Mechanics and General Properties of Matter

IIT-JAM-2005

Q1. A solid sphere of mass m and radius a is rolling with a linear speed v on a flat surface without slipping. The magnitude of the angular momentum of the sphere on the surface is

(a) $\frac{2}{5}mav$ (b) $\frac{7}{5}mav$ (c) mav (d) $\frac{3}{2}mav$

Ans.: (b)

Solution: $L = I\omega + mva \Rightarrow \frac{2}{5}ma^2\frac{v}{a} + mva = \frac{2}{5}mva + mva = \frac{7mva}{5}$

IIT-JAM-2007

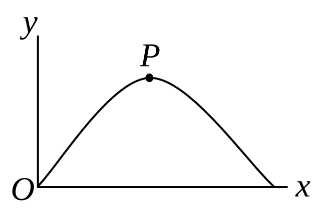
Q2. In terms of the basic units of mass M, length L, time T and charge Q, the dimensions of magnetic permeability of vacuum μ_0 are

(a) MLQ^{-2} (b) $ML^2T^{-1}Q^{-2}$ (c) LTQ^{-1} (d) $LT^{-1}Q^{-1}$

Ans.: (a)

Solution: $\vec{F} = I\int d\vec{l} \times \vec{B}, \quad B \approx \frac{\mu_0 I}{d} \Rightarrow F \propto I^2\mu_0 \Rightarrow MLT^{-2} = \left(\frac{Q}{T}\right)^2 \mu_0 \Rightarrow \mu_0 = MLQ^{-2}$

Q3. A projectile is fired from the origin O at an angle of 45^0 from the horizontal. At the highest point P of its trajectory the radial and transverse components of its acceleration in terms of the gravitational acceleration g are



(a) $a_r = \frac{2g}{\sqrt{5}}, \quad a_\theta = \frac{g}{\sqrt{5}}$ (b) $a_r = \frac{-2g}{\sqrt{5}}, \quad a_\theta = \frac{-g}{\sqrt{5}}$

(c) $a_r = \frac{g}{\sqrt{5}}, \quad a_\theta = \frac{2g}{\sqrt{5}}$ (d) $a_r = \frac{-g}{\sqrt{5}}, \quad a_\theta = \frac{-2g}{\sqrt{5}}$

Ans.: (d)

Solution: Maximum $h_{\max} = \frac{v^2\sin^2\theta}{2g}$ and range $R = \frac{v^2\sin 2\theta}{g}$ where $\theta = \frac{\pi}{4}$

$$\tan\alpha = \frac{h_{\max}}{\frac{R}{2}} = \frac{1}{2}$$

From the figure $a_r = -g\cos(90-\alpha) = -g\sin\alpha = \frac{-g}{\sqrt{5}}$

$$a_\theta = -g\cos\alpha == \frac{-2g}{\sqrt{5}}$$

Q4. A satellite moves around a planet in a circular orbit at a distance R from its centre. The time period of revolution of the satellite is T. If the same satellite is taken to an orbit of radius $4R$ around the same planet, the time period would be

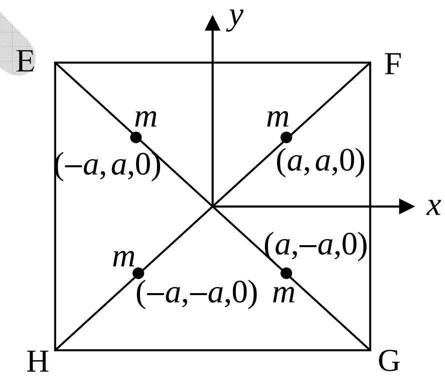
(a) $8T$ (b) $4T$ (c) $\frac{T}{4}$ (d) $\frac{T}{8}$

Ans.: (a)

Solution: $T^2 \propto R^3 \Rightarrow \frac{T_2}{T} = \left(\frac{4R}{R}\right)^{\frac{3}{2}} = 8 \Rightarrow T_2 = 8T$

IIT-JAM-2008

Q5. EFGH is a thin square plate of uniform density σ and side $4a$. Four point masses, each of mass m, are placed on the plate as shown in the figure. In the moment of inertia matrix I of the composite system,



(a) only I_{xy} is zero

(b) only I_{xz} and I_{yz} are zero

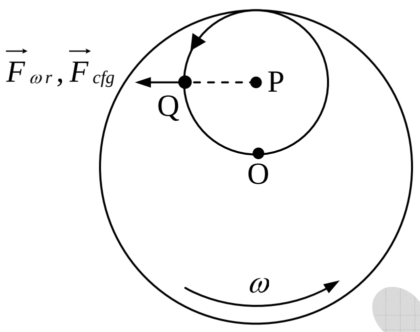
(c) all the product of inertia terms are zero

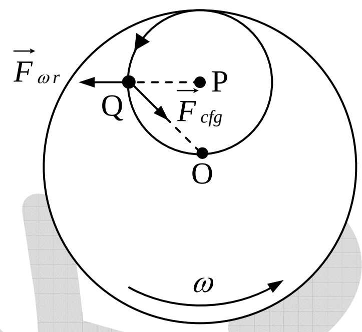
(d) none of the product of inertia terms is zero

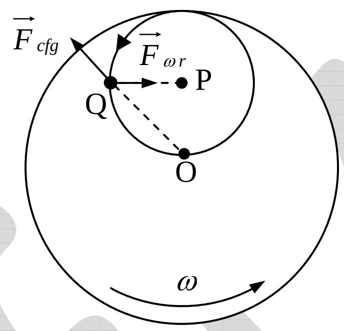
Ans.: (c)

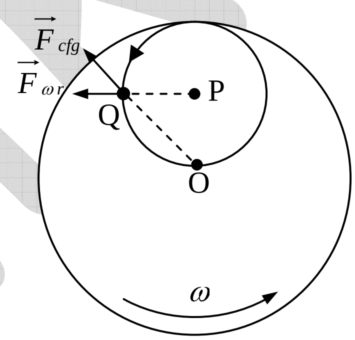
Solution: $I_{xy} = -\sum_i m_i x_i y_i = 0, I_{xz} = -\sum_i m_i x_i z_i = 0, I_{yz} = -\sum_i m_i y_i z_i = 0$

Q6. A circular disc (in the horizontal xy-plane) is spinning about a vertical axis through its center O with a constant angular velocity $\vec{\omega}$. As viewed from the reference frame of the disc, a particle is observed to execute uniform circular motion, in the anticlockwise sense, centered at P. When the particle is at the point Q, which of the following figures correctly represents the directions of the Coriolis force $\vec{F}_{cor}$ and the centrifugal force $\vec{F}_{cfg}$?

(a)


(b)


(c)


(d)


Ans.: (c)

Solution: $\vec{F}_{centifugal} = \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{r}) = \omega_1 \hat{z} \times (\omega_1 \hat{z} \times r\hat{r}) \propto -\hat{r}$

$$\vec{F}_{corr} = -2m(\vec{\omega} \times \vec{v}) = -2m\left(\omega\hat{z} \times \vec{v}\hat{\theta}\right) \propto \hat{r}$$

IIT-JAM-2009

Q7. A space crew has a life support system that can last only for 1000 hours. What minimum speed would be required for safe travel of the crew between two space stations separated by a fixed distance of $1.08\times10^{12}\ km$?

(a) $\frac{c}{\sqrt{3}}$ (b) $\frac{c}{\sqrt{2}}$ (c) $\frac{c}{2}$ (d) $\frac{c}{\sqrt{5}}$

Ans.: (b)

Solution: $\frac{1000\times3600\times v}{\sqrt{1-\frac{v^2}{c^2}}}=1.08\times10^{12}\times1000$

$$\frac{10^5\times36\times v}{\sqrt{1-\frac{v^2}{c^2}}}=1080\times10^{12}\Rightarrow\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}=c\Rightarrow v=\frac{c}{\sqrt{2}}$$

Q8. A particle is moving in space with O as the origin. Some possible expression for its position, velocity and acceleration in cylindrical coordinates (ρ,ϕ,z) are given below. Which one of these is correct?

(a) Position vector $\vec{r}=\rho\hat{\rho}+\varphi\hat{\varphi}+z\hat{z}$ and velocity $\vec{v}=\frac{d\rho}{dt}\hat{\rho}+\rho\frac{d\varphi}{dt}\hat{\varphi}+\frac{dz}{dt}\hat{z}$

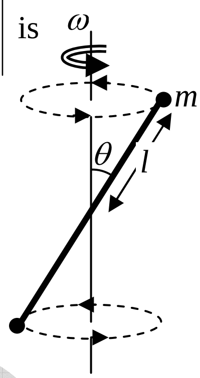
(b) Velocity $\vec{v}=\frac{d\rho}{dt}\hat{\rho}+\rho\frac{d\varphi}{dt}\hat{\varphi}+\frac{dz}{dt}\hat{z}$ and acceleration $\vec{a}=\frac{d^2\rho}{dt^2}\hat{\rho}+\frac{d}{dt}\left(\rho\frac{d\varphi}{dt}\hat{\varphi}\right)+\frac{d^2z}{dt^2}\hat{z}$

(c) Position vector $\vec{r}=\rho\hat{\rho}+z\hat{z}$ and velocity $\vec{v}=\frac{d\rho}{dt}\hat{\rho}+\rho\frac{d\varphi}{dt}\hat{\varphi}+\frac{dz}{dt}\hat{z}$

