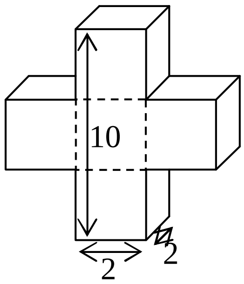
NET/JRF JUNE-14

PART 'A'

Q1. The following diagram shows 2 perpendicularly inter-grown prismatic crystals (twins) of identical shape and size. What is the volume of the object shown (units are arbitrary)?



(a) 60 (b) 65 (c) 72 (d) 80

Q2. Suppose in a box there are 20 red , 30 black, 40 blue and 50 white balls. What is the minimum number of balls to be drawn, without replacement, so that you are certain about getting 4 red, 5 black, 6 blue and 7 white balls?

(a) 140 (b) 97 (c) 104 (d) 124

Q3. In the growing years of a child, the height increases as the square root of the age while the weight increases in direct proportion to the age. The ratio of the weight to the square of the height in this phase of growth

(a) is constant

(b) reduces with age

(c) increases with age

(d) is constant only if the weight and height at birth are both zero.

Q4. Students in group A obtained the following marks: 40, 80, 70, 50, 60, 90, 30. Students in group B obtained 40, 80, 35, 70, 85, 45, 50, 75, 60 marks. Define dispersion (D) = (maximum marks – minimum marks) and $\text{Relative dispresion}\left(\text{RD}\right)=\dfrac{\text{dispersion}}{\text{mean}}$. Then,

(a) RD of group A = RD of group B (b) RD of group A > RD of group B

(c) RD of group A < RD of group B (d) D of group A < D of group B

Q5. In 450 g of pure coffee powder 50 g of chicory is added. A person buys 100 g of this mixture and adds 5 g of chicory to that. What would be the rounded-off percentage of chicory in this final mixture?

(a) 10 (b) 5 (c) 14 (d) 15

Q6. The time gap between the two instants, one before and one after 12.00 noon, when the angle between the hour hand and the minute hand is 66, is

(a) 12 min (b) 16 min (c) 18 min (d) 24 min

Q7. Suppose

$$x\Delta y = (x-y)^2$$
$$x\,o\,y = (x+y)^2$$
$$x * y = (x\times y)^{-1}$$
$$x\cdot y = x\times y$$

+, – and × have their usual meanings. What is the value of

$$\{(197o315)-(197\Delta 315)\}\cdot(197*315)?$$

(a) 118 (b) 512 (c) 2 (d) 4

Q8. If $A\times B = 24, B\times C = 32, C\times D = 48$ then $A\times D$

(a) cannot be found (b) is a perfect square

(c) is a perfect cube (d) is odd

Q9. If all horses are donkeys, some donkeys are monkeys, and some monkeys are men, then which statement must be true?

(a) All donkeys are men (b) Some donkeys may be men

(c) Some horses are men (d) All horses are also monkeys

Q10. A rectangular area of sides 9 and 6 units is to be covered by square tiles of sides 1, 2, and 5 units. The minimum number of tiles needed for this is

(a) 3 (b) 11 (c) 12 (d) 15

Q11. Suppose n is a positive integer. Then $(n^2+n)(2n+1)$

(a) may not be divisible by 2

(b) is always divisible by 2 but may not be divisible by 3

(c) is always divisible by 3 but may not be divisible by 6

(d) is always divisible by 6

Q12. There is a train of length 500 m, in which a man is standing at the rear end. At the instant the rear end crosses a stationary observer on a platform, the man starts walking from the rear to the front and the front to the rear of the train at a constant speed of 3 km/hr. the speed of the train is 80 km/hr. The distance of the man from the observer at the end of 30 minutes is

(a) 41.5 km (b) 40.5 km (c) 40.0 km (d) 41.0 km

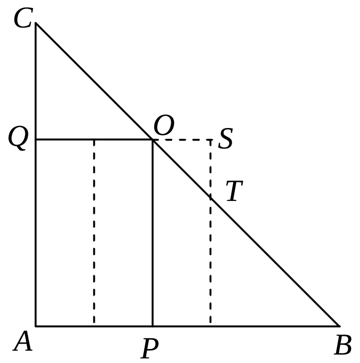
Q13. Three identical flat equilateral-triangular plates of side 5 cm each are placed together such that they form a trapezium. The length of the longer of the two parallel sides of this trapezium is

(a) $5\sqrt{\frac{3}{4}}$ cm (b) $5\sqrt{2}$ cm (c) 10 cm (d) $10\sqrt{3}$ cm

Q14. An archer climbs to the top of a 10 m high building and aims at a bird atop a tree 17 m away. The line of sight from the archer to the bird makes an angle of 45 to the horizontal. What is the height of the tree?

