NET December 2014 (Booklet Code A)

Q1. Lunch-dinner pattern of a person for m days is given below. He has a choice of a VEG or a NON-VEG meal for his lunch-dinner

(i) If he takes a NON-VEG lunch, he will have only VEG for dinner

(ii) He takes NON-VEG dinner for exactly 9 days

(iii) He takes VEG lunch for exactly 15 days

(iv) He takes a total of 14 NON-VEG meals

What is m ?

(a) 18 (b) 24 (c) 20 (d) 38

Q2. Two locomotives are running towards each other with speeds of 60 and 40 km/h. An object keeps on flying to and fro from the front tip of one locomotive to the front tip of the other with a speed of 70 km/h. After 30 minutes, the two locomotives collide and the object is crushed. What distance did the object cover before being crushed?

(a) 50 km (b) 45 km (c) 35 km (d) 10 km

Q3. A sphere is made up of very thin concentric shells of increasing radii (leaving no gaps). The mass of an arbitrarily chosen shell is

(a) equal to the mass of the preceding shell

(b) proportional to its volume

(c) proportional to its radius

(d) proportional to its surface area

Q4. Find the missing letter:

A	?	Q	E
C	M	S	C
E	K	U	A
G	I	W	Y

(a) L (b) Q (c) N (d) O

Q5. A person sells two objects at Rs. 1035/- each. On the first object he suffers a loss of 10% while on the second he gains 15%. What is his net loss/gain percentage?

(a) 5% gain (b) < 1% gain (c) < 1% loss (d) no loss, no gain

Q6. A bank offers a scheme wherein deposits made for 1600 days are doubled in value, the interest being compounded daily. The interest accrued on a deposit of Rs. 1000/- over the first 400 days would be Rs.

(a) 250 (b) 183 (c) 148 (d) 190

Q7. The least significant bit of an 8-bit binary number is zero. A binary number whose value is 8 times the previous number has

(a) 12 bits ending with three zeros (b) 11 bits ending with four zeros

(c) 11 bits ending with three zeros (d) 12 bits ending with four zeros

Q8. What is the next number of the following sequence?

2, 3, 4, 7, 6, 11, 8, 15, 10, ……

(a) 12 (b) 13 (c) 17 (d) 19

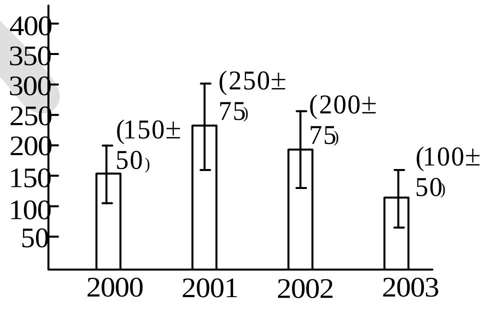
Q9. 20% of students of a particular course get jobs within one year of passing. 20% of the remaining students get jobs by the end of second year of passing. If 16 students are still jobless, how many students had passed the course?

(a) 32 (b) 64 (c) 25 (d) 100

Q10. A rectangle of length d and breadth $d/2$ is revolved once completely around its length and once around its breadth. The ratio of volumes swept in the two cases is

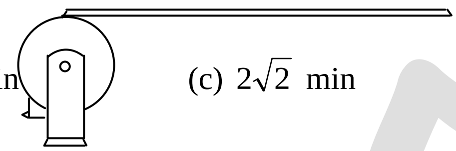
(a) 1:1 (b) 1:2 (c) 1:3 (d) 1:4

Q11. Average yield of a product in different years is shown in the histogram. If the vertical bars indicate variability during the year, then during which year was the percent variability over the average of that year the least?



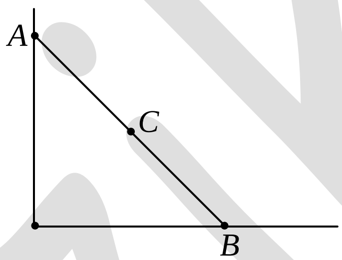
(a) 2000 (b) 2001 (c) 2002 (d) 2003

Q12. A long ribbon is wound around a spool up to a radius R. Holding the tip of the ribbon, a boy runs away from the spool with a constant speed maintaining the unwound portion of the ribbon horizontal. In 4 minutes, the radius of the wound portion becomes $\frac{R}{\sqrt{2}}$. In what further time, it will become $R/2$?



