Electricity and Magnetism

IIT-JAM-2005

OBJECTIVE QUESTIONS

Q1. A small loop of wire of area $A = 0.01\, m^2$, $N = 40$ turns and resistance $R = 20\,\Omega$ is initially kept in a uniform magnetic field B in such a way that the field is normal to the loop. When it is pulled out of the magnetic field, a total charge of $Q = 2\times10^{-5}\, C$ flows through the coil. The magnitude of magnetic field B is

(a) $1\times10^{-3}\, T$ (b) $4\times10^{-3}\, T$

(c) zero (d) unobtainable, as the data is insufficient

Ans.: (a)

Solution: Magnetic flux through the loop $\phi = NBA$

Induced e.m.f $\varepsilon = -\dfrac{d\phi}{dt}$ and induced current $i = -\dfrac{1}{R}\dfrac{d\phi}{dt} = \dfrac{dQ}{dt} \Rightarrow -\dfrac{1}{R}d\phi = dQ$.

Thus, $\dfrac{1}{20}\times\left(40\times B\times 0.01\right) = 2\times10^{-5} \Rightarrow B = 1\times10^{-3}\, T$.

Q2. Two point charges $+q_1$ and $+q_2$ are fixed with a finite distance d between them. It is desired to put a third charge q_3 in between these two charges on the line joining them so that the charge q_3 is in equilibrium. This is

(a) possible only if q_3 is positive

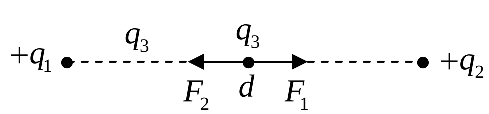
(b) possible only if q_3 is negative

(c) possible irrespective of the sign of q_3

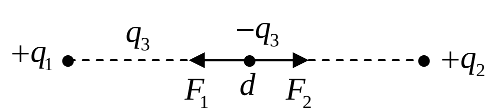
(d) not possible at all

Ans. : (c)

Solution: If q_3 is positive,



If q_3 is negative,



In both case there is possibility that charge q_3 may be in equilibrium.

IIT-JAM-2006

Q3. Two electric dipoles P_1 and P_2 are placed at $(0,0,0)$ and $(1,0,0)$ respectively with both of them pointing in the $+z$ direction. Without changing the orientations of the dipoles P_2 is moved to $(0,2,0)$. The ratio of the electrostatic potential energy of the dipoles after moving to that before moving is

(a) $\frac{1}{16}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Ans. : (d)

Solution: Electrostatic potential energy $U \propto \frac{1}{r^3} \Rightarrow \frac{U_2}{U_1} = \frac{r_1^3}{r_2^3} = \frac{1}{8}$

Q4. A small magnetic dipole is kept at the origin in the x-y plane. One wire L_1 is located at $z = -a$ in the x-z plane with a current I flowing in the positive x direction. Another wire L_2 is at $z = +a$ in y-z plane with the same current I as in L_1, flowing in the positive y-direction. The angle ϕ made by the magnetic dipole with respect to the positive x-axis is

(a) 225^0 (b) 120^0 (c) 45^0 (d) 270^0

Ans.: (a)

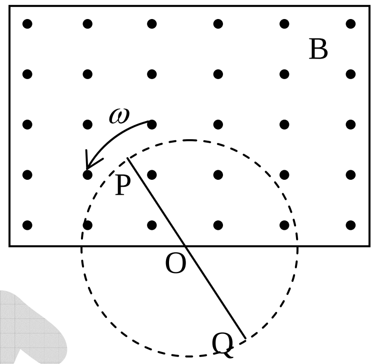
Solution: Magnetic field at $z = 0$ due to wire at $z = -a$ is $\vec{B} = -B\hat{y}$.

Magnetic field at $z = 0$ due to wire at $z = +a$ is $\vec{B} = -B\hat{x}$.

Resultant magnetic field at $z = 0$ makes an angle of 45^0 with $-\hat{x}$ and 225^0 with $\hat{x}$.

IIT-JAM-2007

Q5. A uniform and constant magnetic field B coming out of the plane of the paper exists in a rectangular region as shown in the figure. A conducting rod PQ is rotated about O with a uniform angular speed ω in the plane of the paper. The emf E_{PQ} induced between P and Q is best represented by the graph



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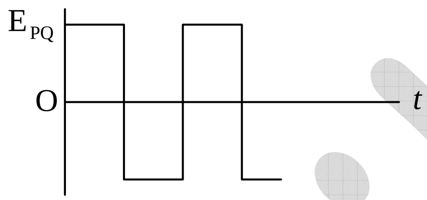
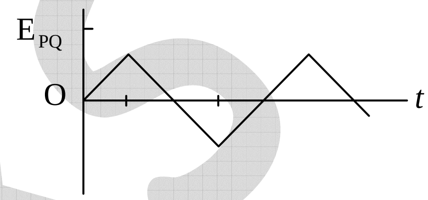


(b)


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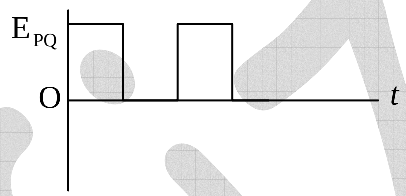

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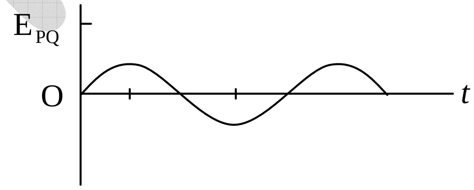


Ans.: (a)

Solution: When point P is inside due to motional emf, potential PQ is positive. When point Q is inside potential QP is positive or potential PQ is negative.

IIT-JAM-2008

Q6. If the electrostatic potential at a point (x, y) is given by $V = (2x + 4y)$ volts, the electrostatic energy density at that point $(in\ J/m^3)$ is

(a) $5\varepsilon_0$ (b) $10\varepsilon_0$ (c) $20\varepsilon_0$ (d) $\frac{1}{2}\varepsilon_0(2x+4y)^2$

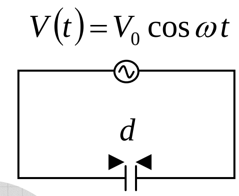
Ans.: (b)

Solution: $\vec{E} = -\vec{\nabla}V = -2\hat{x} - 4\hat{y} \Rightarrow |\vec{E}| = \sqrt{20}V/m$

Electrostatic energy density $= \frac{1}{2}\varepsilon_0 |\vec{E}|^2 = \frac{1}{2}\varepsilon_0 \times 20 = 10\varepsilon_0 \; J/m^3$

IIT-JAM-2009

Q7. An oscillating voltage $V(t) = V_0 \cos\omega t$ is applied across a parallel plate capacitor having a plate separation d. The displacement current density through the capacitor is



(a) $\frac{\varepsilon_0 \omega V_0 \cos\omega t}{d}$ (b) $\frac{\varepsilon_0 \mu_0 \omega V_0 \cos\omega t}{d}$

(c) $-\frac{\varepsilon_0 \mu_0 \omega V_0 \cos\omega t}{d}$ (d) $-\frac{\varepsilon_0 \omega V_0 \sin\omega t}{d}$

Ans.: (d)

Solution: Displacement current density $J_d = \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\varepsilon_0}{d}\frac{\partial V(t)}{\partial t} = -\frac{\varepsilon_0 \omega V_0 \sin\omega t}{d}$

Q8. An electric field $\vec{E}(\vec{r}) = \left(\alpha\hat{r} + \beta \sin\theta \cos\phi \hat{\phi}\right)$ exists in space. What will be the total charge enclosed in a sphere of unit radius centered at the origin?

(a) $4\pi\varepsilon_0\alpha$ (b) $4\pi\varepsilon_0(\alpha+\beta)$ (c) $4\pi\varepsilon_0(\alpha-\beta)$ (d) $4\pi\varepsilon_0\beta$

Ans.: (a)

Solution: $Q_{enc} = \varepsilon_0 \oint \vec{E}\cdot d\vec{a} = \varepsilon_0 \int \left(\alpha\hat{r} + \beta \sin\theta \cos\phi\hat{\phi}\right)\cdot\left(r^2 \sin\theta d\theta d\phi \hat{r}\right) = 4\pi\alpha\varepsilon_0$

IIT-JAM-2010

Q9. The magnetic field associated with the electric field vector $\vec{E} = E_0 \sin(kz - \omega t)\hat{j}$ is given by

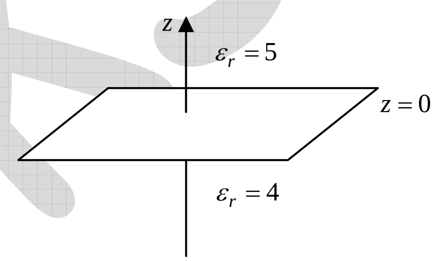
(a) $\vec{B} = -\frac{E_0}{c}\sin(kz - \omega t)\hat{i}$ (b) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{i}$

(c) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{j}$ (d) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{k}$

Ans.: (a)

Solution: $\vec{B} = \frac{\vec{k}\times\vec{E}}{\omega} = \frac{k\hat{z}\times E_0 \sin(kz - \omega t)\hat{j}}{\omega} = -\frac{kE_0}{\omega}\sin(kz - \omega t)\hat{i} = -\frac{E_0}{c}\sin(kz - \omega t)\hat{i}$

