

QUANTUM MECHANICS SOLUTIONS

GATE- 2010

Q1. Which of the following is an allowed wavefunction for a particle in a bound state? N is a constant and $\alpha, \beta > 0$.

(a) $\psi = N \frac{e^{-\alpha r}}{r^3}$

(b) $\psi = N(1 - e^{-\alpha r})$

(c) $\psi = N e^{-\alpha x} e^{-\beta(x^2+y^2+z^2)}$

(d) $\psi = \begin{cases} \text{non - zero constant} & \text{if } r < R \\ 0 & \text{if } r > R \end{cases}$

Ans: (c)

Q2. A particle of mass m is confined in the potential

$$V(x) = \begin{cases} \frac{1}{2} m \omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \leq 0 \end{cases}$$

Let the wavefunction of the particle be given by

$$\psi(x) = -\frac{1}{\sqrt{5}} \psi_0 + \frac{2}{\sqrt{5}} \psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is

(a) $\frac{31}{10} \hbar \omega$

(b) $\frac{25}{10} \hbar \omega$

(c) $\frac{13}{10} \hbar \omega$

(d) $\frac{11}{10} \hbar \omega$

Ans: (a)

Solution: For half parabolic potential

$$E_0 = \frac{3}{2} \hbar \omega, E_1 = \frac{7}{2} \hbar \omega \Rightarrow \langle E \rangle = \frac{1}{5} \times \frac{3}{2} + \frac{4}{5} \times \frac{7}{2} = \frac{31}{10} \hbar \omega.$$

Q3. For a spin- s particle, in the eigen basis of \vec{S}^2 , S_z the expectation value $\langle sm | S_z^2 | sm \rangle$ is

(a) $\frac{\hbar^2 \{s(s+1) - m^2\}}{2}$

(b) $\hbar^2 \{s(s+1) - 2m^2\}$

(c) $\hbar^2 \{s(s+1) - m^2\}$

(d) $\hbar^2 m^2$

Ans: (a)

Solution: $\langle sm | S_z^2 | sm \rangle = \frac{1}{2} \langle sm | (S_+ + S_-)^2 | sm \rangle = \frac{1}{2} \langle sm | S_+^2 + S_-^2 + S_+ S_- + S_- S_+ | sm \rangle$
 $= \frac{1}{2} \langle sm | S_+ S_- + S_- S_+ | sm \rangle = \frac{\hbar^2}{2} [s(s+1) - m^2]$

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Q7. If L_x , L_y and L_z are respectively the x , y and z components of angular momentum operator L . The commutator $[L_x L_y, L_z]$ is equal to

- (a) $i\hbar(L_x^2 + L_y^2)$ (b) $2i\hbar L_z$ (c) $i\hbar(L_x^2 - L_y^2)$ (d) 0

Ans: (c)

Solution: $[L_x L_y, L_z] = L_x [L_y, L_z] + [L_x, L_z] L_y = i\hbar(L_x^2 - L_y^2)$

Q8. The normalized ground state wavefunction of a hydrogen atom is given by

$$\psi(r) = \frac{1}{\sqrt{4\pi}} \frac{2}{a^{3/2}} e^{-r/a}, \text{ where } a \text{ is the Bohr radius and } r \text{ is the distance of the electron}$$

from the nucleus, located at the origin. The expectation value $\left\langle \frac{1}{r^2} \right\rangle$ is

- (a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

Ans: (d)

Solution: $\left\langle \frac{1}{r^2} \right\rangle = \frac{1}{4\pi a^3} \int_0^\infty \frac{1}{r^2} r^2 e^{-\frac{2r}{a}} dr \int_0^\pi \int_0^{2\pi} \sin\theta d\theta d\phi = \frac{2}{a^2}$

Q9. The normalized eigenstates of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

are given by $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$, where $n = 1, 2, 3, \dots$

The particle is subjected to a perturbation

$$V'(x) = \begin{cases} V_o \cos\left(\frac{\pi x}{a}\right) & \text{for } 0 \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

Q10. The shift in the ground state energy due to the perturbation, in the first order perturbation theory,

- (a) $\frac{2V_o}{3\pi}$ (b) $\frac{V_o}{3\pi}$ (c) $-\frac{V_o}{3\pi}$ (d) $-\frac{2V_o}{3\pi}$

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Ans: (a)

$$\text{Solution: } E_1^1 = \int_0^{a/2} \psi_1^* V'(x) \psi_1 dx = \frac{2}{a} \int_0^{a/2} \sin^2\left(\frac{\pi x}{a}\right) V_0 \cos\left(\frac{\pi x}{a}\right) dx = \frac{2}{a} V_0 \left. \frac{\sin^3 \frac{\pi x}{a}}{3 \frac{\pi}{a}} \right|_0^{a/2} = \frac{2V_0}{3\pi}$$

Common data questions Q-10 and Q-11

Q11. In a one-dimensional harmonic oscillator, ϕ_0 , ϕ_1 and ϕ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by

$$\psi_1 = \phi_0 - 2\phi_1 + 3\phi_2, \quad \psi_2 = \phi_0 - \phi_1 + \alpha\phi_2$$

where α is a constant

Q12. The value of α which ψ_2 is orthogonal to ψ_1 is

- (a) 2 (b) 1 (c) -1 (d) -2

Ans: (c)

Solution: For orthogonal condition scalar product $(\psi_2, \psi_1)=0$ so $1 + 2 + 3\alpha = 0 \Rightarrow \alpha = -1$.

