. Of the following term symbols of the  $np^2$  atomic configurations,  $^1S_0$ ,  $^3P_0$ ,  $^3P_1$ ,  $^3P_2$  and  $^1D_2$  which is the ground state?

- (a)  $^3P_0$                       (b)  $^1S_0$                       (c)  $^3P_2$                       (d)  $^3P_1$

Ans. (a)

Solution: According to Hund's rules

- (i) State with highest multiplicity has lowest energy
- (ii) State with same multiplicity, the state with highest  $L$  will have lowest energy
- (iii) State with same multiplicity and  $L$  value. The state with lowest  $J$  has lowest energy (only if subshell is less than half filled) from the given states  $^1S_0$ ,  $^3P_0$ ,  $^3P_1$ ,  $^3P_2$ ,  $^1D_2$

The  $^3P_0$  will have the lowest energy

Q27. A diatomic molecule has vibrational states with energies  $E_v = \hbar\omega \left( v + \frac{1}{2} \right)$  and rotational states with energies  $E_j = B j(j+1)$ , where  $v$  and  $j$  are non-negative integers. Consider the transitions in which both the initial and final states are restricted to  $v \leq 1$  and  $j \leq 2$  and subject to the selection rules  $\Delta v = \pm 1$  and  $\Delta j = \pm 1$ . Then the largest allowed energy of transition is

- (a)  $\hbar\omega - 3B$                       (b)  $\hbar\omega - B$                       (c)  $\hbar\omega + 4B$                       (d)  $2\hbar\omega + B$

Ans. (c)

Solution:  $E = \hbar\omega \left( v + \frac{1}{2} \right) + B J(J+1)$

For vibrational transition with  $\Delta v = \pm 1$  and rotational transition with  $\Delta J = \pm 1$  we get

$\Delta E = E_{\text{initial}} - E_{\text{final}} = \hbar\omega + 2B(J+1)$

where  $J$  is lowest quantum number

According to question in rotational states, two transition is possible, one for  $J = 0 \rightarrow J = 1$  and second for  $J = 1 \rightarrow J = 2$

But, second transition will have photon of higher energy

$\therefore$  For  $J = 1 \rightarrow J = 2$

$$\Delta E = \hbar\omega + 2B(1+1) = \hbar\omega + 4B$$

### NET/JRF (DEC-2015)

Q28. The  $LS$  configurations of the ground state of  $^{12}Mg$ ,  $^{13}Al$ ,  $^{17}Cl$  and  $^{18}Ar$  are respectively,

(a)  $^3S_1$ ,  $^2P_{1/2}$ ,  $^2P_{1/2}$  and  $^1S_0$

(b)  $^3S_1$ ,  $^2P_{3/2}$ ,  $^2P_{3/2}$  and  $^3S_1$

(c)  $^1S_0$ ,  $^2P_{1/2}$ ,  $^2P_{3/2}$  and  $^1S_0$

(d)  $^1S_0$ ,  $^2P_{3/2}$ ,  $^2P_{1/2}$  and  $^3S_1$

Ans.: (c)

Solution:  $^{12}Mg : 1s^2 2s^2 2p^6 3s^2$

Ground state:  $^1S_0$

$^{13}Al : 1s^2 2s^2 2p^6 3s^2 3p^1$

the terms are  $2p_{1/2}$  and  $2p_{3/2}$ . Since its less than half filled. Thus

Ground state:  $2p_{1/2}$

$^{17}Cl : 1s^2 2s^2 2p^6 3s^2 3p^5$

The terms are  $2p_{1/2}$  and  $2p_{3/2}$ . Since it shell is more than half filled.

Thus Ground state:  $2p_{3/2}$

$^{18}Ar : 1s^2 2s^2 2p^6 3s^2 3p^6$

Ground state:  $^1S_0$

Q29. For a two level system, the population of atoms in the upper and lower levels are  $3 \times 10^{18}$  and  $0.7 \times 10^{18}$ , respectively. If the coefficient of stimulated emission is  $3.0 \times 10^5 m^3 / W \cdot s^3$  and the energy density is  $9.0 J / m^3 \cdot Hz$ , the rate of stimulated emission will be

(a)  $6.3 \times 10^{16} s^{-1}$

(b)  $4.1 \times 10^{16} s^{-1}$

(c)  $2.7 \times 10^{16} s^{-1}$

(d)  $1.8 \times 10^{16} s^{-1}$

Ans.: None of the answer is matching.