(d) Position vector $\vec{r}=\rho\hat{\rho}+\rho\varphi\hat{\varphi}+z\hat{z}$ and velocity $\vec{v}=\frac{d\rho}{dt}\hat{\rho}+\frac{d}{dt}(\rho\varphi\hat{\varphi})+\frac{dz}{dt}\hat{z}$

Ans.: (d)

Q9. A thin massless rod of length $2l$ has equal point masses m attached at its ends (see figure). The rod is rotating about an axis passing through its centre and making angle θ with it. The magnitude of the rate of change of its angular momentum $\left|\frac{d\vec{L}}{dt}\right|$ is



(a) $2ml^2\omega^2 \sin\theta\cos\theta$ (b) $2ml^2\omega^2 \sin\theta$

(c) $2ml^2\omega^2 \sin^2\theta$ (d) $2ml^2\omega^2 \cos^2\theta$

Ans.: (a)

Solution: $J = m\left(\vec{r}\times\left(\vec{\omega}\times\vec{r}\right)\right) + m\left(\vec{r}\times\left(\vec{\omega}\times\vec{r}\right)\right) = 2m\left(\vec{r}\times l\omega\sin\right) = 2m\omega l^2 \sin\theta$

$$Torque = \frac{dL}{dt} = \vec{\omega}\times\vec{J} = 2m\omega l^2 \sin\left(90-\theta\right)\sin\theta = 2m\omega l^2 \cos\theta\sin\theta$$

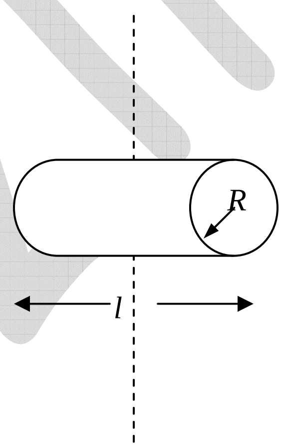
Q10. Moment of inertia of a solid cylinder of mass M, height l and radius R about an axis (shown in the figure by dashed line) passing through its centre of mass and perpendicular to its symmetry axis is

(a) $\frac{1}{4}MR^2 + \frac{1}{12}Ml^2$

(b) $\frac{1}{2}MR^2 + \frac{1}{8}Ml^2$

(c) $\frac{1}{2}MR^2 + \frac{1}{12}Ml^2$

(d) $\frac{1}{2}MR^2 + \frac{1}{4}Ml^2$



Ans.: (a)

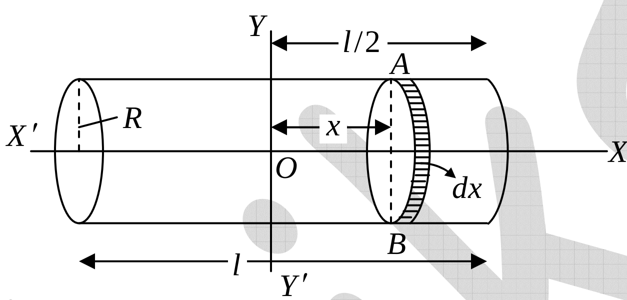
Solution: If R be the radius, l, the length and M, the mass of the solid cylinder, supposed to be uniform and of a homogeneous composition, we have its mass per unit length = $\frac{M}{l}$.

Now, imagining the cylinder to be made up of a number of discs each of radius R, placed adjacent to each other, and considering one such disc of thickness dx and at a distance x from the centre O of the cylinder, (figure), we have

Mass of the disc = $\left(\frac{M}{l}\right)dx$ and radius = R

And $\therefore$ M.I. of the disc about its diameter $AB = \frac{M}{l}dx \cdot \frac{R^2}{4}$ and its M.I. about the parallel axis YOY', passing through the centre O of the cylinder and perpendicular to its axis of cylindrical symmetry (or its length), in accordance with the principle of parallel axes,

$$= \frac{M}{l}dx\frac{R^2}{4} + \frac{M}{l}dx \cdot x^2$$



Hence, M.I. of the whole cylinder about this axis, i.e. $I =$ twice the integral of the above expression between the limits $x = 0$ and $x = \frac{l}{2}$,

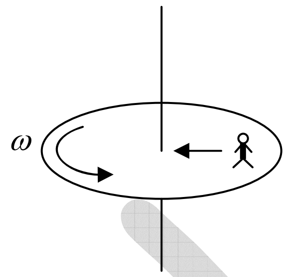
i.e., $$I = 2\int_0^{l/2}\left(\frac{M}{l}\cdot\frac{R^2}{4}dx + \frac{M}{l}x^2dx\right) = \frac{2M}{l}\int_0^{l/2}\left(\frac{R^2}{4}dx + x^2dx\right)$$

$$= \frac{2M}{l}\left[\frac{R^2x}{4} + \frac{x^3}{3}\right]_0^{l/2}$$

or $$I = \frac{2M}{l}\left[\frac{R^2}{4}\cdot\frac{l}{2} + \frac{l^3}{8\times 3}\right] = \frac{2M}{l}\left(\frac{R^2l}{8} + \frac{l^3}{24}\right) = M\left(\frac{R^2}{4} + \frac{l^2}{12}\right)$$

IIT-JAM-2010

Q11. A circular platform is rotating with a uniform angular speed ω counterclockwise about an axis passing through its centre and perpendicular to its plane as shown in the figure. A person of mass m walks radially inward with a uniform speed v on the platform. The magnitude and the direction of the Coriolis force (with respect to the direction along which the person walks) is



(a) $2m\omega v$ towards his left
(b) $2m\omega v$ towards his front
(c) $2m\omega v$ towards his right
(d) $2m\omega v$ towards his back

Ans.: (c)

Solution: $F = -2m\vec{\omega}\times\vec{v} = -2m\omega v\left(\hat{z}\times-\hat{r}\right) = 2m\omega v\hat{\phi}$

$2m\omega v$ towards his right

Q12. A particle of mass m, moving with a velocity $\vec{v} = v_0\left(\hat{i}+\hat{j}\right)$, collides elastically with another particle of mass $2m$ which is at rest initially. Here, v_0 is a constant. Which of the following statements is correct?

(a) The direction along which the centre of mass moves before collision is $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(b) The speed of the particle of mass m before collision in the center of mass frame is $\sqrt{2}v_0$.

(c) After collision the speed of the particle with mass $2m$ in the centre of mass frame is $\frac{\sqrt{2}}{3}v_0$.

(d) The speed of the particle of mass $2m$ before collision in the center of mass frame is $\sqrt{2}v_0$.

Ans. : (c)

Solution: Velocity of center of mass is $\frac{mv_0\left(\hat{i}+\hat{j}\right)+2m\times 0}{m+3m}=\frac{v_0\left(\hat{i}+\hat{j}\right)}{3}$ so option a is wrong

Velocity of mass m with respect to center of mass before collision $\vec{u}_1-\vec{v}_{cm}=v_0\left(\hat{i}+\hat{j}\right)-\frac{v_0\left(\hat{i}+\hat{j}\right)}{3}=\frac{2v_0\left(\hat{i}+\hat{j}\right)}{3}$ so speed is $v_0\sqrt{\frac{4}{9}+\frac{4}{9}}=v_0\sqrt{\frac{8}{9}}=$

Velocity of mass $2m$ with respect to center of mass before collision $\vec{u}_2-v_{cm}=0-\frac{v_0\left(\hat{i}+\hat{j}\right)}{3}=-\frac{v_0\left(\hat{i}+\hat{j}\right)}{3}$ so speed is $v_0\sqrt{\frac{1}{9}+\frac{1}{9}}=v_0\sqrt{\frac{2}{9}}\Rightarrow\frac{\sqrt{2}v_0}{3}$ which is also speed after the collision with respect to center of mass

IIT-JAM-2011

Q13. A rain drop falling vertically under gravity gathers moisture from the atmosphere at a rate given by $\frac{dm}{dt}=kt^2$, where m is the instantaneous mass, t is time and k is a constant. The equation of motion of the rain drop is $m\frac{dv}{dt}+v\frac{dm}{dt}=mg$

If the drop starts falling at $t=0$, with zero initial velocity and initial mass m_0 ($m_0=2\,gm$, $k=12\ gm/s^3$ and $g=1000\ cm/s^2$), the velocity v of the drop after one second is

(a) $250\,cm/s$ (b) $500\,cm/s$ (c) $750\,cm/s$ (d) $1000\,cm/s$

Ans.: (b)

Solution: $\frac{dm}{dt}=kt^2\Rightarrow m=\frac{kt^3}{3}+2$

$$m\frac{dv}{dt}+v\frac{dm}{dt}=mg\Rightarrow m\frac{dv}{dt}+vkt^2=mg$$

$\frac{dv}{dt}+\frac{kt^2}{m}v=g\Rightarrow\frac{dv}{dt}+\frac{3kt^2}{kt^3+6}v=g$ which is linear differential equation

So $I.F=\exp\int\frac{3kt^2}{3kt^3+6}dt=(kt^3+6)$

$v\left(kt^3+6\right)=\int g\left(kt^3+6\right)dt+c \quad t=0, v=0 \Rightarrow c=0$

$$v=\frac{g\left(kt^4+24t\right)}{4\left(kt^3+6\right)}$$

Put 2gm, k = 12 gm/s^3 and g = 1000 cm/s^2),and $t=1$

$$v=\frac{1000(12+24)}{4(12+6)}=500 cm/\sec$$

Q14. A particle of mass m is moving in a potential $V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$ where ω_0 and a are positive constants. The angular frequency of small oscillations for the simple harmonic motion of the particle about a stable minimum of the potential $V(x)$ is