Q10. Assume that $z = 0$ plane is the interface between two linear and homogeneous dielectrics (see figure). The relative permittivities are $\varepsilon_r = 5$ for $z > 0$ and $\varepsilon_r = 4$ for $z < 0$. The electric field in the region $z > 0$ is $\vec{E}_1 = (3\hat{i} - 5\hat{j} + 4\hat{k})kV/m$. If there are no free charges on the interface, the electric field in the region $z < 0$ is given by



(a) $\vec{E}_2 = \left(\frac{3}{4}\hat{i} - \frac{5}{4}\hat{j} + \hat{k}\right)kV/m$ (b) $\vec{E}_2 = (3\hat{i} - 5\hat{j} + \hat{k})kV/m$

(c) $\vec{E}_2 = (3\hat{i} - 5\hat{j} - 5\hat{k})kV/m$ (d) $\vec{E}_2 = (3\hat{i} - 5\hat{j} + 5\hat{k})kV/m$

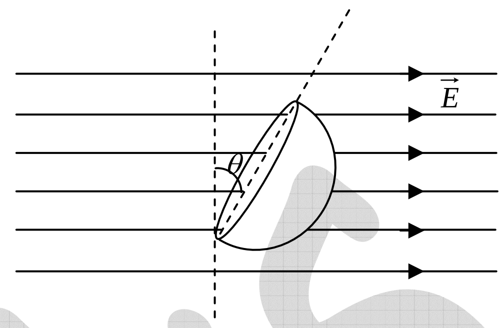
Ans.: (d)

Solution: $\because E_1^{\parallel} = E_2^{\parallel} \Rightarrow E_2^{\parallel} = 3\hat{i} - 5\hat{j}$

and $\sigma_f = 0 \Rightarrow D_1^{\perp} = D_2^{\perp} \Rightarrow E_2^{\perp} = \frac{\varepsilon_1}{\varepsilon_2}E_1^{\perp} = \frac{5}{4}(+4\hat{k}) = 5\hat{k}$

$\Rightarrow \vec{E}_2 = (3\hat{i} - 5\hat{j} + 5\hat{k})kV/m$

Q11. A closed Gaussian surface consisting of a hemisphere and a circular disc of radius R, is placed in a uniform electric field $\vec{E}$, as shown in the figure. The circular disc makes an angle $\theta = 30^0$ with the vertical. The flux of the electric field vector coming out of the curved surface of the hemisphere is



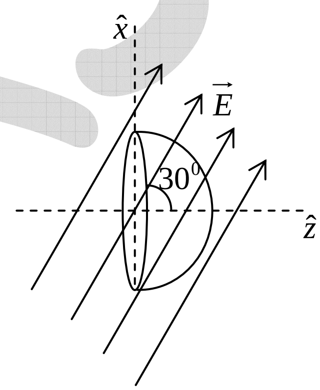
(a) $\frac{1}{2}\pi R^2 E$

(b) $\frac{\sqrt{3}}{2}\pi R^2 E$

(c) $\pi R^2 E$

(d) $2\pi R^2 E$

Ans.: (b)



Solution: $\vec{E} = E\cos 30\hat{z} + E\sin 30\hat{x} = \frac{\sqrt{3}}{2}E\hat{z} + \frac{1}{2}E\hat{x}$

$$\phi_E = \int_S \vec{E}\cdot d\vec{a} = \int\int\left(\frac{\sqrt{3}}{2}E\hat{z} + \frac{1}{2}E\hat{x}\right)\cdot\left(R^2\sin\theta d\theta d\phi\hat{r}\right)$$

$$\phi_E = R^2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\left(\frac{\sqrt{3}}{2}E\cos\theta + \frac{1}{2}E\sin\theta\cos\phi\right)\left(\sin\theta d\theta d\phi\right)$$

$$\phi_E = \frac{\sqrt{3}}{2}ER^2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\left(\cos\theta\sin\theta\right)d\theta d\phi + \frac{1}{2}ER^2\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi}\left(\sin^2\theta\cos\phi\right)d\theta d\phi$$

$$\phi_E = \frac{\sqrt{3}}{2}ER^2\times 2\pi\times\frac{1}{2} + 0 = \frac{\sqrt{3}}{2}\pi R^2 E$$

OR

$$\phi_E = \int_S \vec{E}\cdot d\vec{a} = E\cos 30^0\times\pi R^2 = \frac{\sqrt{3}}{2}\pi R^2 E$$

IIT-JAM-2011

Q12. Equipotential surface corresponding to a particular charge distribution are given by $4x^2+(y-2)^2+z^2=V_i$, where the values of V_i are constants. The electric field $\vec{E}$ at the origin is

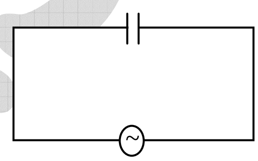
(a) $\vec{E}=0$ (b) $\vec{E}=2\hat{x}$ (c) $\vec{E}=4\hat{y}$ (d) $\vec{E}=-4\hat{y}$

Ans.: (d)

Solution: $\vec{E}=-\vec{\nabla}V=8x\hat{x}+2(y-2)\hat{y}+2z\hat{z}\Rightarrow\vec{E}(0,0,0)=-4\hat{y}$

IIT-JAM-2012

Q13. A parallel plate air-gap capacitor is made up of two plates of area $10\,cm^2$ each kept at a distance of $0.88\,mm$. A sine wave of amplitude $10V$ and frequency $50\,Hz$ is applied across the capacitor as shown in the figure. The amplitude of the displacement current density (in mA/m^2) between the plates will be closest to



(a) 0.03 (b) 0.30 (c) 3.00 (d) 30.00

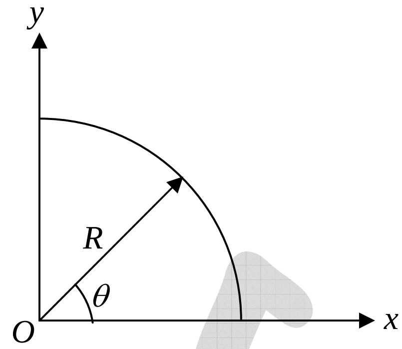
Ans.: (a)

Solution: Displacement current density, $J_d=\varepsilon_0\frac{\partial E}{\partial t}=\frac{\varepsilon_0}{d}\frac{\partial V(t)}{\partial t}=-\frac{\varepsilon_0\omega V_0\sin\omega t}{d}$

Amplitude of the displacement current density (in mA/m^2) , $J_{0d}=\frac{\varepsilon_0\omega V_0}{d}=\frac{2\pi\varepsilon_0 fV_0}{d}$

$$\Rightarrow J_{0d}=4\pi\varepsilon_0\frac{fV_0}{2d}=\frac{1}{9\times10^9}\frac{50\times10}{2\times88\times10^{-5}}=0.03\ mA/m^2$$

Q14. A segment of a circular wire of radius R, extending from $\theta = 0$ to $\pi/2$, carries a constant linear charge density λ. The electric field at origin O is



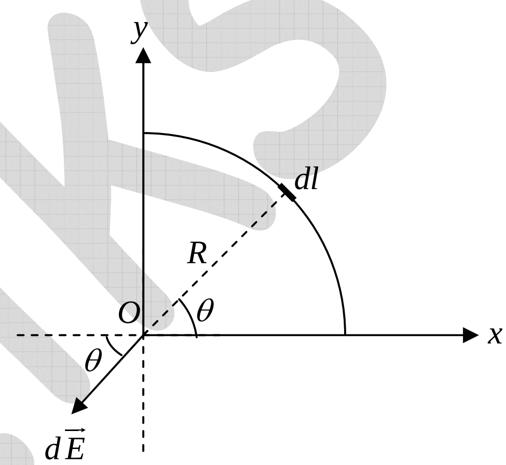
(a) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\hat{x}-\hat{y}\right)$

(b) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{\sqrt{2}}\hat{x}-\frac{1}{\sqrt{2}}\hat{y}\right)$

(c) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{2}\hat{x}-\frac{1}{2}\hat{y}\right)$

(d) 0

Ans.: (a)



Solution: $\vec{E} = -E_x\hat{x} - E_y\hat{y}$

where $E_x = \int_{line} dE\cos\theta$, $E_y = \int_{line} dE\sin\theta$.

and $dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}$.

$$E_x = \int_{line}\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{\pi/2}\cos\theta\frac{Rd\theta}{R^2}$$

$$\Rightarrow E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\left[\sin\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R}$$

Similarly $E_y = \int_{line}\frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{R^2}\sin\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{\pi/2}\sin\theta\frac{Rd\theta}{R^2}$

$$\Rightarrow E_y = \frac{\lambda}{4\pi\varepsilon_0 R}\left[-\cos\theta\right]_0^{\pi/2} = \frac{\lambda}{4\pi\varepsilon_0 R}$$

Thus $\vec{E} = -E_x\hat{x} - E_y\hat{y} = \frac{\lambda}{4\pi\varepsilon_0 R}\left(-\hat{x}-\hat{y}\right)$

IIT-JAM-2014

Q15. A particle of mass m carrying charge q is moving in a circle in a magnetic field B. According to Bohr's model, the energy of the particle in the n^{th} level is

