

CLASSICAL MECHANICS SOLUTIONSGATE- 2010

Q1. For the set of all Lorentz transformations with velocities along the x -axis consider the two statements given below:

P: If L is a Lorentz transformation then, L^{-1} is also a Lorentz transformation.

Q: If L_1 and L_2 are Lorentz transformations then, L_1L_2 is necessarily a Lorentz transformation.

Choose the correct option

- (A) P is true and Q is false (B) Both P and Q are true
(C) Both P and Q are false (D) P is false and Q is true

Ans: (b)

Q2. A particle is placed in a region with the potential $V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$, where $k, \lambda > 0$.

Then,

- (A) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable equilibrium
(B) $x = 0$ is a point of stable equilibrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium
(C) $x = 0$ and $x = \frac{k}{\lambda}$ are points of unstable equilibrium
(D) There are no points of stable or unstable equilibrium

Ans: (b)

$$\text{Solution: } V = \frac{1}{2}kx^2 - \frac{\lambda x^3}{3} \Rightarrow \frac{\partial V}{\partial x} = kx - \lambda x^2 = 0 \Rightarrow x = 0, x = \frac{k}{\lambda}.$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} = k - 2\lambda x$$

$$\Rightarrow \text{At } x = 0, \frac{\partial^2 V}{\partial x^2} = +ve \text{ (Stable) and } \Rightarrow \text{At } x = \frac{k}{\lambda}, \frac{\partial^2 V}{\partial x^2} = -ve \text{ (unstable)}$$

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Q3. A π^0 meson at rest decays into two photons, which move along the x -axis. They are both detected simultaneously after a time, $t = 10$ s. In an inertial frame moving with a velocity $V = 0.6c$ in the direction of one of the photons, the time interval between the two detections is

- (A) 15 s (B) 0 s (C) 10 s (D) 20 s

Ans: (a)

$$\begin{aligned} \text{Solution: } t_1 &= t_0 \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} = 10 \sqrt{\frac{1 + 0.6}{1 - 0.6}} = 10 \times 2 = 20 \text{ sec}, t_2 = t_0 \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \\ &= 10 \sqrt{\frac{1 - 0.6}{1 + 0.6}} = 10 \times \frac{1}{2} = 5 \text{ sec} \\ \Rightarrow t_1 - t_2 &= 15 \text{ sec} \end{aligned}$$

Statement for Linked Answer Questions 4 and 5:

The Lagrangian for a simple pendulum is given by $L = \frac{1}{2} ml^2 \dot{\theta}^2 - mgl(1 - \cos \theta)$

Q4. Hamilton's equations are then given by

- (A) $\dot{p}_\theta = -mgl \sin \theta; \quad \dot{\theta} = \frac{p_\theta}{ml^2}$ (B) $\dot{p}_\theta = mgl \sin \theta; \quad \dot{\theta} = \frac{p_\theta}{ml^2}$
 (C) $\dot{p}_\theta = -m\ddot{\theta}; \quad \dot{\theta} = \frac{p_\theta}{m}$ (D) $\dot{p}_\theta = -\left(\frac{g}{l}\right)\theta; \quad \dot{\theta} = \frac{p_\theta}{ml}$

Ans: (b)

Solution: $H = \frac{p_\theta^2}{2ml^2} + mgl(1 - \cos \theta) \Rightarrow \frac{\partial H}{\partial \theta} = -\dot{p}_\theta \Rightarrow \dot{p}_\theta = mgl \sin \theta; \quad \frac{\partial H}{\partial p_\theta} = \dot{\theta} \Rightarrow \dot{\theta} = \frac{p_\theta}{ml^2}$.Q5. The

Poisson bracket between θ and $\dot{\theta}$ is

- (A) $\{\theta, \dot{\theta}\} = 1$ (B) $\{\theta, \dot{\theta}\} = \frac{1}{ml^2}$
 (C) $\{\theta, \dot{\theta}\} = \frac{1}{m}$ (D) $\{\theta, \dot{\theta}\} = \frac{g}{l}$

Ans: (b)

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$$\{\theta, \dot{\theta}\} = \left\{ \theta, \frac{P_\theta}{ml^2} \right\} \text{ where } \dot{\theta} = \frac{P_\theta}{ml^2} \Rightarrow \frac{1}{ml^2} \left(\frac{\partial \theta}{\partial \theta} \frac{\partial \theta}{\partial P_\theta} - \frac{\partial \theta}{\partial P_\theta} \frac{\partial P_\theta}{\partial \theta} \right) = 1 \cdot \frac{1}{ml^2} - 0 = \frac{1}{ml^2}.$$

GATE- 2011

Q6. A particle is moving under the action of a generalized potential $V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$. The

magnitude of the generalized force is

(A) $\frac{2(1+\dot{q})}{q^3}$ (B) $\frac{2(1-\dot{q})}{q^3}$ (C) $\frac{2}{q^3}$ (D) $\frac{\dot{q}}{q^3}$