(a) $\sqrt{2}\omega_0$ (b) $2\,\omega_0$ (c) $4\,\omega_0$ (d) $4\sqrt{2}\omega_0$

Ans.: (b)

Solution: $V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$

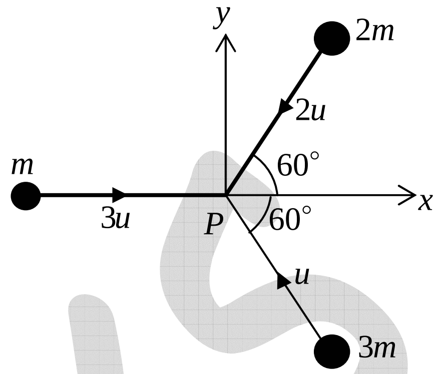
$$\frac{dV}{dx}=m\omega_0^2x-\frac{a}{mx^3}=0\Rightarrow x^4=\frac{a}{m^2\omega_0^2}$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=x_0}=m\omega_0^2+\frac{3a}{mx^4}\Rightarrow m\omega_0^2+\frac{3m^2\omega_0^2}{m}=4m\omega_0^2$$

$$\omega=\sqrt{\frac{\left.\frac{d^2V}{dx^2}\right|_{x=x_0}}{m}}=\sqrt{\frac{4m\omega_0^2}{m}}=2\omega_0$$

IIT-JAM-2012

Q15. Three masses m, $2m$ and $3m$ are moving in xy - plane with speeds $3u, 2u$ and u, respectively, as shown in the figure. The three masses collide at the same time at P and stick together. The velocity of the resulting mass would be



(a) $\frac{u}{12}\left(\hat{x}+\sqrt{3}\hat{y}\right)$

(b) $\frac{u}{12}\left(\hat{x}-\sqrt{3}\hat{y}\right)$

(c) $\frac{u}{12}\left(-\hat{x}+\sqrt{3}\hat{y}\right)$

(d) $\frac{u}{12}\left(-\hat{x}-\sqrt{3}\hat{y}\right)$

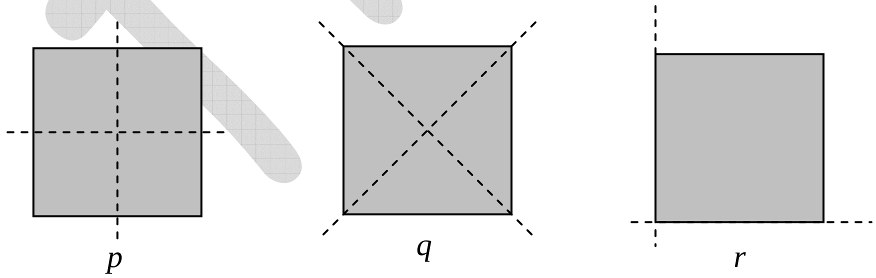
Ans.: (d)

Solution: $u_{cm}\hat{i}=\frac{m3u-2m.2u\cos 60-3m\,u\cos 60}{6}=-\frac{u}{12}\hat{x}$

$$u_{cm}\hat{i}=\frac{m0-2m.2u\sin 60-3m\,u\sin 60}{6m}=-\frac{\sqrt{3}u}{12}\hat{y}$$

The combined mass will move with same velocity as initial center of mass is moving $v=\frac{u}{12}\left(-\hat{x}-\sqrt{3}\hat{y}\right)$

Q16. The figure shows a thin square sheet of metal of uniform density along with possible choices for a set of principal axes (indicated by dashed lines) of the moment of inertia, lying in the plane of the sheet. The correct choice(s) for the principal axes would be



(a) p, q and r (b) p and r (c) p and q (d) p only

Ans.: (c)

Solution: I_z about center of mass due to p and q are same

So, p and q are correct choice.

IIT-JAM-2013

Q17. A particle is released at $x=1$ in a force field $\vec{F}(x)=\left(\frac{2}{x^2}-\frac{x^2}{2}\right)\hat{e}_x, x\geq 0$. Which one of the following statements is FALSE?

(a) $\vec{F}(x)$ is conservative

(b) The angular momentum of the particle about the origin is constant

(c) The particle moves towards $x=\sqrt{2}$

(d) The particle moves towards the origin

Ans.: (c)

Solution: $\vec{F}(x)=\left(\frac{2}{x^2}-\frac{x^2}{2}\right)\hat{e}_x, x\geq 0$

$\vec{\nabla}\times\vec{F}=0$ are zero so force is conservative

$\vec{r}\times\vec{F}=0$ so angular momentum is conserved.

For equilibrium point $F=0\Rightarrow F=\left(\frac{2}{x^2}-\frac{x^2}{2}\right)=0\Rightarrow x=-\sqrt{2},\sqrt{2}\quad x>0$ so equilibrium point is $\sqrt{2}$

To check stable and unstable equilibrium point $\frac{dF}{dx}=0\Rightarrow\frac{-4}{x^3}-\frac{2x}{2}=0$

At $x=\sqrt{2}$, $\frac{dF}{dx}=0\Rightarrow\frac{-4}{x^3}-\frac{2x}{2}=-\frac{4}{2\sqrt{2}}-\sqrt{2}=-ve$ so it is unstable point so particle moves away from point $x=\sqrt{2}$ so it is obvious that it will moves towards origin

Q18. If the dimensions of mass, length, time and charge are M,L,T and C respectively, the dimensions of the magnetic induction field $\vec{B}$ is

(a) $ML^2T^{-1}C^{-1}$ (b) $MT^{-1}C^{-1}$ (c) $L^2T^{-1}C$ (d) $L^{-1}T^{-1}C$

Ans.: (b)

Solution: $\vec{F}=I\int d\vec{l}\times\vec{B}, \Rightarrow MLT^{-2}=\left(\frac{C}{T}\right)LB\Rightarrow B=MT^{-1}C^{-1}$

Q19. The path of a particle of mass m, moving under the influence of a central force, in plane polar coordinates is given by $r = r_0 e^{k\theta}$, where r_0 and k are positive constants of appropriate dimensions. The angular momentum of the particle is L and its total energy is zero. The potential energy functions $V(r)$, in terms of m, L and k is……………..

Ans.:

Solution: $-\frac{L^2u^2}{m}\left(\frac{d^2u}{d\theta^2}+u\right)=f\left(\frac{1}{u}\right)$

$$r = r_0 e^{k\theta}, u = \frac{1}{r_0}e^{-k\theta},\ \frac{d^2u}{d\theta^2} = \frac{1}{r_0}(-k)(-k)e^{-k\theta} = \frac{k^2e^{-k\theta}}{r_0}$$

$$-\frac{L^2u^2}{m}\left(\frac{d^2u}{d\theta^2}+u\right)=f\left(\frac{1}{u}\right)$$

$$\frac{-L^2u^2}{m}\left(\frac{k^2e^{-k\theta}}{r_0}+\frac{k}{r_0}e^{-k\theta}\right)\Rightarrow\frac{-L^2}{m}.\frac{e^{-2k\theta}}{r_0^2}\left(\frac{k^2e^{-k\theta}}{r_0}+\frac{k}{r_0}e^{-k\theta}\right)\Rightarrow\frac{L^2e^{-3k\theta}}{mr_0^3}\left(k^2+k\right)=f\left(\frac{1}{u}\right)$$

$$f(1/u) = -\frac{L^2}{m}u^3\left(k(k+1)\right)$$

$$f(r) = -\frac{L^2k(k+1)}{m}\frac{1}{r^3}$$

$$-\frac{dV}{dr} = -\frac{L^2k(k+1)}{m}\cdot\frac{1}{r^3}\Rightarrow dV = \frac{L^2k(k+1)}{m}\int\frac{1}{r^3}dr$$

$$V = -\frac{L^2k(k+1)}{2}\frac{1}{r^2}$$

Total energy, $E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} - \frac{L^2k(k+1)}{2}\cdot\frac{1}{r^2}$

Also, $r = r_0e^{k\theta} \Rightarrow \dot{r} = kr_0e^{k\theta}\cdot\dot{\theta} = kr_0e^{k\theta}\cdot\frac{L}{mr^2} = \frac{kL}{mr}$

$$E = 0 = \frac{1}{2}m\frac{k^2L^2}{m^2r^2} + \frac{L^2}{2mr^2} - \frac{L^2k(k+1)}{2}\times\frac{1}{r^2}$$

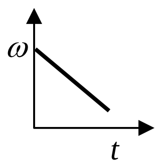
$$r^2 = \frac{k^2L^2}{2m} + \frac{L^2}{2m} - \frac{L^2k(k+1)\times m}{2\times m} = \frac{k^2L^2 + L^2 - mL^2k(k+1)}{2m}$$

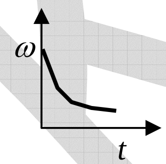
$$V = \frac{L^2 k(k+1)}{2} \cdot \frac{2m}{L^2(k^2+1) - mL^2 k(k+1)}$$

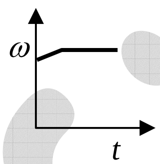
$$V = \frac{mk(k+1)}{mk(k+1) - (k^2+1)}$$

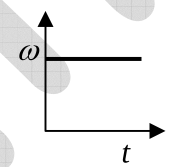
IIT-JAM-2014

Q20. A spherical ball of ice has radius R_0 and is rotating with an angular speed ω about an axis passing through its centre. At time $t = 0$, it starts acquiring mass because the moisture (at rest) around it starts to freeze on it uniformly. As a result its radius increases as $R(t) = R_0 + \alpha t$, where α is a constant. The curve which best describes its angular speed with time is