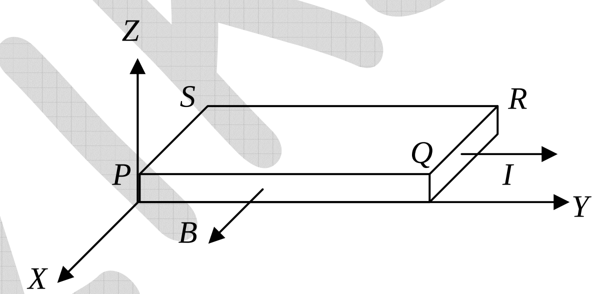
(a) $\frac{1}{n^2}\left(\frac{hqB}{\pi m}\right)$ (b) $n\left(\frac{hqB}{\pi m}\right)$ (c) $n\left(\frac{hqB}{2\pi m}\right)$ (d) $n\left(\frac{hqB}{4\pi m}\right)$

Ans.: (d)

Solution: $E_n = \frac{q^2B^2r_n^2}{2m}$ $\quad \because\ mv_nr_n = n\hbar \ \ and \ \ r_n = \frac{mv_n}{qB} \Rightarrow r_n = \frac{m}{qB}\frac{n\hbar}{mr_n} \Rightarrow r_n^2 = \frac{n\hbar}{qB}$

$$\Rightarrow E_n = \frac{q^2B^2r_n^2}{2m} = \frac{q^2B^2}{2m}\times\frac{n\hbar}{qB} = n\left(\frac{qBh}{4\pi m}\right)$$

Q16. A conducting slab of copper $PQRS$ is kept on the x-y plane in a uniform magnetic field along x- axis as indicted in the figure. A steady current I flows through the cross section of the slab along the y- axis. The direction of the electric field inside the slab, arising due to the applied magnetic field is along the

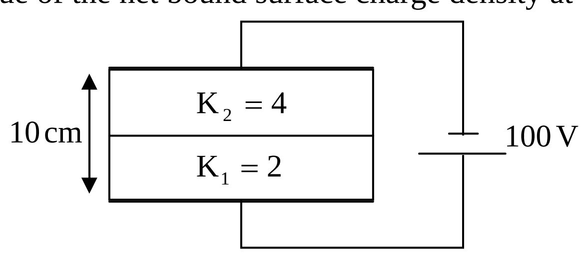


(a) negative Y direction (b) positive Y direction

(c) negative Z direction (d) positive Z direction

Ans.: (c)

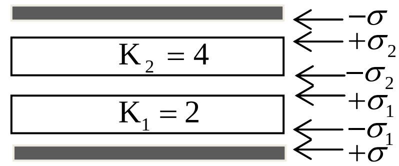
Q17. In a parallel plate capacitor the distance between the plates is $10\,cm$. Two dielectric slabs of thickness $5\,cm$ each and dielectric constants $K_1 = 2$ and $K_2 = 4$ respectively, are inserted between the plates. A potential of $100\,V$ is applied across the capacitor as shown in the figure. The value of the net bound surface charge density at the interface of the two dielectrics is



(a) $-\frac{2000}{3}\varepsilon_0$ (b) $-\frac{1000}{3}\varepsilon_0$ (c) $-250\varepsilon_0$ (d) $\frac{2000}{3}\varepsilon_0$

Ans.: (a)

Solution: $V = E_1 d + E_2 d = \frac{\sigma}{\varepsilon_1} d + \frac{\sigma}{\varepsilon_2} d = \frac{\sigma}{2\varepsilon_0} d + \frac{\sigma}{4\varepsilon_0} d = \frac{3\sigma}{4\varepsilon_0} d$



$V = 100\ volts,\ d = 5\times 10^{-2}\, cm$

$\Rightarrow \sigma = \frac{4\varepsilon_0}{3d} V = \frac{4\varepsilon_0}{3\times 5\times 10^{-2}} \times 100 = \frac{4\times 10^4}{15}\varepsilon_0$

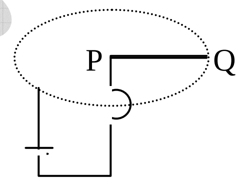
$\vec{P_1} = \varepsilon_0 \chi_e \vec{E_1} = \varepsilon_0 (K_1 - 1)\vec{E_1} \Rightarrow \sigma_1 = \varepsilon_0 \times \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{2}$

$\vec{P_2} = \varepsilon_0 \chi_e \vec{E_2} = \varepsilon_0 (K_2 - 1)\vec{E_2} \Rightarrow \sigma_2 = 3\varepsilon_0 \times \frac{\sigma}{4\varepsilon_0} = \frac{3\sigma}{4}$

$\Rightarrow \sigma = \sigma_1 - \sigma_2 = \frac{\sigma}{2} - \frac{3\sigma}{4} = -\frac{\sigma}{4} = -\frac{1}{4} \times \frac{4\times 10^4}{15}\varepsilon_0 = -\frac{2000}{3}\varepsilon_0$

Q18. A rigid uniform horizontal wire PQ of mass M, pivoted at P, carries a constant current I.

It rotates with a constant angular speed in a uniform vertical magnetic field B. If the current were switched off, the angular acceleration of the wire, in terms of B, M and I would be



(a) 0 (b) $\frac{2BI}{3M}$ (c) $\frac{3BI}{2M}$ (d) $\frac{BI}{M}$

Ans.: (c)

Solution: Torque $\vec{\tau} = I_m \alpha = \vec{r} \times \vec{F}$

$I_m = \frac{ML^2}{3}$ (Moment of inertia about point P)

$dF = IBdl$ ($\vec{F} = I\int d\vec{l} \times \vec{B}$)

$d\tau = \vec{r} \times d\vec{F} = l \times IBdl$

$\tau = IB\int_0^L l dl = \frac{IBL^2}{2} \Rightarrow \alpha = \frac{3}{2}\frac{BI}{M}$

Q19. A steady current in a straight conducting wire produces a surface charge on it. Let E_{out} and E_{in} be the magnitudes of the electric fields just outside and just inside the wire, respectively. Which of the following statements is true for these fields?

(a) E_{out} is always greater than E_{in}

(b) E_{out} is always smaller than E_{in}

(c) E_{out} could be greater or smaller than E_{in}

(d) E_{out} is equal to E_{in}

Ans.: (a)

Solution: In this case $E_{in} = 0, E_{out} \neq 0$. So $E_{out} > E_{in}$

Q20. A small charged spherical shell of radius $0.01m$ is at a potential of $30V$. The electrostatic energy of the shell is

(a) 10^{-10} J (b) 5×10^{-10} J (c) 5×10^{-9} J (d) 10^{-9} J

Ans.: (b)

Solution: $V = \frac{q}{4\pi\varepsilon_0 R}$ and $W = \frac{q^2}{8\pi\varepsilon_0 R}$.

Thus, $W = \frac{(4\pi\varepsilon_0 VR)^2}{8\pi\varepsilon_0 R} = \frac{4\pi\varepsilon_0 V^2 R}{2} = \frac{900\times10^{-2}}{9\times10^9\times2} = 0.5\times10^{-9} = 5\times10^{-10}$ Joules

Q21. A ring of radius R carries a linear charge density λ. It is rotating with angular speed ω. The magnetic field at its center is

(a) $\frac{3\mu_0\lambda\omega}{2}$ (b) $\frac{\mu_0\lambda\omega}{2}$ (c) $\frac{\mu_0\lambda\omega}{\pi}$ (d) $\mu_0\lambda\omega$

Ans.: (b)

Solution: $B = \frac{\mu_0 I}{2R}$, where $I = \lambda v = \lambda R\omega$. Thus $B = \frac{\mu_0\lambda\omega}{2}$.

IIT-JAM-2015

Q22. The electric field of a light wave is given by $\vec{E} = E_0\left[\hat{i}\sin(\omega t - kz) + \hat{j}\sin\left(\omega t - kz - \frac{\pi}{4}\right)\right]$.

The polarization state of the wave is

(a) Left handed circular (b) Right handed circular

(c) Left handed elliptical (d) Right handed elliptical

Ans.: (c)

Solution: $E_x = E_0 \sin(\omega t - kz)$, $E_y = E_0 \sin\left(\omega t - kz - \frac{\pi}{4}\right)$.

Thus resultant is elliptically polarized wave.

At $z = 0$, $E_x = E_0 \sin(\omega t)$, $E_y = E_0 \sin\left(\omega t - \frac{\pi}{4}\right)$

When $\omega t = 0$, $E_x = 0$, $E_y = -\frac{E_0}{\sqrt{2}}$ and when $\omega t = \frac{\pi}{4}$, $E_x = \frac{E_0}{\sqrt{2}}$, $E_y = 0$

Q23. A charge q is at the center of two concentric spheres. The outward electric flux through the inner sphere is ϕ, while that through the outer sphere is 2ϕ. The amount of charge contained in the region between the two spheres is

(a) $2q$ (b) q (c) $-q$ (d) $-2q$

Ans.: (b)

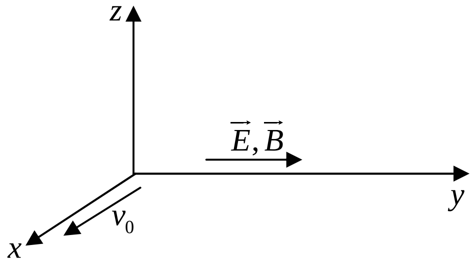
Solution: $\phi = \frac{q}{\varepsilon_0}$, $\phi' = 2\phi = \frac{q+q'}{\varepsilon_0} \Rightarrow q' = q$

Q24. A positively charged particle, with a charge q, enters a region in which there is a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$, both directed parallel to the positive y-axis. At $t = 0$, the particle is at the origin and has a speed v_0 directed along the positive x- axis. The orbit of the particle, projected on the x-z plane, is a circle. Let T be the time taken to complete one revolution of this circle. The y-coordinate of the particle at $t = T$ is given by