(a) $\sqrt{2}$ min (b) 2 min (c) $2\sqrt{2}$ min (d) 4 min

Q13. A ladder rests against a wall as shown. The top and the bottom ends of the ladder are marked A and B. The base B slips. The central point C of the ladder falls along



(a) a parabola (b) the arc of a circle

(c) a straight line (d) a hyperbola

Q14. Binomial theorem in algebra gives $(1+x)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_n x^n$ where $a_0, a_1, \ldots\ldots, a_n$ are constants depending on n. What is the sum $a_0 + a_1 + a_2 + \ldots.. + a_n$?

(a) 2^n (b) n (c) n^2 (d) $n^2 + n$

Q15. Continue the sequence

2, 5, 10, 17, 28, 41, _, _, _

(a) 58, 77, 100 (b) 64, 81, 100 (c) 43, 47, 53 (d) 55, 89, 113

Q16. A code consists of at most two identical letters followed by at most four identical digits. The code must have at least one letter and one digit. How many distinct codes can be generated using letters A to Z and digits 1 to 9?

(a) 936 (b) 1148 (c) 1872 (d) 2574

Q17. Two solid iron spheres are heated to $100^0 C$ and then allowed to cool. One has size of a football; the other has the size of a pea. Which sphere will attain the room temperature (constant) first?

(a) The bigger sphere (b) The smaller sphere

(c) Both spheres will take the same time (d) It will depend on the room temperature

Q18. Weights (in kg) of 13 persons are given below:

70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94

Two new persons having weights 100 kg and 79 kg join the group. The average weight of the group increases by

(a) 0 kg (b) 1 kg (c) 1.6 kg (d) 1.8 kg

Q19. If n is a positive integer, then

$$n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)$$

is divisible by

(a) 3 but not 7 (b) 3 and 7 (c) 7 but not 3 (d) neither 3 nor 7

Q20. The area (in m^2) of a triangular park of dimensions 50 m, 120 m and 130 m is

(a) 3000 (b) 3250 (c) 5550 (d) 7800

PART B

Q21. Let $\vec{r}$ denote the position vector of any point in three-dimensional space, and $r = |\vec{r}|$. Then

(a) $\vec{\nabla}\cdot\vec{r} = 0$ and $\vec{\nabla}\times\vec{r} = \vec{r}/r$

(b) $\vec{\nabla}\cdot\vec{r} = 0$ and $\nabla^2 r = 0$

(c) $\vec{\nabla}\cdot\vec{r} = 3$ and $\nabla^2\vec{r} = \vec{r}/r^2$

(d) $\vec{\nabla}\cdot\vec{r} = 3$ and $\vec{\nabla}\times\vec{r} = 0$

Q22. The column vector $\begin{pmatrix} a \\ b \\ a \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

(a) $b = 0$ or $a = 0$

(b) $b = a$ or $b = -2a$

(c) $b = 2a$ or $b = -a$

(d) $b = a/2$ or $b = -a/2$

Q23. The principal value of the integral $\int_{-\infty}^{\infty} \frac{\sin(2x)}{x^3}dx$ is

(a) -2π

(b) $-\pi$

(c) π

(d) 2π

Q24. The Laurent series expansion of the function $f(z) = e^z + e^{1/z}$ about $z = 0$ is given by

(a) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ for all $|z| < \infty$

(b) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}$ only if $0 < |z| < 1$

(c) $\sum_{n=0}^{\infty} \left(z^n + \frac{1}{z^n}\right)\frac{1}{n!}$ for all $0 < |z| < \infty$

(d) $\sum_{n=-\infty}^{\infty} \frac{z^n}{n!}$ only if $|z| < 1$

Q25. Two independent random variables m and n, which can take the integer values $0, 1, 2, \ldots, \infty$, follow the Poisson distribution, with distinct mean values μ and ν respectively. Then

(a) the probability distribution of the random variable $l = m + n$ is a binomial distribution.