Solution: The rate of stimulated emission is

$$\frac{dN_2}{dt} = N_2 B_{21} u(\omega)$$

$$\text{where } N_2 = 3 \times 10^{18}, B_{21} = 3 \times 10^5 \frac{m^3}{W - s^3} = 3 \times 10^5 \frac{m^3}{J - s^2}$$

$$\text{and } u(\omega) = 9.0 \text{ J} / m^3 - \text{Hz} = 9.0 \frac{J - s}{m^3}$$

$$\frac{dN_2}{dt} = 3 \times 10^{18} \times 3 \times 10^5 \frac{m^3}{J - s^2} \times 9 \frac{J - s}{m^3} = 8.1 \times 10^{24} s^{-1}$$

Q30. The first ionization potential of  $K$  is  $4.34 \text{ eV}$ , the electron affinity of  $Cl$  is  $3.82 \text{ eV}$  and the equilibrium separation of  $KCl$  is  $0.3 \text{ nm}$ . Then energy required to dissociate a  $KCl$  molecule into a  $K$  and a  $Cl$  atom is

- (a)  $8.62 \text{ eV}$                       (b)  $8.16 \text{ eV}$                       (c)  $4.28 \text{ eV}$                       (d)  $4.14 \text{ eV}$

Ans.: (c)

Solution: Energy required to dissociate  $KCl$  is  $(KCl \rightarrow K^+ + Cl^-)$

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}} = \left( 9 \times 10^9 \frac{Nm^2}{c^2} \right) \frac{(1.6 \times 10^{-19} c)^2}{0.3 \times 10^{-9} m} = 7.7 \times 10^{-19} \text{ J} = 4.79 \text{ eV}$$

The bond dissociation energy is the energy required to dissociate a molecule into its component atom  $KCl \rightarrow K + Cl$

To find the energy required to dissociate  $KCl$  into  $K$  and  $Cl$ , we must add an electron to the  $K^+$  ion, which releases the atomic potassium ionization energy. Remove one electron from  $Cl^-$  ion which requires the atomic chlorine electron affinity energy

Given ionization energy of  $K = E_{ie} = 4.34 \text{ eV}$

Electron affinity of  $Cl = E_{ai} = 3.82 \text{ eV}$

Thus the energy gained in the transformation from ion to atom is

$$= E_{ie} - E_{ai} = 4.34 - 3.82 = 0.52 \text{ eV}$$

Thus dissociation energy  $= 4.79 - 0.52 = 4.27 \text{ eV}$

**NET/JRF (JUNE-2016)**

Q31. The ground state electronic configuration of  $^{22}\text{Ti}$  is  $[\text{Ar}]3d^24s^2$ . Which state, in the standard spectroscopic notations, is not possible in this configuration?

- (a)  $^1F_3$                       (b)  $^1S_0$                       (c)  $^1D_2$                       (d)  $^3P_0$

Ans: (a)

Solution: The spectroscopic term for  $d^2$  are obtained as

$$l_1 = 2, l_2 = 2 \Rightarrow L = 4, 3, 2, 1, 0$$

$$s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \Rightarrow S = 0, 1$$

Now,  $S = 0, L = 4 \Rightarrow J = 4, ^1G_4$

$$S = 0, L = 2 \Rightarrow J = 2, ^1D_2$$

$$S = 0, L = 0 \Rightarrow J = 0, ^1S_0$$

$$S = 1, L = 3 \Rightarrow J = 4, 3, 2, ^3F_4, ^3F_3, ^3F_2$$

$$S = 1, L = 1 \Rightarrow J = 2, 1, 0, ^3P_2, ^3P_1, ^3P_0$$

Thus  $^1F_3$  is not possible spectroscopic term of  $[\text{Ar}]3d^24s^2$

Q32. In a normal Zeeman Effect experiment using a magnetic field of strength  $0.3\text{ T}$ , the splitting between the components of a  $660\text{ nm}$  spectral line is

- (a)  $12\text{ pm}$                       (b)  $10\text{ pm}$                       (c)  $8\text{ pm}$                       (d)  $6\text{ pm}$

Ans: (d)

Solution: 
$$\Delta\lambda = \frac{\lambda^2}{c} \frac{eB}{4\pi m} = \frac{(660 \times 10^{-9})^2}{3 \times 10^8} \times \frac{1.6 \times 10^{-19} \times 0.3}{4\pi \times 9.1 \times 10^{-31}} = 6.09 \times 10^{-12}\text{ m} = 6\text{ pm}$$

Q33. The separation between the energy levels of a two-level atom is  $2\text{ eV}$ . Suppose that  $4 \times 10^{20}$  atoms are in the ground state and  $7 \times 10^{20}$  atoms are pumped into the excited state just before lasing starts. How much energy will be released in a single laser pulse?