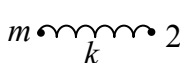
Ans: (c)

Solution: $\frac{d}{dt} \left(\frac{\partial V}{\partial \dot{q}} \right) - \frac{\partial V}{\partial q} = F_q \Rightarrow F_q = \frac{2}{q^3}.$

Q7. Two bodies of mass m and $2m$ are connected by a spring constant k . The frequency of the normal mode is

(A) $\sqrt{3k/2m}$ (B) $\sqrt{k/m}$ (C) $\sqrt{2k/3m}$ (D) $\sqrt{k/2m}$

Ans: (a)

Solution:  $\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{k}{\frac{2m}{3}}} = \sqrt{\frac{3k}{2m}}$ where reduce mass $\mu = \frac{2mm}{2m+m} = \frac{2m}{3}.$

Q8. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$$Q = q^\alpha \cos(\beta p), P = q^\alpha \sin(\beta p)$$

is canonical for

(A) $\alpha = 2, \beta = 1/2$ (B) $\alpha = 2, \beta = 2$ (C) $\alpha = 1, \beta = 1$ (D) $\alpha = 1/2, \beta = 2$

Ans: (d)

Solution: $\frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q} = 1$

$$\Rightarrow \alpha q^{\alpha-1} \cos(\beta p) \times q^\alpha \beta \cos(\beta p) - q^\alpha \beta (-\sin(\beta p)) \times \alpha q^{\alpha-1} \sin(\beta p) = 1$$

$$\alpha q^{2\alpha-1} \beta (\cos^2 \beta p + \sin^2 \beta p) = 1 \Rightarrow \alpha \beta q^{2\alpha-1} = 1 \Rightarrow \alpha = \frac{1}{2}, \beta = 2.$$

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- Q9. Two particles each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is
- (A) $5m / 4$ (B) $2m$ (C) $5m / 2$ (D) $25 m / 8$

Ans: (c)

Solution: From conservation of energy

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = m_1c^2 \Rightarrow \frac{2mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = m_1c^2$$

Since $v = 0.6c \Rightarrow m_1 = 5m / 2$

GATE- 2012

- Q10. In a central force field, the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by,

$$\frac{1}{r} = \frac{m}{L^2}(1 + \varepsilon \cos \theta)$$

where, ε is the eccentricity of the particle's motion. Which one of the following choice for ε gives rise to a parabolic trajectory?

- (a) $\varepsilon = 0$ (b) $\varepsilon = 1$ (c) $0 < \varepsilon < 1$ (d) $\varepsilon > 1$

Ans: (b)

Solution: $\frac{l}{r} = \frac{m}{l^2}(1 + \varepsilon \cos \theta)$ for parabolic trajectory $\varepsilon = 1$.

- Q11. A particle of unit mass moves along the x -axis under the influence of a potential, $V(x) = x(x - 2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is

- (a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

Ans: (b)

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$$V(x) = x(x-2)^2 \Rightarrow \frac{\partial V}{\partial x} = (x-2)^2 + 2x(x-2) = 0 \Rightarrow x = 2, x = \frac{2}{3}$$

$$\frac{\partial^2 V}{\partial x^2} = 2(x-2)^2 + 2(x-2) + 2x \Rightarrow \frac{\partial^2 V}{\partial x^2} = 2 \times 2 = 4$$

$$\Rightarrow \omega = \sqrt{\left. \frac{\partial^2 V}{\partial x^2} \right|_{x=2}} \Rightarrow \omega = \frac{2\pi}{T} = 2 \Rightarrow T = \pi$$

Q12. A rod of proper length l_0 oriented parallel to the x -axis moves with speed $2c/3$ along the x -axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the x -axis with speed $c/2$ with respect to the S-frame. The length of the rod as measured by the observer is

- (a) $0.35l_0$ (b) $0.48l_0$ (c) $0.87l_0$ (d) $0.97l_0$

Ans: (d)

Solution: $l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = 0.97 l_0$

Q13. A particle of mass m is attached to fixed point O by a weightless inextensible string of length a . It is rotating under the gravity as shown in the figure. The Lagrangian of the particle is

$$L(\theta, \phi) = \frac{1}{2} ma^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) - mga \cos \theta \quad \text{where } \theta \text{ and } \phi \text{ are the}$$

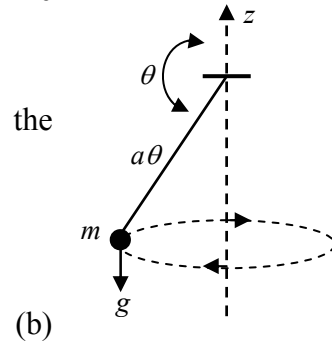
polar angles. The Hamiltonian of the particles is