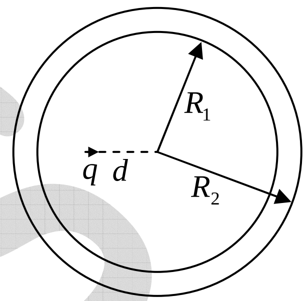
(a) $\frac{\pi^2 mE}{2qB^2}$ (b) $\frac{2\pi^2 mE}{qB^2}$ (c) $\frac{\pi^2 mE}{qB^2} + \frac{v_0 \pi m}{qB}$ (d) $\frac{2\pi m v_0}{qB}$

Ans.: (b)

Solution: $y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = \frac{1}{2}\frac{qE}{m}\left(\frac{2\pi m}{qB}\right)^2 = \frac{2\pi^2 mE}{qB^2}$



Q25. A hollow, conducting spherical shell of inner radius R_1 and outer radius R_2 encloses a charge q inside, which is located at a distance $d(< R_1)$ from the centre of the spheres. The potential at the centre of the shell is



(a) Zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{d}$

(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d} - \frac{q}{R_1}\right)$

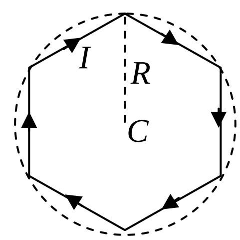
(d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d} - \frac{q}{R_1} + \frac{q}{R_2}\right)$

Ans.: (d)

Solution: Charge induced on inner surface is $-q$ and charge induced on outer surface is $+q$.

Thus, $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d} - \frac{q}{R_1} + \frac{q}{R_2}\right)$.

Q26. A conducting wire is in the shape of a regular hexagon, which is inscribed inside an imaginary circle of radius R, as shown. A current I flows through the wire. The magnitude of the magnetic field at the center of the circle is



(a) $\frac{\sqrt{3}\mu_0 I}{2\pi R}$

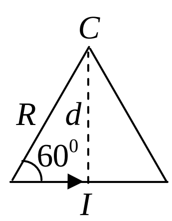
(b) $\frac{\mu_0 I}{2\sqrt{3}\pi R}$

(c) $\frac{\sqrt{3}\mu_0 I}{\pi R}$

(d) $\frac{3\mu_0 I}{2\pi R}$

Ans.: (c)

Solution: $d = R\cos 30^0 = \frac{\sqrt{3}}{2}R$



$\because B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)$

$\Rightarrow B_1 = \frac{\mu_0 I}{4\pi d} 2\sin 30^0 = \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{2}R} 2\sin 30^0 = \frac{\mu_0 I}{2\sqrt{3}\pi R}$

The magnitude of the magnetic field at center of the circle is

$$\Rightarrow B = 6B_1 = 6\times\frac{\mu_0 I}{2\sqrt{3}\pi R} = \frac{3\mu_0 I}{\sqrt{3}\pi R} = \frac{\sqrt{3}\mu_0 I}{\pi R}$$

MSQ

Q27. For an electromagnetic wave traveling in free space, the electric field is given by $\vec{E} = 100\cos\left(10^8 t + kx\right)\hat{j}\frac{V}{m}$. Which of the following statements are true?

(a) The wavelength of the wave in meter is 6π

(b) The corresponding magnetic field is directed along the positive z direction

(c) The Poynting vector is directed along the positive z direction

(d) The wave is linearly polarized

Ans.: (a) and (d)

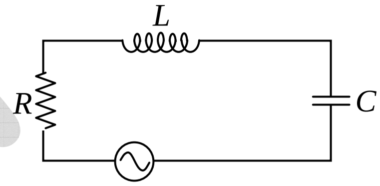
Solution: $\vec{E} = 100\cos\left(10^8 t + kx\right)\hat{j}\ V/m$

$\omega = 10^8 \Rightarrow \frac{2\pi c}{\lambda} = 10^8 \Rightarrow \lambda = \frac{2\pi\times 3\times 10^8}{10^8} = 6\pi$. Option (a) is true

$\vec{B} \propto \left(\hat{k}\times\vec{E}\right) \propto \left(-\hat{x}\times\hat{y}\right) \propto -\hat{z}$. Option (b) is wrong

$\vec{S} \propto \hat{k} \propto -\hat{x}$. Option (c) is wrong. Option (d) is true.

Q28. Consider the circuit, consisting of an AC function generator $V(t) = V_0 \sin 2\pi\nu t$ with $V_0 = 5V$ an inductor $L = 8.0mH$, resistor $R = 5\Omega$ and a capacitor $C = 100\mu F$. Which of the following statements are true if we vary the frequency?



(a) The current in the circuit would be maximum at $\nu = 178Hz$

(b) The capacitive reactance increases with frequency

(c) At resonance, the impedance of the circuit is equal to the resistance in the circuit

(d) At resonance, the current in the circuit is out of phase with the source voltage

Ans.: (a) and (c)

Solution: $\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{\left(8\times10^{-3}\right)\left(100\times10^{-6}\right)}} = 178\,Hz$. Option (a) is true.

$X_C = \frac{1}{\omega C} \Rightarrow X_C \downarrow \text{ as } \omega \uparrow$. Option (b) is wrong

Option (c) is true

Option (d) is wrong

Q29. A unit cube made of a dielectric material has a polarization $\vec{P} = 3\hat{i} + 4\hat{j}$ units. The edges of the cube are parallel to the Cartesian axes. Which of the following statements are true?

(a) The cube carries a volume bound charge of magnitude 5 units

(b) There is a charge of magnitude 3 units on both the surfaces parallel to the $y - z$ plane

(c) There is a charge of magnitude 4 units on both the surfaces parallel to the $x - z$ plane

(d) There is a net non-zero induced charge on the cube

Ans.: (b) and (c)

Solution: $\because \vec{P} = 3\hat{i} + 4\hat{j} \Rightarrow \rho_b = -\vec{\nabla}.\vec{P} = 0$. Option (a) is wrong

At $x = 0$, $\sigma_b = \vec{P}.\hat{n} = \left(3\hat{i} + 4\hat{j}\right).\left(-\hat{i}\right) = -3$, At $x = 1$, $\sigma_b = \vec{P}.\hat{n} = \left(3\hat{i} + 4\hat{j}\right).\left(\hat{i}\right) = 3$

Option (b) is true

At $y = 0$, $\sigma_b = \vec{P}.\hat{n} = \left(3\hat{i} + 4\hat{j}\right).\left(-\hat{j}\right) = -4$, At $y = 1$, $\sigma_b = \vec{P}.\hat{n} = \left(3\hat{i} + 4\hat{j}\right).\left(\hat{j}\right) = 4$

Option (c) is true.

Option (d) is wrong

Q30. The power radiated by sun is $3.8\times10^{26}W$ and its radius is $7\times10^5\,km$. The magnitude of the Poynting vector (in $\frac{W}{cm^2}$) at the surface of the sun is……………..

Ans.: 6174

Solution: $I = \frac{P}{A} = \frac{3.8\times10^{26}}{4\pi\times\left(7\times10^{10}\right)^2} W/cm^2 = 6174\ W/cm^2$

Q31. In an experiment on charging of an initially uncharged capacitor, an RC circuit is made with the resistance $R = 10k\Omega$ and the capacitor $C = 1000\mu F$ along with a voltage source of $6V$. The magnitude of the displacement current through the capacitor (in μA), 5 seconds after the charging has started, is…………………

Ans.: 364

Solution: $I = \frac{V}{R}e^{-t/RC} = \frac{6}{10\times10^3}e^{-5/10\times10^3\times1000\times10^{-6}} = \frac{6}{10^4}e^{-5/10} = \frac{6}{\sqrt{e}\times10^4} = \frac{6}{1.65\times10^4} = 364\,\mu A$

Q32. In a region of space, a time dependent magnetic field $B(t) = 0.4t$ tesla points vertically upwards. Consider a horizontal, circular loop of radius $2\,cm$ in this region. The magnitude of the electric field (in mV/m) induced in the loop is……………

Ans.: 4

Solution: $\left|\vec{E}\right|\times 2\pi r = -\frac{\partial B}{\partial t}\times \pi r^2 \Rightarrow \left|\vec{E}\right| = \frac{r}{2}\frac{\partial B}{\partial t} = \frac{2\times10^{-2}}{2}0.4 = 4\;\; mV/m$

Q33. A plane electromagnetic wave of frequency $5\times10^{14}\,Hz$ and amplitude $10^3\,V/m$ traveling in a homogeneous dielectric medium of dielectric constant 1.69 is incident normally at the interface with a second dielectric medium of dielectric constant 2.25. The ratio of the amplitude of the transmitted wave to that of the incident wave is………………

Ans.: 0.93

Solution: $E_{0T} = \left(\frac{2n_1}{n_1+n_2}\right)E_{0I} \Rightarrow \frac{E_{0T}}{E_{0I}} = \left(\frac{2\sqrt{\varepsilon_{r_1}}}{\sqrt{\varepsilon_{r_1}}+\sqrt{\varepsilon_{r_2}}}\right) = \left(\frac{2\sqrt{1.69}}{\sqrt{1.69}+\sqrt{2.25}}\right) = 0.93$

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Q34. For an infinitely long wire with uniform line-charge density, λ along the z - axis, the electric field at a point $(a,b,0)$ away from the origin is

($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian – coordinate system)

