

## Solid State Physics

### JEST-2012

Q1. A beam of X-rays is incident on a BCC crystal. If the difference between the incident and scattered wavevectors is  $\vec{K} = n\hat{x} + k\hat{y} + l\hat{z}$  where  $\hat{x}, \hat{y}, \hat{z}$  are the unit vectors of the associated cubic lattice, the necessary condition for the scattered beam to give a Laue maximum is

- (a)  $h + k + l = \text{even}$  (b)  $h = k = l$   
 (c)  $h, k, l$  are all distinct (d)  $h + k + l = \text{odd}$

Ans.: (a)

Solution: In BCC basis  $(0, 0, 0), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

Crystal structure factor ( $F$ ) is defined as

$$F = f \cdot S = f \sum_{n=1}^{n_{\text{eff}}} e^{2\pi i(hu_n + kv_n + l\omega_n)} = f \left[ e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}\right)[h+k+l]} \right] = f \left[ 1 + e^{\pi i(h+k+l)} \right]$$

$$F_{110} = 2f \Rightarrow I = 4f^2, F_{111} = 0 \Rightarrow I = 0, F_{200} = 2f \Rightarrow I = 4f^2$$

Thus, if  $h + k + l = \text{even}$ , then plane will be present.

If  $h + k + l = \text{odd}$ , then plane will be absent.

Q2. The second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees to the crystal plane. The distance between the lattice planes is

- (a) 1 Angstrom (b) 2 Angstrom (c) 4 Angstrom (d) 8 Angstrom

Ans.: (c)

Solution:  $2d \sin \theta = n\lambda \Rightarrow 2d \sin \theta = 2\lambda \Rightarrow 2 \times d \times \sin 30^\circ = 2 \times 0.2 \times 10^{-9} m$

$$d = 2 \times 0.2 \times 10^{-9} m = 0.4 \times 10^{-9} m = 4 \times 10^{-10} m = 4 \text{ \AA}$$

Q3. The Dulong –Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye temperature is

- (a)  $\gg 300 \text{ K}$  (b)  $\sim 300 \text{ K}$  (c)  $\ll 300 \text{ K}$  (d) 0 K

Ans.: (a)

## JEST-2013

Q4. A flat surface is covered with non-overlapping disks of same size. What is the largest fraction of the area that can be covered?

- (a)  $\frac{3}{\pi}$                       (b)  $\frac{5\pi}{6}$                       (c)  $\frac{6}{7}$                       (d)  $\frac{\pi}{2\sqrt{3}}$

Ans.: (d)

Solution: In closed packed hexagonal lattice,

$$n_{eff} = \frac{1}{3}n_c + \frac{1}{2}n_f + 1 \times n_i = \frac{1}{3} \times 6 + 1 = 3 \text{ and } a = 2r$$

$$\text{Now, largest fraction of area i.e., packing fraction} = \frac{n_{eff} \times A}{6 \times \frac{\sqrt{3}}{4} \times a^2} = \frac{3 \times \pi r^2}{6 \times \frac{\sqrt{3}}{4} \times (2r)^2} = \frac{\pi}{2\sqrt{3}}$$

Q5. A metal suffers a structural phase transition from face-centered cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbor distances  $d_{fcc}$  and  $d_{sc}$  for the FCC and the SC

structures respectively are in the ratio  $\left(\frac{d_{fcc}}{d_{sc}}\right)$  [Given  $2^{\frac{1}{3}} = 1.26$ ]

- (a) 1.029                      (b) 1.122                      (c) 1.374                      (d) 1.130

Ans. : (c)

Solution: Nearest neighbour in SC is  $a$  and  $C.N = 6$

Nearest neighbour in FCC is  $\frac{a}{\sqrt{2}}$  and  $C.N = 12$

$$\frac{d_{fcc}}{d_{sc}} = \frac{\frac{a}{\sqrt{2}}}{a} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.707$$

### JEST-2014

- Q6. Circular discs of radius 1 m each are placed on a plane so as to form a closely packed triangular lattice. The number of discs per unit area is approximately equal to
- (a)  $0.86 m^{-2}$       (b)  $0.43 m^{-2}$       (c)  $0.29 m^{-2}$       (d)  $0.14 m^{-2}$

Ans.: (c)

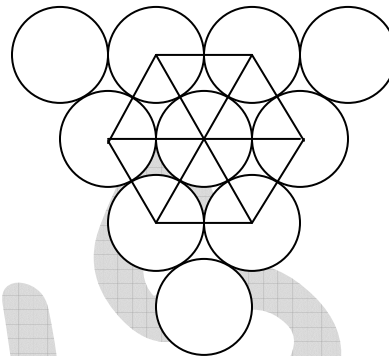
Solution: For closely packed triangular lattice,

$$a = 2r, \quad r = 1 \quad n_{eff} = \frac{1}{6} \times n_c + \frac{1}{2} \times n_f + 1 \times n_l$$

$$\Rightarrow n_{eff} = \frac{1}{6} \times 3 + 0 \times \frac{1}{2} + 1 \times 0 \Rightarrow n_{eff} = 0.5$$

$$\text{Occupancy} = \frac{n_{eff}}{A} \quad (\because a = 2)$$

$$\Rightarrow \frac{0.5}{\frac{\sqrt{3}}{4} \times 2} = \frac{0.5}{\sqrt{3}} = 0.29 m^{-2}$$