(a) $H = \frac{1}{2ma^2} \left(p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta} \right) - mga \cos \theta$

$$H = \frac{1}{2ma^2} \left(p_\theta^2 + \frac{p_\phi^2}{\sin^2 \theta} \right) + mga \cos \theta$$

(c) $H = \frac{1}{2ma^2} (p_\theta^2 + p_\phi^2) - mga \cos \theta$

(d) $H = \frac{1}{2ma^2} (p_\theta^2 + p_\phi^2) + mga \cos \theta$



Ans: (b)

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Solution: $H = P_\theta \dot{\theta} + P_\phi \dot{\phi} - L = P_\theta \dot{\theta} + P_\phi \dot{\phi} - \frac{1}{2} ma^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) + mga \cos \theta$

$$\frac{\partial L}{\partial \dot{\theta}} = P_\theta \Rightarrow ma^2 \dot{\theta} = P_\theta \Rightarrow \dot{\theta} = \frac{P_\theta}{ma^2} \quad \text{and} \quad P_\phi = \frac{\partial L}{\partial \dot{\phi}} = ma^2 \sin^2 \theta \dot{\phi} \Rightarrow \dot{\phi} = \frac{P_\phi}{ma^2 \sin^2 \theta}$$

Put the value of $\dot{\theta}$ and $\dot{\phi}$

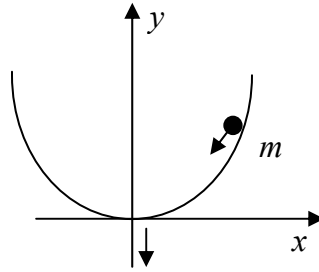
$$H = P_\theta \times \frac{P_\theta}{ma^2} + P_\phi \times \frac{P_\phi}{ma^2 \sin^2 \theta} - \frac{1}{2} ma^2 \left(\left(\frac{P_\theta}{ma^2} \right)^2 + \sin^2 \theta \left(\frac{P_\phi}{ma^2 \sin^2 \theta} \right)^2 \right) + mga \cos \theta$$

$$H = \frac{P_\theta^2}{ma^2} - \frac{P_\theta^2}{2ma^2} + \frac{P_\phi^2}{ma^2 \sin^2 \theta} - \frac{P_\phi^2}{2ma^2 \sin^2 \theta} + mga \cos \theta$$

$$H = \frac{1}{2ma^2} \left(P_\theta^2 + \frac{P_\phi^2}{\sin^2 \theta} \right) + mga \cos \theta$$

Statement for Linked Answer Questions 14 and 15:

Q14. A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$ axis shown in the figure. Here a is a constant.



The Lagrangian for this particle is given by

(a) $L = \frac{1}{2} m \dot{x}^2 - mgax^2$

(b) $L = \frac{1}{2} m(1 + 4a^2 x^2) \dot{x}^2 - mgax^2$

(c) $L = \frac{1}{2} m \dot{x}^2 + mgax^2$

(d) $L = \frac{1}{2} m(1 + 4a^2 x^2) \dot{x}^2 + mgax^2$

Ans: (d)

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Solution: Equation of constrain is given by $y = ax^2$, K.E $T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2)$

$$\dot{y} = 2ax\dot{x} \Rightarrow T = \frac{1}{2} m(\dot{x}^2 + 4ax^2\dot{x}^2) = \frac{1}{2} m\dot{x}^2(1 + 4ax^2)$$

$V = -mgy = -mgax^2$. Since particle is moving downward direction so potential V is negative.

$$\therefore L = T - V \Rightarrow L = \frac{1}{2} m(1 + 4a^2x^2)\dot{x}^2 + mgax^2$$

Q15. The Lagrange's equation of motion of the particle for above question is given by

- (a) $\ddot{x} = 2gax$ (b) $m(1 + 4a^2x^2)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$
 (c) $m(1 + 4a^2x^2)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$ (d) $\ddot{x} = -2gax$

Ans: (c)

Solution: $\frac{d}{dt} \left(\frac{dL}{dx} \right) = \frac{dL}{dx} \Rightarrow m(1 + 4a^2x^2)\ddot{x} = 4ma^2x\dot{x}^2 + 2mgax$

GATE- 2013

Q16. In the most general case, which one of the following quantities is NOT a second order tensor?

- (a) Stress (b) Strain
 (c) Moment of inertia (d) Pressure

Ans: (b)

Solution: Strain is not a tensor.