(b) the probability distribution of the random variable $r = m - n$ is also a Poisson distribution.

(c) the variance of the random variable $l = m + n$ is equal to $\mu + \nu$

(d) the mean value of the random variable $r = m - n$ is equal to 0.

Q26. The equation of motion of a system described by the time-dependent Lagrangian

$$L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right] \text{ is}$$

(a) $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$ (b) $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$

(c) $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$ (d) $m\ddot{x} + \frac{dV}{dx} = 0$

Q27. A particle of mass m is moving in the potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$ where a, b are positive constants. The frequency of small oscillations about a point of stable equilibrium is

(a) $\sqrt{a/m}$ (b) $\sqrt{2a/m}$ (c) $\sqrt{3a/m}$ (d) $\sqrt{6a/m}$

Q28. The radius of Earth is approximately 6400 km. The height h at which the acceleration due to Earth's gravity differs from g at the Earth's surface by approximately 1% is

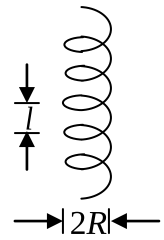
(a) 64 km (b) 48 km (c) 32 km (d) 16 km

Q29. According to the special theory of relativity, the speed v of a free particle of mass m and total energy E is:

(a) $v = c\sqrt{1 - \frac{mc^2}{E}}$ (b) $v = \sqrt{\frac{2E}{m}}\left(1 + \frac{mc^2}{E}\right)$

(c) $v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2}$ (d) $v = c\left(1 + \frac{mc^2}{E}\right)$

Q30. A charged particle moves in a helical path under the influence of a constant magnetic field. The initial velocity is such that the component along the magnetic field is twice the component in the plane normal to the magnetic field. The ratio ℓ/R of the pitch ℓ to the radius R of the helical path is



(a) $\pi/2$ (b) 4π (c) 2π (d) π

Q31. A parallel beam of light of wavelength λ is incident normally on a thin polymer film with air on both sides. If the film has a refractive index $n>1$, then second-order bright fringes can be observed in reflection when the thickness of the film is

(a) $\lambda/4n$ (b) $\lambda/2n$ (c) $3\lambda/4n$ (d) λ/n

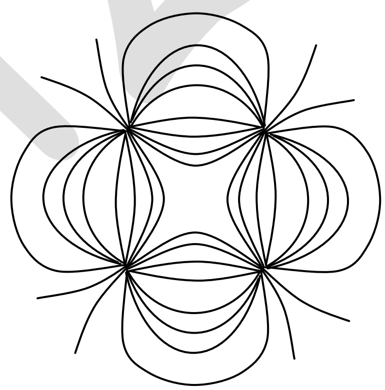
Q32. A solid sphere of radius R has a charge density, given by

$$\rho(r)=\rho_0\left(1-\frac{ar}{R}\right)$$

where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r=R/2$ is 1.25 times that at $r=R$, then the value of a is

(a) 2 (b) 1 (c) $1/2$ (d) $1/4$

Q33. The electrostatic lines of force due to a system of four point charges is sketched below.



At a large distance r, the leading asymptotic behaviour of the electrostatic potential is proportional to

(a) r (b) r^{-1} (c) r^{-2} (d) r^{-3}

Q34. Suppose Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is

(a) $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$ (b) $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$

(c) $-i\hbar\omega x\frac{\partial}{\partial x}$ (d) $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

Q35. Let ψ_1 and ψ_2 denote the normalized eigenstates of a particle with energy eigenvalues E_1 and E_2 respectively, with $E_2 > E_1$. At time $t = 0$ the particle is prepared in a state

$$\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

The shortest time T at which $\Psi(t=T)$ will be orthogonal to $\Psi(t=0)$ is

(a) $\frac{2\hbar\pi}{(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{(E_2 - E_1)}$ (c) $\frac{\hbar\pi}{2(E_2 - E_1)}$ (d) $\frac{\hbar\pi}{4(E_2 - E_1)}$

Q36. Consider the normalized wavefunction

$$\phi = a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1-1}$$

where ψ_{lm} is a simultaneous normalized eigenfunction of the angular momentum operators L^2 and L_z, with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. If ϕ is an eigenfunction of the operator L_x with eigenvalue $\hbar$, then

