CONDENSED MATTER PHYSICS

NET/JRF (JUNE-2011)

Q1. A narrow beam of *X*-rays with wavelength 1.5 Å is reflected from an ionic crystal with an *fcc* lattice structure with a density of 3.32 gcm^{-3}. The molecular weight is 108 AMU (1AMU = 1.66 × 10^{-24} g)

A. The lattice constant is

(a) 6.00 Å (b) 4.56 Å (c) 4.00 Å (d) 2.56 Å

Ans: (a)

Solution: Given $n_{eff} = 4$, $M = 108$ kg, $\rho = 3.32$gm cm^{-3} = 3320 kgm^{-3},

$N_A = 6.023\times10^{+26}$ atoms |kmd

$$a^3 = \frac{n_{eff}\times M}{N_A\times\rho} = \frac{4\times108}{6.023\times10^{26}\times3320} = 6.00\times10^{-30}\,m^3 = 6.00\times10^{-10} = 6.00\,A^0$$

B. The sine of the angle corresponding to (111) reflection is

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Ans: (b)

Solution: According to Bragg's law

$$2d\sin\theta = \lambda, \sin\theta = \frac{\lambda}{2d} \text{ where } d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a}{\sqrt{3}} \text{ for (111) plane}$$

$$\therefore \sin\theta = \frac{\sqrt{3}\lambda}{2a} = \frac{\sqrt{3}\times1.5A^0}{2\times6A^0} = \frac{\sqrt{3}\times3}{2\times6\times2} = \frac{\sqrt{3}}{8}.$$

Q2. A flux quantum (fluxoid) is approximately equal to 2×10^{-7} gauss-cm^2. A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is $\frac{2}{\pi}\times10^5$ gauss.

A. The penetrating depth of this superconductor is

(a) 100 Å (b) 10Å (c) 1000Å (d) 314Å

Ans: (a)

Solution: Given Fluxoid $(\phi)_0 = 2\times10^{-7}$ gauss –cm^2

First Critical field $\left(H_{c1}\right)=\frac{2}{\pi}\times 10^{5}$ gauss

The relation between first critical field and penetration depth is

$$H_{c1}=\frac{\phi_0}{\pi\lambda^2}\ \therefore \lambda^2=\frac{\phi_0}{\pi H_{c1}}=\frac{2.10^{-7}}{\pi\times\frac{2}{\pi}\times 10^{5}}=10^{-12}\,cm^2\Rightarrow \lambda=10^{-6}\,cm=100\,A^0$$

B. The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi}\times 10^{5}$ gauss. The correlation length of the superconductor is

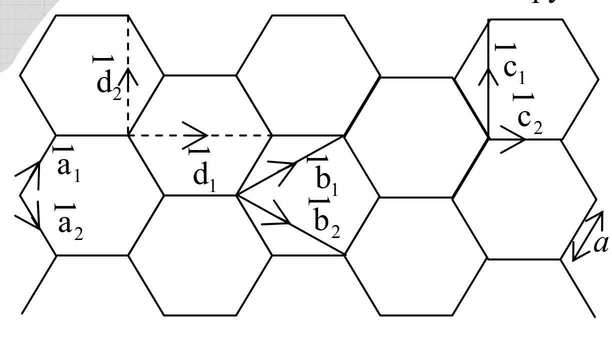
(a) 20 Å (b) 200 Å (c) 628 Å (d) 2000 Å

Ans: None of the options is matched.

Solution: Given second critical field $\left(H_{c2}\right)=\frac{8}{\pi}\times 10^{5}$ gauss. The relation between second critical field and correlation length is $H_{c2}=\frac{\phi_0}{\pi\varepsilon^2}$.

$$\therefore \varepsilon^2=\frac{\phi_0}{\pi H_{c2}}=\frac{2\times 10^{-7}}{\pi\times\frac{8}{\pi}\times 10^{5}}=\frac{1}{4}\times 10^{-12}\,cm^2\Rightarrow \varepsilon=\frac{1}{2}\times 10^{-6}\,cm=\frac{100}{2}\times 10^{-10}\,m\ =50\,A^0$$

Q3. The two dimensional lattice of graphene is an arrangement of Carbon atoms forming a honeycomb lattice of lattice spacing a, as shown below. The Carbon atoms occupy the vertices.



(A). The Wigner-Seitz cell has an area of

(a) $2a^2$ (b) $\frac{\sqrt{3}}{2}a^2$

(c) $6\sqrt{3}a^2$ (d) $\frac{3\sqrt{3}}{2}a^2$

Ans: (d)

Solution: Primitive lattice vectors are $\vec{b}_1$ and $\vec{b}_2$

$$\vec{b}_1=\sqrt{3}a\cos 30^0\,\hat{i}+\sqrt{3}a\cos 60^0\,\hat{j}=\frac{\sqrt{3}}{2}a\left(\sqrt{3}\hat{i}+\hat{j}\right)$$

$$\vec{b}_2=\frac{\sqrt{3}}{2}a\left(\sqrt{3}i-\hat{j}\right),\ \Rightarrow A=\left|\vec{b}_2\times\vec{b}_1\right|=\frac{3\sqrt{3}}{2}a^2$$

(B). The Bravais lattice for this array is a

(a) rectangular lattice with basis vectors $\vec{d}_1$ and $\vec{d}_2$

(b) rectangular lattice with basis vectors $\vec{c}_1$ and $\vec{c}_2$

(c) hexagonal lattice with basis vectors $\vec{a}_1$ and $\vec{a}_2$

(d) hexagonal lattice with basis vectors $\vec{b}_1$ and $\vec{b}_2$

Ans: (c)

Solution: The Bravaiss lattice for this array is the Hexagonal lattice with basis vectors $\vec{a}_1$ and $\vec{a}_2$

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Q4. The potential of a diatomic molecule as a function of the distance r between the atoms is given by $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. The value of the potential at equilibrium separation between the atoms is:

(a) $-4a^2/b$ (b) $-2a^2/b$ (c) $-a^2/2b$ (d) $-a^2/4b$

Ans: (d)

Solution: Given $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. At equilibrium radius, $\left.\frac{dV(r)}{dr}\right|_{r=r_0} = 0$

$$\therefore \frac{dV(r)}{dr} = +\frac{6a}{r_0^7} - \frac{12b}{r_0^{13}} = 0 \Rightarrow \frac{r_0^{13}}{r_0^7} = \frac{12b}{6a} = \frac{2b}{a} \Rightarrow r_0^6 = \frac{2b}{a}$$

$\therefore$ The value of potential at equilibrium is $V(r_0) = -\frac{a}{r_0^6} + \frac{b}{r_0^{12}} = -\frac{a^2}{2b} + \frac{a^2}{4b} = \frac{-a^2}{4b}$.