Q17. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is

- (a) c (b) $-c$
 (c) $0.15c$ (d) $-0.15c$

Ans: (b)

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Q18. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

- (a) $p_q = 2\beta q$ and it is a conserved quantity.
- (b) $p_q = 2\beta q$ and it is not a conserved quantity.
- (c) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity.
- (d) $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

Ans: (d)

Solution: $\frac{\partial L}{\partial \dot{q}} = p_q$ but $\frac{\partial L}{\partial q} \neq 0$

Q19. The relativistic form of Newton's second law of motion is

- (a) $F = \frac{mc}{\sqrt{c^2 - v^2}} \frac{dv}{dt}$
- (b) $F = \frac{m\sqrt{c^2 - v^2}}{c} \frac{dv}{dt}$
- (c) $F = \frac{mc^2}{c^2 - v^2} \frac{dv}{dt}$
- (d) $F = m \frac{c^2 - v^2}{c^2} \frac{dv}{dt}$

Ans:

Solution: $P = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow F = \frac{dP}{dt} = m \frac{dv}{dt} \cdot \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + mv \left(-\frac{1}{2} \right) \cdot \frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \cdot \frac{-2v}{c^2} \frac{dv}{dt}$

$$\Rightarrow F = m \frac{dv}{dt} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(1 + \frac{1}{2} \frac{\frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)} \right) = m \frac{dv}{dt} \left(\frac{1 - \frac{v^2}{2c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \right)$$

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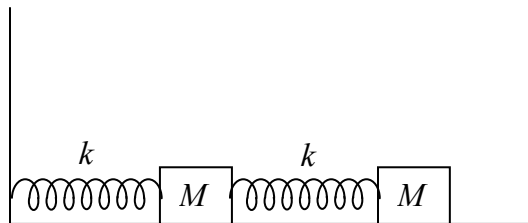
Q20. Consider two small blocks, each of mass M , attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k . The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,

(a) $\sqrt{\frac{3+\sqrt{2}}{2}} \sqrt{\frac{k}{M}}$

(b) $\sqrt{\frac{3+\sqrt{3}}{2}} \sqrt{\frac{k}{M}}$

(c) $\sqrt{\frac{3+\sqrt{5}}{2}} \sqrt{\frac{k}{M}}$

(d) $\sqrt{\frac{3+\sqrt{6}}{2}} \sqrt{\frac{k}{M}}$



Ans: (c)

Solution: $T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2,$

$$V = \frac{1}{2}kx_1^2 + \frac{1}{2}k(x_2 - x_1)^2 = \frac{1}{2}kx_1^2 + \frac{1}{2}k(x_2^2 + x_1^2 - 2x_2x_1) = \frac{1}{2}k(2x^2 + x^2 - 2x_2x_1)$$

$$T = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}; \quad V = \begin{pmatrix} 2k & -k \\ -k & k \end{pmatrix}$$

$$\begin{vmatrix} 2k - \omega^2 m & -k \\ -k & k - \omega^2 m \end{vmatrix} = 0 \Rightarrow (2k - \omega^2 m)(k - \omega^2 m) - k^2 = 0 \Rightarrow \omega = \sqrt{\frac{3+\sqrt{5}}{2}} \sqrt{\frac{k}{m}}$$

GATE- 2014

Q21. If the half-life of an elementary particle moving with speed $0.9c$ in the laboratory frame is $5 \times 10^{-8} s$, then the proper half-life is _____ $\times 10^{-8} s.$ ($c = 3 \times 10^8 m/s$)

Ans: 2.18

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Solution: $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, $t_0 = t \times \sqrt{1 - \frac{v^2}{c^2}} = t_0 = 5 \times 10^{-8} \times \sqrt{.19} \Rightarrow 2.18 \times 10^{-8} s$

Q22. Two masses m and $3m$ are attached to the two ends of a massless spring with force constant K . If $m = 100g$ and $K = 0.3N/m$, then the natural angular frequency of oscillation is _____ Hz.

Ans: 0.318

Solution: $f = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}}$ $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{3m \cdot m}{4m} = \frac{3m}{4}$ $\omega = \sqrt{\frac{4k}{3m}} = 2 = 0.318 \text{ Hz}$

Q23. The Hamilton's canonical equation of motion in terms of Poisson Brackets are

(a) $\dot{q} = \{q, H\}; \dot{p} = \{p, H\}$

(b) $\dot{q} = \{H, q\}; \dot{p} = \{H, p\}$

(c) $\dot{q} = \{H, p\}; \dot{p} = \{H, p\}$

(d) $\dot{q} = \{p, H\}; \dot{p} = \{q, H\}$

Ans: (a)

Solution: $\frac{df}{dt} = \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial t} + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{\partial t} + \frac{\partial f}{\partial t}$

$\frac{df}{dt} = \frac{\partial f}{\partial q} \cdot \frac{\partial H}{\partial p} - \frac{\partial f}{\partial p} \cdot \frac{\partial H}{\partial q} + \frac{\partial f}{\partial t} \Rightarrow \frac{df}{dt} = \{f, H\} + \frac{\partial f}{\partial t}$