(a) 17 m (b) 27 m (c) 37 m (d) 47 m

Q15. Consider a right-angled triangle ABC where AB = AC = 3. A rectangle APOQ is drawn inside it, as shown, such that the height of the rectangle is twice its width. The rectangle is moved horizontally by a distance 0.2 as shown schematically in the diagram (not to scale). What is the value of the ratio $\frac{\text{area of } \Delta \text{ ABC}}{\text{area of } \Delta \text{ OST}}$



(a) 625 (b) 400

(c) 225 (d) 125

Q16. 80 gsm paper is cut into sheets of 200 mm $\times$ 300 mm size and assembled in packets of 500 sheets. What will be the weight of a packet? (gsm = g/m^2)

(a) 1.2 kg (b) 2.4 kg (c) 3.6 kg (d) 4.8 kg

Q17. Find the missing letter

A	B	C	D
F	I	L	O
K	P	U	Z
P	W	D	?

(a) P (b) K (c) J (d) L

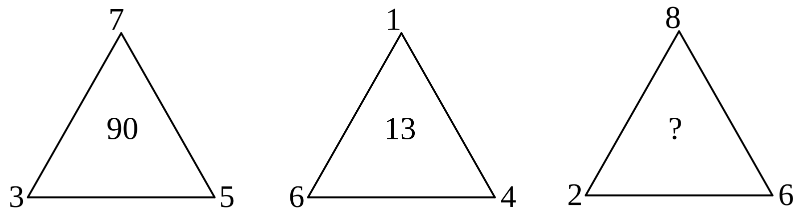
Q18. A merchant buys equal numbers of shirts and trousers and pays Rs 38000. if the cost of 3 shirts is Rs. 800 and that of a trouser is Rs. 1000, then how many shirts were bought?

(a) 60 (b) 30 (c) 15 (d) 10

Q19. Consider the set of numbers $\{17^1, 17^2, ...17^{300}\}$. How many of these numbers end with the digit 3?

(a) 60 (b) 75 (c) 100 (d) 150

Q20. Find the missing number in the triangle,



(a) 16 (b) 96 (c) 50 (d) 80

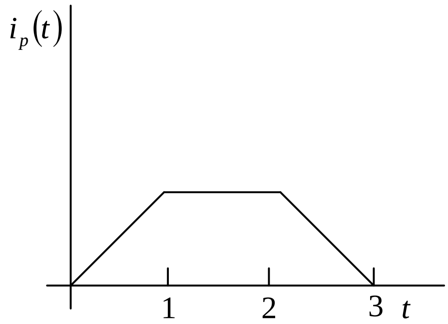
PART 'B'

Q21. A time-dependent current $\vec{I}(t) = Kt\hat{z}$ (where K is a constant) is switched on at $t = 0$ in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance a from the wire is given (for time $t > a/c$) by

(a) $\hat{z}\dfrac{\mu_0 K}{4\pi c}\displaystyle\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \dfrac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$ (b) $\hat{z}\dfrac{\mu_0 K}{4\pi}\displaystyle\int_{-ct}^{ct} dz \dfrac{t}{\left(a^2+z^2\right)^{1/2}}$

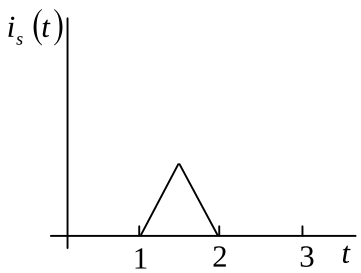
(c) $\hat{z}\dfrac{\mu_0 K}{4\pi c}\displaystyle\int_{-ct}^{ct} dz \dfrac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$ (d) $\hat{z}\dfrac{\mu_0 K}{4\pi}\displaystyle\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \dfrac{t}{\left(a^2+z^2\right)^{1/2}}$

Q22. A current i_p flows through the primary coil of a transformer. The graph of $i_p(t)$ as a function of time t is shown in the figure below.