Q5. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temperature will

(a) increase by a factor of 4 (b) decrease by a factor of 4

(c) increase by a factor of 8 (d) decrease by a factor of 8

Ans: (a)

Solution: The relation between Fermi energy and electron density is $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$.

$$\Rightarrow E_F^{'} = \frac{\hbar^2}{2m}\left(3\pi^2 \times 8n\right)^{2/3} = 4E_F.$$

Q6. The excitations of a three-dimensional solid are bosonic in nature with their frequency ω and wave-number k are related by $\omega \propto k^2$ in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to

(a) $T^{1/2}$ (b) T (c) $T^{3/2}$ (d) T^3

Ans: (c)

Solution: If the dispersion relation is $\omega \propto k^s$ in large wavelength. Then the specific heat is

$C_v \propto T^{3/s}$. Given $\omega \propto k^2 \therefore C_v \propto T^{3/2}$

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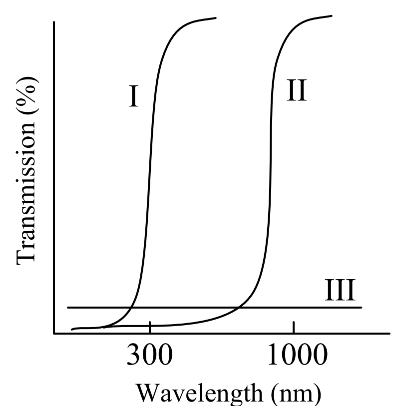
Q7. Consider a system of non-interacting particles in d dimensional obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, k is the wavevector, s is an integer and A is constant. The density of states, $N(\varepsilon)$, is proportional to

(a) $\varepsilon^{\frac{s}{d}-1}$ (b) $\varepsilon^{\frac{d}{s}-1}$ (c) $\varepsilon^{\frac{d}{s}+1}$ (d) $\varepsilon^{\frac{s}{d}+1}$

Ans: (b)

Q8. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond, respectively, to



(a) insulator, semiconductor and metal
(b) semiconductor, metal and insulator
(c) metal, semiconductor and insulator
(d) insulator, metal and semiconductor

Ans: (a)

Q9. The energy required to create a lattice vacancy in a crystal is equal to 1 eV. The ratio of the number densities of vacancies $n(1200\text{ K})/n(300\text{ K})$ when the crystal is at equilibrium at 1200 K and 300 K, respectively, is approximately

(a) $\exp(-30)$ (b) $\exp(-15)$ (c) $\exp(15)$ (d) $\exp(30)$

Ans: (d)

Solution: The equation for number density of vacancies $n = Ne^{-E/2k_BT}$ where E: Energy required to form vacancies, N: density of lattice sites

$$\therefore \frac{n_1}{n_2} = \frac{e^{-E/2k_BT_1}}{e^{-E/2k_BT_2}} = e^{\frac{+E}{2k_B}\left[\frac{1}{T_2}-\frac{1}{T_1}\right]},\ \frac{n(1200K)}{n(300K)} = e^{\frac{E}{2k_B}\left[\frac{1}{300}-\frac{1}{1200}\right]} = e^{\frac{E}{2k_B}\left[\frac{1}{400}\right]} = e^{30}$$

Q10. The dispersion relation of phonons in a solid is given by

$$\omega^2(k) = \omega_0^2(3 - \cos k_x a - \cos k_y a - \cos k_z a)$$

The velocity of the phonons at large wavelength is

(a) $\omega_0 a/\sqrt{3}$ (b) $\omega_0 a$ (c) $\sqrt{3}\omega_0 a$ (d) $\omega_0 a/\sqrt{2}$

Ans: (d)

Solution: For large λ, $(k_x a, k_y a, k_z a)$ are small.

$$\omega^2(k) = \omega_0^2\left[3 - \left(1 - \frac{k_x^2 a^2}{2}\right) - \left(1 - \frac{k_y^2 a^2}{2}\right) - \left(1 - \frac{k_z^2 a^2}{2}\right)\right] = \frac{\omega_0^2 a^2}{2}(k_x^2 + k_y^2 + k_z^2)$$

$$\omega^2(k) == \frac{\omega_0^2 a^2}{2}k^2 \Rightarrow \omega = \frac{\omega_0 a}{\sqrt{2}}k \Rightarrow v_g = \frac{d\omega}{dk} = \frac{\omega_0 a}{\sqrt{2}}.$$

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Q11. A magnetic field sensor based on the Hall Effect is to be fabricated by implanting As into a Si film of thickness 1 μm. The specifications require a magnetic field sensitivity of 500 mV/Tesla at an excitation current of 1 mA. The implantation dose is to be adjusted such that the average carrier density, after activation, is

(a) 1.25×10^{26} m^{-3} (b) 1.25×10^{22} m^{-3}

(c) 4.1×10^{21} m^{-3} (d) 4.1×10^{20} m^{-3}

Ans: (b)

Solution: $n = \frac{IB}{teV_H} = \frac{10^3}{10^{-6} \times 1.6 \times 10^{-19}} \times \frac{1}{500 \times 10^{-3}} = 1.25 \times 10^{22} m^{-3}$ where $\frac{V_H}{B} = 500 \times 10^{-3} V/T$.

Q12. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta(\cos k_x a + \cos k_y a + \cos k_z a),$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zone is

(a) $\frac{2\hbar^2}{5\beta a^2}$ (b) $\frac{4\hbar^2}{5\beta a^2}$ (c) $\frac{\hbar^2}{2\beta a^2}$ (d) $\frac{\hbar^2}{3\beta a^2}$

Ans: (d)

Solution: $\varepsilon_k = \beta(\cos k_x a + \cos k_y a + \cos k_z a)$, Effective mass $m^* = \dfrac{\hbar^2}{\left(\dfrac{d^2\varepsilon_k}{d^2k}\right)}$

Brillouin zone boundary is at $k_x = \pm\dfrac{\pi}{a}, k_y = \pm\dfrac{\pi}{a}, k_z = \pm\dfrac{\pi}{a}$.

Hence $\left(\dfrac{d^2\varepsilon_k}{d^2k}\right)\Bigg|_{\frac{\pi}{a},\frac{\pi}{a},\frac{\pi}{a}} = 3\beta a^2 \Rightarrow m^* = \dfrac{\hbar^2}{3\beta a^2}$.

Q13. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3, is

(a) $\left(\dfrac{12\pi^2}{a^3}\right)^{1/3}$ (b) $\left(\dfrac{3\pi^2}{a^3}\right)^{1/3}$ (c) $\left(\dfrac{\pi^2}{a^3}\right)^{1/3}$ (d) $\dfrac{1}{a}$

Ans: (a)

Solution: Radius of Fermi sphere is $k_F = \left(\dfrac{3\pi^2 N}{V}\right)^{1/3}$, $E_F = \left(\dfrac{\hbar^2}{2m}\right)\left(3\pi^2 n\right)^{2/3} = \left(\dfrac{\hbar^2 k_F^2}{2m}\right)$

For fcc solid $\dfrac{N}{V} = \dfrac{4}{a^3} \Rightarrow k_F = \left(\dfrac{12\pi^2}{a^3}\right)^{1/3}$.

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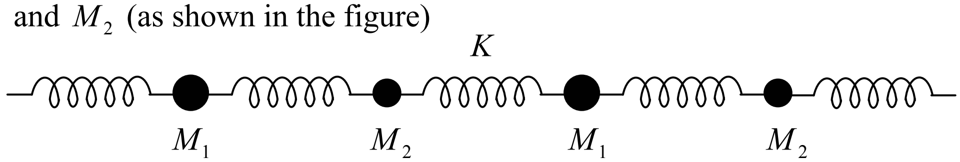
Q14. Using the frequency-dependent Drude formula, what is the effective kinetic inductance of a metallic wire that is to be used as a transmission line? [In the following, the electron mass is m, density of electrons is n, and the length and cross-sectional area of the wire ℓ and A respectively.]