(a) $a_1 = -a_3 = \frac{1}{2},\ a_2 = \frac{1}{\sqrt{2}}$ (b) $a_1 = a_3 = \frac{1}{2},\ a_2 = \frac{1}{\sqrt{2}}$

(c) $a_1 = a_3 = \frac{1}{2},\ a_2 = -\frac{1}{\sqrt{2}}$ (d) $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

Q37. Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x, p] = i$ in natural units $(\hbar = 1)$. Then the commutator $[x,\ pe^{-p}]$ is

(a) $i(1-p)e^{-p}$ (b) $i(1-p^2)e^{-p}$ (c) $i(1-e^{-p})$ (d) ipe^{-p}

Q38. The pressure P of a fluid is related to its number density ρ by the equation of state

$$P = a\rho + b\rho^2$$

where a and b are constants. If the initial volume of the fluid is V_0, the work done on the system when it is compressed so as to increase the number density from an initial value of ρ_0 to $2\rho_0$ is

(a) $a\rho_0 V_0$ (b) $(a + b\rho_0)\rho_0 V_0$

(c) $\left(\frac{3a}{2} + \frac{7\rho_0 b}{3}\right)\rho_0 V_0$ (d) $(a\ln 2 + b\rho_0)\rho_0 V_0$

Q39. The Hamiltonian of a classical particle moving in one dimension is

$$H = \frac{p^2}{2m} + \alpha q^4$$

where α is a positive constant and p and q are its momentum and position respectively. Given that its total energy $E \le E_0$ the available volume of phase space depends on E_0 as

(a) $E_0^{3/4}$ (b) E_0

(c) $\sqrt{E_0}$ (d) is independent of E_0

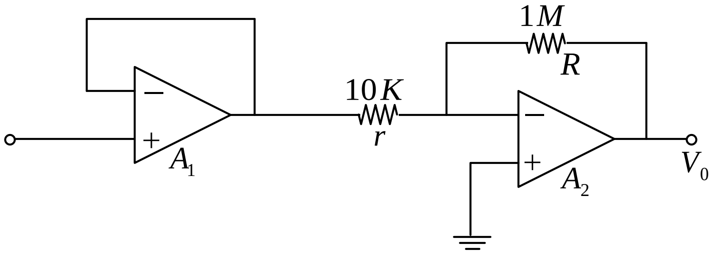
Q40. An ideal Bose gas is confined inside a container that is connected to a particle reservoir. Each particle can occupy a discrete set of single-particle quantum states. If the probability that a particular quantum state is unoccupied is 0.1, then the average number of bosons in that state is

(a) 8 (b) 9 (c) 10 (d) 11

Q41. In low density oxygen gas at low temperature, only the translational and rotational modes of the molecules are excited. The specific heat per molecule of the gas is

(a) $\frac{1}{2}k_B$ (b) k_B (c) $\frac{3}{2}k_B$ (d) $\frac{5}{2}k_B$

Q42. Consider the amplifier circuit comprising of the two op-amps A_1 and A_2 as shown in the figure.



If the input ac signal source has an impedance of $50k\Omega$, which of the following statements is true?

(a) A_1 is required in the circuit because the source impedance is much greater than r

(b) A_1 is required in the circuit because the source impedance is much less than R

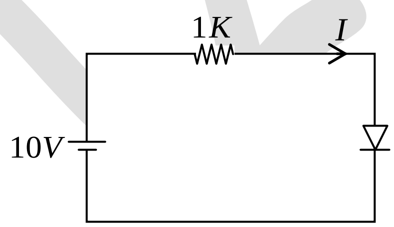
(c) A_1 can be eliminated from the circuit without affecting the overall gain

(d) A_1 is required in the circuit if the output has to follow the phase of the input signal

Q43. The $I-V$ characteristics of the diode in the circuit below is given by

$$I = \begin{cases} (V-0.7)/500 & \text{for} \quad V \geq 0.7 \\ 0 & \text{for} \quad V < 0.7 \end{cases}$$

where V is measured in volts and I is measured in amperes.