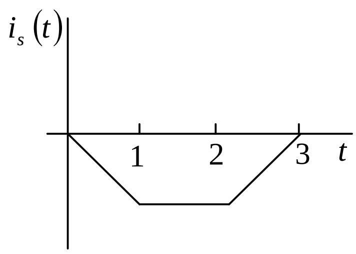


Which of the following graphs represents the current i_S in the secondary coil?

(a)



(b)



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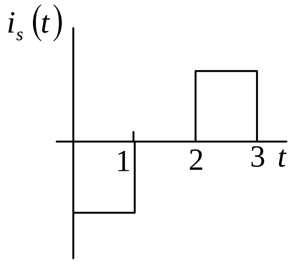
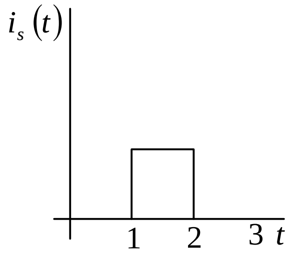
(c) 

(d) 

Q23. If the electrostatic potential in spherical polar coordinates is

$$\varphi(r) = \varphi_0 e^{-r/r_0}$$

where φ_0 and r_0 are constants, then the charge density at a distance $r = r_0$ will be

(a) $\frac{\varepsilon_0 \varphi_0}{e r_0^2}$ (b) $\frac{e \varepsilon_0 \varphi_0}{2 r_0^2}$ (c) $-\frac{\varepsilon_0 \varphi_0}{e r_0^2}$ (d) $-\frac{2e \varepsilon_0 \varphi_0}{r_0^2}$

Q24. If $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by $z = 1$, with the centre on the z - axis, then the value of the integral $\oint_C \vec{A} \cdot d\vec{\ell}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

Q25. The time period of a simple pendulum under the influence of the acceleration due to gravity g is T. The bob is subjected to an additional acceleration of magnitude $\sqrt{3}g$ in the horizontal direction. Assuming small oscillations, the mean position and time period of oscillation, respectively, of the bob will be

(a) 0° to the vertical and $\sqrt{3}T$ (b) 30° to the vertical and $T/2$

(c) 60° to the vertical and $T/\sqrt{2}$ (d) 0° to the vertical and $T/\sqrt{3}$

Q26. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

with the initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$. The solution $x(t)$ attains its maximum value when t is

(a) 1/2 (b) 1 (c) 2 (d) ∞

Q27. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are

(a) $-5, -2, 7$ (b) $-7, 0, 7$ (c) $-4i, 2i, 2i$ (d) $2, 3, 6$

Q28. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral

$$\oint_C \frac{dz}{\sin^2 z}$$

is

(a) ∞ (b) $2\pi i$ (c) 0 (d) πi

Q29. Given

$\sum_{n=0}^{\infty} P_n(x) t^n = \left(1 - 2xt + t^2\right)^{-1/2}$, for $|t| < 1$, the value of $P_5(-1)$ is

(a) 0.26 (b) 1 (c) 0.5 (d) –1

Q30. Consider an electromagnetic wave at the interface between two homogenous dielectric media of dielectric constants ε_1 and ε_2. Assuming $\varepsilon_2 > \varepsilon_1$ and no charges on the surface, the electric field vector $\vec{E}$ and the displacement vector $\vec{D}$ in the two media satisfy the following inequalities

(a) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (b) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

(c) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (d) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

Q31. Consider a system of two non-interacting identical fermions, each of mass m in an infinite square well potential of width a. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy $E=\frac{5\pi^2\hbar^2}{2ma^2}$ is

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)-\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)+\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right)-\sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right)-\sin\left(\frac{\pi x_2}{a}\right)\cos\left(\frac{\pi x_2}{a}\right)\right]$

Q32. A particle of mass m in the potential $V(x,y)=\frac{1}{2}m\omega^2\left(4x^2+y^2\right)$, is in an eigenstate of energy $E=\frac{5}{2}\hbar\omega$. The corresponding un-normalized eigen function is

(a) $y\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$

(b) $x\exp\left[-\frac{m\omega}{2\hbar}\left(2x^2+y^2\right)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$

(d) $xy\exp\left[-\frac{m\omega}{2\hbar}\left(x^2+y^2\right)\right]$

Q33. A particle of mass m and coordinate q has the Lagrangian $L=\frac{1}{2}m\dot{q}^2-\frac{\lambda}{2}q\dot{q}^2$ where λ is a constant. The Hamiltonian for the system is given by