Q13. For the value of α determined in Q30, the expectation value of energy of the oscillator in the state ψ_2 is

- (a) $\hbar\omega$ (b) $3\hbar\omega/2$ (c) $3\hbar\omega$ (d) $9\hbar\omega/2$

Ans: (b)

$$\text{Solution: } \psi_2 = \phi_0 - \phi_1 + \alpha\phi_2 \text{ put } \alpha = -1, \langle H \rangle = \frac{\langle \psi_2 | H | \psi_2 \rangle}{\langle \psi_2 | \psi_2 \rangle} = \frac{\frac{\hbar\omega}{2} + \frac{3\hbar\omega}{2} + \frac{5\hbar\omega}{2}}{3} = \frac{3}{2} \hbar\omega$$

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GATE- 2012

Q14. A particle of mass m is confined in a two dimensional square well potential of dimension a . This potential $V(x, y)$ is given by

$$V(x, y) = 0 \text{ for } -a < x < a \text{ and } -a < y < a \\ = \infty \text{ elsewhere}$$

The energy of the first excited state for this particle is given by,

(a) $\frac{\pi^2 \hbar^2}{ma^2}$ (b) $\frac{2\pi^2 \hbar^2}{ma^2}$ (c) $\frac{5\pi^2 \hbar^2}{8ma^2}$ (d) $\frac{4\pi^2 \hbar^2}{ma^2}$

Ans: (c)

Solution: $E = (n_x^2 + n_y^2) \frac{\pi^2 \hbar^2}{2m(2a)^2} = (n_x^2 + n_y^2) \frac{\pi^2 \hbar^2}{8ma^2} = \frac{5\pi^2 \hbar^2}{8ma^2} \quad \because n_x = 1, n_y = 2.$

Q15. Consider the wavefunction $\psi = \psi(\vec{r}_1, \vec{r}_2) \chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} [\phi_1(\vec{r}_1) \phi_2(\vec{r}_2) + \phi_2(\vec{r}_1) \phi_1(\vec{r}_2)]$$

where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha(+1/2)$ and $\beta(-1/2)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$ (c) $\alpha\alpha$ (d) $\beta\beta$

Ans: (b)

Solution: Since $\psi(r_1, r_2)$ is symmetric the total wavefunction must be antisymmetric for fermions so spin part must be antisymmetric.

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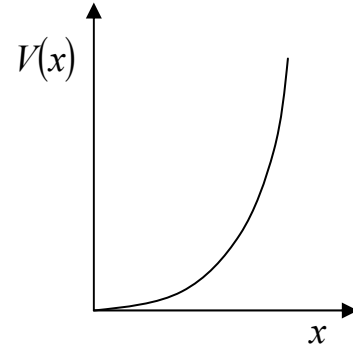
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Q16. A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is CORRECT?

- (a) The parity of the first excited state is even
- (b) The parity of the ground state is even
- (c) the ground state energy is $\frac{1}{2} \hbar \omega$
- (d) The first excited state energy is $\frac{7}{2} \hbar \omega$



Ans: (d)

Solution: There is only odd parity. Ground state is $\frac{3}{2} \hbar \omega$ and first excited = $\frac{7}{2} \hbar \omega$

Q17. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The system is subjected to a perturbation of the form $H' = \begin{pmatrix} \delta & \delta \\ \delta & \delta \end{pmatrix}$, when $\delta \ll 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are

- (a) 1 and $(1 + 2\delta)$
- (b) $(1 + \delta)$ and $(1 - \delta)$
- (c) $(1 + 2\delta)$ and $(1 - 2\delta)$
- (d) $(1 + \delta)$ and $(1 - 2\delta)$

Ans: (a)

Solution: $H_0 + H'$, H_0 is degenerate so after using degenerate perturbation through diagonalized

$$H' \text{ one will get } H' = \delta \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, H = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \delta \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}.$$

So $E = 1 + 2\delta$ and $1 + 0\delta$.

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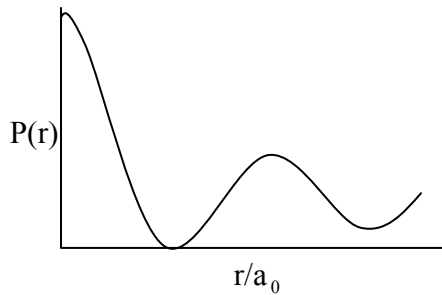
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Q18. The ground state wavefunction for the hydrogen atom is given by

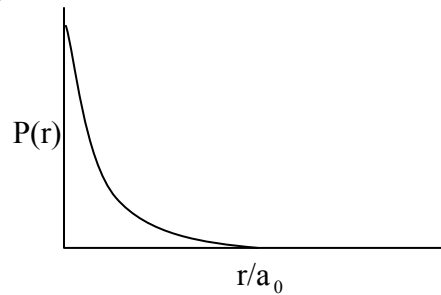
$$\psi_{100} = \frac{1}{\sqrt{4\pi}} \left(\frac{1}{a_0} \right)^{3/2} e^{-r/a_0}, \text{ where } a_0 \text{ is the Bohr radius. The plot of the radial probability}$$

density, $P(r)$ for the hydrogen atom in the ground state is

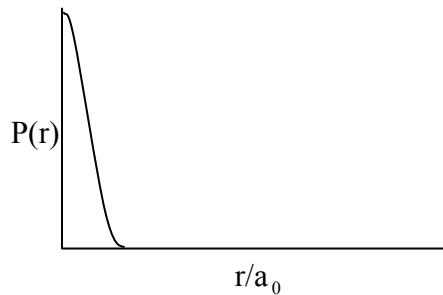
(a)