(a) 

(b) 

(c) 

(d) 

Ans.: (b). $I(t)\omega(t) = \text{constant}$

$M(t)R(t) = \text{constant}$

$$\omega(t) = \frac{\text{constant}}{M(t)[R_0 + \alpha t]^2} \Rightarrow \omega(t) \propto \frac{1}{t^2}$$

Q21. The moment of inertia of a disc about one of its diameters is I_M. The mass per unit area of the disc is proportional to the distance from its centre. If the radius of the disc is R and its mass is M, the value of I_M is

(a) $\frac{1}{2}MR^2$ (b) $\frac{2}{5}MR^2$ (c) $\frac{3}{10}MR^2$ (d) $\frac{3}{5}MR^2$

Ans.: (c)

Solution: Mass density is $\sigma = cr$, $M = \int_0^r cr.rdrd\theta \Rightarrow c = \frac{3M}{2\pi R^3}$

$I_z = \int_0^R r^2 dm \Rightarrow \int_0^R r^2 cr.rdrd\theta = \frac{c2\pi R^5}{5}$ put value of c $I_z = \frac{3}{5}MR^2$

From perpendicular axis theorem $I_z = I_x + I_y$ and $I_x = I_y$ so $I_x = I_y = I_M = \frac{3}{10}MR^2$

Q22. Two points N and S are located in the northern and southern hemisphere, respectively, on the same longitude. Projectiles P and Q are fired from N and S, respectively, towards each other. Which of the following options is correct for the projectiles as they approach the equator?

(a) Both P and Q will move towards the east

(b) Both P and Q will move towards the west

(c) P will move towards the east and Q towards the west

(d) P will move towards the west and Q towards the east

Ans.: (b)

Solution: Coriolis force

$$F = -2m(\vec{\omega} \times \vec{v})$$

$$\vec{\omega} = \omega \hat{z}$$

For both particles $\vec{V} = V_r \hat{r} + V_z \hat{z}$

So, $F = -2m[\omega \hat{z} \times v_r \hat{r} + \omega \hat{z} \times v_z \hat{z}] = -2m[\omega v_r \hat{\theta}]$

So both will move towards $-\hat{\theta}$

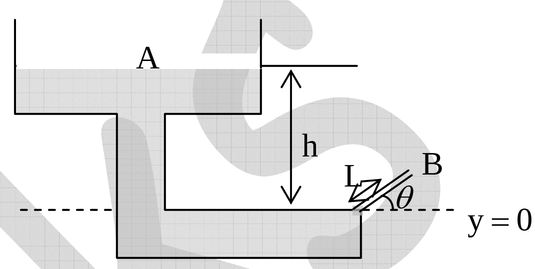
Earth revolves from west to east. So the particle will move towards west.

Q23. Two particles A and B of mass m and one particle C of mass M are kept on the x axis in the order ABC. Particle A is given a velocity $v\hat{i}$. Consequently there are two collisions, both of which are completely inelastic. If the net energy loss because of these collisions is $\frac{7}{8}$ of the initial energy, the value of M is (ignore frictional losses)

(a) $8m$ (b) $6m$ (c) $4m$ (d) $2m$

Ans.: (b)

Q24. What is the maximum height above the dashed line attained by the water stream coming out at B from a thin tube of the water tank assembly shown in the figure? Assume $h = 10\,m$, $L = 2\,m$ and $\theta = 30°$.



(a) $10\,m$ (b) $2\,m$ (c) $1.2\,m$ (d) $3.2\,m$

Ans.: (d)

Solution: Velocity at the end of thin tube $mgh = \frac{1}{2}mv^2$

Distance at the end of thin tube $= 10 - L\sin 30^0$

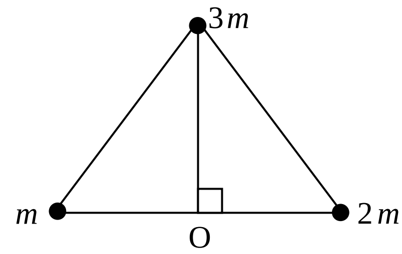
$h = 9m$

$mg \times 9 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{18g}$

$$h = \frac{v^2 \sin^2\theta}{2g} = \frac{18\times 10\times 1/4}{2\times 10} = 2\cdot 25\,m$$

Height from the dashed line $= 2.25m + 2\sin 30^0 = 3.25\,m$

Q25. At an instant shown, three point masses m, $2m$ and $3m$, rest on a horizontal surface and are at the vertices of an equilateral triangle of unit side length. Assuming that G is the gravitational constant, the magnitude and direction of the torque on the mass $3m$, about the point O, at that instant is



(a) Zero

(b) $\frac{3}{2}G\sqrt{3}m^2$, going into the paper

(c) $3G\sqrt{3}m^2$, coming out of the paper

(d) $\frac{3}{4}G\sqrt{3}m^2$, going into the paper

Ans.: (d) Force on $3m$ mass

$$\vec{F} = \left(6Gm^2 \cdot \sin 30^0 - 3Gm^2 \cdot \sin 30^0\right)\hat{x} + \left(-6Gm^2 \cos 30^0 - 3Gm^2 \cos 30^0\right)\hat{y}$$

$$\vec{F} = \frac{3}{2}Gm^2\hat{x} - \frac{9\sqrt{3}}{2}Gm^2\hat{y}$$

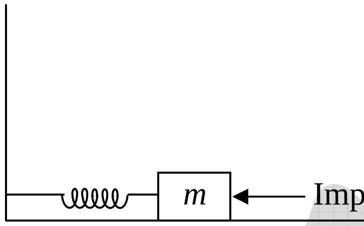
Radius vector for $3m$ mass

$$\vec{r} = 0\hat{x} + \frac{\sqrt{3}}{2}\hat{y}$$

$$\tau = \vec{r} \times \vec{f} = \frac{\sqrt{3}}{2} \times \frac{3}{2}Gm^2\hat{y} \times \hat{x} - 0 = -\frac{3\sqrt{3}}{4}Gm^2\hat{z}.$$

IIT-JAM-2015

Q26. A mass m, lying on a horizontal, frictionless surface, is connected to one end of a spring. The other end of the spring is connected to a wall, as shown in the figure. At $t = 0$, the mass is given an impulse.



The time dependence of the displacement and the velocity of the mass (in terms of non-zero constants A and B) are given by

(a) $x(t) = A\sin\omega t,\ v(t) = B\cos\omega t$ (b) $x(t) = A\sin\omega t,\ v(t) = B\sin\omega t$

(c) $x(t) = A\cos\omega t,\ v(t) = B\sin\omega t$ (d) $x(t) = A\cos\omega t,\ v(t) = B\cos\omega t$

Ans.: (a)

Solution: At time $t = 0$, the mass 'm' is at rest. Thus, displacement will be zero at time $t = 0$.

$$\therefore \quad x = A\sin(\omega t)$$

Velocity is $v = \frac{dx}{dt} = A\omega\cos\omega t = B\cos\omega t$

Thus, $x = A\sin\omega t$ and $V(t) = B\cos\omega t$

Q27. A satellite moves around the earth in a circular orbit of radius R centered at the earth. A second satellite moves in an elliptic orbit of major axis $8R$, with the earth at one of the foci. If the former takes 1 day to complete a revolution, the latter would take

(a) 21.6 days (b) 8 days (c) 3 hours (d) 1.1 hour

Ans.: (a)

Solution: $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{8R}\right)^3 \Rightarrow T_2 = (8)^{3/2} T_1 \approx 22\ days$

Q28. An observer is located on a horizontal, circular turntable which rotates about a vertical axis passing through its center, with a uniform angular speed of $2\,\text{rad/sec}$. A mass of 10 grams is sliding without friction on the turntable. At an instant when the mass is at a distance of $8\ cm$ from the axis it is observed to move towards the center with a speed of $6\ cm/\sec$. The net force on the mass, as seen by the observer at that instant, is

(a) $0.0024\,N$ (b) $0.0032\,N$ (c) $0.004\,N$ (d) $0.006\,N$

Ans.: (c)

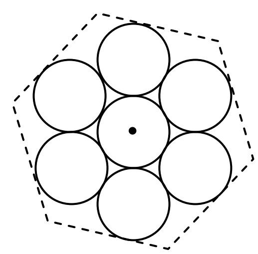
Solution: Two forces will act on the particle

First coriolis force $F_c = -2m(\omega \times v) = -240 \times 10^{-5}\,N$ (in tangential direction)

Another force is centrifugal force $F_r = m\omega^2 r = 320 \times 10^{-5}\,N$ (in radial direction)

Total force $F = \sqrt{F_c^2 + F_{c\,r}^2} = 0.04N$

Q29. Seven uniform disks, each of mass m and radius r, are inscribed inside a regular hexagon as shown. The moment of inertia of this system of seven disks, about an axis passing through the central disk and perpendicular to the plane of the disks, is



(a) $\frac{7}{2}mr^2$ (b) $7mr^2$

(c) $\frac{13}{2}mr^2$ (d) $\frac{55}{2}mr^2$

Ans.: (d)

Solution: $\frac{mr^2}{2} + 6 \times \left(\frac{mr^2}{2} + 4mr^2\right) = \frac{mr^2}{2} + \frac{54mr^2}{2} = \frac{55mr^2}{2}$

(MSQ)

Q30. A particle of mass m is moving in $x-y$ plane. At any given time t, its position vector is given by $\vec{r}(t)=A\cos\omega t\,\hat{i}+B\sin\omega t\,\hat{j}$ where A,B and ω are constants with $A\neq B$. Which of the following statements are true?