The current I in the circuit is

(a) 10.0 mA (b) 9.3 mA (c) 6.2 mA (d) 6.7 mA

Q44. A junction is made between a metal of work function W_M, and a doped semiconductor of work function W_S with $W_M > W_S$. If the electric field at the interface has to be increased by a factor of 3, then the dopant concentration in the semiconductor would have to be

(a) increased by a factor of 9 (b) decreased by a factor of 3

(c) increased by a factor of 3 (d) decreased by a factor of $\sqrt{3}$

Q45. In a measurement of the viscous drag force experienced by spherical particles in a liquid, the force is found to be proportional to $V^{1/3}$ where V is the measured volume of each particle. If V is measured to be $30\,mm^3$, with an uncertainty of $2.7\,mm^3$, the resulting relative percentage uncertainty in the measured force is

(a) 2.08 (b) 0.09 (c) 6 (d) 3

PART C

Q46. Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{a}$ and $\vec{b}$ are two arbitrary constant vectors in three dimensions, the commutator $\left[\vec{a}\cdot\vec{\sigma}, \vec{b}\cdot\vec{\sigma}\right]$ is equal to (in the following I is the identity matrix)

(a) $\left(\vec{a}\cdot\vec{b}\right)(\sigma_1+\sigma_2+\sigma_3)$ (b) $2i\left(\vec{a}\times\vec{b}\right)\cdot\vec{\sigma}$

(c) $\left(\vec{a}\cdot\vec{b}\right)I$ (d) $|\vec{a}|\left|\vec{b}\right|I$

Q47. Consider the function $f(z) = \frac{1}{z}\ln(1-z)$ of a complex variable $z = re^{i\theta}\,(r \geq 0, -\infty < \theta < \infty)$. The singularities of $f(z)$ are as follows:

(a) branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ only for $0 \leq \theta < 2\pi$

(b) branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all θ other than $0 \leq \theta < 2\pi$

(c) branch points at $z = 1$ and $z = \infty$; and a pole at $z = 0$ for all θ

(d) branch points at $z = 0$, $z = 1$ and $z = \infty$.

Q48. The function $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)!}\left(\frac{x}{2}\right)^{2n+1}$ satisfies the differential equation

(a) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2+1)f = 0$ (b) $x^2\frac{d^2f}{dx^2} + 2x\frac{df}{dx} + (x^2-1)f = 0$

(c) $x^2\frac{d^2f}{dx^2} + x\frac{df}{dx} + (x^2-1)f = 0$ (d) $x^2\frac{d^2f}{dx^2} - x\frac{df}{dx} + (x^2-1)f = 0$

Q49. Let α and β be complex numbers. Which of the following sets of matrices forms a group under matrix multiplication?

(a) $\begin{pmatrix} \alpha & \beta \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}$, where $\alpha\beta \neq 1$

(c) $\begin{pmatrix} \alpha & \alpha^* \\ \beta & \beta^* \end{pmatrix}$, where $\alpha\beta^*$ is real

(d) $\begin{pmatrix} \alpha & \beta \\ -\beta^* & \alpha^* \end{pmatrix}$, where $|\alpha|^2 + |\beta|^2 = 1$

Q50. The expression

$$\sum_{i,j,k=1}^{3} \in_{ijk} \{x_i, \{p_j, L_k\}\}$$

(where $\in_{ijk}$ is the Levi-Civita symbol, $\vec{x}, \vec{p}, \vec{L}$ are the position, momentum and angular momentum respectively, and $\{A, B\}$ represents the Poisson Bracket of A and B) simplifies to

(a) 0

(b) 6

(c) $\vec{x}, (\vec{p} \times \vec{L})$

(d) $\vec{x} \times \vec{p}$

Q51. A mechanical system is described by the Hamiltonian $H(q,p) = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$. As a result of the canonical transformation generated by $F(q, Q) = -\frac{Q}{q}$, the Hamiltonian in the new coordinate Q and momentum P becomes

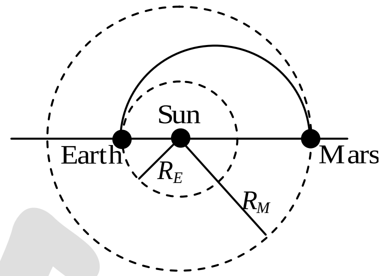
(a) $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}Q^2$