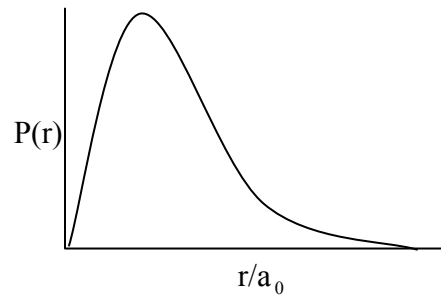
(b)



(c)



(d)

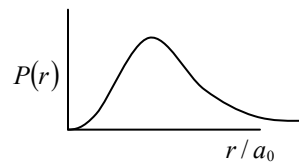


Ans: (d)

Solution: The ground state is given by $\psi_{100} = \frac{1}{\sqrt{4\pi}} \left(\frac{1}{a_0} \right)^{3/2} e^{-r/a_0}$

Radial probability function

$$P(r) = |\psi|^2 r^2 = \frac{1}{4\pi} \frac{1}{a_0^3} r^2 e^{-2r/a_0}$$



Common Data for Questions 19–20

The wavefunction of particle moving in free space is given by, $\psi = e^{ikx} + 2e^{-ikx}$

Q19. The energy of the particle is

(a) $\frac{5\hbar^2 k^2}{2m}$

(b) $\frac{3\hbar^2 k^2}{4m}$

(c) $\frac{\hbar^2 k^2}{2m}$

(d) $\frac{\hbar^2 k^2}{m}$

Ans: (c)

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Solution: $H\psi = E\psi$, $H\psi = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{-\hbar^2}{2m} (ik)(ik)e^{ikx} + 2(-ik)(-ik)e^{-ikx}$

$$\Rightarrow H\psi = \frac{\hbar^2 k^2}{2m} (e^{ikx} + 2e^{-2ikx}) = \frac{\hbar^2 k^2}{2m} \psi$$

Q20. The probability current density for the real part of the wavefunction is

- (a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

Ans: (d)

Solution: The real part of the wave function $\psi_{real} = \cos kx + 2 \cos kx$

Current density for real part of wave function = 0

GATE- 2013

Q21. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings.

- (a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = i\hbar L_z$
 (c) $[L_z, L_+] = \hbar L_+$ (d) $[L_z, L_-] = \hbar L_-$

Ans: (d)

Q22. A proton is confined to a cubic box, whose sides have length $10^{-12} m$. What is the minimum kinetic energy of the proton? The mass of proton is $1.67 \times 10^{-27} kg$ and Planck's constant is $6.63 \times 10^{-34} Js$.

- (a) $1.1 \times 10^{-17} J$ (b) $3.3 \times 10^{-17} J$ (c) $9.9 \times 10^{-17} J$ (d) $6.6 \times 10^{-17} J$

Ans: (c)

Solution: $\frac{3\pi^2 \hbar^2}{2ma^2} = 9.9 \times 10^{-17}$

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Q23. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z then what is the expectation value up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? Here, symbols have their usual meanings. _____

Ans: 76

$$\text{Solution: } |\psi\rangle = .8|\uparrow\rangle + .6|\downarrow\rangle = .8\begin{pmatrix} 1 \\ 0 \end{pmatrix} + .6\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} .8 \\ .6 \end{pmatrix}$$

$$\text{Operator } A = 10\sigma_z + 5\sigma_x = 10\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 5\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix}$$

$$\langle A \rangle = \langle \psi | A | \psi \rangle = (.8 \ .6) \begin{pmatrix} 10 & 5 \\ 5 & 10 \end{pmatrix} \begin{pmatrix} .8 \\ .6 \end{pmatrix} = (88 - 12) = 76$$

Q24. Consider the wave function $Ae^{ikr}(r_0/r)$, where A is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2 \hbar k / m)$ is. _____

Ans:

$$\text{Solution: } \vec{J} = |\psi|^2 \frac{\hbar k}{m} = |A|^2 \left| \frac{r_0}{r} \right|^2 \frac{\hbar k}{m} \Rightarrow J = |A|^2 \left| \frac{r_0}{2r_0} \right|^2 \frac{\hbar k}{m} \Rightarrow J = |A|^2 \frac{\hbar k}{4m}$$

Common data questions 25 and 26

To the given unperturbed Hamiltonian
$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

we add a small perturbation given by
$$\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$
 where ε is small quantity.

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Q25. The ground state eigenvector of the unperturbed Hamiltonian is

- (a) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$
 (c) $(0, 0, 1)$ (d) $(1, 0, 0)$

Ans: (c)

$$H_0 = \begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}, H_p = \varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Eigen value of H_0 is $E_1 = 2, E_2 = 3, E_3 = 7$ and the Eigen vector corresponds

$$\text{to } |\phi_1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, |\phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, |\phi_3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$$

Q26. A pair of eigenvalues of the perturbed Hamiltonian, using first order perturbation theory, is

- (a) $3 + 2\varepsilon, 7 + 2\varepsilon$ (b) $3 + 2\varepsilon, 2 + \varepsilon$
 (c) $3, 7 + 2\varepsilon$ (d) $3, 2 + 2\varepsilon$

Ans: (c)