(a) Orbit of the particle is an ellipse

(b) Speed of the particle is constant

(c) At any given time t the particle experiences a force towards origin

(d) The angular momentum of the particle is $m\omega AB\hat{k}$

Ans.: (a), (c) and (d)

Solution: (a) $\vec{r}(t)=A\cos\omega t\,\hat{i}+B\sin\omega t\,\hat{j}\Rightarrow x=A\cos\omega t,\ y=B\sin\omega t$

$$\Rightarrow\frac{x}{A}=\cos\omega t,\ \frac{y}{B}=\sin\omega t\Rightarrow\frac{x^2}{A^2}+\frac{y^2}{B^2}=1\ \text{(Ellipse)}$$

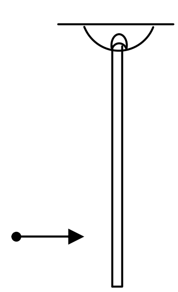
(b) $\dfrac{d\vec{r}}{dt}=-A\omega\sin\omega t\,\hat{i}+B\omega\cos\omega t\,\hat{j}$

Speed $=\left|\dfrac{d\vec{r}}{dt}\right|=\sqrt{A^2\omega^2\sin^2\omega t+B^2\omega^2\cos^2\omega t}$. Speed is function of time, so not constant.

(c) $\dfrac{d\vec{r}^2}{dt^2}=-A\omega^2\cos\omega t\,\hat{i}-B\omega^2\sin\omega t\,\hat{j}=-\omega^2\vec{r}$. Force act towards origin.

(d) $L=(\vec{r}\times\vec{p})=m\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ A\cos\omega t & B\sin\omega t & 0\\ -A\omega\sin\omega t & B\omega\cos\omega t & 0\end{pmatrix}\Rightarrow L=m\omega AB\hat{k}$

Q31. A rod is hanging vertically from a pivot. A partic1e traveling in horizontal direction, collides with the rod as shown in the figure. For the rod-particle system, consider the linear momentum and the angular momentum about the pivot .Which of the following statements are **NOT** true?



(a) Both linear momentum and angular momentum are conserved

(b) Linear momentum is conserved but angular momentum is not

(c) Linear momentum is not conserved but angular momentum is conserved

(d) Neither linear momentum nor annular momentum are conserved

Ans.: (b), (c) and (d)

(NAT)

Q32. A rod is moving with a speed of $0.8c$ in a direction at 60° to its own length. The percentage contraction in the length of the rod is…………

Ans.: 9

Solution: $l_x = l_{0x}\sqrt{1-\frac{v^2}{c^2}} = l_0\cos\theta\sqrt{1-(0.8)^2} \Rightarrow l_x = l_0\times\frac{1}{2}\times 0.6 = 0.3l_0$ and $l_y = l_0\sin\theta = \frac{l_0\sqrt{3}}{2}$

New length $l = \sqrt{(0.3l_0)^2+\left(\frac{\sqrt{3}l_0}{2}\right)^2} = l_0\sqrt{0.09+\frac{3}{4}} = 0.916\ l_0$

% change in length $\frac{(1-0.91)l_0}{l_0}\times 100 = 0.09\times 100 = 9\%$

Q33. A uniform disk of mass m and radius R rolls, without slipping, down a fixed plane inclined at an angle 30° to the horizontal. The linear acceleration of the disk (in $m/\sec^2$) is………………

Ans.: 3.266

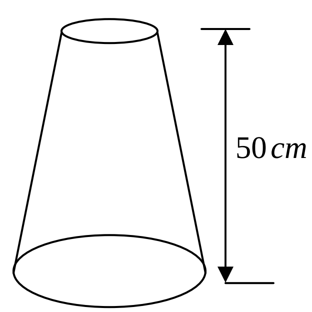
Solution: Equation of Motion $mg\sin\theta - f = ma$

Torque$=$ $fR = I\alpha$

$mg\sin\theta - \frac{I\alpha}{R} = ma,\quad \left(\alpha = \frac{a}{R}, I = \frac{mR^2}{2}\right)$

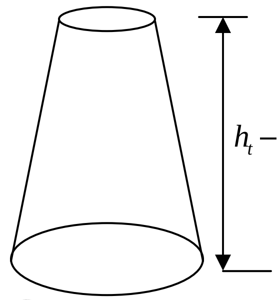
$a = \frac{g}{3} = 3.266$

Q34. A nozzle is in the shape of a truncated cone, as shown in the figure. The area at the wide end is $25\,cm^2$ and the narrow end has an area of $1\ cm^2$. Water enters the wider end at a rate of $500\,gm/\sec$. The height of the nozzle is $50\,cm$ and it is kept vertical with the wider end at the bottom. The magnitude of the pressure difference in kPa ($1kPa = 10^3\ N/m^2$) between the two ends of the nozzle is…………



Ans.: 17.5

Solution: According to Bernoulli's equation



$$P_b + \rho g h_b + \frac{1}{2}\rho V_b^2 = P_t + \rho g h_t + \frac{1}{2}\rho V_t^2$$

$$\Rightarrow P_b - P_t = \rho g\left(h_t - h_b\right) + \frac{1}{2}\rho\left(V_t^2 - V_b^2\right)$$

Now given $\rho A_t V_t = 500\ gm/\sec$

$$\Rightarrow V_t = \frac{500\times10^{-3}\,kg/\sec}{\rho\times A_t} = \frac{500\times10^{-3}\,kg/\sec}{1000\,kg/m^3\times10^{-4}\,m^2}$$

$$\Rightarrow V_t = 5\ m/\sec$$

According to equation of continuity

$$A_t V_t = A_b V_b \Rightarrow V_b = \frac{A_t}{A_b}V_t \Rightarrow V_b = \frac{1cm^2}{25\,cm^2}\times 5\,m/\sec = 0.2\,m/\sec$$

$$\therefore \Delta P = P_b - P_t = 1000\times10\times50\times10^{-2} + \frac{1}{2}\times1000\times\left[5^2 - (0.2)^2\right]$$

$$\Rightarrow \Delta P = 5000 + 500\left(25 - 0.04\right) = 5000 + 12480 = 17480\ N/m^2 \Rightarrow \Delta P = 17.5\,kP_a$$

Q35. A block of mass $2\,kg$ is at rest on a horizontal table The coefficient of friction between the block and the table is 0.1. A horizontal force $3\,N$ is applied to the block. The speed of the block (in m/s) after it has moved a distance $10\,m$ is…………….

Ans.: 3.225

Solution: $f_r = \mu N = 0.1\times2\times10 = 2N$ $\quad\because m = 2\,kg$

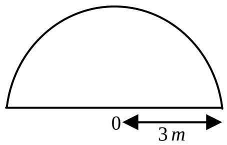
Applied force is more than friction

$$ma = F - \mu N = 3 - 2 = 1 \Rightarrow a = \frac{1}{m} = \frac{1}{2} = 0.5\ m/s^2$$

$$\because v = u^2 + 2as \ \Rightarrow v = \sqrt{2as} = \sqrt{2\times0.5\times10} = \sqrt{10} = 3.225\ m/s$$

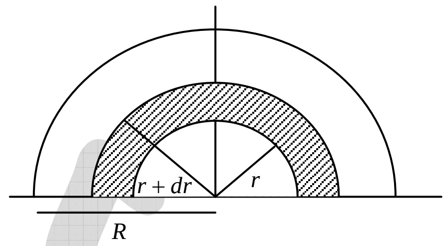
$$\because u = 0,\ s = 10m$$

Q36. A homogeneous semi-circular plate of radius $R = 3m$ is shown in the figure. The distance of the center of mass of the p1ate (in meter) from the point O is.........



Ans.: 1.3

Solution: In problem $R = 3m$



The area of the shaded part is $\pi r dr$. The area of the plate is $\pi R^2/2$. As the plate is uniform, the mass per unit area is $\frac{M}{\pi R^2/2}$. Hence the mass of the semicircular element

$$dm = \frac{M}{\pi R^2/2} r dr d\theta$$

The x - coordinate of the centre of mass is zero by symmetry.