$\frac{dq}{dt} = \{q, H\}$ and $\frac{dp}{dt} = \{p, H\}$

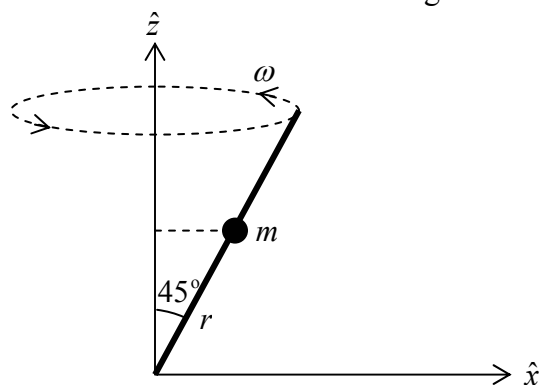
Q24. A bead of mass m can slide without friction along a mass less rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω . At any instant r is the distance of the bead from the origin. The momentum conjugate to r is

(a) $m\dot{r}$

(b) $\frac{1}{\sqrt{2}} m\dot{r}$

(c) $\frac{1}{2} m\dot{r}$

(d) $\sqrt{2} m\dot{r}$



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Ans: (a)

Solution: $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2 \theta \dot{\phi}^2) - mgr \cos \theta$

equation of constrain is $\theta = \frac{\pi}{4}$ and it is given $\dot{\phi} = \omega$

$$L = \frac{1}{2}m(\dot{r}^2 + \frac{1}{2}r^2\omega^2) - \frac{1}{\sqrt{2}}mgr$$

the momentum conjugate to r is $p_r = \frac{\partial L}{\partial \dot{r}} = p_r = m\dot{r}$

Q25. A particle of mass m is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$$

when a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

Ans: (a)

Solution: $V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$ for equilibrium $\frac{\partial V}{\partial r} = \frac{a}{r^2} - \frac{3ar_0^2}{3r^4} = 0$ $r = \pm r_0$

$$\frac{\partial^2 V}{\partial r^2} = -\frac{2a}{r^3} + \frac{4ar_0^2}{r^5} \Big|_{r_0} = -\frac{2a}{r_0^3} + \frac{4ar_0^2}{r_0^5} = \frac{2a}{r_0^3}$$

$$\omega = \sqrt{\frac{\partial^2 V}{\partial r^2} \Big|_{r_0}} \Rightarrow T = 2\pi\sqrt{\frac{mr_0^3}{2a}}$$

Q26. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential

$V(r) = -\frac{k}{r}$, where k is a positive constant. The orbit angular momentum of the planet is

(a) $2r_0 km$ (b) $\sqrt{2r_0 km}$ (c) $r_0 km$ (d) $\sqrt{r_0 km}$

Ans: (d)

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Solution: $V_{\text{effective}} = \frac{J^2}{2mr^2} - \frac{k}{r} \Rightarrow \frac{dV_{\text{effective}}}{dr} = -\frac{J^2}{mr^3} + \frac{k}{r^2} = 0$ at $r = r_0$

so $J = \sqrt{r_0 km}$

Q27. Given that the linear transformation of a generalized coordinate q and the corresponding momentum p ,

$$Q = q + 4ap$$

$$P = q + 2p$$

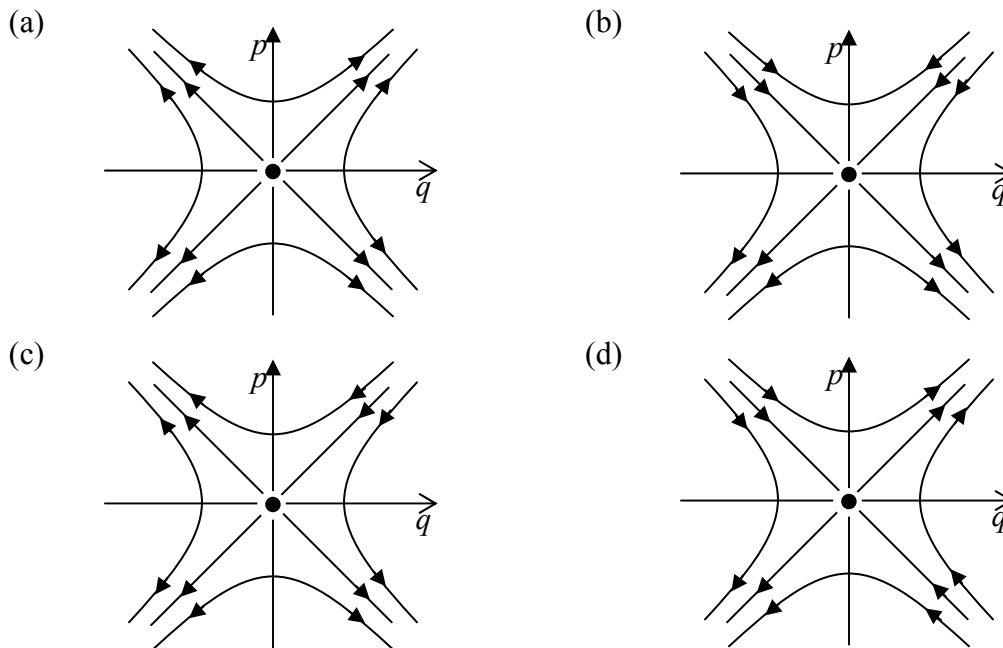
is canonical, the value of the constant a is _____