Solution: $E'_1 = \langle \phi_1 | H_p | \phi_1 \rangle = 1\varepsilon \Rightarrow E_1 = 2 + 1\varepsilon$

$$E'_2 = \langle \phi_2 | H_p | \phi_2 \rangle = \frac{1}{\sqrt{2}} (1 \quad -1 \quad 0) \cdot \varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} (0 \quad 0 \quad 1) \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0$$

$$E'_3 = \langle \phi_3 | H_p | \phi_3 \rangle = \frac{1}{\sqrt{2}} (1 \quad 1 \quad 0) \cdot \varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \varepsilon \cdot \frac{1}{2} (2 \quad 2 \quad 0) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow E'_3 = \frac{1}{2} (4) \varepsilon = 2\varepsilon$$

$$E_1 = 2 + 1\varepsilon, E_2 = 3 + 0\varepsilon, E_3 = 7 + 2\varepsilon.$$

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GATE- 2014

Q27. The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength 1500nm , and it is p_B when it emits a photon of visible wavelength 500nm . The ratio

$$\frac{p_A}{p_B} \text{ is}$$

- (a) 1 : 1 (b) $1 : \sqrt{3}$ (c) 1 : 3 (d) 3 : 2

Ans: (c)

$$\text{Solution: } p = \frac{h}{\lambda}, \quad \frac{p_A}{p_B} = \frac{\lambda_B}{\lambda_A}, \quad \frac{\lambda_B}{\lambda_A} = \frac{500}{1500} = 1 : 3$$

Q28. The ground state and first excited state wave function of a one dimensional infinite potential well are ψ_1 and ψ_2 respectively. When two spin-up electrons are placed in this potential which one of the following with x_1 and x_2 denoting the position of the two electrons correctly represents the space part of the ground state wave function of the system?

- (a) $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)]$ (b) $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)]$
 (c) $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)]$ (d) $\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$

Ans: (d)

Solution: From the given information only possible spin configuration is symmetric in nature so space part will anti symmetric

$$\frac{1}{\sqrt{2}} [\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)]$$

Q29. If \vec{L} is the orbital angular momentum and \vec{S} is the spin angular momentum, then $\vec{L} \cdot \vec{S}$ does not commute with

- (a) S_z (b) L^2 (c) S^2 (d) $(\vec{L} + \vec{S})^2$

Ans: (d)

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Q30. An electron in the ground state of the hydrogen atom has the wave function

$$\psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\left(\frac{r}{a_0}\right)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r \cos \theta$ is

(Hint: $\int_0^\infty e^{-ar} r^n dr = \frac{n!}{a^{n+1}} = \frac{(n-1)!}{a^{n+1}}$)

- (a) $\frac{-a_0^2}{2}$ (b) $-a_0^2$ (c) $\frac{-3a_0^2}{2}$ (d) $-2a_0^2$

Ans: (d)

Solution: $\langle \hat{Q} \rangle = \langle z^2 \rangle - \langle r^2 \rangle \Rightarrow a_0^2 - 3a_0^2 = -2a_0^2$

Q31. A particle of mass m is subjected to a potential

$$V(x, y) = \frac{1}{2} m \omega^2 (x^2 + y^2), -\infty \leq x \leq \infty, -\infty \leq y \leq \infty$$

The state with energy $4\hbar\omega$ is g -fold degenerate. The value of g is _____

Ans: 4

Solution: This is two isotropic dimensional harmonic oscillator the energy eigen value for n th state is $E_n = (n+1)\hbar\omega$ with degeneracy $g_n = (n+1)$ so degeneracy for $4\hbar\omega$ is 4.

Q32. A hydrogen atom is in the state

$$\psi = \sqrt{\frac{8}{21}} \psi_{200} - \sqrt{\frac{3}{7}} \psi_{310} + \sqrt{\frac{4}{21}} \psi_{321}$$

where n, l, m in ψ_{nlm} denote the principal, Orbit and magnetic quantum numbers, respectively. If \vec{L} is the angular momentum operator, the average value of L^2 is

_____ \hbar^2

Ans: 2

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Solution: If L^2 will measure on state ψ the measurement is $0\hbar^2$, $2\hbar^2$ and $6\hbar^2$ with probability

$$\frac{8}{21}, \frac{3}{7}, \frac{4}{21} \text{ and } \text{so } \langle L^2 \rangle = 2\hbar^2 \times \frac{3}{7} + 6\hbar^2 \times \frac{4}{21} = 2\hbar^2$$

Q33. ψ_1 and ψ_2 are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the

system is in the state ψ_2 its probability to be in the spin-up state is _____

Ans: $\frac{2}{3}$

Solution: If is $\psi_1 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{2}{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then $\psi_2 = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sqrt{\frac{1}{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

so probability that ψ_2 is in up state is $\frac{2}{3}$

Q34. A particle is confined to a one dimensional potential box, with the potential

$$V(x) = 0, \quad 0 < x < a \\ = \infty, \quad \text{otherwise}$$

If particle is subjected to a perturbation within the box. $W = \beta x$. Where β is small constant, the first order correction to the ground state energy is

- (a) 0 (b) $a\beta/4$ (c) $a\beta/2$ (d) $a\beta$

Ans: (c)

Solution: First order energy correction is $\langle W \rangle = \beta \langle x \rangle$ the average value position in ground state

is $\langle x \rangle = \frac{a}{2}$ so ans is $a\beta/2$

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Q35. A one dimensional harmonic oscillator is in the superposition of number state $|n\rangle$ given by

$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _____ $\hbar\omega$.