(b) $\frac{1}{2m}Q^2P^2 + \frac{m\omega^2}{2}P^2$

(c) $\frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$

(d) $\frac{1}{2m}Q^2P^4 + \frac{m\omega^2}{2}P^{-2}$

Q52. The probe *Mangalyaan* was sent recently to explore the planet Mars. The inter-planetary part of the trajectory is approximately a half-ellipse with the Earth (at the time of launch), Sun and Mars (at the time the probe reaches the destination) forming the major axis. Assuming that the orbits of Earth and mars are approximately circular with radii R_E and R_M, respectively, the velocity (with respect to the Sun) of the probe during its voyage when it is at a distance $r\left(R_E << r << R_M\right)$ from the Sun, neglecting the effect of Earth and Mars, is



(a) $\sqrt{2GM\dfrac{\left(R_E+R_M\right)}{r\left(R_E+R_M-r\right)}}$ (b) $\sqrt{2GM\dfrac{\left(R_E+R_M-r\right)}{r\left(R_E+R_M\right)}}$

(c) $\sqrt{2GM\dfrac{R_E}{rR_M}}$ (d) $\sqrt{\dfrac{2GM}{r}}$

Q53. A plane electromagnetic wave incident normally on the surface of a material is partially reflected. Measurements on the standing wave in the region in front of the interface show that the ratio of the electric field amplitude at the maxima and the minima is 5. The ratio of the reflected intensity to the incident intensity is

(a) $4/9$ (b) $2/3$ (c) $2/5$ (d) $1/5$

Q54. The scalar and vector potentials $\varphi(\vec{x},t)$ and $\vec{A}(\vec{x},t)$ are determined up to a gauge transformation $\varphi \to \varphi' = \varphi - \frac{\partial \xi}{\partial t}$ and $\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\xi$ where ξ is an arbitrary continuous and differentiable function of $\vec{x}$ and t. If we further impose the Lorenz gauge condition

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0$$

then a possible choice for the gauge function $\xi(\vec{x},t)$ is (where $\omega, \vec{k}$ are nonzero constants with $\omega = c\left|\vec{k}\right|$)

(a) $\cos\omega t\ \cosh\vec{k}\cdot\vec{x}$
(b) $\sinh\omega t\ \cos\vec{k}\cdot\vec{x}$
(c) $\cosh\omega t\ \cos\vec{k}\cdot\vec{x}$
(d) $\cosh\omega t\ \cosh\vec{k}\cdot\vec{x}$

Q55. A non-relativistic particle of mass m and charge e, moving with a velocity $\vec{v}$ and acceleration $\vec{a}$, emits radiation of intensity I. What is the intensity of the radiation emitted by a particle of mass $m/2$, charge $2e$, velocity $\vec{v}/2$ and acceleration $2\vec{a}$?

(a) $16\ I$
(b) $8\ I$
(c) $4\ I$
(d) $2\ I$

Q56. The ground state energy of the attractive delta function potential

$$V(x) = -b\delta(x),$$

where $b > 0$, is calculated with the variational trial function

$$\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for} \quad -a < x < a, \\ 0, & \text{otherwise,} \end{cases}$$

is

(a) $-\frac{mb^2}{\pi^2\hbar^2}$
(b) $-\frac{2mb^2}{\pi^2\hbar^2}$
(c) $-\frac{mb^2}{2\pi^2\hbar^2}$
(d) $-\frac{mb^2}{4\pi^2\hbar^2}$

Q57. Let $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ (where c_0 and c_1 are constants with $c_0^2 + c_1^2 = 1$) be a linear combination of the wavefunctions of the ground and first excited states of the one-dimensional harmonic oscillator. For what value of c_0 is the expectation value $\langle x\rangle$ a maximum?