(a) $\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+b^2}}\left(\hat{e}_x+\hat{e}_y\right)$ (b) $\frac{\lambda}{2\pi\varepsilon_0\left(a^2+b^2\right)}\left(a\hat{e}_x+b\hat{e}_y\right)$

(c) $\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+b^2}}\hat{e}_x$ (d) $\frac{\lambda}{2\pi\varepsilon_0\sqrt{a^2+b^2}}\hat{e}_z$

Ans.: (b)

Solution: $\vec{E}=\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}=\frac{\lambda}{2\pi\varepsilon_0 r^2}\vec{r}=\frac{\lambda}{2\pi\varepsilon_0\left(a^2+b^2\right)}\left(a\hat{e}_x+b\hat{e}_y\right)$ $\because r=\sqrt{a^2+b^2}$

Q35. A 1 W point source at origin emits light uniformly in all the directions. If the units for both the axes are measured in centimeter, then the Poynting vector at the point $(1,1,0)$ in $\frac{W}{cm^2}$ is

(a) $\frac{1}{8\pi\sqrt{2}}\left(\hat{e}_x+\hat{e}_y\right)$ (b) $\frac{1}{16\pi}\left(\hat{e}_x+\hat{e}_y\right)$

(c) $\frac{1}{16\pi\sqrt{2}}\left(\hat{e}_x+\hat{e}_y\right)$ (d) $\frac{1}{4\pi\sqrt{2}}\left(\hat{e}_x+\hat{e}_y\right)$

Ans.: (a)

Solution: $I=<\vec{S}>=\frac{P}{A}\hat{r}=\frac{P}{4\pi r^2}\frac{\vec{r}}{r}=\frac{P}{4\pi r^3}\vec{r}=\frac{1}{4\pi\times 2\sqrt{2}}\left(\hat{e}_x+\hat{e}_y\right)=\frac{1}{8\pi\sqrt{2}}\left(\hat{e}_x+\hat{e}_y\right)$

$\because r=\sqrt{1^2+1^2}=\sqrt{2}$

Q36. A charged particle in a uniform magnetic field $\vec{B} = B_0\hat{e}_z$ starts moving from the origin with velocity $\vec{v} = (3\hat{e}_x + 2\hat{e}_z)\, m/s$. The trajectory of the particle and the time t at which it reaches 2 meters above the xy - plane are

($\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian-coordinate system)

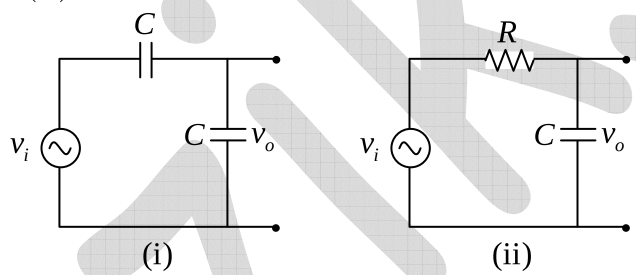
(a) Helical path; $t = 1s$ (b) Helical path; $t = 2/3s$

(c) Circular path; $t = 1s$ (d) Circular path; $t = 2/3s$

Ans.: (a)

Solution: $v_\perp = 3\ m/s$ and $v_\parallel = 2\ m/s$, thus $t = \frac{2m}{v_\parallel} = 1$ sec

Q37. The phase difference (δ) between input and output voltage for the following circuits (i) and (ii)



will be

(a) 0 and 0 (b) $\pi/2$ and $0 < \delta \le \pi/2$ respectively

(c) $\pi/2$ and $\pi/2$ (d) 0 and $0 < \delta \le \pi/2$ respectively

Ans.: (d)

Solution: (i) $v_o = \frac{X_C}{X_C + X_C} v_i \Rightarrow \frac{v_o}{v_i} = \frac{1}{2}$, phase difference (δ) is 0.

(ii) $v_o = \frac{X_C}{R + X_C} v_i \Rightarrow \frac{v_o}{v_i} = \frac{1}{1 + R/X_C} = \frac{1}{1 + i\omega CR} = \frac{1}{\sqrt{1 + (\omega CR)^2}} e^{-i\omega CR}$

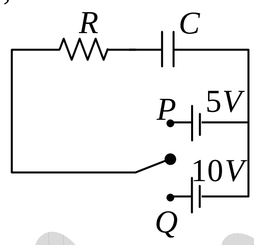
Phase difference (δ) is $0 < \delta \le \pi/2$.

Q38. In the following RC circuit, the capacitor was charged in two different ways.

(i) The capacitor was first charged to $5V$ by moving the toggle switch to position P and then it was charged to $10V$ by moving the toggle switch to position Q.

(ii) The capacitor was directly charged to $10V$, by keeping the toggle switch at position Q.

Assuming the capacitor to be ideal, which one of the following statements is correct?



(a) The energy dissipation in cases (i) and (ii) will be equal and non-zero

(b) The energy dissipation for case (i) will be more than that for case (ii)

(c) The energy dissipation for case (i) will be less than that for case (ii)

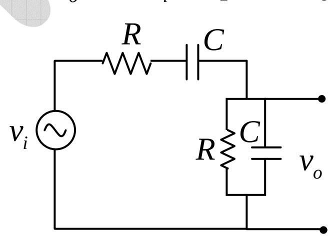
(d) The energy will not be dissipated in either case.

Ans.: (c)

Solution: The energy dissipation in cases (i) is $=\frac{1}{2}C(5)^2+\frac{1}{2}C(10-5)^2=25C$

The energy dissipation in cases (ii) is $=\frac{1}{2}C(10)^2=50C$

Q39. In the following RC network, for an input signal frequency $f=\frac{1}{2\pi RC}$, the voltage gain $\left|\frac{v_o}{v_i}\right|$ and the phase angle ϕ between v_o and v_i respectively are



(a) $\frac{1}{2}$ and 0 (b) $\frac{1}{3}$ and 0 (c) $\frac{1}{2}$ and $\frac{\pi}{2}$ (d) $\frac{1}{3}$ and $\frac{\pi}{2}$

Ans.: (b)

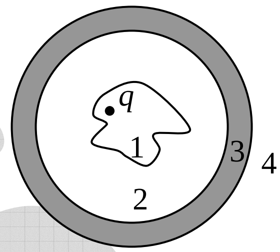
Solution: $\because f=\frac{1}{2\pi RC}$, then $X_C=\frac{1}{j2\pi fC}=-jR$

$$Z_P=\frac{RX_C}{R+X_C}=\frac{-jR^2}{R-jR}=\frac{-jR}{1-j}=\frac{-j(1+j)R}{2} \text{ and } Z_S=R+X_C=R-jR=R(1-j)$$

$$v_o = \frac{Z_P}{Z_P + Z_S} v_i \Rightarrow \frac{v_o}{v_i} = \frac{1}{1 + \frac{Z_S}{Z_P}} = \frac{1}{1 + \frac{R(1-j)}{\frac{-j(1+j)R}{2}}} = \frac{1}{1 - \frac{2R(1-j)}{j(1+j)R}} = \frac{j(1+j)R}{jR - R - 2R(1-j)}$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{j(1+j)R}{jR - R - 2R(1-j)} = \frac{j(1+j)R}{3jR - 3R} = \frac{(j-1)}{3(j-1)} = \frac{1}{3}, \text{ and phase angle } \phi = 0$$

Q40. An arbitrarily shaped conductor encloses a charge q and is surrounded by a conducting hollow sphere as shown in the figure. Four different regions of space $1, 2, 3$ and 4 are indicated in the figure. Which one of the following statements is correct?



(a) The electric field lines in region 2 are not affected by the position of the charge q

(b) The surface charge density on the inner wall of the hollow sphere is uniform

(c) The surface charge density on the outer surface of the sphere is always uniform irrespective of the position of charge q in region 1

(d) The electric field in region 2 has a radial symmetry

Ans.: (c)

Solution: From the given statement only option (c) is correct.

Q41. Consider a small bar magnet undergoing simple harmonic motion (SHM) along the x - axis. A coil whose plane is perpendicular to the x - axis is placed such that the magnet passes in and out of it during its motion. Which one of the following statements is correct? Neglect damping effects.

(a) Induced e.m.f. is minimum when the center of the bar magnet crosses the coil

(b) The frequency of the induced current in the coil is half of the frequency of the SHM

(c) Induced e.m.f. in the coil will not change with the velocity of the magnet

(d) The sign of the e.m.f. depends on the pole (N or S) face of the magnet which enters into the coil

Ans.: (a)

Solution: From the given statement only option (a) is correct.

Q42. Consider a spherical dielectric material of radius 'a' centered at origin. If the polarization vector, $\vec{P} = P_0\hat{e}_x$, where P_0 is a constant of appropriate dimensions, then ($\hat{e}_x, \hat{e}_y$, and $\hat{e}_z$ are unit vectors in Cartesian- coordinate system)

(a) the bound volume charge density is zero.

(b) the bound surface charge density is zero at $(0,0,a)$.

(c) the electric field is zero inside the dielectric

(d) the sign of the surface charge density changes over the surface.