Ans: 3.25

Solution: Average energy will $\frac{\frac{1}{4} \cdot \frac{5\hbar\omega}{2} + \frac{3}{4} \cdot \frac{7\hbar\omega}{2}}{\frac{1}{4} + \frac{3}{4}} = 3.25\hbar\omega$

Q36. If L_+ and L_- are the angular momentum ladder operators then the expectation value of $(L_+L_- + L_-L_+)$ in the state $|l=1, m=1\rangle$ of an atom is _____ \hbar^2

Ans: 2

Solution: $(L_+L_- + L_-L_+) = 2(L^2 - L_z^2) = 2(l(l+1) - m^2)\hbar^2 = 2\hbar^2$

GATE- 2015

Q37. An operator for a spin $-\frac{1}{2}$ particle is given by $\hat{A} = \lambda \vec{\sigma} \cdot \vec{B}$, where

$\vec{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{y})$, $\vec{\sigma}$ denotes Pauli matrices and λ is a constant. The eigenvalues of \hat{A} are

- (a) $\pm \frac{\lambda B}{\sqrt{2}}$ (b) $\pm \lambda B$ (c) $0, \lambda B$ (d) $0, -\lambda B$

Ans.: (b)

Solution: $\hat{A} = \lambda \vec{\sigma} \cdot \vec{B}$, $\vec{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{y})$

$$\hat{A} = \lambda(\sigma_x B_x + \sigma_y B_y + \sigma_z B_z) \Rightarrow \hat{A} = \lambda[\sigma_x B_x + \sigma_y B_y]$$

$$\hat{A} = \lambda \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{B}{\sqrt{2}} + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{B}{\sqrt{2}} \right] \Rightarrow \hat{A} = \frac{\lambda B}{\sqrt{2}} \begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}$$

$$|A - \lambda'I| = 0 \Rightarrow \frac{\lambda B}{\sqrt{2}} \begin{bmatrix} -\lambda & 1-i \\ 1+i & -\lambda \end{bmatrix} = 0 \Rightarrow \lambda' = \pm \lambda B$$

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Q38. The Pauli matrices for three spin $-\frac{1}{2}$ particles are $\vec{\sigma}_1, \vec{\sigma}_2$ and $\vec{\sigma}_3$, respectively. The

dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is _____

Ans.: 8

Solution: $\sigma_2 \times \sigma_3$ has dimension of 4 and $\sigma_1 \cdot \sigma_2 \times \sigma_3$ has dimension of $2 \times 4 = 8$

Q39. Let \vec{L} and \vec{p} be the angular and linear momentum operators, respectively, for a particle.

The commutator $[L_x, p_y]$ gives

- (a) $-i\hbar p_z$ (b) 0 (c) $i\hbar p_x$ (d) $i\hbar p_z$

Ans.: (d)

Solution: $[L_x, p_y] = [yp_z - zp_y, p_y] = [yp_z, p_y] - [zp_y, p_y] = [y, p_y] p_z$

$$\because [p_y, p_y] = 0 \text{ and } [z, p_y] = 0$$

$$\Rightarrow [L_x, p_y] = i\hbar p_z \quad \because [y, p_y] = i\hbar$$

Q40. Consider a system of eight non-interacting, identical quantum particles of spin $-\frac{3}{2}$ in a

one dimensional box of length L . The minimum excitation energy of the system, in units

of $\frac{\pi^2 \hbar^2}{2mL^2}$ is _____

Ans.: 5

Solution: spin $\frac{3}{2} \Rightarrow$ degeneracy $= (2S+1) = \left(2 \times \frac{3}{2} + 1\right) = 4$

$$E_{\text{ground}} = 4 \times \frac{\pi^2 \hbar^2}{2mL^2} + 4 \times \frac{4\pi^2 \hbar^2}{2mL^2} = \frac{20\pi^2 \hbar^2}{2mL^2}$$

$$E_{\text{excited}}^{1st} = 4 \times \frac{\pi^2 \hbar^2}{2mL^2} + 3 \times 4 \times \frac{\pi^2 \hbar^2}{2mL^2} + 1 \times 9 \times \frac{\pi^2 \hbar^2}{2mL^2} = 25 \frac{\pi^2 \hbar^2}{2mL^2}$$

$$\text{Now minimum excitation energy } \Delta E = E_{\text{excited}}^{1st} - E_{\text{ground}} = 25 \frac{\pi^2 \hbar^2}{2mL^2} - 20 \frac{\pi^2 \hbar^2}{2mL^2} = 5 \frac{\pi^2 \hbar^2}{2mL^2}$$

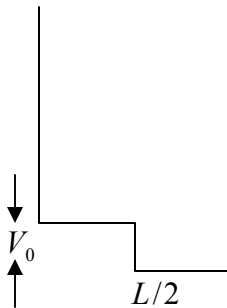
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Q41. A particle is confined in a box of length L as shown below.