(a) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}, \quad c_0 = \frac{1}{\sqrt{2}}$

(b) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}, \quad c_0 = \frac{1}{2}$

(c) $\langle x\rangle = \sqrt{\frac{\hbar}{2m\omega}}, \quad c_0 = \frac{1}{\sqrt{2}}$

(d) $\langle x\rangle = \sqrt{\frac{\hbar}{m\omega}}, \quad c_0 = \frac{1}{2}$

Q58. Consider a particle of mass m in the potential $V(x) = a|x|, a > 0$. The energy eigen-values $E_n (n = 0, 1, 2, ...)$, in the WKB approximation, are

(a) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{1/3}$

(b) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{2/3}$

(c) $\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)$

(d) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n+\frac{1}{2}\right)\right]^{4/3}$

Q59. The Hamiltonian H_0 for a three-state quantum system is given by the matrix $H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. When perturbed by $H' = \in \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ where $\in << 1$, the resulting shift in the energy eigenvalue $E_0 = 2$ is

(a) $\in, -2\in$

(b) $-\in, 2\in$

(c) $\pm\in$

(d) $\pm 2\in$

Q60. When a gas expands adiabatically from volume V_1 to V_2 by a quasi-static reversible process, it cools from temperature T_1 to T_2. If now the same process is carried out adiabatically and irreversibly, and T_2' is the temperature of the gas when it has equilibrated, then

(a) $T_2' = T_2$

(b) $T_2' > T_2$

(c) $T_2' = T_2\left(\frac{V_2 - V_1}{V_2}\right)$

(d) $T_2' = \frac{T_2 V_1}{V_2}$

Q61. A random walker takes a step of unit length in the positive direction with probability $2/3$ and a step of unit length in the negative direction with probability $1/3$. The mean displacement of the walker after n steps is

(a) $n/3$ (b) $n/8$ (c) $2n/3$ (d) 0

Q62. A collection N of non-interacting spins S_i, $i = 1, 2, ..., N$, $(S_i = \pm 1)$ is kept in an external magnetic field B at a temperature T. The Hamiltonian of the system is $H = -\mu B \Sigma_i S_i$. What should be the minimum value of $\frac{\mu B}{k_B T}$ for which the mean value $\langle S_i \rangle \geq \frac{1}{3}$?

(a) $\frac{1}{2} N \ln 2$ (b) $2 \ln 2$ (c) $\frac{1}{2} \ln 2$ (d) $N \ln 2$

Q63. A large MOS transistor consists of N individual transistors connected in parallel. If the only form of noise in each transistor is $1/f$ noise, then the equivalent voltage noise spectral density for the MOS transistor is

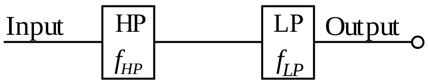
(a) $1/N$ times that of a single transistor

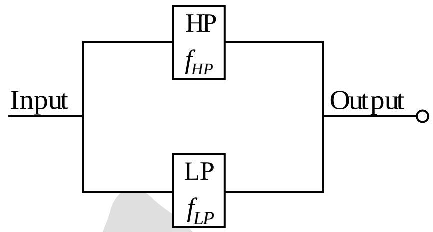
(b) $1/N^2$ times that of a single transistor

(c) N times that of a single transistor

(d) N^2 times that of a single transistor

Q64. Consider a Low Pass (LP) and a High Pass (HP) filter with cut-off frequencies f_{LP} and f_{HP}, respectively, connected in series or in parallel configurations as shown in the Figures A and B below.

(A) 

(B) 

Which of the following statements is correct?

(a) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B acts as a band Reject filter

(b) For $f_{HP} > f_{LP}$, A stops the signal from passing through and B passes the signal without filtering

(c) For $f_{HP} < f_{LP}$, A acts as a Band Pass filter and B passes the signal without filtering

(d) For $f_{HP} > f_{LP}$, A passes the signal without filtering and B acts as a Band Reject filter

Q65. When laser light of wavelength λ falls on a metal scale with 1 mm engravings at a grazing angle of incidence, it is diffracted to form a vertical chain of diffraction spots on a screen kept perpendicular to the scale. If the wavelength of the laser is increased by 200 nm, the angle of the first-order diffraction spot changes from 5^0 to

(a) 6.60^0 (b) 5.14^0 (c) 5.018^0 (d) 5.21^0

Q66. The power density of sunlight incident on a solar cell is $100\ mW/cm^2$. Its short circuit current density is $30\ mA/cm^2$ and the open circuit voltage is $0.7\ V$. If the fill factor of the solar cell decreases from 0.8 to 0.5 then the percentage efficiency will decrease from