Ans: 0.5

Solution: $\frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q} = 0 \Rightarrow 1.2 - 4a.1 = 0 \Rightarrow a = 0.5$

Q28. The Hamiltonian of particle of mass m is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. which one of the

following figure describes the motion of the particle in phase space?



Ans: (d)

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GATE- 2015

Q29. A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

- (a) $V = -2T; E = -T$ (b) $V = -T; E = 0$
 (c) $V = -\frac{T}{2}; E = \frac{T}{2}$ (d) $V = \frac{-3T}{2}; E = \frac{-T}{2}$

Ans.: (a)

Solution: From Virial theorem $\langle T \rangle = \frac{n+1}{2} \langle V \rangle$ where $V \propto r^{n+1}$

$$\because V = \frac{-k}{r} \Rightarrow V \propto \frac{1}{r} \Rightarrow n = -2 \Rightarrow \langle V \rangle = -2 \langle T \rangle$$

Q30. In an inertial frame S , two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame S' moving with a velocity $0.6c\hat{y}$ with respect to S are given by

- (a) $ct'_A = 0; ct'_B = -\frac{3}{2}$ (b) $ct'_A = 0; ct'_B = 0$
 (c) $ct'_A = 0; ct'_B = \frac{3}{2}$ (d) $ct'_A = 0; ct'_B = \frac{1}{2}$

Ans.: (a)

Solution: Velocity of S' with respect to S is $v = .6c$

$$t'_A = \frac{t_A - \frac{v}{c^2}y}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{for event A } t_A = 0, y = 0 \text{ so } ct'_A = 0$$

$$t'_B = \frac{t_B - \frac{v}{c^2}y}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{for event B } t_B = 0, y = 2 \text{ so } ct'_B = -\frac{3}{2}$$

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Q31. The Lagrangian for a particle of mass m at a position \vec{r} moving with a velocity \vec{v} is given by $L = \frac{m}{2}\vec{v}^2 + C\vec{r}\cdot\vec{v} - V(r)$, where $V(r)$ is a potential and C is a constant. If \vec{p}_c is the canonical momentum, then its Hamiltonian is given by

- (a) $\frac{1}{2m}(\vec{p}_c + C\vec{r})^2 + V(r)$ (b) $\frac{1}{2m}(\vec{p}_c - C\vec{r})^2 + V(r)$
 (c) $\frac{p_c^2}{2m} + V(r)$ (d) $\frac{1}{2m}p_c^2 + C^2r^2 + V(r)$

Ans.: (b)

Solution: $L = \frac{m}{2}\vec{v}^2 + C\vec{r}\cdot\vec{v} - V(r)$ where $\vec{v} = \dot{r}$

$$H = \sum \dot{r} p_c - L = \dot{r} p_c - L \quad \text{where } L = \frac{m}{2}\dot{r}^2 + Cr\dot{r} - V(r)$$

$$\Rightarrow \frac{\partial L}{\partial \dot{r}} = p_c = (m\dot{r} + Cr) \Rightarrow \dot{r} = \frac{p_c - Cr}{m}$$

$$\Rightarrow H = \left(\frac{p_c - Cr}{m}\right)p_c - \frac{m}{2}\left(\frac{p_c - Cr}{m}\right)^2 - cr\left(\frac{p_c - Cr}{m}\right) + V(r)$$

$$\Rightarrow H = \left(\frac{p_c - Cr}{m}\right)(p_c - Cr) - \frac{m}{2}\left(\frac{p_c - Cr}{m}\right)^2 + V(r)$$

$$\Rightarrow H = \frac{(p_c - Cr)^2}{m} - \frac{(p_c - Cr)^2}{2m} + V(r) \quad \Rightarrow H = \frac{1}{2m}(p_c - Cr)^2 + V(r)$$

Q32. The Hamiltonian for a system of two particles of masses m_1 and m_2 at \vec{r}_1 and \vec{r}_2 having velocities \vec{v}_1 and \vec{v}_2 is given by $H = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2} \hat{z} \cdot (\vec{r}_1 \times \vec{r}_2)$, where C is constant. Which one of the following statements is correct?