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

(a) $E = \frac{\hbar^2 \pi^2}{2mL^2} + V_0$

(b) $E = \frac{\hbar^2 \pi^2}{2mL^2} - \frac{V_0}{2}$

(c) $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{4}$

(d) $E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{2}$

Ans.: (d)

Solution: $E_0^1 = \int_0^{\frac{L}{2}} V_0 \sin^2 \frac{\pi x}{L} dx + \int_{\frac{L}{2}}^L 0 \times \sin^2 \frac{\pi x}{L} dx$

$$\Rightarrow E_0^1 = \frac{V_0}{2} \int_0^{\frac{L}{2}} \left(1 - \cos \frac{2\pi x}{L}\right) dx = \frac{V_0}{2} \left[x - \sin\left(\frac{2\pi x}{L}\right) \frac{L}{2\pi x} \right]_0^{\frac{L}{2}}$$

$$\Rightarrow E_0^1 = \frac{V_0}{2} \Rightarrow E = \frac{\hbar^2 \pi^2}{2mL^2} + \frac{V_0}{2}$$

Q42. Let the Hamiltonian for two spin- $\frac{1}{2}$ particles of equal masses m , momenta \vec{p}_1 and \vec{p}_2 and positions \vec{r}_1 and \vec{r}_2 be $H = \frac{1}{2m} p_1^2 + \frac{1}{2m} p_2^2 + \frac{1}{2} m \omega^2 (r_1^2 + r_2^2) + k \vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar \omega = 0.1 eV$ and $k = 0.2 eV$. If the ground state has net spin zero, then the energy (in eV) is _____

Ans.: -0.3

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Solution: $H = \frac{1}{2m} p_1^2 + \frac{1}{2m} p_2^2 + \frac{1}{2} m\omega^2 (r_1^2 + r_2^2) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$

$$\vec{\sigma} = \vec{\sigma}_1 + \vec{\sigma}_2 \Rightarrow \vec{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + 2\vec{\sigma}_1 \cdot \vec{\sigma}_2 \Rightarrow 2\vec{\sigma}_1 \cdot \vec{\sigma}_2 = \vec{\sigma}^2 - \sigma_1^2 - \sigma_2^2$$

$$\Rightarrow 2\vec{\sigma}_1 \cdot \vec{\sigma}_2 = 0 - 3I - 3I = -6I \Rightarrow \vec{\sigma}_1 \cdot \vec{\sigma}_2 = -3$$

Now energy $E = 2 \times \frac{3}{2} \hbar\omega + k(-3) = 3 \times (0.1) + (0.2)(-3) = -0.3 \text{ eV}$

Q43. Suppose a linear harmonic oscillator of frequency ω and mass m is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[|\psi_0\rangle + e^{i\frac{\pi}{2}} |\psi_1\rangle \right] \text{ at } t=0 \text{ where } |\psi_0\rangle \text{ and } |\psi_1\rangle \text{ are the ground and the first}$$

excited states, respectively. The value of $\langle \psi | x | \psi \rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t=0$ is _____

Ans. : 0

Solution: $|\Psi\rangle = \frac{1}{\sqrt{2}} \left[|\Psi_0\rangle + e^{i\frac{\pi}{2}} |\Psi_1\rangle \right]$

$$\langle \Psi | x | \Psi \rangle = \langle \Psi | \sqrt{\frac{\hbar}{2m\omega}} (a + a^+) | \Psi \rangle \Rightarrow \langle \Psi | x | \Psi \rangle = \sqrt{\frac{\hbar}{2m\omega}} (\langle \Psi | a | \Psi \rangle + \langle \Psi | a^+ | \Psi \rangle)$$

$$\because a |\Psi\rangle = \frac{1}{\sqrt{2}} e^{i\frac{\pi}{2}} |\Psi_0\rangle \text{ and } a^+ |\Psi\rangle = \frac{1}{\sqrt{2}} \left(|\Psi_0\rangle + \sqrt{2} e^{i\frac{\pi}{2}} |\Psi_2\rangle \right)$$

$$\Rightarrow \langle \Psi | x | \Psi \rangle = \frac{\hbar}{2m\omega} \left(\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} e^{i\frac{\pi}{2}} \langle \Psi_0 | \Psi_0 \rangle + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} e^{-i\frac{\pi}{2}} \langle \Psi_1 | \Psi_1 \rangle \right)$$

$$\Rightarrow \langle \Psi | x | \Psi \rangle = \frac{\hbar}{2m\omega} (0) = 0$$

GATE-2016

Q44. Which of the following operators is Hermitian?

- (a) $\frac{d}{dx}$ (b) $\frac{d^2}{dx^2}$ (c) $i \frac{d^2}{dx^2}$ (d) $\frac{d^3}{dx^3}$

Ans.: (b)

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Q45. The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresponding Born approximation is known as the first Born approximation. Such an approximation is valid for

- (a) large incident energies and weak scattering potentials.
- (b) large incident energies and strong scattering potentials.
- (c) small incident energies and weak scattering potentials.
- (d) small incident energies and strong scattering potentials.

Ans.: (a)

Q46. Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90° and the magnitude of the incident wave vector is equal to $\sqrt{2\pi} \text{ fm}^{-1}$ then the total scattering cross section in units of fm^2 is _____.

Ans.: 2

Solution: $\sigma = \frac{4\pi}{k^2} \sum_{l=0} (2l+1) \sin^2 \delta_l$ for $l=0$ it is given $\delta_0 = 90^\circ$ and $k = \sqrt{2\pi} \text{ fm}^{-1}$

$$\sigma = \frac{4\pi}{2\pi} \sin^2 90 = 2$$

Q47. A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E} = E_0 \hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent n is _____.