Ans.: (a), (b), (d)

Solution: $\rho_b = -\vec{\nabla}.\vec{P} = 0$

$$\sigma_b = \vec{P}.\hat{n} = \left(P_0\hat{e}_x\right).\hat{r} = P_0 \sin\theta\cos\phi = 0 \text{ at } \left(0,0,a\right) \because \theta = 0.$$

Q43. For an electric dipole with momentum $\vec{P} = p_0\hat{e}_z$ placed at the origin, (p_0 is a constant of appropriate dimensions and $\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit vectors in Cartesian coordinate system)

(a) potential falls as $\frac{1}{r^2}$, where r is the distance from origin

(b) a spherical surface centered at origin is an equipotential surface

(c) electric flux through a spherical surface enclosing the origin is zero

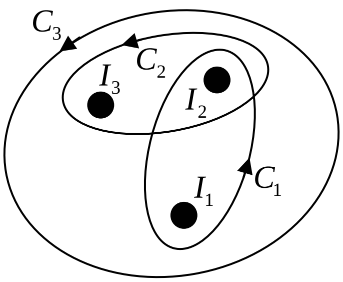
(d) radial component of $\vec{E}$ is zero on the xy- plane.

Ans.: (a), (c), (d)

Solution: $V_{dip}\left(r,\theta\right) = \dfrac{\hat{r}.\vec{p}}{4\pi\varepsilon_o r^2} = \dfrac{p\cos\theta}{4\pi\varepsilon_o r^2}$.

$$\vec{E}_{dip}\left(r,\theta\right) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\hat{\theta}\right).$$

Q44. Three infinitely-long conductors carrying currents I_1, I_2 and I_3 lie perpendicular to the plane of the paper as shown in the figure. If the value of the integral $\oint_C \vec{B}.\vec{dl}$ for the loops C_1, C_2 and C_3 are $2\mu_0, 4\mu_0$ and μ_0 in the units of $\frac{N}{A}$ respectively, then



(a) $I_1 = 3A$ into the paper (b) $I_2 = 5A$ out of the paper

(c) $I_3 = 0$. (d) $I_3 = 1A$ out of the paper

Ans.: (a), (b)

Solution: $\because \oint_C \vec{B}.\vec{dl} = \mu_0 I_{enc}$

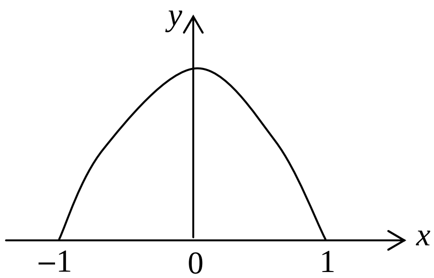
$\Rightarrow I_1 + I_2 = 2,\ I_2 + I_3 = 4, I_1 + I_2 + I_3 = 1$

$\Rightarrow I_1 = -3A,\ I_2 = 5A$ and $I_3 = -1A$.

Q45. The shape of a dielectric lamina is defined by the two curves $y = 0$ and $y = 1 - x^2$. If the charge density of the lamina $\sigma = 15y\, C/m^2$, then the total charge on the lamina is................C.

Ans.: 8

Solution: Total charge on the lamina is



$$Q = \int_S \sigma da = \int_{-1}^{1}\int_0^{1-x^2} 15y dx dy = \frac{15}{2}\int_{-1}^{1}\left(1-x^2\right)^2 dx$$

$$\Rightarrow Q = \frac{15}{2}\int_{-1}^{1}\left(1+x^4-2x^2\right)dx = \frac{15}{2}\left[x+\frac{x^5}{5}-2\frac{x^3}{3}\right]_{-1}^{1}$$

$$\Rightarrow Q = \frac{15}{2}\left[1+\frac{1}{5}-\frac{2}{3}-\left(-1-\frac{1}{5}+\frac{2}{3}\right)\right] = \frac{15}{2}\left[2+\frac{2}{5}-\frac{4}{3}\right]$$

$$\Rightarrow Q = \frac{15}{2}\times\frac{16}{15} = 8\ C$$

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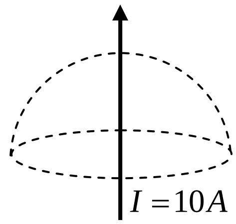
Q46. A current $I = 10A$ flows in an infinitely long wire along the axis of hemisphere (see figure). The value of $\int \left(\vec{\nabla}\times\vec{B}\right)\cdot\overrightarrow{ds}$ over the hemispherical surface as shown in the figure is:

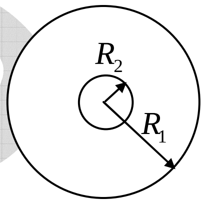


(a) $10\,\mu_0$ (b) $5\,\mu_0$ (c) 0 (d) $7.5\,\mu_0$

Ans. : (a)

Solution: $\int \left(\vec{\nabla}\times\vec{B}\right)\cdot\overrightarrow{ds} = \oint \vec{B}.d\vec{l} = |B|\times 2\pi r = \dfrac{\mu_0 I}{2\pi r}\times 2\pi r = \mu_0 I = 10\mu_0$

Q47. Consider two, single turn, co-planar, concentric coils of radii R_1 and R_2 with $R_1 \gg R_2$. The mutual inductance between the two coils is proportional to

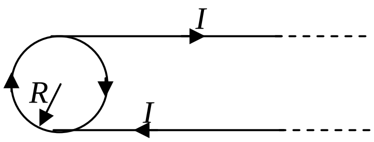


(a) $\dfrac{R_1}{R_2}$ (b) $\dfrac{R_2}{R_1}$ (c) $\dfrac{R_2^2}{R_1}$ (d) $\dfrac{R_1^2}{R_2}$

Ans. : (c)

Solution: $\phi_2 = M_{21}I_1 \Rightarrow M_{21} = \dfrac{\phi_2}{I_1} = \dfrac{B_1\times\pi R_2^2}{I_1} = \dfrac{\dfrac{\mu_0 I_1}{2\pi R_1}\times\pi R_2^2}{I_1} \propto \dfrac{R_2^2}{R_1}$

Q48. Consider a thin long insulator coated conducting wire carrying current I . It is now wound once around an insulating thin disc of radius R to bring the wire back on the same side, as shown in the figure. The magnetic field at the centre of the disc is equal to:



(a) $\dfrac{\mu_0 I}{2R}$ (b) $\dfrac{\mu_0 I}{4R}\left[3+\dfrac{2}{\pi}\right]$ (c) $\dfrac{\mu_0 I}{4R}\left[1+\dfrac{2}{\pi}\right]$ (d) $\dfrac{\mu_0 I}{2R}\left[1+\dfrac{1}{\pi}\right]$

Ans. : (d)

Solution: From R.H.R. magnetic field is pointing inwards, $B = 2\times\dfrac{\mu_0 I}{4\pi R}+\dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 I}{2R}\left[1+\dfrac{1}{\pi}\right]$

Q49. The electric field of an electromagnetic wave is given by

$$\vec{E} = \left(2\hat{k} - 3\hat{j}\right) \times 10^{-3} \sin\left[10^7 \left(x + 2y + 3z - \beta t\right)\right].$$

The value of β is (c is the speed of light):

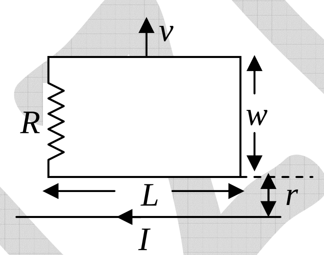
(a) $\sqrt{14}\, c$ (b) $\sqrt{12}\, c$ (c) $\sqrt{10}\, c$ (d) $\sqrt{7}\, c$

Ans. : (a)

Solution: $\vec{E} = \left(2\hat{k} - 3\hat{j}\right) \times 10^{-3} \sin\left[10^7 \left(x + 2y + 3z - \beta t\right)\right]$

$$\vec{k} = 10^7 \left(\hat{x} + 2\hat{y} + 3\hat{z}\right) \Rightarrow \left|\vec{k}\right| = 10^7 \sqrt{14},\ \omega = 10^7 \beta,\ c = \frac{\omega}{\left|\vec{k}\right|} = \frac{10^7 \beta}{10^7 \sqrt{14}} \Rightarrow \beta = \sqrt{14} c$$

Q50. A rectangular loop of dimension L and width w moves with a constant velocity v away from an infinitely long straight wire carrying a current I in the plane of the loop as shown in the figure below. Let R be the resistance of the loop. What is the current in the loop at the instant the near –side is at a distance r from the wire?



(a) $\dfrac{\mu_0 IL}{2\pi R} \dfrac{wv}{r[r+2w]}$ (b) $\dfrac{\mu_0 IL}{2\pi R} \dfrac{wv}{[2r+w]}$

(c) $\dfrac{\mu_0 IL}{2\pi R} \dfrac{wv}{r[r+w]}$ (d) $\dfrac{\mu_0 IL}{2\pi R} \dfrac{wv}{2r[r+w]}$

Ans. : (c)

Solution: $\phi_B = \int_S \vec{B}.d\vec{a} = \int_r^{r+\omega} \dfrac{\mu_0 I}{2\pi r} L dr = \dfrac{\mu_0 IL}{2\pi} \ln\left(\dfrac{r+w}{r}\right)$

$$\Rightarrow I = -\frac{1}{R}\frac{d\phi_B}{dt} = \frac{-\mu_0 IL}{2\pi R}\left[\frac{1}{r+w} - \frac{1}{r}\right]\frac{dr}{dt} = \frac{\mu_0 ILwv}{2\pi Rr(r+w)}$$

Q51. For a point dipole of dipole moment $\vec{p} = p\hat{z}$ located at the origin, which of the following is (are) correct?

(a) The electric field at $(0,b,0)$ is zero

(b) The work done in moving a charge q from $(0,b,0)$ to $(0,0,b)$ is $\frac{qp}{4\pi \epsilon_0 b^2}$

(c) The electrostatic potential at $(b,0,0)$ is zero

(d) If a charge q is kept at $(0,0,b)$ it will exert a force of magnitude $\frac{qp}{4\pi \epsilon_0 b^3}$ on the dipole.