- (a) The total energy and total momentum are conserved
 (b) Only the total energy is conserved
 (c) The total energy and the z - component of the total angular momentum are conserved
 (d) The total energy and total angular momentum are conserved

Ans.: (c)

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Solution: Lagrangian is not function of time so energy is conserve and component of $(\vec{r}_1 \times \vec{r}_2)$ are

Only in \hat{z} direction means potential is symmetric under ϕ so L_z is conserve.

Q33. A particle of mass 0.01 kg falls freely in the earth's gravitational field with an initial velocity $(0) = 10 \text{ ms}^{-1}$. If the air exerts a frictional force of the form, $f = -kv$, then for $k = 0.05 \text{ Nm}^{-1}\text{s}$, the velocity (in ms^{-1}) at time $t = 0.2 \text{ s}$ is _____ (upto two decimal places). (use $g = 10 \text{ ms}^{-2}$ and $e = 2.72$)

Ans.: Data given is incorrect

$$\begin{aligned} \text{Solution: } m \frac{dv}{dt} &= mg - kv \Rightarrow \frac{dv}{dt} = g - \frac{k}{m}v \Rightarrow \frac{dv}{g - \frac{k}{m}v} = dt \Rightarrow \int_{10}^u \frac{dv}{g - \frac{k}{m}v} = \int_0^{0.2} dt \\ &\Rightarrow -\frac{m}{k} \left[\ln \left[g - \frac{k}{m}v \right] \right]_{10}^u = [t]_0^{0.2} \Rightarrow -\frac{m}{k} \left\{ \left[\ln \left(g - \frac{k}{m}u \right) \right] - \ln \left(g - \frac{10k}{m} \right) \right\} = 0.2 \\ &\Rightarrow -\frac{m}{k} \left\{ \ln \left(10 - \frac{0.05}{0.01}u \right) - \ln \left(10 - 10 \times \frac{.05}{.01} \right) \right\} = 0.2 \\ &\Rightarrow -\frac{m}{k} \left\{ \ln(10 - 5u) - \ln(-40) \right\} = 0.2 \end{aligned}$$

$\therefore \ln(-40)$ can not be defined. So given data are not correct.

Q34. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If \vec{F}_c and \vec{F}_{Co} denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

- (a) \vec{F}_c is radially outward and $\vec{F}_{Co} = \vec{F}_c$
- (b) \vec{F}_c is radially inward and $\vec{F}_{Co} = -2\vec{F}_c$
- (c) \vec{F}_c is radially outward and $\vec{F}_{Co} = -2\vec{F}_c$
- (d) \vec{F}_c is radially outward and $\vec{F}_{Co} = 2\vec{F}_c$

Ans.: (b)

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Q35. A particle with rest mass M is at rest and decays into two particles of equal rest masses

$\frac{3}{10}M$ which move along the z axis. Their velocities are given by

- (a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$
 (c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z}; \vec{v}_2 = (-0.8c)\hat{z}$

Ans.: (b)

Solution: $M \rightarrow \frac{3}{10}M + \frac{3}{10}M$

From momentum conservation

$$0 = \vec{P}_1 + \vec{P}_2 \Rightarrow \vec{P}_1 = -\vec{P}_2 \Rightarrow |P_1| = |P_2|$$

From energy conservation $E = E_1 + E_2$

$$\Rightarrow Mc^2 = \frac{3}{10} \frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{3}{10} \frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow Mc^2 = \frac{3}{5} \frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(1 - \frac{v^2}{c^2}\right) = \frac{9}{25} \Rightarrow \frac{v^2}{c^2} = \frac{16}{25} \Rightarrow v = 0.8c$$

GATE-2016

Q36. The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of m_0c , where c is the speed of light in vacuum, is _____.

(Give your answer upto two decimal places)

Ans. : 1.73

Solution: $\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = 2m_0c^2 \Rightarrow E$

$$E^2 = p^2c^2 + m_0^2c^4 \Rightarrow 4m_0^2c^4 - m_0^2c^4 = p^2c^2 \Rightarrow p = \sqrt{3}m_0c = 1.732m_0c$$

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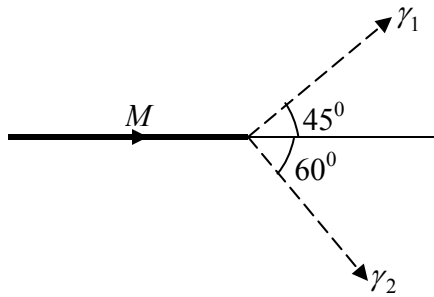
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Ans.: (a)

Solution: φ is cyclic coordinate so $\frac{\partial L}{\partial \dot{\varphi}} = p_{\varphi} \Rightarrow ml^2 \sin^2 \dot{\varphi}$ is constant hence m, l and g are constants. Then $\dot{\varphi} \sin^2 \theta$