(a) $\frac{p^2}{2m}+\frac{\lambda q p^2}{2m^2}$

(b) $\frac{p^2}{2(m-\lambda q)}$

(c) $\frac{p^2}{2m}+\frac{\lambda q p^2}{2(m-\lambda q)^2}$

(d) $\frac{p\dot{q}}{2}$

Q34. A light source is switched on and off at a constant frequency f. An observer moving with a velocity u with respect to the light source will observe the frequency of the switching to be

(a) $f\left(1-\frac{u^2}{c^2}\right)^{-1}$ (b) $f\left(1-\frac{u^2}{c^2}\right)^{-1/2}$ (c) $f\left(1-\frac{u^2}{c^2}\right)$ (d) $f\left(1-\frac{u^2}{c^2}\right)^{1/2}$

Q35. A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance $d/2$ from the plane. At what distance from the plane will the particle reach the speed $2u$?

(a) $d/6$ (b) $d/3$ (c) $d/4$ (d) $d/5$

Q36. A particle of mass m in three dimensions is in the potential

$$V(r)=\begin{cases}0 & r<a\\ \infty & r>a\end{cases}$$

Its ground state energy is

(a) $\frac{\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

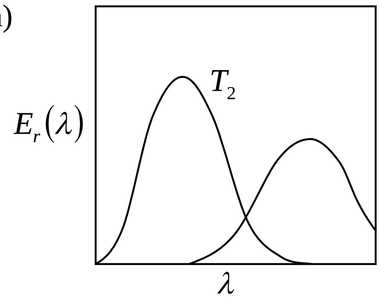
Q37. Given that $\hat{p}_r=-i\hbar\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)$, the uncertainty Δp_r in the ground state.

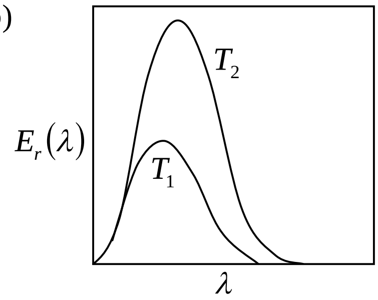
$$\psi_0(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$

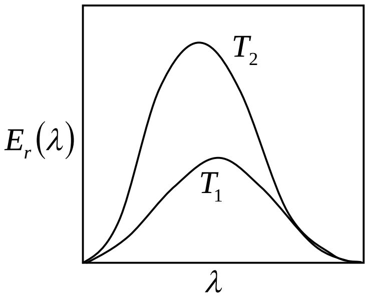
of the hydrogen atom is

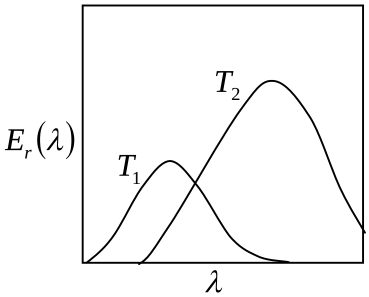
(a) $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$ (c) $\frac{\hbar}{2a_0}$ (d) $\frac{2\hbar}{a_0}$

Q38. Which of the graphs below gives the correct qualitative behaviour of the energy density $E_r(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and $T_2 (T_1 < T_2)$?

(a)


(b)


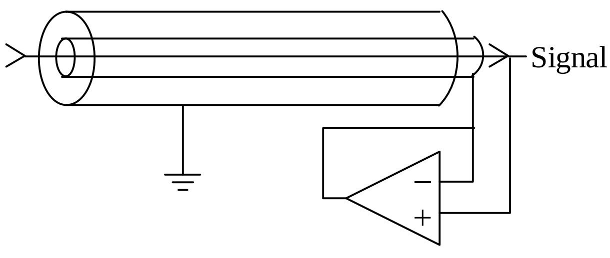
(c)


(d)


Q39. One gram of salt is dissolved in water that is filled to a height of 5 cm in a beaker of diameter 10 cm. The accuracy of length measurement is 0.01 cm while that of mass measurement is 0.01 mg. When measuring the concentration C, the fractional error $\Delta C / C$ is

(a) 0.8% (b) 0.14% (c) 0.5% (d) 0.28%

Q40. The inner shield of a triaxial conductor is driven by an (ideal) op-amp follower circuit as shown. The effective capacitance between the signal-carrying conductor and ground is



(a) unaffected (b) doubled (c) halved (d) made zero

Q41. An op-amp based voltage follower

(a) is useful for converting a low impedance source into a high impedance source.