The y coordinate of centre of mass

$$Y = \frac{1}{M}\int r\sin\theta dm = \frac{1}{M}\int_0^{\pi}\int_0^{R} r\sin\theta \frac{2M}{\pi R^2} r dr d\theta$$

$$= \frac{2}{\pi R^2}\int_0^R r^2 dr \int_0^{\pi} \sin\theta d\theta = \frac{2}{\pi R^2}\frac{R^3}{3}\times 2 = \frac{4R}{3\pi} = 1.3$$

IIT-JAM-2016

Q37. A particle is moving in a plane with a constant radial velocity of $12\, m/s$ and constant angular velocity of $2\, rad/s$. When the particle is at a distance $r = 8\,m$ from the origin, the magnitude of the instantaneous velocity of the particle in m/s is

(a) $8\sqrt{15}$ (b) 20 (c) $2\sqrt{37}$ (d) 10

Ans.: (b)

Solution: $v_r = 12\, m/s$ $v_\theta = \omega r \Rightarrow 2\times 8 = 16 m/\sec$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{144+256} = \sqrt{400} = 20 m/\sec$$

Q38. A cylindrical rod of length L has a mass density distribution given by $\rho(x)=\rho_0\left(1+\frac{x}{L}\right)$, where x is measured from one end of the rod and ρ_0 is a constant of appropriate dimensions. The centre of mass of the rod is

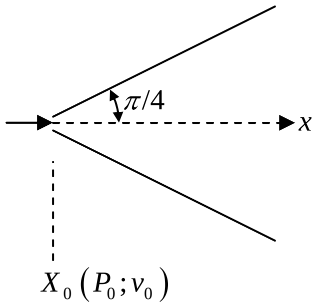
(a) $\frac{5}{9}L$ (b) $\frac{4}{9}L$ (c) $\frac{1}{9}L$ (d) $\frac{1}{2}L$

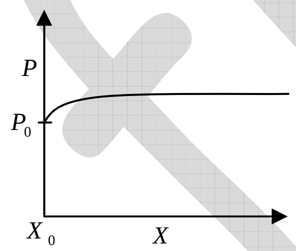
Ans.: (a)

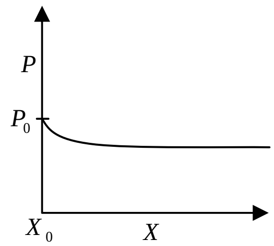
Solution: $$x_{cm}=\frac{\int_0^L x\,dm}{\int_0^L dm}=\frac{\int_0^L x\rho\,dx}{\int_0^L \rho\,dx}=\frac{\int_0^L x\rho_0\left(1+\frac{x}{L}\right)dx}{\int_0^L \rho_0\left(1+\frac{x}{L}\right)dx}=\frac{\int_0^L \frac{x^2}{2}+\frac{x^3}{3L}}{\int_0^L x+\frac{x^2}{2L}}=\frac{\frac{L^2}{2}+\frac{L^3}{3L}}{L+\frac{L^2}{2L}}=\frac{L^2\left(\frac{1}{2}+\frac{1}{3}\right)}{L\left(1+\frac{1}{2}\right)}$$

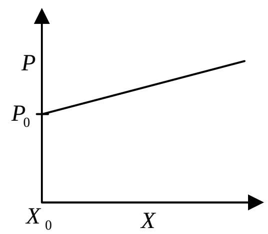
$$\Rightarrow x_{cm}=\frac{\frac{5}{6}L}{\frac{3}{2}}=\frac{5}{9}L$$

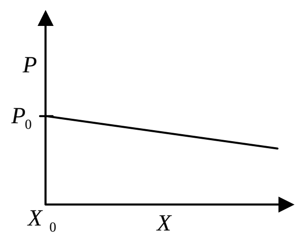
Q39. An incompressible, non-viscous fluid is injected into a conical pipe at its orifice as schematically shown in the figure. The pressure at the orifice of area A_0 is P_0. Neglecting the effect of gravity and assuming streamline flow, which one of the following plots correctly predicts the pressure along axis of the cone?



(a)


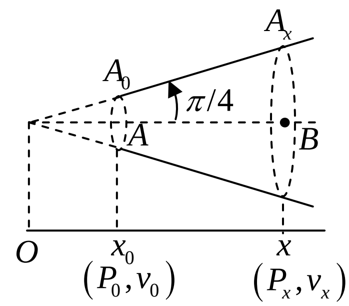
(b)


(c)


(d)


Ans. : (a)

Solution: The area cross-section at B is



$$A_x = \pi r^2 = \pi\left(x\tan\frac{\pi}{4}\right)^2$$

$$A_x = \pi x^2$$

Velocity at B is $A_0 v_0 = A_x v_x \Rightarrow v_x = \frac{A_0 v_0}{A_x} = \frac{A_0 v_0}{\pi x^2} \Rightarrow v_x = \frac{A_0 v_0}{\pi x^2}$

According to Bernauli equation,

$$P_0 + \frac{1}{2}\rho v_0^2 = P_x + \frac{1}{2}\rho v_x^2 \Rightarrow P_x = P_0 + \frac{1}{2}\rho\left(v_0^2 - v_x^2\right) = P_0 + \frac{1}{2}\rho\left(v_0^2 - \frac{A_0^2 v_0^2}{\pi^2 x^4}\right)$$

$$P_x = P_0 + \frac{1}{2}\rho v_0^2\left(1 - \frac{A_0^2}{\pi^4 x^4}\right)$$

Graph (a) correctly represent the variation of P_x w.r.t. x

Q40. A particle moves in a circular path in the xy - plane centered at the origin. If the speed of the particle is constant, then its angular momentum

(a) about the origin is constant both in magnitude and direction

(b) about $(0,0,1)$ is constant in magnitude but not in direction

(c) about $(0,0,1)$ varies both in magnitude and direction

(d) about $(0,0,1)$ is constant in direction but not in magnitude

Ans.: (a) and (b)

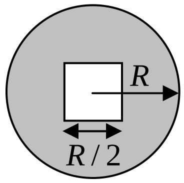
Solution: Angular momentum will be constant in $x-y$ plane

IIT-JAM 2017

Q41. Consider a uniform thin circular disk of radius R and mass M. A concentric square of side $R/2$ is cut out from the disk (see figure). What is the moment of inertia of the resultant disk about an axis passing through the centre of the disk and perpendicular to it?

(a) $I = \frac{MR^2}{4}\left[1 - \frac{1}{48\pi}\right]$ (b) $I = \frac{MR^2}{2}\left[1 - \frac{1}{48\pi}\right]$

(c) $I = \frac{MR^2}{4}\left[1 - \frac{1}{24\pi}\right]$ (d) $I = \frac{MR^2}{2}\left[1 - \frac{1}{24\pi}\right]$



Ans. : (b)

Solution: $I = I_{disc} - I_{square} = \frac{MR^2}{2} - \frac{M'\left(a^2 + a^2\right)}{12}$

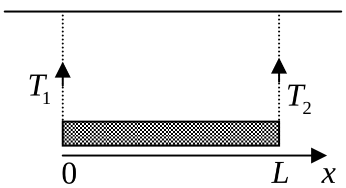
$M' = \frac{M}{\pi R^2} \times \frac{R}{2} \times \frac{R}{2} = \frac{M}{4\pi}$ and $a = \frac{R}{2}$

$I = \frac{MR^2}{2}\left[1 - \frac{1}{48\pi}\right]$

Q42. The linear mass density of a rod of length L varies from one end to the other as $\lambda_0\left(1 + \frac{x^2}{L^2}\right)$, where x is the distance from one end with tensions T_1 and T_2 in them (see figure), and λ_0 is a constant. The rod is suspended from a ceiling by two massless strings. Then, which of the following statement(s) is (are) correct?

(a) The mass of the rod is $\frac{2\lambda_0 L}{3}$

(b) The centre of gravity of the rod is located at $x = \frac{9L}{16}$



(c) The tension T_1 in the left string is $\frac{7\lambda_0 Lg}{12}$

(d) The tension T_2 in the right string is $\frac{3\lambda_0 Lg}{2}$

Ans. : (b), (c)

Solution: The mass of rod is $m = \int_0^L \lambda_0 \left(1 + \frac{x^2}{L^2}\right) dx = \frac{4\lambda_0 L}{3}$ so (a) is wrong

The centre of gravity of the rod is located at $x_{cm} = \frac{\int_0^L x\lambda_0 \left(1+\frac{x^2}{L^2}\right)dx}{\int_0^L \lambda_0 \left(1+\frac{x^2}{L^2}\right)dx} = \frac{9L}{16}$

Force equation $T_1 + T_2 = \frac{4\lambda_0 Lg}{3}$ and torque equation

$$T_1 \times \frac{9L}{16} = T_2 \times \left(L - \frac{9L}{16}\right) \Rightarrow T_1 \times \frac{9L}{16} = T_2 \times \frac{7L}{16} \Rightarrow T_2 = T_1 \times \frac{9}{7}$$

put value of $T_2 = \frac{9}{7}T_1$ in equation $T_1 + T_2 = \frac{4\lambda_0 Lg}{3}$

$$\frac{16}{7}T_1 = \frac{4\lambda_0 Lg}{3} \Rightarrow T_1 = \frac{7\lambda_0 Lg}{12}$$

$$T_2 = \frac{9}{7} \times T_1 = \frac{9}{7} \times \frac{7\lambda_0 Lg}{12} = \frac{9\lambda_0 Lg}{12}$$

Q43. An object of mass m with non-zero angular momentum ℓ is moving under the influence of gravitational force of a much larger mass (ignore drag). Which of the following statement(s) is (are) correct?

(a) If the total energy of the system is negative, then the orbit is always circular

(b) The motion of m always occurs in a two-dimensional plane

(c) If the total energy of the system is 0, then the orbit is a parabola

(d) If the area of the particle's bound orbit is S, then its time period is $2mS/\ell$

Ans. : (b), (c) and (d)

Solution: The eccentricity of curve $e = \sqrt{1 + \frac{2El^2}{mk^2}}$ where $k = Gm_1m_2$ and E is energy .