Closely packed hexagonal

- Q7. An ideal gas of non-relativistic fermions in 3-dimensions is at 0K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor
- (a)  $2^{1/2}$       (b) 1      (c)  $2^{1/3}$       (d)  $2^{-1/3}$

Ans.: (d)

$$\text{Solution: } E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \quad \text{at } T = 0 K$$

$$\because n' = 2n \text{ and } m' = 2m \Rightarrow E'_F = \frac{\hbar^2}{4m} (3\pi^2 2n)^{2/3} = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \times 2^{-1/3}$$

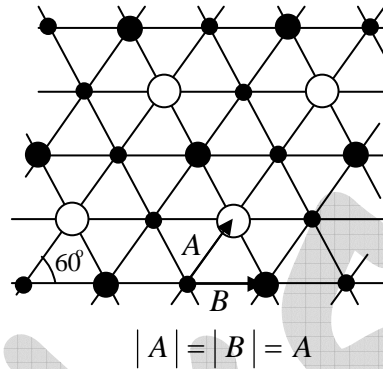
- Q8. When two different solids are brought in contact with each other, which one of the following is true?
- (a) Their Fermi energies become equal  
 (b) Their band gaps become equal  
 (c) Their chemical potentials become equal  
 (d) Their work functions become equal

Ans.: (c)

JEST-2015

Q9. What is the area of the irreducible Brillouin zone of the crystal structure as given in the figure?

- (a)  $\frac{2\pi^2}{\sqrt{3}A^2}$
- (b)  $\frac{\sqrt{3}\pi^2}{2A^2}$
- (c)  $\frac{2\pi^2}{A^2}$
- (d)  $\frac{\pi^2}{\sqrt{3}A^2}$



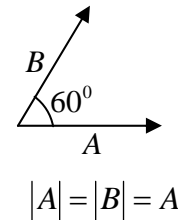
Ans.: (a)

Solution: Area of the Brillouin zone can be related to the area of normal cell as

$$\text{Area of B.Z.} = \frac{\pi^2}{\text{Area of cell}} = \frac{\pi^2}{|\vec{A} \times \vec{B}|}$$

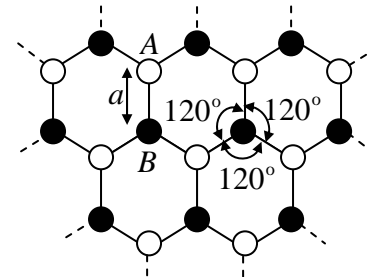
$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}|\sin\theta = A^2 \sin(60^\circ) = \frac{\sqrt{3}}{2} A^2$$

$$\therefore \text{Area of Brillouin zone} = \frac{2\pi^2}{\sqrt{3}A^2}$$



Q10. For a 2 - dimensional honeycomb lattice as shown in the figure, the first Bragg spot occurs for the grazing angle  $\theta_1$ , while sweeping the angle from  $0^\circ$ . The next Bragg spot is obtained at  $\theta_2$  given by

- (a)  $\sin^{-1}(3 \sin \theta_1)$
- (b)  $\sin^{-1}\left(\frac{3}{2} \sin \theta_1\right)$
- (c)  $\sin^{-1}\left(\frac{\sqrt{3}}{2} \sin \theta_1\right)$
- (d)  $\sin^{-1}(\sqrt{3} \sin \theta_1)$



Ans.: (c)

Solution: According to Bragg's law, the condition for first Bragg spot and second spot is

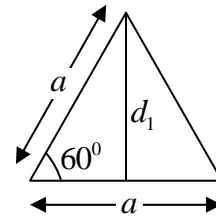
$$2d_1 \sin \theta_1 = n\lambda \quad \text{and} \quad 2d_2 \sin \theta_2 = n\lambda$$

$$\therefore 2d_1 \sin \theta_1 = 2d_2 \sin \theta_2 \Rightarrow \theta_2 = \sin^{-1} \left( \frac{d_1}{d_2} \sin \theta_1 \right)$$

For 2 - dimensional honeycomb lattice, the lattice constant 'a' and interplanar spacing 'd' is linked as

$$d_1^2 = a^2 - \left( \frac{a}{2} \right)^2 \Rightarrow d_1 = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2} a \quad \text{and} \quad d_2 = a$$

$$\therefore \theta_2 = \sin^{-1} \left( \frac{\sqrt{3}}{2} \sin \theta_1 \right)$$



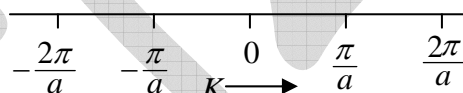
Q11. Given the tight binding dispersion relation  $E(k) = E_0 + A \sin^2 \left( \frac{ka}{2} \right)$ , where  $E_0$  and  $A$  are constants and  $a$  is the lattice parameter. What is the group velocity of an electron at the second Brillouin zone boundary?