(a) $mA/(ne^2\ell)$ (b) zero (c) $m\ell/(ne^2A)$ (d) $m\sqrt{A}/(ne^2\ell^2)$

Ans: (c)

Q15. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



is given by

$$\omega^2(q) = K\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

where a is the lattice parameter and K is the spring constant. The velocity of sound is

(a) $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}}a$ (b) $\sqrt{\frac{K}{2(M_1+M_2)}}a$

(c) $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}}a$ (d) $\sqrt{\frac{KM_1M_2}{2(M_1+M_2)^3}}a$

Ans: (b)

Solution: For small value of q (i.e. long wavelength approximation limit).

We have $\sin\left(\frac{qa}{2}\right) \approx \frac{qa}{2}$

$$\therefore \omega^2(q) = \text{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

$$\Rightarrow \omega^2(q) = \text{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\left(\frac{qa}{2}\right)^2}\right]$$

$$\Rightarrow \omega^2(q) = \text{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \left(1-\frac{1}{2}\times\frac{4M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{4}\right)\right]$$

$$\Rightarrow \omega^2(q) = \text{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1 \pm \left(1-\frac{M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{2}\right)\right]$$

For Acoustical branch: $\omega^2(q) = \text{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1-\left(1-\frac{M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{2}\right)\right]$

$$\Rightarrow \omega^2(q) = \text{K}\left(\frac{M_1+M_2}{M_1M_2}\right)\left(\frac{M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{2}\right) = \frac{\text{K}a^2}{2(M_1+M_2)}q^2$$

$$\therefore \omega(q) = \sqrt{\frac{\text{K}}{2(M_1+M_2)}}aq$$

Velocity of sound is $v_g = \frac{\omega}{q} = \sqrt{\frac{\text{K}}{2(M_1+M_2)}}a$

Q16. The electron dispersion relation for a one-dimensional metal is given by

$$\varepsilon_k = 2\varepsilon_0\left[\sin^2\frac{ka}{2} - \frac{1}{6}\sin^2 ka\right]$$

where k is the momentum, a is the lattice constant, ε_0 is a constant having dimensions of energy and $|ka| \le \pi$. If the average number of electrons per atom in the conduction band is 1/3, then the Fermi energy is

(a) $\varepsilon_0/4$ (b) ε_0 (c) $2\varepsilon_0/3$ (d) $5\varepsilon_0/3$

Ans: (a)

Q17. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states $D(E)$ depends on the energy as

(a) $1/\sqrt{E}$ (b) $\sqrt{E}$ (c) E (d) constant

Ans: (c)

Solution: In two dimensional system, the number of allowed k-states in range k and $k+dk$ is

$$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k dk .$$

Given dispersion relation is $E = u\hbar k \quad \therefore k = \frac{E}{u\hbar} \Rightarrow dk = \frac{dE}{u\hbar}$

$$\therefore g(E)dE = \left(\frac{L}{2\pi}\right)^2 2\pi \times \frac{E}{u\hbar} \times \frac{dE}{u\hbar} = \left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{(u\hbar)^2} EdE$$

$$\Rightarrow \rho(E) = \frac{g(E)dE}{dE} = \frac{1}{(u\hbar)^2}\frac{L^2}{2\pi}E .$$

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Q18. The physical phenomenon that cannot be used for memory storage applications is

(a) large variation in magnetoresistance as a function of applied magnetic field

(b) variation in magnetization of a ferromagnet as a function of applied magnetic field

(c) variation in polarization of a ferroelectric as a function of applied electric field

(d) variation in resistance of a metal as a function of applied electric field

Ans: (d)

Q19. The energy of an electron in a band as a function of its wave vector k is given by $E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$, where E_0, B and a are constants. The effective mass of the electron near the bottom of the band is

(a) $\frac{2\hbar^2}{3Ba^2}$ (b) $\frac{\hbar^2}{3Ba^2}$ (c) $\frac{\hbar^2}{2Ba^2}$ (d) $\frac{\hbar^2}{Ba^2}$

Ans: (d)

Solution: Near the bottom of the band the $k \to 0$

$$\cos k_x a \approx 1 - \frac{1}{2}(k_x a)^2, \cos k_y a \approx 1 - \frac{1}{2}(k_y a)^2, \cos k_z a \approx 1 - \frac{1}{2}(k_z a)^2$$

$$E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a) = E_0 - B\left(1 - \frac{1}{2}(k_x a)^2 + 1 - \frac{1}{2}(k_y a)^2 + 1 - \frac{1}{2}(k_z a)^2\right)$$

$$= E_0 - B\left(3 - \frac{1}{2}a^2(k_x + k_x + k_x)^2\right) = E_0 - 3B - \frac{1}{2}Ba^2k^2$$

Effective mass of the electron is $m^* = \frac{\hbar^2}{d^2E/dk^2} = \frac{\hbar^2}{Ba^2}$

Q20. A DC voltage V is applied across a Josephson junction between two superconductors with a phase difference ϕ_0. If I_0 and k are constants that depend on the properties of the junction, the current flowing through it has the form

(a) $I_0 \sin\left(\frac{2eVt}{\hbar} + \phi_0\right)$ (b) $kV \sin\left(\frac{2eVt}{\hbar} + \phi_0\right)$

(c) $kV \sin\phi_0$ (d) $I_0 \sin\phi_0 + kV$

Ans: (a)

Q21. A uniform linear monoatomic chain is modeled by a spring-mass system of masses m separated by nearest neighbour distance a and spring constant $m\omega_0^2$. The dispersion relation for this system is

(a) $\omega(k) = 2\omega_0\left(1 - \cos\left(\frac{ka}{2}\right)\right)$ (b) $\omega(k) = 2\omega_0 \sin^2\left(\frac{ka}{2}\right)$

(c) $\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$ (d) $\omega(k) = 2\omega_0 \tan\left(\frac{ka}{2}\right)$

Ans: (c)

Solution: The dispersion relation for uniform linear mono-atomic chain of atoms is

$$\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$$

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Q22. The pressure of a nonrelativistic free Fermi gas in three-dimensions depends, at $T=0$, on the density of fermions n as

(a) $n^{5/3}$ (b) $n^{1/3}$ (c) $n^{2/3}$ (d) $n^{4/3}$

Ans: (a)

Solution: The Fermi energy in three dimension is defined as

$$E_F=\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}=\frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

Where, n is the electron concentration or density of free Fermi gas.

The total energy of free Fermi gas in 3D is

$$E=\frac{3}{5}NE_F=\frac{3}{5}N\times\frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

The pressure of a nonrelativistic free Fermi gas is defined as

$$p_F=-\left(\frac{\partial E}{\partial V}\right)_N=-\frac{3}{5}N\times\frac{\hbar^2}{2m}\left(3\pi^2 N\right)^{2/3}\times\left(-\frac{2}{3}\right)V^{-5/3}$$

$$=\frac{2}{5}nE_F=\frac{2}{5}n\times\frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}=\frac{2}{5}\frac{\hbar^2}{2m}\left(3\pi^2\right)^{2/3}n^{5/3}$$

Q23. Consider an electron in b.c.c. lattice with lattice constant a. A single particle wavefunction that satisfies the Bloch theorem will have the form $f(\vec{r})\exp\left(i\vec{k}.\vec{r}\right)$, with $f(\vec{r})$ being

(a) $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$

(b) $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

(c) $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$

(d) $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

Ans: (b)

Solution: The primitive translational vector for BCC is

$\vec{a}' = \frac{a}{2}\left(-\hat{i}+\hat{j}+\hat{k}\right),\ \vec{b}' = \frac{a}{2}\left(\hat{i}-\hat{j}+\hat{k}\right),\ \vec{c}' = \frac{a}{2}\left(\hat{i}+\hat{j}-\hat{k}\right)$

Bloch function defined as

$\psi_k(\vec{r}) = u_k(\vec{r})e^{i\vec{k}.\vec{r}} = f(\vec{r})e^{i\vec{k}.\vec{r}}$

Here $f(\vec{r})$ is atomic wavefunction, which has the periodicity of the lattice i.e.