If total energy is negative then orbit can be either elliptical or circular so (a) is wrong

In two body central force problem motion is confine in plane so (b) is correct

If the total energy of the system is 0, then the orbit is a parabola one can calculate $e = 1$ so (c) is correct

From second law of thermodynamics $\frac{dA}{dt}=\frac{l}{2m}\Rightarrow\frac{S}{T}=\frac{l}{2m}\Rightarrow T=\frac{2mS}{l}$ so (d) is correct

Q44. A particle of mass m fixed in space is observed from a frame rotating about its z - axis with angular speed ω. The particle is in the frame's xy plane at a distance R from its origin. If the Coriolis and centrifugal forces on the particle are $\vec{F}_{COR}$ and $\vec{F}_{CFG}$, respectively, then (all the symbols have their standard meaning and refer to the rotating frame),

(a) $\vec{F}_{COR}+\vec{F}_{CFG}=0$ (b) $\vec{F}_{COR}+\vec{F}_{CFG}=-m\omega^2 R\hat{r}$

(c) $\vec{F}_{COR}=-2m\omega^2 R\hat{r}$ (d) $\vec{F}_{CFG}=-m\omega^2 R\hat{r}$

Ans. : (b), (c)

Solution: $\vec{F}_{corr}+\vec{F}_{centipetal}=-mr\omega^2 R\hat{r}$

$F_{cor}=-2m\left(\vec{\omega}\times\vec{v}\right),v=\omega r\Rightarrow -2m\omega^2 R$

Q45. A particle of unit mass is moving in a one-dimensional potential $V\left(x\right)=x^2-x^4$. The minimum mechanical energy (in the same units as $V\left(x\right)$) above which the motion of the particle cannot be bounded for any given initial condition is..............

(Specify your answer to two digits after the decimal point)

Ans. : 0.25

Solution: $V\left(x\right)=x^2-x^4$

The potential is mimima at $x=0$ and maxima at $x=\pm\sqrt{\frac{1}{2}}$ fig

The value $V\left(x\right)=0\ \ x=0$

The value $V\left(x\right)=0.25\ \ x=\pm\sqrt{\frac{1}{2}}$

minimum mechanical energy above which the motion of the particle cannot be bounded is $E_f=.25$

Q46. Sand falls on a conveyor belt at the rate of $1.5\,kg/s$. If the belt is moving with a constant speed of $7\,m/s$, the power needed to keep the conveyor belt running is……………
(Specify your answer in Watts to two digits after the decimal point)

Ans. : 73.4

Solution: $P=\dfrac{dW}{dt}\Rightarrow\dfrac{d\left(\frac{1}{2}mv^2\right)}{dt}=\dfrac{1}{2}.v^2.\dfrac{dm}{dt}+\dfrac{2}{2}mv\dfrac{dv}{dt}$ if v is constant $\dfrac{dv}{dt}=0$

$$P_1=\frac{1}{2}.\frac{dm}{dt}.v^2=\frac{1}{2}\times1.5\times49=36.7\,watt$$

Work done due to friction $W=\mu mg.d$, $d=\dfrac{1}{2}at^2$, $v=at\Rightarrow t=\dfrac{v}{a}=\dfrac{v}{\mu g}$

$$d=\frac{1}{2}\mu g\left(\frac{v}{\mu g}\right)^2$$

$$W=\mu mg.\left(\frac{1}{2}\mu g\left(\frac{v}{\mu g}\right)^2\right)=\frac{1}{2}mv^2$$

$$P_2=\frac{1}{2}.\frac{dm}{dt}.v^2=\frac{1}{2}\times1.5\times49=36.7\,watt$$

$$P=P_1+P_2=36.7\,watt+36.7\,watt=73.4\ watt.$$

Q47. In planar polar co-ordinates, an object's position at time t is given as $(r,\theta)=\left(e^t m,\sqrt{8}\,t\text{ rad}\right)$. The magnitude of its acceleration in m/s^2 at $t=0$ (to the nearest integer) is……………….

Ans. : 9

Solution: $a_r=\ddot{r}-\dot{\theta}^2r\Rightarrow e^t-8\times e^t$ at $t=0$,$1-8\times1=-7\,m/\sec^2$

$$a_\theta=r\ddot{\theta}+2\dot{r}\dot{\theta}\Rightarrow e^t\times0+2e^t\sqrt{8}=2\sqrt{8}\quad |a|=\sqrt{(-7)^2+4\times8}=\sqrt{49+32}=\sqrt{81}=9\,m/\sec^2$$

Q48. At $t=0$, a particle of mass m having velocity v_0 starts moving through a liquid kept in a horizontal tube and experiences a drag force $\left(F_d = -k\frac{dx}{dt}\right)$. It covers a distance L before coming to rest. If the times taken to cover the distances $L/2$ and $L/4$ are t_2 and t_4 respectively, then the ratio t_2/t_4 (ignoring gravity) is..........

(Specify your answer to two digits after the decimal point)

Ans. : 2.41

Solution: Using Newton's second law we obtain

$$-k\frac{dx}{dt} = m\frac{d^2x}{dt^2} \Rightarrow m\frac{d^2x}{dt^2} + k\frac{dx}{dt} = 0 \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}\frac{dx}{dt} = 0$$

The characteristic equation is

$$\lambda^2 + \frac{k}{m}\lambda = 0 \Rightarrow \lambda\left(\lambda + \frac{k}{m}\right) = 0,\ \lambda = 0 \text{ and } \lambda = -\frac{k}{m}$$

General solution:

$$x(t) = c_1 + c_2 e^{-\frac{k}{m}t} \qquad \text{(i)}$$

Using the condition $x=0$, we obtain

$$c_1 + c_2 = 0 \qquad \text{(ii)}$$

$$v(t) = -\frac{k}{m}c_2 e^{-\frac{k}{m}t} \qquad \text{(iii)}$$

Using the condition $v(0) = v_0$

$$v_0 = -\frac{k}{m}c_2 \Rightarrow c_2 = -\frac{m}{k}v_0 \text{ and } c_1 = \frac{m}{k}v_0$$

$$x(t) = \frac{mv_0}{k}\left(1 - e^{-\frac{k}{m}t}\right) \qquad \text{(iv)}$$

$$v(t) = -\frac{mv_0}{k}\left(-\frac{k}{m}\right)e^{-\frac{k}{m}t} = v_0 e^{-\frac{k}{m}t} \Rightarrow e^{-\frac{k}{m}t} = \frac{v}{v_0} \qquad \text{(v)}$$

Using equations (iv) and (v), we obtain $x(t)=\frac{mv_0}{k}\left[1-\frac{v}{v_0}\right]$,

$$L=\frac{mv_0}{k}(1)\Rightarrow L=\frac{mv_0}{k}$$

From the question

$$\frac{L}{2}=\frac{mv_0}{k}\left[1-e^{\frac{-k}{m}t_2}\right]\Rightarrow e^{-\frac{k}{m}t_2}=1-\frac{kL}{2mv_0} \qquad \text{(vi)}$$

$$\frac{L}{4}=\frac{mv_0}{k}\left[1-e^{-\frac{k}{m}t_4}\right]\Rightarrow e^{-\frac{k}{m}t_4}=1-\frac{kL}{4mv_0} \qquad \text{(vii)}$$

Taking Logarithm on both sides of equations (vi) and (vii)

$-\frac{k}{m}t_2=\ln\left(1-\frac{kL}{2mv_0}\right)$ and $-\frac{k}{m}t_4=\ln\left(1-\frac{kL}{4mv_0}\right)$

Thus, $\frac{t_2}{t_4}=\frac{\ln\left(1-\frac{k}{2mv_0}\cdot\frac{mv_0}{k}\right)}{\ln\left(1-\frac{k}{4mv_0}\cdot\frac{mv_0}{k}\right)}=\frac{\ln(1/2)}{\ln(3/4)}=2\cdot 41$.

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Q49. There are three planets in circular orbits around a star at distances $a, 4a$ and $9a$, respectively. At time $t = t_0$, the star and the three planets are in a straight line. The period of revolution of the closest planet is T. How long after t_0 will they again be in the same straight line?

(a) $8T$ (b) $27T$ (c) $216T$ (d) $512T$

Ans. : (c)

Solution: $T_1 = ka^{3/2} = T$, $T_2 = k(4a)^{3/2} = 8T$, $T_3 = k(9a)^{3/2} = 27T$

Common time that all three star will meet again is $t_0 = T_1 \times T_2 \times T_3 = 216T$, which is LCM of all time period.

Q50. A disc of radius R_1 having uniform surface density has a concentric hole of radius $R_2 < R_1$. If its mass is M, the principal moments of inertia are

(a) $\frac{M(R_1^2 - R_2^2)}{2}, \frac{M(R_1^2 - R_2^2)}{4}, \frac{M(R_1^2 - R_2^2)}{4}$

(b) $\frac{M(R_1^2 + R_2^2)}{2}, \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{4}$

(c) $\frac{M(R_1^2 + R_2^2)}{2}, \frac{M(R_1^2 + R_2^2)}{4}, \frac{M(R_1^2 + R_2^2)}{8}$

(d) $\frac{M(R_1^2 - R_2^2)}{2}, \frac{M(R_1^2 - R_2^2)}{4}, \frac{M(R_1^2 - R_2^2)}{8}$

Ans. : (b)

Solution: $I_{zz} = \int_{R_1}^{R_2} dm\, r^2 = \frac{M}{\pi(R_2^2 - R_1^2)} \int_{R_1}^{R_2} 2\pi r . r^2 dr = I_{ZZ} = \frac{M(R_2^2 + R_1^2)}{2}$

$I_{xx} + I_{yy} = I_{zz}$

By symmetry $I_{xx} = I_{yy}$. Therefore, $I_{xx} = I_{yy} = \frac{M(R_2^2 + R_1^2)}{4}$

Q51. A raindrop falls under gravity and captures water molecules from atmosphere. Its mass charges at the rate $\lambda m(t)$, where λ is a positive constant and $m(t)$ is the instantaneous mass. Assume that acceleration due to gravity is constant and water molecules are at rest with respect to earth before capture. Which of the following statements is correct?