Ans. : (b) and (c)

Solution: $V = \frac{p\cos\theta}{4\pi \epsilon_0 r^2}$ and $\vec{E} = \frac{p}{4\pi \epsilon_0 r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$

(a) At$(0,b,0)$; $\theta = \frac{\pi}{2} \Rightarrow \vec{E} \neq 0$

(b) The work done in moving a charge q from $(0,b,0)$ to $(0,0,b)$

$$W = q\left[V(0,0,b) - V(0,b,0)\right] = q\left[\frac{p}{4\pi \epsilon_0 b^2} - 0\right] = \frac{qp}{4\pi \epsilon_0 b^2}$$

(c) The electrostatic potential at $(b,0,0)$ is $V(b,0,0) = 0$

(d) At$(0,0,b)$; $\theta = 0 \Rightarrow \vec{E} = \frac{2p}{4\pi \epsilon_0 b^3}\hat{r}$

If a charge q is kept at $(0,0,b)$ it will exert a force of magnitude $\frac{2qp}{4\pi \epsilon_0 b^3}$.

Q52. A dielectric sphere of radius R has constant polarization $\vec{P} = P_0\hat{z}$, so that the field inside the sphere is $\vec{E}_{in} = -\frac{P_0}{3\epsilon_0}\hat{z}$. Then, which of the following is (are) correct?

(a) The bound surface charge density is $P_0\cos\theta$

(b) The electric field at a distance r on the z - axis varies as $\frac{1}{r^2}$ for $r >> R$

(c) The electric potential at a distance $2R$ on the z - axis is $\frac{P_0 R}{12\epsilon_0}$

(d) The electric field outside is equivalent to that of a dipole at the origin

Ans. : (a), (c) and (d)

Solution: $\sigma_b = \vec{P}.\hat{n} = (P_0\hat{z}).\hat{r} = P_0\cos\theta$

$$V_{dip} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\hat{r}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{4\pi R^3}{3}\frac{\vec{P}.\hat{r}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{4\pi R^3}{3}\frac{(P_0\hat{z}).\hat{z}}{(2R)^2} = \frac{P_0R}{12\epsilon_0}$$

Q53. Consider a circular parallel plate capacitor of radius R with separation d between the plates $(d << R)$. The plates are placed symmetrically about the origin. If a sinusoidal voltage $V = V_0\sin\omega t$ is applied between the plates, which of the following statement(s) is (are) true?

(a) The maximum value of the Poynting vector at $r = R$ is $\frac{V_0^2\varepsilon_0\omega R}{4d^2}$

(b) The average energy per cycle flowing out of the capacitor is zero

(c) The magnetic field inside the capacitor is constant

(d) The magnetic field lines inside the capacitor are circular with the curl being independent of r.

Ans. : (a), (b) and (d)

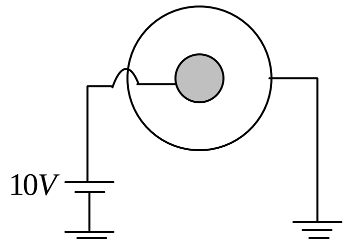
Solution: $E = \frac{V}{d} = \frac{V_0\sin\omega t}{d}$ and $B = \frac{\mu_0 I_d}{2\pi R} = \frac{\mu_0}{2\pi R}\varepsilon_0\frac{\partial E}{\partial t}\times\pi R^2 = \frac{\mu_0\varepsilon_0 R}{2}\frac{\omega V_0\cos\omega t}{d}$

$$S = \frac{1}{\mu_0}EB = \frac{\varepsilon_0 R}{2}\frac{\omega V_0\cos\omega t}{d}\times\frac{V_0\sin\omega t}{d} = \frac{\varepsilon_0\omega RV_0^2\sin\omega t\cos\omega t}{2d^2} = \frac{\varepsilon_0\omega RV_0^2}{4d^2}\sin 2\omega t$$

$$S_{\max} = \frac{\varepsilon_0\omega RV_0^2}{4d^2};\quad \langle S\rangle = 0,\quad B = \frac{\mu_0 I_d r}{2\pi R^2},\ \text{inside}$$

Q54. In a coaxial cable, the radius of the inner conductor is $2\,mm$ and that of the outer one is $5\,mm$. The inner conductor is at a potential of $10\,V$, while the outer conductor is grounded. The value of the potential at a distance of $3.5\,mm$ from the axis is…………

(Specify your answer in volts to two digits after the decimal point)



Ans. : 3.8

Solution: $\because \nabla^2 V = 0$

In Cylindrical coordinate system, $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \Rightarrow V = A\ln r + B$

Thus $10 = A\ln 2 + B$ and $0 = A\ln 5 + B$

$\Rightarrow 10 = A\ln 2 - A\ln 5 \Rightarrow A = -\frac{10}{\ln(5/2)} = -10.86$ and $\Rightarrow B = \frac{10\ln 5}{\ln(5/2)} = 17.39$

$\Rightarrow V(r = 3.5) = A\ln 3.5 + B = 3.8\ V$

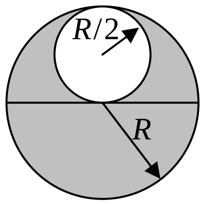
Q55. The wave number of an electromagnetic wave incident on a metal surface is $(20\pi + 750i)m^{-1}$ inside the metal, where $i = \sqrt{-1}$. The skin depth of the wave in the metal is.........(Specify your answer in mm to two digits after the decimal point).

Ans. : 1.33

Solution: $\tilde{k} = k + i\kappa = (20\pi + 750i)m^{-1}$

Skin depth, $d = \frac{1}{\kappa} = \frac{1}{750}m = \frac{1000}{750}mm = 1.33\ mm$

Q56. A sphere of radius R has a uniform charge density ρ. A sphere of smaller radius $R/2$ is cut out from the original sphere, as shown in the figure below. The center of the cut out sphere lies at $z = R/2$. After the smaller sphere has been cut out, the magnitude of the electric field at $z = -R/2$ is $\rho R / n \in_0$. The value of the integer n is..............



Ans. : 8

Solution: Electric field inside a uniformly charge solid sphere of radius R is $\vec{E} = \frac{\rho r}{3\in_0}\hat{r}$

Electric field outside a uniformly charge solid sphere of radius R is $\vec{E} = \frac{\rho R^3}{3\in_0 r^2}\hat{r}$

Electric field at $z = -\frac{R}{2}$ is $E = \frac{\rho R/2}{3\in_0} - \frac{\rho(R/2)^3}{3\in_0 R^2} = \frac{\rho R}{8\in_0} \Rightarrow n = 8$

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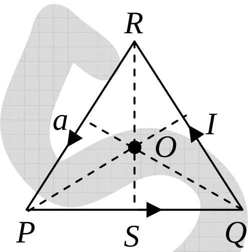
Q57. A current I is flowing through the sides of an equilateral triangle of side a. The magnitude of the magnetic field at the centroid of the triangle is

(a) $\frac{9\mu_0 I}{2\pi a}$ (b) $\frac{\mu_0 I}{\pi a}$ (c) $\frac{3\mu_0 I}{2\pi a}$ (d) $\frac{3\mu_0 I}{\pi a}$

Ans.: (a)

Solution: $RS = \sqrt{a^2 - a^2/4} = \frac{\sqrt{3}}{2}a$ and $OS = \frac{RS}{3} = \frac{\sqrt{3}}{6}a$



For segment PQ

$$B_{PQ} = \frac{\mu_0 I}{4\pi\left(\frac{\sqrt{3}}{6}a\right)} \times 2\sin 60^0 = \frac{3\mu_0 I}{2\pi a} = B_{QR} = B_{RP}$$

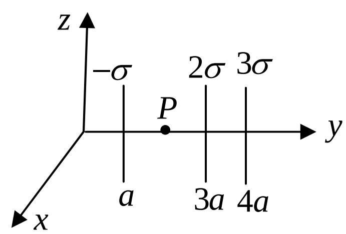
$$B = 3B_{PQ} = \frac{9\mu_0 I}{2\pi a}$$

Q58. Three infinite plane sheets carrying uniform charge densities $-\sigma, 2\sigma, 3\sigma$ are parallel to the $x-z$ plane at $y = a, 3a, 4a$, respectively. The electric field at the point $(0, 2a, 0)$ is

(a) $\frac{4\sigma}{\varepsilon_0}\hat{j}$ (b) $-\frac{3\sigma}{\varepsilon_0}\hat{j}$ (c) $-\frac{2\sigma}{\varepsilon_0}\hat{j}$ (d) $\frac{\sigma}{\varepsilon_0}\hat{j}$

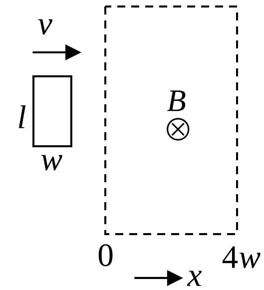
Ans.: (b)