$u_k(\vec{r}+a) = u_k(\vec{r})$

Given Bloch function

$$f(\vec{r}) = 1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$$

$$f(\vec{r}+\vec{a}') = 1+\cos\left[\frac{2\pi}{a}\left(x+y-\frac{a}{2}+\frac{a}{2}\right)\right]+\cos\left[\frac{2\pi}{a}\left(y+z+\frac{a}{2}+\frac{a}{2}\right)\right]+\cos\left[\frac{2\pi}{a}\left(z+x+\frac{a}{2}-\frac{a}{2}\right)\right]$$

$$f(\vec{r}+\vec{a}') = 1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)+2\pi\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$$

$$f(\vec{r}+\vec{a}') = 1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right] = f(\vec{r})$$

$f(\vec{r}+\vec{a}') = f(\vec{r})$

Similarly,

$f(\vec{r}+\vec{b}') = f(\vec{r})$ and $f(\vec{r}+\vec{c}') = f(\vec{r})$

Other functions do not satisfy the periodicity

Q24. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation, by

$$\varepsilon(k) = -4\varepsilon_0\left[\cos\frac{k_x a}{2}\cos\frac{k_y a}{2}+\cos\frac{k_y a}{2}\cos\frac{k_z a}{2}+\cos\frac{k_z a}{2}\cos\frac{k_x a}{2}\right]$$

where a is the lattice constant and ε_0 is a constant with the dimension of energy. The x-component of the velocity of the electron at $\left(\frac{\pi}{a},0,0\right)$ is

(a) $-2\varepsilon_0 a/\hbar$ (b) $2\varepsilon_0 a/\hbar$ (c) $-4\varepsilon_0 a/\hbar$ (d) $4\varepsilon_0 a/\hbar$

Ans: (d)

Solution: Group velocity of electron in dispersive medium is expressed as

$$\vec{v}=\frac{1}{\hbar}\frac{d\varepsilon}{dk}=\frac{1}{\hbar}\left[\frac{d\varepsilon}{dk_x}\hat{i}+\frac{d\varepsilon}{dk_y}\hat{j}+\frac{d\varepsilon}{dk_z}\hat{k}\right]=\vec{v}_x\hat{i}+\vec{v}_y\hat{j}+\vec{v}_z\hat{k}$$

$$\vec{v}=\frac{2\varepsilon_0 a}{\hbar}\left[\begin{array}{l}\left(\sin\frac{k_x a}{2}\cos\frac{k_y a}{2}+\cos\frac{k_z a}{2}\sin\frac{k_x a}{2}\right)\hat{i}+\left(\cos\frac{k_x a}{2}\sin\frac{k_y a}{2}+\sin\frac{k_y a}{2}\cos\frac{k_z a}{2}\right)\hat{j}+\\ \left(\sin\frac{k_z a}{2}\cos\frac{k_y a}{2}+\cos\frac{k_x a}{2}\sin\frac{k_z a}{2}\right)\hat{k}\end{array}\right]$$

At $\left(\frac{\pi}{a},0,0\right)$

$$\vec{v}=\frac{2\varepsilon_0 a}{\hbar}\left[\left(\sin\frac{\pi}{2}\cos 0+\cos 0\sin\frac{\pi}{2}\right)\hat{i}+\left(\cos\frac{\pi}{2}\sin 0+\sin 0\cos 0\right)\hat{j}+\left(\cos 0\sin 0+\sin 0\cos\frac{\pi}{2}\right)\hat{k}\right]$$

$$\vec{v}=\frac{4\varepsilon_0 a}{\hbar}\left[\hat{i}+0\hat{j}+0\hat{k}\right]=\left[0\hat{i}+0\hat{j}+0\hat{k}\right]=\vec{v}_x\hat{i}+\vec{v}_y\hat{j}+\vec{v}_z\hat{k}$$

$$\vec{v}_x=\frac{4\varepsilon_0 a}{\hbar},\ \vec{v}_y=0,\ \vec{v}_z=0$$

The x - component of velocity is $v_x=\frac{4\varepsilon_0 a}{\hbar}$

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Q25. When laser light of wavelength λ falls on a metal scale with 1 mm engravings at a grazing angle of incidence, it is diffracted to form a vertical chain of diffraction spots on a screen kept perpendicular to the scale. If the wavelength of the laser is increased by 200 nm, the angle of the first-order diffraction spot changes from 5^0 to

(a) 6.60^0 (b) 5.14^0 (c) 5.018^0 (d) 5.21^0

Ans: (c)

Solution: The condition of maxima peak in grating is

$$b\sin\theta=m\lambda;\qquad m=0,1,2,3,....$$

where b is the width of slit or width of engraving, whereas 'm' is the order of diffraction and θ is the angle of diffraction

For 1st order diffraction: $b\sin\theta=\lambda$ (i)

When wavelength of incident light increased to $(\lambda+200)\,nm$, let's assume the 1st order peak appears at θ' $\quad \therefore b\sin\theta' = \lambda+200$ (ii)

Subtracting equation (i) from equation (ii), we get

$$b\sin\theta - b\sin\theta' = 200$$

$$\Rightarrow \sin\theta' - \sin\theta = \frac{200}{b} \Rightarrow \sin\theta' = \frac{200\times10^{-9}}{10^{-3}} + \sin\theta$$

$$\Rightarrow \sin\theta' = 2\times10^{-4} + \sin\theta = 2\times10^{-4} + \sin 5^0 = 2\times10^{-4} + 0.087196 = 0.08736$$

$$\Rightarrow \theta' = \sin^{-1}(0.08736) \Rightarrow \theta' = 5.01^0$$

Q26. Consider the crystal structure of sodium chloride which is modeled as a set of touching spheres. Each sodium atom has a radius r_1 and each chlorine atom has a radius r_2. The centres of the spheres from a simple cubic lattice. The packing fraction of this system is

(a) $\pi\left[\left(\frac{r_1}{r_1+r_2}\right)^3 + \left(\frac{r_2}{r_1+r_2}\right)^3\right]$ (b) $\frac{2\pi}{3}\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$

(c) $\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$ (d) $\pi\frac{r_1^3+r_2^3}{2(r_1+r_2)^3}$

Ans: (b)

Solution: This question can only be solved by solving each option by assuming $r_1 = r_2$ and comparing result with the packing fraction of simple cubic which is $\frac{\pi}{6}$.