(a) The speed of the raindrop increases linearly with time

(b) The speed of the raindrop increases exponentially with time

(c) The speed of the raindrop approaches a constant value when $\lambda t \gg 1$

(d) The speed of the raindrop approaches a constant value when $\lambda t \ll 1$

Ans.: (c)

Solution: Applying impulse momentum m equation.

$$mg \times dt = (m + dm)(v + dv) - mv \Rightarrow mg = \frac{mdv}{dt} + v\frac{dm}{dt}$$

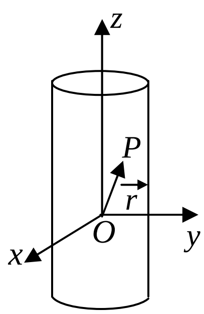
$$\text{or, } g = \frac{dv}{dt} + v\frac{(dm)}{mdt} \Rightarrow \frac{dv}{dt} - g + \lambda v = 0, \lambda = \frac{dm}{mdt}$$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{g - \lambda v} = \int_0^t dt \text{ at } t = 0, V = V_0 \Rightarrow \ln(g - \lambda v) - \ln|g - \lambda v_0| = -\lambda t$$

$$\Rightarrow \frac{g - \lambda v}{g - \lambda v_0} = e^{-\lambda t} \Rightarrow V = \frac{g}{\lambda} + \left(V_0 - \frac{g}{\lambda}\right)e^{-\lambda t}$$

As, $\lambda t >> 1, e^{-\lambda t} \to 0$, so $V \to \frac{g}{\lambda}$ (which is constant)

Q52. A particle P of mass m is constrained to move on the surface of cylinder under a force $-k\vec{r}$ as shown in figure (k is the positive constant). Which of the following statements is correct? (Neglect friction.)



(a)Total energy of the particle is not conserved.

(b) The motion along z direction is simple harmonic.

(c) Angular momentum of the particle about O increases with time.

(d) Linear momentum of the particle is conserved.

Ans. : (b)

Solution: $\vec{F} = -k\vec{r} = -k\left(r\hat{r} + z\hat{z}\right)$

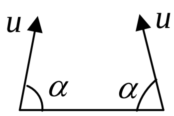
$F_r = -kr$

$F_\theta = 0$

$F_z = -kz \Rightarrow m\ddot{z} = -kz$ the motion along z is simple harmonic motion.

(MSQ)

Q53. Two projectiles of identical mass are projected from the ground with same initial angle (α) with respect to earth surface and same initial velocity (u) in the same plane. They collide at the highest point of their trajectories and stick to each other. Which of the following statements is (are) correct?



(a) The momentum of the combined object immediately after collision is zero.

(b) Kinetic energy is conserved in the collision

(c) The combined object moves vertically downward.

(d) The combined object moves in a parabolic path.

Ans. : (a), (c)

Solution: At the highest point there is only Horizontal velocity. In horizontal direction there is not any External force. So momentum in the horizontal direction is conserved.

(c) After collision whole system will full under gravitation.

Q54. A particle of mass m is moving along the positive x direction under a potential $V(x) = \frac{1}{2}kx^2 + \frac{\lambda}{2x^2}$ (k and λ are positive constants). If the particle is slightly displaced from its equilibrium position, it oscillates with an angular frequency (ω)______.

(Specify your answer in units of $\sqrt{\frac{k}{m}}$ as an integer.)

Ans. : 2

Solution: $V(x) = \frac{kx^2}{2} + \frac{\lambda}{2x^2}$

$$\frac{\partial V}{\partial x} = kx - \frac{\lambda}{x^3} = 0 \Rightarrow x^4 = +\left(\frac{\lambda}{x}\right) \Rightarrow x_0 = \pm\left(\frac{\lambda}{x}\right)^{1/4}$$

$$\frac{\partial^2 V}{\partial x^2} = k + \frac{3\lambda}{x^4} = 4k \quad = \omega = \sqrt{\frac{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=x_0}}{m}} = \sqrt{\frac{4k}{m}} = 2\omega = 2$$

Q55. A planet has average density same as that of the earth but it has only $\frac{1}{8}$ of the mass of the Earth. If the acceleration due to gravity at the surface is g_p and g_e for the planet and Earth, respectively, then $\frac{g_p}{g_e} =$ ____________.

(Specific your answer upto one digit after the decimal point.)

Ans. : 0.5

Solution: $g = \frac{GM}{R^2}$ and $\frac{M}{V} = \rho$

$$V = \frac{M}{\rho} \Rightarrow R^3 \propto \frac{M}{\rho} \Rightarrow R \propto \left(\frac{M}{\rho}\right)^{1/3}$$

$$g \propto \frac{M}{(M/\rho)^{2/3}} \Rightarrow g \propto M^{1/3}$$

$$\frac{g_p}{g_e} = \left(\frac{M_p}{M_e}\right)^{1/3} = \left(\frac{M_e}{8M_e}\right)^{1/3} = \frac{1}{2} = 0.5 = \left(\frac{1}{2}\right) = 0.5$$

Q56. A body of mass 1 kg is moving under a central force in an elliptic orbit with semi major axis 1000 m and semi minor axis $100\,m$. The orbital angular momentum of the body is 100 $kg\, m^2 s^{-1}$. The time period of motion of the body is _________ hours.

(Specific your answer upto two digits after the decimal point)

Ans. : 1.74

Solution: $\frac{dA}{dt} = \frac{L}{2m} \Rightarrow T = \pi ab \cdot \frac{2m}{L}$

$$= 3.14 \times 1000 \times 100 \times \frac{2 \times 1}{100} = 6\cdot 28 \times 1000 = \frac{6280}{3600} = 1\cdot 744\, hr$$

Q57. The moon moves around the earth in a circular orbit with a period of 27 days. The radius of the earth (R) is $6.4\times10^{6}\ m$ and the acceleration due to gravity on the earth surface is $9.8\ ms^{-2}$. If D is the distance of the moon from the center of the earth, the value of $\frac{D}{R}$ will be _________. (Specific your answer upto two digits after the decimal point)

Ans. : 59.6

Solution: For circular orbit $m\omega^2 D = \frac{GMm}{D^2}$

$$D^3 = \frac{GM}{\omega^2} = \frac{GMT^2}{(2\pi)^2}\frac{R^2}{R^2}$$

$$D^3 = \frac{GM}{R^2}\frac{T^2}{4\pi^2}R^2 = \frac{gT^2R^2\pi R}{4\pi^2}$$

$$\left(\frac{D}{R}\right)^3 = \frac{gT^2}{4\pi^2 R} = \frac{9.8\times(27\times24\times60\times60)^2}{4\pi^2\times6.4\times10^6}$$

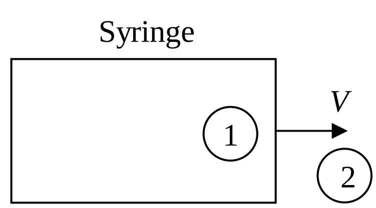
$$\Rightarrow \frac{D}{R} = \left(\frac{5.34\times10^{13}}{4\pi^2\times6.4\times10^6}\right)^{\frac{1}{3}} = 59.5$$

Q58. A syringe is used to exert 1.5 atmospheric pressure to release water horizontally. The speed of water immediately after ejection is________ (take 1 atmospheric pressure $=10^5$ Pascal, density of water $=10^3 kg\ m^{-3}$) (Specify your answer in ms^{-1} as an integer)

Ans: 10

Solution: Apply Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \rho_2 + \frac{1}{2}e v_2^2 + \rho g h_2$$



$$h_1 = h_2, v_1 \approx 0$$

$$P_1 = 1.5 \text{ atm}, P_2 = 1 \text{ atm}$$

$$\Rightarrow v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} = \sqrt{\frac{2\times(1\cdot5-0.5)\times10^5}{1000}}$$

$$v_2 = 10\ m/s$$

Q59. A particle of mass m is moving in a circular orbit given by $x = R\cos\omega t$; $y = R\sin(\omega t)$, as observed in an inertial frame S_1. Another inertial frame S_2 moves with uniform velocity $\vec{v} = \omega R\hat{i}$ with respect to S_1.S_1 and S_2 are related by Galilean transformation, such that the origins coincide at $t = 0$. The magnitude of the angular momentum of the particle at $t = \frac{2\pi}{\omega}$, as observed in S_2 about its origin is expressed as $(mR^2\omega)x$. Then x is _______.

(Specify your answer upto two digits after the decimal point)

Ans. : 5.28

Solution: From S_2 Frame

$$\vec{r}' = (x - vt)\hat{i} + y\hat{i} = (R\cos\omega t - vt)\hat{i} + R\sin\omega t\hat{i}$$

$$= \vec{v}' = (\dot{x} - v)\hat{i} + \dot{y}\hat{j} = (-R\omega\sin t - v)\hat{i} + R\omega\cos\omega t\hat{j}$$

$$\vec{L} = \vec{r}' \times \vec{P} = m\left[(R\cos\omega t - vt)\hat{i} + R\sin\omega t\hat{i}\right] \times \left[(-R\omega\sin t - v)\hat{i} + R\omega\cos\omega t\hat{j}\right]$$

$$= m\left(\omega R^2\cos^2\omega t - vtR\cos\omega t + R^2\omega\sin^2\omega t + vR\sin\omega t\right)\hat{k}$$

$$= m\left(R^2\omega - vtR\omega\cos\omega t + vR\sin\omega t\right)$$

at $t = \frac{2\pi}{\omega}$, $\vec{L} = m\omega R^2(1 - 2\pi) = -5.28m\omega R^2 \Rightarrow |\vec{L}| = 5.28\ m\omega R^2$