- (a)  $24.6\text{ J}$                       (b)  $22.4\text{ J}$                       (c)  $98\text{ J}$                       (d)  $48\text{ J}$

Ans: (d)

Solution:  $N_2 - N_1 = 3 \times 10^{20}$

Energy of laser pulse, 
$$E = \left( \frac{N_2 - N_1}{2} \right) h\nu = \frac{3 \times 10^{20}}{2} \times 2 \times 1.6 \times 10^{-19}\text{ J} \Rightarrow E = 48\text{ J}$$





Q36. A two level system in a thermal (black body) environment can decay from the excited state by both spontaneous and thermally stimulated emission. At room temperature ( $300\text{ K}$ ), the frequency below which thermal emission dominates over spontaneous emission is nearest to

- (a)  $10^{13}\text{ Hz}$                       (b)  $10^8\text{ Hz}$                       (c)  $10^5\text{ Hz}$                       (d)  $10^{11}\text{ Hz}$

Ans. : (d)

Solution: At thermal equilibrium, the ratio of the number of spontaneous to stimulated emission is given by

$$\frac{A_{21}}{B_{21}u(\omega)} = e^{\frac{h\omega}{kT}} - 1; \quad \text{where } \frac{h}{kT} = \frac{1.054 \times 10^{-34}\text{ J.S}}{1.38 \times 10^{-23}\text{ J/K} \times 300\text{ K}} = 2.55 \times 10^{-14}\text{ sec}$$

Now, for  $\nu = 10^{13}\text{ Hz} \Rightarrow \omega = 6.3 \times 10^{13}\text{ rad/sec}$ ,

$$\frac{A_2 1}{B_{21}u(\omega)} = e^{1.6} - 1 = 5 - 1 \cong 4$$

For  $\nu = 10^{11}\text{ Hz} \Rightarrow \omega = 6.3 \times 10^{11}\text{ rad/sec}$ ,

$$\therefore \frac{A_{21}}{B_{21}u(\omega)} = e^{1.6 \times 10^{-3}} - 1 = 1.03 - 1 \cong 0.03$$

This ratio is less than one, thus stimulated thermal emission dominates over spontaneous emission below all frequency of  $10^{11}\text{ Hz}$

### NET/JRF (JUNE-2017)

Q37. An atomic spectral line is observed to split into nine components due to Zeeman shift. If the upper state of the atom is  $^3D_2$  then the lower state will be

- (a)  $^3F_2$                       (b)  $^3F_1$                       (c)  $^3P_1$                       (d)  $^3P_2$

Ans. : (c)

Solution: The possible zeeman component for different transitions are:

(i)  $^3D_2 \rightarrow ^3F_2$  : Total zeeman components are 12

(ii)  $^3D_2 \rightarrow ^3P_1$  : Total zeeman components are 9

(iii)  $^3D_2 \rightarrow ^3P_2$  : Total zeeman components are 12

$^3F_1$  state is not allowed because for  $^3F$ , possible 'J' values are 2, 3 and 4.

Only  $^3D_2 \rightarrow ^3P_1$  transition gives 9 components. Therefore, ground state is  $^3P_1$ .

Thus, correct option is (c).

- Q38. If the coefficient of stimulated emission for a particular transition is  $2.1 \times 10^{19} m^3 W^{-1} s^{-3}$  and the emitted photon is at wavelength  $3000 \text{ \AA}$ , then the lifetime of the excited state is approximately
- (a)  $20 ns$                       (b)  $40 ns$                       (c)  $80 ns$                       (d)  $100 ns$

Ans. : (c)

Solution:  $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$ . Life time  $= \tau = \frac{1}{A_{21}} = \frac{c^3}{8\pi h\nu^3 B_{21}} = \frac{\lambda^3}{8\pi h B_{21}}$

$$\Rightarrow \tau = \frac{(3000 \times 10^{-10})^3}{8\pi (6.6 \times 10^{-34} J.s.) (2.1 \times 10^{19} m^3 W^{-1} s^{-3})}$$

$$= \frac{2.7 \times 10^{-20}}{3.5 \times 10^{-13}} \approx 7.7 \times 10^{-8} \text{ sec} \approx 80 ns$$

- Q39. If the binding energies of the electron in the  $K$  and  $L$  shells of silver atom are  $25.4 keV$  and  $3.34 keV$ , respectively, then the kinetic energy of the Auger electron will be approximately
- (a)  $22 keV$                       (b)  $9.3 keV$                       (c)  $10.5 keV$                       (d)  $18.7 keV$

Ans. : (d)

Solution:  $K.E.$  Auger electron is  $K.E = (E_K - E_L) - E_L$

$$= E_K - 2E_L$$

$$= (25.4 - 2 \times 3.34) keV$$

$$= 18.7 keV$$