Option (a): $\pi\left[\left(\frac{r_1}{r_1+r_2}\right)^3 + \left(\frac{r_2}{r_1+r_2}\right)^3\right] = \pi\left[\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3\right] = \frac{\pi}{4}$

Option (b): $\frac{2\pi}{3}\frac{r_1^3+r_2^3}{(r_1+r_2)^3} = \frac{2\pi}{3}\times\frac{2r^3}{8r^3} = \frac{2\pi}{3}\times\frac{1}{4} = \frac{\pi}{6}$

Option (c): $\frac{r_1^3+r_2^3}{(r_1+r_2)^3} = \frac{2r^3}{8r^3} = \frac{1}{4}$

Option (d): $\frac{\pi}{2}\frac{r_1^3+r_2^3}{(r_1+r_2)^3} = \pi\frac{2r^3}{2\times8r^3} = \frac{\pi}{8}$

Thus, correct option is (b)

Q27. Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the c-direction is half the effective spring constant between atoms in the a and b directions. At low temperatures, the behaviour of the lattice contribution to the specific heat will depend as a function of temperature T as

(a) T^2 for the tetragonal solid, but as T^3 for the simple cubic solid

(b) T for the tetragonal solid, and as T^3 for the simple cubic solid

(c) T for both solids

(d) T^3 for both solids

Ans: (d)

Solution: The specific heat of solid in three dimensions is proportional to T^3 and it is independent of crystal structure.

In $3D$: $C_V \propto T^3$

In $2D$: $C_V \propto T^2$

In $1D$: $C_V \propto T$

Q28. A superconducting ring carries a steady current in the presence of a magnetic field $\vec{B}$ normal to the plane of the ring. Identify the **incorrect** statement.

(a) The flux passing through the superconductor is quantized in units of hc/e

(b) The current and the magnetic field in the superconductor are time independent.

(c) The current density $\vec{j}$ and $\vec{B}$ are related by the equation $\vec{\nabla} \times \vec{j} + \Lambda^2 \vec{B} = 0$, where Λ is a constant

(d) The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

Ans: (a)

Solution: The flux quantization in superconducting ring is $\phi = n\phi_o$

where $\phi_o = \dfrac{hc}{2e}$ in CGS units and $\phi_o = \dfrac{h}{2e}$ in MKS units.

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Q29. X-ray of wavelength $\lambda = a$ is reflected from the (111) plane of a simple cubic lattice. If the lattice constant is a, the corresponding Bragg angle (in radian) is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{8}$

Ans. (c)

Solution: According to Bragg's Law $2d\sin\theta = \lambda$

where $d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{a}{\sqrt{1+1+1}} = \frac{a}{\sqrt{3}}$ for $(1\,1\,1)$ plane

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{2d} = \frac{a}{2\times\frac{a}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

Q30. The critical magnetic fields of a superconductor at temperatures $4K$ and $8K$ are $11\,mA/m$ and $5.5\,mA/m$ respectively. The transition temperature is approximately

(a) 8.4 K (b) 10.6 K (c) 12.9 K (d) 15.0 K

Ans. (b)

Solution: The relation between critical field and critical temperature is

$$H_C(T) = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Let at $T = T_1$, $H_C(T_1)$, $T = T_2$, $H_C(T) = H_C(T_2)$

Thus we get $H_C(T_1) = H_0\left[1-\left(\frac{T_1}{T_C}\right)^2\right]$, $H_C(T_2) = H_0\left[1-\left(\frac{T_2}{T_C}\right)^2\right]$

$$\frac{H_C(T_1)}{H_C(T_2)} = \frac{1-\left(\frac{T_1}{T_C}\right)^2}{1-\left(\frac{T_2}{T_C}\right)^2} \Rightarrow T_C = \sqrt{\frac{\frac{H_C(T_1)}{H_C(T_2)}T_2^2 - T_1^2}{\frac{H_C(T_1)}{H_C(T_2)}-1}} \Rightarrow T_C = \sqrt{\frac{2(8)^2-(4)^2}{2-1}} \approx 10.6$$

where $T_1 = 4k$, $T_2 = 8k$, $H_C(T_1) = 11\ mA/m$ and $H_C(T_2) = 5.5\ mA/m$

Q31. The low-energy electronic excitations in a two-dimensional sheet of grapheme is given by $E(\vec{k}) = \hbar v k$, where v is the velocity of the excitations. The density of states is proportional to

(a) E (b) $E^{\frac{3}{2}}$ (c) $E^{\frac{1}{2}}$ (d) E^{2}

Ans. (a)

Solution: The number of k - states in range k and $k + dk$ in two dimension is

$$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k dk$$

$$\therefore E = \hbar v k \Rightarrow dE = \hbar v dk \Rightarrow g(E)dE = \left(\frac{L}{2\pi}\right)^2 2\pi \times \frac{E}{\hbar v} \times \frac{dE}{\hbar v} = \left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{(\hbar v)^2} E dE$$

The density of state is

$$\rho(E) = \frac{g(E)dE}{dE} = \left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{(\hbar v)^2} E \Rightarrow \rho(E) \propto E$$

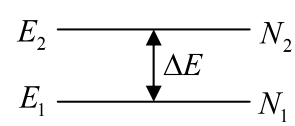
Q32. A $He - Ne$ laser operates by using two energy levels of Ne separated by $2.26\ eV$. Under steady state conditions of optical pumping, the equivalent temperature of the system at which the ratio of the number of atoms in the upper state to that in the lower state will be $\frac{1}{20}$, is approximately (the Boltzmann constant $k_B = 8.6 \times 10^{-5}\ eV/K$)

(a) $10^{10}\ K$ (b) $10^{8}\ K$ (c) $10^{6}\ K$ (d) $10^{4}\ K$

Ans. (d)

Solution: According to Boltzmann relation

$$\frac{N_2}{N_1} = \exp\left(-\frac{\Delta E}{kT}\right) \Rightarrow \frac{N_1}{N_2} = \exp\left(\frac{kT}{\Delta E}\right) \Rightarrow T = \frac{\Delta E}{k \ln\left(\frac{N_2}{N_1}\right)}$$



$$\Delta E = 2.26\ eV, k_B = 8.6 \times 10^{-5}\ eV/K, \frac{N_1}{N_2} = 20 \Rightarrow T = \frac{2.26}{8.6 \times 10^{-5} \ln\left(\frac{1}{20}\right)} \approx 0.877 \times 10^4 K$$

$$\Rightarrow T = 10^4\ K$$

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Q33. The first order diffraction peak of a crystalline solid occurs at a scattering angle of 30^0 when the diffraction pattern is recorded using an x-ray beam of wavelength $0.15\,nm$. If the error in measurements of the wavelength and the angle are $0.01\,nm$ and 1^0 respectively, then the error in calculating the inter-planar spacing will approximately be

(a) $1.1\times10^{-2}\,nm$ (b) $1.3\times10^{-4}\,nm$ (c) $2.5\times10^{-2}\,nm$ (d) $2.0\times10^{-3}\,nm$

Ans.: (a)

Solution: Bragg's Law for $n=1,\ \lambda=2d\sin\theta \Rightarrow d=\frac{\lambda}{2\sin\theta} \Rightarrow \frac{\partial d}{\partial\lambda}=\frac{1}{2\sin\theta}, \frac{\partial d}{\partial\theta}=\frac{-\lambda\cos\theta}{2\sin^2\theta}$