- (a) 0                      (b)  $\frac{a}{h}$                       (c)  $\frac{2a}{h}$                       (d)  $\frac{a}{2h}$

Ans.: (a)

Solution: Group velocity is defined as,  $v_g = \frac{1}{\hbar} \frac{dE}{dk}$

$$\text{Since } E = E_0 + A \sin^2 \left( \frac{ka}{2} \right) \Rightarrow \frac{dE}{dk} = aA \sin \left( \frac{ka}{2} \right) \cos \left( \frac{ka}{2} \right) = \frac{aA}{2} \sin ka$$



In one dimension, the Brillouin zone boundary is

The 1<sup>st</sup> Brillouin zone boundaries lie at  $\pm \frac{\pi}{a}$

The 2<sup>nd</sup> Brillouin zone boundaries lie at  $\pm \frac{2\pi}{a}$

Thus, the group velocity at the second Brillouin zone boundary is

$$v_g \Big|_{\pm \frac{2\pi}{a}} = \frac{aA}{2} \sin \left( \frac{2\pi}{a} \times a \right) = \frac{aA}{2} \sin 2\pi \Rightarrow v_g = 0$$

Q12. The total number of  $Na^+$  and  $Cl^-$  ions per unit cell of  $NaCl$  is,

- (a) 2                      (b) 4                      (c) 6                      (d) 8

Ans.: (d)

Solution: Total number of  $Na^+$  and  $Cl^-$  ions per unit (d) is

$$N_{Cl^-} = \frac{1}{8}n_c + \frac{1}{2}n_f, \quad N_{Na^+} = \frac{1}{4}n_e + 1 \times n_i$$

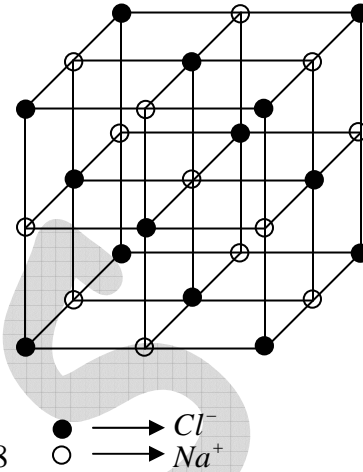
where  $n_c$  = number of ions at corner

$n_f$  = number of ions at face

$n_e$  = number of ions at edges

$n_i$  = number of ions inside

$$N = N_{Cl^-} + N_{Na^+} = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 + \frac{1}{4} \times 12 + 1 \times 1 = 1 + 3 + 3 + 1 = 8$$



Q13. For non-interacting Fermions in  $d$  – dimensions, the density of states  $D(E)$  varies as

$E^{\left(\frac{d-1}{2}\right)}$ . The Fermi energy  $E_F$  of an  $N$  particle system in 3–, 2– and 1– dimensions will scale respectively as,

- (a)  $N^2, N^{2/3}, N$                       (b)  $N, N^{2/3}, N^2$   
 (c)  $N, N^2, N^{2/3}$                       (d)  $N^{2/3}, N, N^2$

Ans.: (d)

## JEST-2016

Q14. If  $\vec{k}$  is the wavevector of incident light ( $|\vec{k}| = \frac{2\pi}{\lambda}$ ,  $\lambda$  is the wavelength of light) and  $\vec{G}$  is a reciprocal lattice vector, then the Bragg's law can be written as:

- (a)  $\vec{k} + \vec{G} = 0$  (b)  $2\vec{k} \cdot \vec{G} + G^2 = 0$   
 (c)  $2\vec{k} \cdot \vec{G} + k^2 = 0$  (d)  $\vec{k} \cdot \vec{G} = 0$

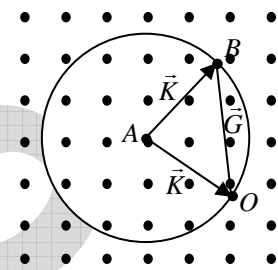
Ans. : (b)

Solution: By means of Ewald construction, we can write the Bragg's law in vector form

$$\vec{G} = \vec{OB}, \vec{K}' = \vec{AO}$$

For diffraction it is necessary that vector  $\vec{K}' + \vec{G}$ , that is vector  $\vec{AB}$  be equal in magnitude to the vector  $\vec{K}$  or

$$(\vec{K} + \vec{G})^2 = K^2 \Rightarrow 2\vec{K} \cdot \vec{G} + G^2 = 0$$



Q15. The number of different Bravais lattices possible in two dimensions is:

- (a) 2 (b) 3 (c) 5 (d) 6

Ans. : (c)

Solution: Five Bravais lattices in 2D are:

- (i) Square lattice
- (ii) Rectangular ( $P$ ) lattice
- (iii) Rectangular ( $C$ ) lattice
- (iv) Hexagonal lattice
- (v) Oblique lattice