Ans.: 2

Solution: First order energy correction is zero $\langle \psi_{1,0,0} | E_0 r \cos \theta | \psi_{1,0,0} \rangle = 0$

So one need to find correction of second $\sum_{n \neq 1} \frac{|\langle \psi_{n,l,m} | E_0 r \cos \theta | \psi_{1,0,0} \rangle|^2}{E_1^0 - E_n^0} \propto E_0^2$

So value of $n = 2$

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Q48. If \vec{s}_1 and \vec{s}_2 are the spin operators of the two electrons of a He atom, the value of $\langle \vec{s}_1 \cdot \vec{s}_2 \rangle$ for the ground state is

- (a) $-\frac{3}{2}\hbar^2$ (b) $-\frac{3}{4}\hbar^2$ (c) 0 (d) $\frac{1}{4}\hbar^2$

Ans.: (b)

Solution: $\vec{s} = \vec{s}_1 + \vec{s}_2$, $s_1 = \frac{1}{2}$, $s_2 = \frac{1}{2}$, $s = 0, 1$, $\langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{s(s+1)\hbar^2 - s_1(s_1+1)\hbar^2 - s_2(s_2+1)\hbar^2}{2}$

$$\text{For } s=1, \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{2\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} = \frac{3}{4}\hbar^2$$

$$s=0, \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{0\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} = -\frac{3}{4}\hbar^2$$

Q49. A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0K$. The mass of the electron is m . The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is _____.

Ans.: 24

$$\text{Solution: } 2 \times \frac{(1^2 + 1^2)\pi^2\hbar^2}{2mL^2} + 4 \times \frac{(2^2 + 1^2)\pi^2\hbar^2}{2mL^2} = \frac{24\pi^2\hbar^2}{2mL^2}$$

Q50. σ_x, σ_y and σ_z are the Pauli matrices. The expression $2\sigma_x\sigma_y + \sigma_y\sigma_x$ is equal to

- (a) $-3i\sigma_z$ (b) $-i\sigma_z$ (c) $i\sigma_z$ (d) $3i\sigma_z$

Ans.: (c)

Solution: $2\sigma_x\sigma_y + \sigma_y\sigma_x \Rightarrow \sigma_x\sigma_y + \sigma_x\sigma_y + \sigma_y\sigma_x \Rightarrow \sigma_x\sigma_y = i\sigma_z$

Q51. If x and p are the x components of the position and the momentum operators of a particle respectively, the commutator $[x^2, p^2]$ is

- (a) $i\hbar(xp - px)$ (b) $2i\hbar(xp - px)$ (c) $i\hbar(xp + px)$ (d) $2i\hbar(xp + px)$

Ans.: (d)

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Solution: $[x^2, p^2] = p[x^2, p] + [x^2, p]p = 2i\hbar px + 2i\hbar xp \Rightarrow 2i\hbar(xp + px)$

Q52. Let $|l, m\rangle$ be the simultaneous eigenstates of L^2 and L_z . Here \vec{L} is the angular momentum operator with Cartesian components (L_x, L_y, L_z) , l is the angular momentum quantum number and m is the azimuthal quantum number. The value of $\langle 1, 0 | (L_x + iL_y) | 1, -1 \rangle$ is

- (a) 0 (b) \hbar (c) $\sqrt{2}\hbar$ (d) $\sqrt{3}\hbar$

Ans.: (c)

Solution: $\langle 1, 0 | (L_x + iL_y) | 1, -1 \rangle = \langle 1, 0 | L_+ | 1, -1 \rangle = \sqrt{2}\hbar \langle 1, 0 | 1, 0 \rangle = \sqrt{2}\hbar$

Q53. For the parity operator P , which of the following statements is **NOT** true?

- (a) $P^\dagger = P$ (b) $P^2 = -P$ (c) $P^2 = I$ (d) $P^\dagger = P^{-1}$

Ans.: (b)

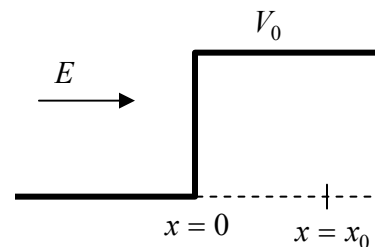
Q54. The state of a system is given by

$$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$$

Where $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is _____. (Give your answer upto two decimal places)

Ans probability that ψ in state $|\phi_2\rangle = \frac{2^2}{1^2 + 2^2 + 3^2} = \frac{4}{1+4+9} = \frac{4}{14} = \frac{2}{7} = 0.28$

Q55. A particle of mass m and energy E , moving in the positive x direction, is incident on a step potential at $x = 0$, as indicated in the figure. The height of the potential is V_0 , where $V_0 > E$. At $x = x_0$,



where $x_0 > 0$, the probability of finding the electron is $\frac{1}{e}$ times the

probability of finding it at $x = 0$. If $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$, the value of x_0 is

- (a) $\frac{2}{\alpha}$ (b) $\frac{1}{\alpha}$ (c) $\frac{1}{2\alpha}$ (d) $\frac{1}{4\alpha}$

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Ans.: (c)

Solution: $\frac{1}{e} = e^{-2\alpha x_0} = e^{-1} = e^{-2\alpha x_0} \Rightarrow x_0 = \frac{1}{2\alpha}$

GATE-2017

Q56. The Compton wavelength of a proton is..... fm. (up to two decimal places).