Error in d can be calculated as

$$\sigma_d^2=\left(\frac{\partial d}{\partial\lambda}\right)^2\sigma_\lambda^2+\left(\frac{\partial d}{\partial\theta}\right)\sigma_\theta^2=\left(\frac{1}{2\sin\theta}\right)^2\sigma_\lambda^2+\left(\frac{-\lambda\cos\theta}{2\sin^2\theta}\right)^2\sigma_\theta^2$$

$$\Rightarrow\frac{\sigma_d^2}{d^2}=\frac{1}{\left(4\sin^2\theta\right)}\times\left(\frac{2\sin\theta}{\lambda}\right)^2\sigma_\lambda^2+\left(\frac{-\lambda}{2\sin\theta}\times\frac{\cos\theta}{\sin\theta}\right)^2\times\left(\frac{2\sin\theta}{\lambda}\right)^2\sigma_\theta^2$$

$$\Rightarrow\frac{\sigma_d^2}{d^2}=\frac{\sigma_\lambda^2}{\lambda^2}+\frac{\sigma_\theta^2}{\tan^2\theta}\Rightarrow\sigma_d=d\left[\left(\frac{\sigma_\lambda}{\lambda}\right)^2+\left(\frac{\sigma_\theta}{\tan\theta}\right)^2\right]^{\frac{1}{2}}$$

where $\theta=30^{\circ},\ \lambda=1.5\times10^{-10}\ m,\ \sigma_\lambda=0.1\times10^{-10}\ m, \sigma_\theta=1^{\circ}$

$$d=\frac{\lambda}{2\sin\theta}=\frac{1.5\times10^{-10}}{2\sin30^{\circ}}=1.5\times10^{-10}\ m$$

Thus, $$\sigma_d=1.5\times10^{-10}\left[\left(\frac{0.1\times10^{-10}}{1.5\times10^{-10}}\right)^2+\left(\frac{\frac{\pi}{180}}{\tan30}\right)^2\right]^{\frac{1}{2}}=1.5\times10^{-10}\left[\left(0.067\right)^2+\left(\frac{\sqrt{3}\pi}{180}\right)^2\right]^{\frac{1}{2}}$$

$$=1.5\times10^{-10}\left[\left(0.067\right)^2+\left(0.03\right)^2\right]^{\frac{1}{2}}=1.5\times10^{-10}\left[0.005389\right]^{\frac{1}{2}}$$

$$\sigma_d=1.5\times10^{-10}\times0.0734=0.11\times10^{-10}=1.1\times10^{-11}\ m=1.1\times10^{-2}\ nm$$

Q34. The dispersion relation of electrons in a 3-dimensional lattice in the tight binding approximation is given by,

$$\varepsilon_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$$

where a is the lattice constant and α, β, γ are constants with dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$ is

(a) $\frac{\hbar^2}{a^2}\begin{pmatrix} -1/\alpha & 0 & 0 \\ 0 & -1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$

(b) $\frac{\hbar^2}{a^2}\begin{pmatrix} -1/\alpha & 0 & 0 \\ 0 & -1/\beta & 0 \\ 0 & 0 & -1/\gamma \end{pmatrix}$

(c) $\frac{\hbar^2}{a^2}\begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$

(d) $\frac{\hbar^2}{a^2}\begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & -1/\gamma \end{pmatrix}$

Ans.: (c)

Solution: The effective mass as a tensor quantity can be written as

$$m_{ij}^* = \begin{bmatrix} m_{xx}^* & m_{xy}^* & m_{xz}^* \\ m_{yx}^* & m_{yy}^* & m_{yz}^* \\ m_{zx}^* & m_{zy}^* & m_{zz}^* \end{bmatrix} \text{ where } m_{ij}^* = \frac{\hbar^2}{\left(\frac{\partial^2 E}{\partial k_i \partial k_j}\right)}$$

since $\in_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$

$$\therefore\ m_{xx}^* = \frac{\hbar^2}{\left(\frac{\partial^2 \in}{\partial k_x \partial k_x}\right)} = \frac{-\hbar^2}{\alpha a^2 \cos k_x a}, \quad m_{yy}^* = \frac{\hbar^2}{\left(\frac{\partial^2 \in}{\partial k_y^2}\right)} = \frac{-\hbar^2}{\beta a^2 \cos k_y a}$$

$$m_{zz}^* = \frac{\hbar^2}{\left(\frac{\partial^2 \in}{\partial k_z^2}\right)} = \frac{-\hbar^2}{\gamma a^2 \cos k_z a}, \text{ other terms are zero}$$

Now, at $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right); m_{xx}^* = \frac{\hbar^2}{\alpha a^2}, m_{yy}^* = \frac{\hbar^2}{\beta a^2}, m_{zz}^* = \frac{\hbar^2}{\gamma a^2} \Rightarrow m_{ij}^* = \frac{\hbar^2}{a^2}\begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{bmatrix}$

Q35. A thin metal film of dimension $2mm \times 2mm$ contains 4×10^{12} electrons. The magnitude of the Fermi wavevector of the system, in the free electron approximation, is

(a) $2\sqrt{\pi}\times10^{7}\ cm^{-1}$ (b) $\sqrt{2\pi}\times10^{7}\ cm^{-1}$ (c) $\sqrt{\pi}\times10^{7}\ cm^{-1}$ (d) $2\pi\times10^{7}\ cm^{-1}$

Ans.: (b)

Solution: This is the case of two dimensional metal box. The Fermi wave vector of electron in $2-D$ is

$$k_F = (2\pi n)^{\frac{1}{2}} = \left(2\pi \frac{N}{L^2}\right)^{\frac{1}{2}};\ L^2 = 2mm\times 2mm = 4\times10^{-2}\ cm^2,$$

$$\Rightarrow k_F = \sqrt{2\pi}\left(\frac{4\times10^{12}}{4\times10^{-2}\ cm^2}\right)^{\frac{1}{2}} = \sqrt{2\pi}\left(10^{14}\ cm^{-2}\right)^{\frac{1}{2}} = \sqrt{2\pi}\times10^{7}\ cm^{-1}$$

Q36. For an electron moving through a one-dimensional periodic lattice of periodicity a, which of the following corresponds to an energy eigenfunction consistent with Bloch's theorem?

(a) $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a}+\cos\left(\frac{\pi x}{2a}\right)\right]\right)$ (b) $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a}+\cos\left(\frac{2\pi x}{a}\right)\right]\right)$

(c) $\psi(x) = A\exp\left(i\left[\frac{2\pi x}{a}+i\cosh\left(\frac{2\pi x}{a}\right)\right]\right)$ (d) $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a}+i\left|\frac{\pi x}{2a}\right|\right]\right)$

Ans.: (b)

Solution: According to block theorem, $\psi(x+a) = \psi(x)$

$$\psi(x+a) = A\exp\left\{i\left[\frac{\pi}{a}(x+a)+\cos\left(\frac{2\pi}{a}(x+a)\right)\right]\right\} = A\exp\left\{i\left[\left(\frac{\pi x}{a}+\pi\right)+\cos\left(\frac{2\pi x}{a}+2\pi\right)\right]\right\}$$

$$= A\exp\left[i\left\{\frac{\pi}{a}(x+a)+\cos\frac{2\pi x}{a}\right\}\right] = A\exp\left[i\left(\frac{\pi x}{a}+\cos\frac{2\pi x}{a}\right)\right]$$

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Q37. Consider electrons in graphene, which is a planar monoatomic layer of carbon atoms. If the dispersion relation of the electrons is taken to be $\varepsilon(k) = ck$ (where c is constant) over the entire k-space, then the Fermi energy ε_F depends on the number density of electrons ρ as