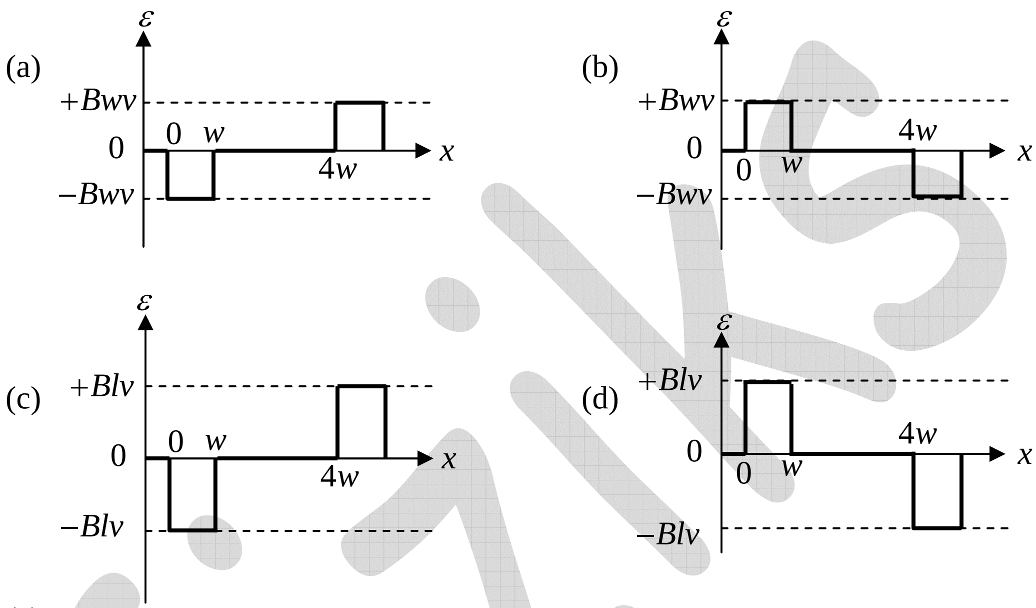
Solution: The electric field at the point $P(0, 2a, 0)$ is



$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0}\right)(-\hat{j}) = -\frac{3\sigma}{\varepsilon_0}\hat{j}$$

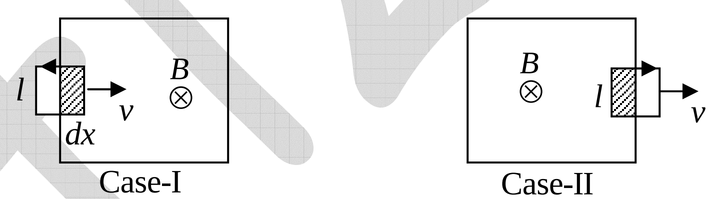
Q59. A rectangular loop of dimensions l and w moves with a constant speed of v through a region containing a uniform magnetic field B directed into the paper and extending a distance of $4w$. Which of the following figures correctly represents the variation of emf(ε) with the position (x) of the front end of the loop?





Ans.: (c)

Solution:



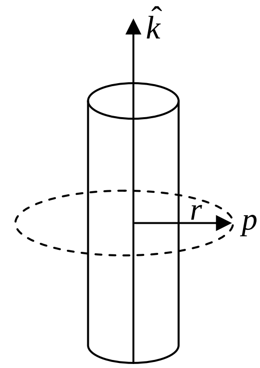
Case-I: at $x = 0, \phi_1 = Blw$ and at, $x = dx, \phi_2 = Bl(w - dx)$

$$\Rightarrow \Delta\phi = Bldx \Rightarrow \varepsilon = -\frac{d\phi}{dt} = Blv$$

Case-II: $|\varepsilon| = Blv$ and direction will be opposite.

When loop is inside there is no flux change so, $\varepsilon = 0$.

Q60. A long solenoid is carrying a time dependent current such that the magnetic field inside has the form $\vec{B}(t) = B_0 t^2 \hat{k}$, where $\hat{k}$ is along the axis of the solenoid. The displacement current at the point P on a circle of radius r in a plane perpendicular to the axis



(a) is inversely proportional to r and radially outward

(b) is inversely proportional to r and tangential

(c) increases linearly with time and is tangential.

(d) is inversely proportional to r^2 and tangential

Ans.: (b)

Solution: $\because \oint \vec{E}\cdot d\vec{l} = -\int \frac{d\vec{B}}{dt}\cdot d\vec{l}$

$$\Rightarrow E\times 2\pi r = -2B_0 t\times \pi R^2 \Rightarrow E = \frac{-B_0 t R^2}{r}$$

$$\because J_d = \varepsilon_0 \frac{\partial E}{\partial t} \Rightarrow J_d = \frac{-\varepsilon_0 B_0 R^2}{r} \Rightarrow J_d \propto \frac{1}{r}$$

Q61. Given a spherically symmetric charge density $\rho(r) = \begin{cases} kr^2, & r < R \\ 0, & r > R \end{cases}$ (k being a constant), the electric field for $r < R$ is (take the total charge as Q)

(a) $\frac{Qr^3}{4\pi\varepsilon_0 R^5}\hat{r}$ (b) $\frac{3Qr^2}{4\pi\varepsilon_0 R^4}\hat{r}$ (c) $\frac{5Qr^3}{8\pi\varepsilon_0 R^5}\hat{r}$ (d) $\frac{Q}{4\pi\varepsilon_0 R^5}\hat{r}$

Ans.: (a)

Solution: $\because \oint_S \vec{E}.d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow |\vec{E}|\times 4\pi r^2 = \frac{1}{\varepsilon_0}\left(\int_0^r kr^2\times 4\pi r^2 dr\right)$

$$\Rightarrow |\vec{E}|\times 4\pi r^2 = \frac{1}{E_0}\times 4\pi k\frac{r^5}{5} \Rightarrow |\vec{E}| = \frac{kr^3}{5\varepsilon_0}$$

$$\because Q = \int_0^R kr^2\times 4\pi r^2 dr = 4\pi k\frac{R^5}{5} \Rightarrow k = \frac{5Q}{4\pi R^5} \Rightarrow |\vec{E}| = \frac{5Q}{4\pi R^5}\times\frac{r^3}{5\varepsilon_0} = \frac{Qr^3}{4\pi\varepsilon_0 R^5}$$

Q62. An infinitely long solenoid, with its axis along $\hat{k}$, carries a current I. In addition there is a uniform line charge density λ along thee axis. If $\vec{S}$ is the energy flux, in cylindrical coordinates $\left(\hat{\rho},\hat{\phi},\hat{k}\right)$, then

(a) $\vec{S}$ is along $\hat{\rho}$

(b) $\vec{S}$ is along $\hat{k}$

(c) $\vec{S}$ has non zero components along $\hat{\rho}$ and $\hat{k}$

(d) $\vec{S}$ is along $\hat{\rho}\times\hat{k}$

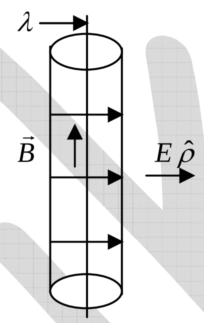
Ans. : (d)

Solution: $\vec{E} = E\hat{\rho}$

$\vec{B} = B\hat{k}$

$\vec{S} \propto \vec{E}\times\vec{B}$

$\vec{S} \propto \hat{\rho}\times\hat{k}$



Q63. Let the electric field in some region R be given by $\vec{E} = e^{-y^2}\hat{i} + e^{-x^2}\hat{j}$. From this we may conclude that

(a) R has a non-uniform charge distribution

(b) R has no charge distribution

(c) R has a time dependent magnetic field.

(d) The energy flux in R is zero everywhere.

Ans.: (b), (c)

Solution: $\because \vec{\nabla}\cdot\vec{E} = 0$ and $\vec{\nabla}\times\vec{E} \neq 0$,

Thus R has no charge distribution and R has a time dependent magnetic field.

Q64. In presence of a magnetic field $B\hat{j}$ and an electric field $(-E)\hat{k}$, a particle moves undeflected . Which of the following statements is (are) correct?

(a) The particle has positive charge, velocity $=-\frac{E}{B}\hat{i}$

(b) The particle has positive charge, velocity $=\frac{E}{B}\hat{i}$

(c) The particle has negative charge, velocity $=-\frac{E}{B}\hat{i}$

(d) The particle has negative charge, velocity $=-\frac{E}{B}\hat{i}$

Ans.: (b), (d)

Solution: $\because \vec{F}=q\left[\vec{E}+\left(\vec{v}\times\vec{B}\right)\right]=0 \quad \Rightarrow |\vec{v}|=\frac{E}{B}$

For $+ve$ charge: $\vec{a}\rightarrow -\hat{k} \Rightarrow \vec{v}=\frac{E}{B}\hat{x}$

For $-ve$ charge: $\vec{a}\rightarrow \hat{k} \Rightarrow \vec{v}=\frac{E}{B}\hat{x}$

Q65. Consider an electromagnetic plane wave $\vec{E}=E_0\left(\hat{i}+b\hat{j}\right)\cos\left[\frac{2\pi}{\lambda}\left\{ct-\left(x-\sqrt{3}y\right)\right\}\right]$, where λ is the wavelength, c is the speed of light and b is a constant. The value of b is __________. (Specific your answer upto two digits after the decimal point)

Ans. : 0.577

Solution: $\vec{E}=E_0\hat{n}\cos\left[\omega t-\hat{k}\cdot\vec{r}\right]\Rightarrow \hat{n}=\left(\hat{i}+b\hat{j}\right)$

$\hat{k}=\frac{2\pi}{\lambda}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

$\because \vec{k}\cdot\hat{n}=0\Rightarrow \frac{2\pi}{\lambda}\left(1-b\sqrt{3}\right)=0\Rightarrow b=\frac{1}{\sqrt{3}}=0.577$