(a) 42.0 to 26.2 (b) 24.0 to 16.8 (c) 21.0 to 10.5 (d) 16.8 to 10.5

Q67. An atomic transition ${}^1P \rightarrow {}^1S$ in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton $\mu_B = 9.27\times10^{-24}\ J/T$, and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately

(a) 0.01 nm (b) 0.1 nm (c) 1.0 nm (d) 10 nm

Q68. If the leading anharmonic correction to the energy of the n-th vibrational level of a diatomic molecule is $-x_e\left(n+\frac{1}{2}\right)^2\hbar\omega$ with $x_e = 0.001$, the total number of energy levels possible is approximately

(a) 500 (b) 1000 (c) 250 (d) 750

Q69. The effective spin-spin interaction between the electron spin $\vec{S}_e$ and the proton spin $\vec{S}_p$ in the ground state of the Hydrogen atom is given by $H' = a\vec{S}_e\cdot\vec{S}_p$. As a result of this interaction, the energy levels split by an amount

(a) $\frac{1}{2}a\hbar^2$ (b) $2a\hbar^2$ (c) $a\hbar^2$ (d) $\frac{3}{2}a\hbar^2$

Q70. Consider the crystal structure of sodium chloride which is modeled as a set of touching spheres. Each sodium atom has a radius r_1 and each chlorine atom has a radius r_2. The centres of the spheres from a simple cubic lattice. The packing fraction of this system is

(a) $\pi\left[\left(\frac{r_1}{r_1+r_2}\right)^3+\left(\frac{r_2}{r_1+r_2}\right)^3\right]$ (b) $\frac{2\pi}{3}\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$

(c) $\frac{r_1^3+r_2^3}{(r_1+r_2)^3}$ (d) $\pi\frac{r_1^3+r_2^3}{2(r_1+r_2)^3}$

Q71. Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the c-direction is half the effective spring constant between atoms in the a and b directions. At low temperatures, the behaviour of the lattice contribution to the specific heat will depend as a function of temperature T as

(a) T^2 for the tetragonal solid, but as T^3 for the simple cubic solid

(b) T for the tetragonal solid, and as T^3 for the simple cubic solid

(c) T for both solids

(d) T^3 for both solids

Q72. A superconducting ring carries a steady current in the presence of a magnetic field $\vec{B}$ normal to the plane of the ring. Identify the **incorrect** statement.

(a) The flux passing through the superconductor is quantized in units of hc/e

(b) The current and the magnetic field in the superconductor are time independent.

(c) The current density $\vec{j}$ and $\vec{B}$ are related by the equation $\vec{\nabla}\times\vec{j}+\Lambda^2\vec{B}=0$, where Λ is a constant

(d) The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

Q73. Consider the four processes

(i) $p^+ \rightarrow n+e^+ +\nu_e$ (ii) $\Lambda^0 \rightarrow p^+ +e^+ +\nu_e$

(iii) $\pi^+ \rightarrow e^+ +\nu_e$ (iv) $\pi^0 \rightarrow \gamma+\gamma$

which of the above is/are forbidden for free particles?

(a) only (ii) (b) (ii) and (iv) (c) (i) and (iv) (d) (i) and (ii)

Q74. In deep inelastic scattering electrons are scattered off protons to determine if a proton has any internal structure. The energy of the electron for this must be at least

(a) $1.25\times10^9 eV$ (b) $1.25\times10^{12} eV$ (c) $1.25\times10^6 eV$ (d) $1.25\times10^8 eV$

Q75. If the binding energy B of a nucleus (mass number A and charge Z) is given by

$$B=a_V A-a_S A^{2/3}-a_{sym}\frac{(2Z-A)^2}{A}-\frac{a_C Z^2}{A^{1/3}}$$

where $a_V=16\,MeV$, $a_S=16\,MeV$, $a_{sym}=24\,MeV$ and $a_C=0.75\,MeV$, then the Z for the most stable isobar for a nucleus with $A=216$ is

(a) 68 (b) 72 (c) 84 (d) 92