(a) $\varepsilon_F \propto \rho^{\frac{1}{2}}$ (b) $\varepsilon_F \propto \rho$ (c) $\varepsilon_F \propto \rho^{\frac{2}{3}}$ (d) $\varepsilon_F \propto \rho^{\frac{1}{3}}$

Ans: (a)

Solution: In $2D$, density of state is

$$g(k)dk = \left(\frac{L}{2\pi}\right) 2\pi k dk \text{, where } \varepsilon = ck \Rightarrow k = \frac{\varepsilon}{c} \text{ and } dk = \frac{d\varepsilon}{c}$$

$$\Rightarrow g(\varepsilon)d\varepsilon = \left(\frac{L}{2\pi}\right)^2 \times 2\pi \times \frac{\varepsilon}{c}.\frac{d\varepsilon}{c} = \frac{L^2}{2\pi c^2}\varepsilon d\varepsilon$$

Now, number electrons at $T = 0K$ is

$$N = \int_0^{\varepsilon_F} g(\varepsilon)d\varepsilon = \frac{L^2}{2\pi c^2}\int_0^{\varepsilon_F} \varepsilon d\varepsilon = \frac{L^2}{4\pi c^2}\varepsilon_F^2 \Rightarrow \varepsilon_F^2 = 4\pi c^2 \frac{N}{L^2} = 4\pi c^2 \rho$$

$$\Rightarrow \varepsilon_F = \sqrt{4\pi c^2}\rho^{1/2} \Rightarrow \varepsilon_F \propto \rho^{1/2}$$

Q38. Suppose the frequency of phonons in a one-dimensional chain of atoms is proportional to the wave vector. If n is the number density of atoms and c is the speed of the phonons, then the Debye frequency is

(a) $2\pi cn$ (b) $\sqrt{2}\pi cn$ (c) $\sqrt{3}\pi cn$ (d) $\frac{\pi cn}{2}$

Ans: (d)

Solution: Given $\omega \propto k \Rightarrow \omega = ck$ (c is velocity of phonon)

$$\text{Now } g(\omega)d\omega = \frac{L}{\pi}\frac{d\omega}{d\omega/dk} = \frac{L}{c\pi}d\omega$$

$$\text{Also } N = \int_0^{\omega_D} g(\omega)d\omega = \frac{L}{c\pi}\int_0^{\omega_D} d\omega \Rightarrow N = \frac{L}{c\pi}\omega_D$$

$$\Rightarrow \omega_D = c\pi\frac{N}{L} = c\pi n, \left(n = \frac{N}{L}\right) \Rightarrow f_D = \frac{cn}{2}.$$ Best answer is (d).

Q39. The band energy of an electron in a crystal for a particular k-direction has the form $\varepsilon(k) = A - B\cos 2ka$, where A and B are positive constants and $0 < ka < \pi$. The electron has a hole-like behaviour over the following range of k:

(a) $\frac{\pi}{4} < ka < \frac{3\pi}{4}$ (b) $\frac{\pi}{2} < ka < \pi$ (c) $0 < ka < \frac{\pi}{4}$ (d) $\frac{\pi}{2} < ka < \frac{3\pi}{4}$

Ans: (a)

Solution: $\varepsilon(k) = A - B\cos 2ka$, $\frac{d\varepsilon}{dk} = 2Ba\sin 2ka$, $\frac{d^2\varepsilon}{dk^2} = 4Ba^2\cos 2ka$

Effective mass $m^* = \frac{\hbar^2}{d^2\varepsilon / dk^2} = \frac{\hbar^2}{4Ba^2\cos 2ka}$

Effective mass of electron (m_e^*) and effective mass of holes (m_h^*) are opposite in sign i.e., $(m_h^* = -m_e^*)$.

Now, in the range $0 < ka < \frac{\pi}{4}$, m^* is positive

While in the range $\frac{\pi}{4} < ka < \frac{3\pi}{4}$, m^* is negative

Thus, electron has hole like behaviour in the region $\frac{\pi}{4} < ka < \frac{3\pi}{4}$

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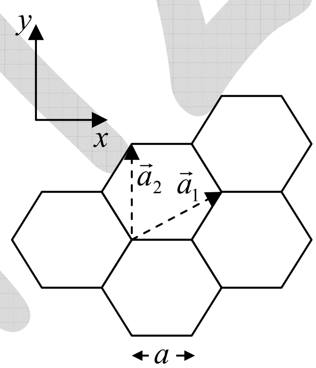
Q40. Consider a hexagonal lattice with basis vectors as shown in the figure below.



If the lattice spacing is $a = 1$, the reciprocal lattice vectors are

(a) $\left(\frac{4\pi}{3}, 0\right), \left(-\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$ (b) $\left(\frac{4\pi}{3}, 0\right), \left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right)$

(c) $\left(0, \frac{4\pi}{3}\right), \left(\pi, \frac{2\pi}{\sqrt{3}}\right)$ (d) $\left(\frac{2\pi}{3}, \frac{2\pi}{\sqrt{3}}\right), \left(-2\pi, \frac{2\pi}{\sqrt{3}}\right)$

Ans. : (a)

Solution: From the figure, we can write

$$\vec{a}_1 = \frac{\sqrt{3}a}{2}\left(\sqrt{3}\hat{x}+\hat{y}\right), \vec{a}_2 = \sqrt{3}a\,\hat{y},\; \vec{a}_3 = a\hat{z} \qquad \text{(let us assume)}$$

Now $V = \vec{a}_1.\left(\vec{a}_2 \times \vec{a}_3\right) = \frac{\sqrt{3}a}{2}\left(\sqrt{3}\hat{x}+\hat{y}\right).\left(\sqrt{3}a\hat{y}\times a\hat{z}\right)$

$$= \frac{\sqrt{3}a}{2}\left(\sqrt{3}\hat{x}+\hat{y}\right).\left(\sqrt{3}a^2\hat{x}\right) = \frac{3\sqrt{3}}{2}a^3$$

Also, $\vec{a}_3 \times \vec{a}_1 = a\hat{z}\times\frac{\sqrt{3}a}{2}\left(\sqrt{3}\hat{x}+\hat{y}\right) = \frac{\sqrt{3}a^2}{2}\left(\sqrt{3}\hat{y}-\hat{x}\right)$

Reciprocal lattice vectors are

$$\vec{a}_1^* = 2\pi\frac{\vec{a}_2\times\vec{a}_3}{V} = 2\pi\frac{\sqrt{3}a^2\hat{x}}{\frac{3\sqrt{3}}{2}a^3} = \frac{4\pi}{3a}\hat{x}+0\hat{y} = \frac{4\pi}{3}\hat{x}+0\hat{y}$$

$$\vec{a}_2^* = 2\pi\frac{\vec{a}_3\times\vec{a}_1}{V} = 2\pi\frac{\frac{\sqrt{3}}{2}a^2\left(\sqrt{3}\hat{y}-\hat{x}\right)}{\frac{3\sqrt{3}}{2}a^3} = \frac{2\pi}{3a}\left(-\hat{x}+\sqrt{3}\,\hat{y}\right) = \left(\frac{-2\pi}{3}\hat{x}+\frac{2\pi}{\sqrt{3}}\hat{y}\right)$$

for $a=1$: $\vec{a}_1^* = \frac{4\pi}{3}\hat{x}+0\hat{y}$, $\vec{a}_2^* = -\frac{2\pi}{3}\hat{x}+\frac{2\pi}{\sqrt{3}}\hat{y}$