Q39. A particle of rest mass M is moving along the positive x -direction. It decays into two photons γ_1 and γ_2 as shown in the figure. The energy of γ_1 is 1 GeV and the energy of γ_2 is 0.82 GeV . The value of M (in units of $\frac{\text{GeV}}{c^2}$) is _____. (Give your answer upto two decimal places)



Ans.: 1.40

Solution: $\sqrt{p^2 c^2 + M^2 c^4} = E_1 + E_2 = 1.82 \text{ GeV}$

$$p = \frac{E_1}{c} \cos \theta_1 + \frac{E_2}{c} \cos \theta_2 = \frac{1 \text{ GeV}}{c} \frac{1}{\sqrt{2}} + \frac{.82 \text{ GeV}}{c} \frac{1}{2} = \frac{1.11 \text{ GeV}}{c}$$

$$\Rightarrow p^2 c^2 + m^2 c^4 = 3.312$$

$$\Rightarrow m^2 c^4 = 3.312 - 1.21 = 2.077$$

$$\Rightarrow m = \sqrt{2.076} = 1.40$$

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GATE- 2017

Q40. If the Lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2q^2$ is modified to $L = L_0 + \alpha q\left(\frac{dq}{dt}\right)$, which one of the following is TRUE?

- (a) Both the canonical momentum and equation of motion do not change
- (b) Canonical momentum changes, equation of motion does not change
- (c) Canonical momentum does not change, equation of motion changes
- (d) Both the canonical momentum and equation of motion change

Ans. : (b)

Solution: For lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2q^2$ canonical momentum is $p = m\dot{q}$ and

$$\text{equation of motion is given by } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \quad m\ddot{q} + m\omega^2q = 0$$

$$\text{For langrangian } L = L_0 + \alpha q\left(\frac{dq}{dt}\right) = L = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2q^2 + \alpha q\dot{q}$$

Canonical momentum is $p = m\dot{q} + \alpha q$

$$\text{Equation of motion } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \quad m\ddot{q} + m\omega^2q = 0$$

Q41. Two identical masses of 10 gm each are connected by a massless spring of spring constant 1N/m. The non-zero angular eigenfrequency of the system is.....rad/s. (up to two decimal places)

Ans. : 14.14

$$\text{Solution: } \omega = \sqrt{\frac{k}{\mu}}, \text{ where } \mu = \frac{m}{2} = \frac{10}{2 \times 1000} = \frac{1}{200} \text{ and } k = 1N/m \quad \omega = 14.14$$

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Q42. The phase space trajectory of an otherwise free particle bouncing between two hard walls elastically in one dimension is a

- (a) straight line (b) parabola
(c) rectangle (d) circle

Ans. : (c)

Solution: $E = \frac{p^2}{2m}$ $p = \pm\sqrt{2mE}$

Q43. The Poisson bracket $[x, xp_y + yp_x]$ is equal to

- (a) $-x$ (b) y (c) $2p_x$ (d) p_y

Ans. : (b)

Solution: $[x, xp_y + yp_x] = [x, xp_y] + [x, yp_x] = 0 + y[x, p_x] = y$

Q44. An object travels along the x -direction with velocity $\frac{c}{2}$ in a frame O . An observer in a frame O' sees the same object travelling with velocity $\frac{c}{4}$. The relative velocity of O' with respect to O in units of c is..... (up to two decimal places).

Ans. : 0.28

Solution: $u'_x = \frac{c}{2}, v = \frac{c}{4}$ $u_x = \frac{u'_x - v}{1 - \frac{u'_x v}{c^2}}$

$$u_x = \frac{\frac{c}{2} - \frac{c}{4}}{1 - \frac{\frac{c}{2} \cdot \frac{c}{4}}{c^2}} = \frac{2c}{7} = 0.28c$$

Q45. A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10 gm and 1 cm respectively, the final linear velocity of the cylinder is..... m/s . (up to two decimal places).

Ans. : 3.33

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Solution: $mv_r = mv_{cm}r + I_{cm}\omega = mv_{cm}r + \frac{1}{2}mr^2 \frac{v_{cm}}{r} = v = \frac{3}{2}v_{cm} \Rightarrow v_{cm} = \frac{2}{3}v = \frac{10}{3} = 3.33m / \text{sec}$

Q46. A person weighs w_p at Earth's north pole and w_e at the equator. Treating the Earth as a perfect sphere of radius 6400 km , the value $100 \times \frac{(w_p - w_e)}{w_p}$ is..... (up to two decimal places). (Take $g = 10 \text{ ms}^{-2}$).

Ans. : 0.33

Solution: $g_p = g$

$$g_e = g - \omega^2 R$$

$$100 \times \frac{w_p - w_e}{w_p} = \frac{\omega^2 R}{g}$$

Put the value of $g = 10 \text{ m} / \text{sec}^2$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600}$$

$$R = 6400 \times 10^3$$

$$\text{Then } 100 \times \frac{w_p - w_e}{w_p} = 0.33$$

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