Ans. : 3×10^8

Solution: ($m_p = 1.67 \times 10^{-27} \text{ kg}, h = 6.626 \times 10^{-34} \text{ Js}, e = 1.602 \times 10^{-19} \text{ C}, c = 3 \times 10^8 \text{ ms}^{-1}$)

Q57. A one dimensional simple harmonic oscillator with Hamiltonian $H_0 = \frac{p^2}{2m} + \frac{1}{2}kx^2$ is subjected to a small perturbation, $H_1 = \alpha x + \beta x^3 + \gamma x^4$. The first order correction to the ground state energy is dependent on

- (a) only β (b) α and γ
 (c) α and β (d) only γ

Ans. : (d)

Solution: $H_1 = \alpha x + \beta x^3 + \gamma x^4$ $E_g^1 = \alpha \langle x \rangle + \beta \langle x^3 \rangle + \gamma \langle x^4 \rangle$

$\langle x \rangle = 0, \langle x^3 \rangle = 0, \langle x^4 \rangle \neq 0$

Q58. For the Hamiltonian $H = a_0 I + \vec{b} \cdot \vec{\sigma}$ where $a_0 \in R, \vec{b}$ is a real vector, I is the 2×2 identity matrix, and $\vec{\sigma}$ are the Pauli matrices, the ground state energy is

- (a) $|b|$ (b) $2a_0 - |b|$ (c) $a_0 - |b|$ (d) a_0

Ans. : (c)

Solution: $a_0 I + \vec{b} \cdot \vec{\sigma} = a_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b_x \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + b_y \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + b_z \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} a_0 + b_z & b_x - ib_y \\ b_x + ib_y & a_0 - b_z \end{pmatrix}$

$H = a_0 I + \vec{b} \cdot \vec{\sigma} = \begin{pmatrix} a_0 + b_z & b_x - ib_y \\ b_x + ib_y & a_0 - b_z \end{pmatrix}$

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For eigen value $\begin{pmatrix} a_0 + b_z - \lambda & b_x - ib_y \\ b_x + ib_y & a_0 - b_z - \lambda \end{pmatrix} = 0$

$$(a_0 + b_z - \lambda)(a_0 - b_z - \lambda) - (b_x^2 + b_y^2) = 0$$

$$\lambda_1 = a_0 - |b|, \lambda_2 = a_0 + |b|$$

Q59. The degeneracy of the third energy level of a 3-dimensional isotropic quantum harmonic oscillator is

- (a) 6 (b) 12 (c) 8 (d) 10

Ans. : (a)

Solution: First energy level is $n = 0$

Second energy level is $n = 1$

Third energy level is $n = 2$

$$\text{Degeneracy of third level } \frac{(n+1)(n+2)}{2} = \frac{3 \times 4}{2} = 6$$

Q60. A free electron of energy $1eV$ is incident upon a one-dimensional finite potential step of height $0.75eV$. The probability of its reflection from the barrier is..... (up to two decimal places).

Ans. : 0.11

$$\text{Solution: } R = \left(\frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}} \right)^2 = \left(\frac{1 - \sqrt{0.25}}{1 + \sqrt{0.25}} \right)^2 = \left(\frac{1 - 0.5}{1 + 0.5} \right)^2 = 0.11$$

Q61. Consider a one-dimensional potential well of width $3nm$. Using the uncertainty principle

$$\left(\Delta x \cdot \Delta p \geq \frac{\hbar}{2} \right), \text{ an estimate of the minimum depth of the well such that it has at least one}$$

bound state for an electron is ($m_e = 9.31 \times 10^{-31} \text{ kg}$, $h = 6.626 \times 10^{-34} \text{ Js}$, $e = 1.602 \times 10^{-19} \text{ C}$)

- (a) $1\mu eV$ (b) $1meV$ (c) $1eV$ (d) $1MeV$

Ans. : (b)

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Solution: $E = \frac{p^2}{2m}$, $\Delta p = \frac{\hbar}{2\Delta x} \Rightarrow \Delta p = \frac{\hbar}{2a}$

so $E = \frac{\hbar^2}{8ma^2} = \frac{\hbar^2}{32\pi^2 ma^2} = \frac{(6.6 \times 10^{-34})^2}{32 \times 10 \times 9.31 \times 10^{-31} \times 9 \times 10^{-18}} = .001 \times 10^{-19} J \approx 1 meV$

Q62. The integral $\int_0^{\infty} x^2 e^{-x^2} dx$ is equal to..... (up to two decimal places).

Ans. : 0.44

Solution: The given integral is $\int_0^{\infty} x^2 e^{-x^2} dx$

Let $x^2 = t$ then $2x dx = dt \Rightarrow dx = \frac{dt}{2\sqrt{t}}$

Thus the given integral can be written as

$$\int_0^{\infty} t e^{-t} \frac{dt}{2\sqrt{t}} = \frac{1}{2} \int_0^{\infty} e^{-t} t^{1/2} dt = \frac{1}{2} \int_0^{\infty} e^{-t} t^{\frac{3}{2}-1} dt$$

$$= \frac{1}{2} \Gamma\left(\frac{3}{2}\right) = \frac{1}{2} \cdot \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{4}$$

Hence the value of the integral up to two decimal places is 0.44 .

Q63. Which one of the following operators is Hermitian?

(a) $i \frac{(p_x x^2 - x^2 p_x)}{2}$

(b) $i \frac{(p_x x^2 + x^2 p_x)}{2}$

(c) $e^{i p_x a}$

(d) $e^{-i p_x a}$

Ans. : (a)

Solution: $A = i \frac{(p_x x^2 - x^2 p_x)}{2}$, $A^\dagger = -i \frac{((p_x x^2)^\dagger - (x^2 p_x)^\dagger)}{2} = i \frac{(p_x x^2 - x^2 p_x)}{2}$

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