Q41. Consider a one-dimensional chain of atoms with lattice constant a. The energy of an electron with wave-vector k is $\varepsilon(k) = \mu - \gamma\cos(ka)$, where μ and γ are constants. If an electric field E is applied in the positive x-direction, the time dependent velocity of an electron is

(In the following B is the constant)

(a) Proportional to $\cos\left(B-\frac{eE}{\hbar}at\right)$

(b) proportional to E

(c) independent of E

(d) proportional to $\sin\left(B-\frac{eE}{\hbar}at\right)$

Ans. : (d)

Solution: In the presence of electric field E, we can write

$$\vec{F} = -e\vec{E} \Rightarrow \frac{d\vec{p}}{dt} = -e\vec{E} \Rightarrow \hbar\frac{dk}{dt} = -eE$$

Integration gives, $k(t) = k(0) - \frac{eE}{\hbar}t$

The group velocity $v = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{\partial \varepsilon(k)}{dk}$

Since, $\varepsilon(k) = \mu - \gamma\cos(ka)$, $\quad \therefore \frac{\partial \varepsilon(k)}{\partial k} = \gamma a \sin ka$

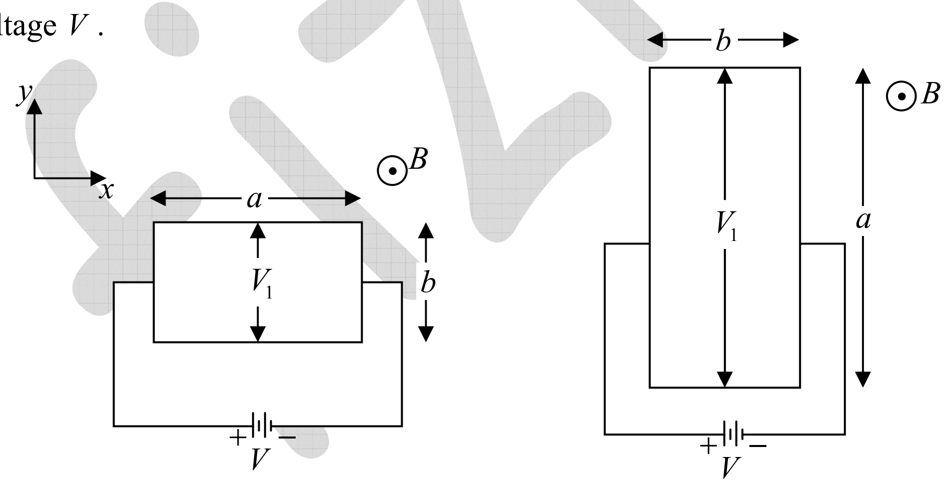
Thus, $v = \frac{\gamma a}{\hbar}\sin(ka)$

Time dependent velocity of electron is

$$v(t) = \frac{\gamma a}{\hbar}\sin\left[k(t)a\right] = \frac{\gamma a}{\hbar}\sin\left[\left(k(0) - \frac{eE}{\hbar}t\right)a\right]$$

$$= \frac{\gamma a}{\hbar}\sin\left[k(0)a - \frac{eE}{\hbar}at\right] \Rightarrow v(t) = \frac{\gamma a}{\hbar}\sin\left[B - \frac{eE}{\hbar}at\right]$$

Q42. A thin rectangular conducting plate of length a and width b is placed in the xy-plane in two different orientations as shown in the figures below. In both cases a magnetic field B is applied in the z-direction and a current flows in the x direction due to the applied voltage V.



If the Hall voltage across the y-direction in the two cases satisfy $V_2 = 2V_1$ the ratio $a:b$ must be

(a) $1:2$ (b) $1:\sqrt{2}$ (c) $2:1$ (d) $\sqrt{2}:1$

Ans. : (d)

Solution: Since, Hall voltage is given by $V_H = \frac{IB}{\rho w}$, where w is width of conducting plate.

Since, in case (I), $V = I_1 R_1$ and $R_1 = \frac{\rho l_1}{A_1} = \frac{\rho a}{a \times b} = \frac{\rho}{b}$

$$V = \frac{I_1 \rho}{b} \Rightarrow I_1 = \frac{bV}{\rho}$$

Then, $V_H = V_1 = \frac{I_1 B}{\rho w} = \frac{bVB}{\rho^2 w} = \frac{bVB}{\rho^2 a}$ $\qquad (\because w = a)$

And also in case (II), $R_2 = \frac{\rho l_2}{A_2} = \frac{\rho b}{a \times b} = \frac{\rho}{a}$

$$V = I_2 R_2 \Rightarrow I_2 = \frac{V}{R_2} = \frac{Va}{\rho}$$

Then, $V_H = V_2 = \frac{I_2 B}{\rho w} = \frac{VaB}{\rho^2 b}$

Since, $V_2 = 2V_1 \Rightarrow \frac{a^2}{b^2} = \frac{2}{1} \Rightarrow a : b = \sqrt{2} : 1$

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Q43. The energy gap and lattice constant of an indirect band gap semiconductor are $1.875\ eV$ and $0.52\,nm$, respectively. For simplicity take the dielectric constant of the material to be unity. When it is excited by broadband radiation, an electron initially in the valence band at $k = 0$ makes a transition to the conduction band. The wavevector of the electron in the conduction band, in terms of the wavevector $k_{\max}$ at the edge of the Brillouin zone, after the transition is closest to

(a) $k_{\max}/10$ (b) $k_{\max}/100$ (c) $k_{\max}/1000$ (d) 0

Ans. : (a)

Solution: The $K-$ value of electron in $C.B.$ is

$$K = \frac{\sqrt{2mE}}{\hbar} = \frac{\left[2\left(9.1\times10^{-31}kg\right)\left(1.875\times1.6\times10^{-19}J\right)\right]^{1/2}}{1.05\times10^{-34}J.S.} \Rightarrow K \approx 7\times10^{9}m^{-1}$$

$K_{\max}$ at the Brillouin Zero is $K_{\max} = \frac{2\pi}{a} = \frac{2\times3.14}{0.52\times10^{-9}m} \approx 1.2\times10^{10}m^{-1} \Rightarrow K \approx \frac{K_{\max}}{10}$

Q44. The electrical conductivity of copper is approximately 95% of the electrical conductivity of silver, while the electron density in silver is approximately 70% of the electron density in copper. In Drude's model, the approximate ratio τ_{Cu}/τ_{Ag} of the mean collision time in copper $\left(\tau_{Cu}\right)$ to the mean collision time in silver $\left(\tau_{Ag}\right)$ is

(a) 0.44 (b) 1.50 (c) 0.33 (d) 0.66

Ans. : (d)

Solution: $\sigma = \dfrac{ne^2\tau}{m} \Rightarrow \dfrac{\sigma_{cu}}{\sigma_{Ag}} = \dfrac{n_{cu}}{n_{Ag}}\dfrac{\tau_{cu}}{\tau_{Ag}} \Rightarrow \dfrac{\tau_{cu}}{\tau_{Ag}} = \dfrac{\sigma_{cu}}{\sigma_{Ag}} \times \dfrac{n_{Ag}}{n_{cu}}$

$$\Rightarrow \frac{\tau_{cu}}{\tau_{Ag}} = \frac{0.95\sigma_{Ag}}{\sigma_{Ag}} \times \frac{0.7n_{cu}}{n_{cu}} \approx 0.66$$