(b) is useful for converting a high impedance source into a low impedance source.

(c) has infinitely high closed loop output impedance

(d) has infinitely high closed loop gain

Q42. A system can have three energy levels: $E = 0, \pm \varepsilon$. The level $E = 0$ is doubly degenerate, while the others are non-degenerate. The average energy at inverse temperature β is

(a) $-\varepsilon \tanh(\beta\varepsilon)$

(b) $\frac{\varepsilon\left(e^{\beta\varepsilon} - e^{-\beta\varepsilon}\right)}{\left(1 + e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right)}$

(c) zero

(d) $-\varepsilon \tanh\left(\frac{\beta\varepsilon}{2}\right)$

Q43. The free energy F of a system depends on a thermodynamic variable ψ as

$$F = -a\psi^2 + b\psi^6$$

with $a, b > 0$. The value of ψ, when the system is in thermodynamic equilibrium, is

(a) zero (b) $\pm(a/6b)^{1/4}$ (c) $\pm(a/3b)^{1/4}$ (d) $\pm(a/b)^{1/4}$

Q44. For a particular thermodynamic system the entropy S is related to the internal energy U and volume V by

$$S = cU^{3/4}V^{1/4}$$

where c is a constant. The Gibbs potential $G = U - TS + pV$ for this system is

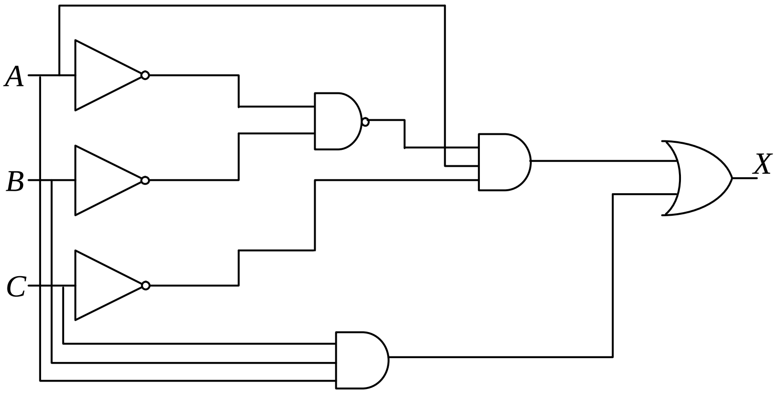
(a) $\frac{3pU}{4T}$ (b) $\frac{cU}{3}$ (c) zero (d) $\frac{US}{4V}$

Q45. An RC network produces a phase-shift of 30°. How many such RC networks should be cascaded together and connected to a Common Emitter amplifier so that the final circuit behaves as an oscillator?

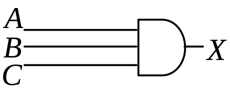
(a) 6 (b) 12 (c) 9 (d) 3

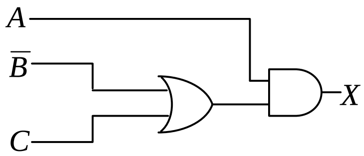
PART ‘C’

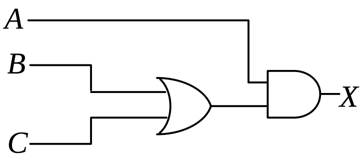
Q46. For the logic circuit shown in the below

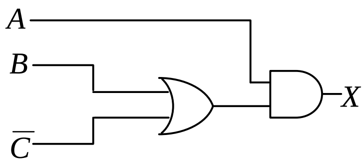


A simplified equivalent circuit is

(a)


(b)


(c)


(d)


Q47. The coordinates and momenta $x_i,\ p_i\,(i=1,2,3)$ of a particle satisfy the canonical Poisson bracket relations $\{x_i,\ p_j\}=\delta_{ij}$. If $C_1=x_2p_3+x_3p_2$ and $C_2=x_1p_2+x_2p_1$ are constants of motion, and if $C_3=\{C_1,\ C_2\}=x_1p_3+x_3p_1$, then

(a) $\{C_2,\ C_3\}=C_1$ and $\{C_3,\ C_1\}=C_2$

(b) $\{C_2,\ C_3\}=-C_1$ and $\{C_3,\ C_1\}=-C_2$

(c) $\{C_2,\ C_3\}=-C_1$ and $\{C_3,\ C_1\}=C_2$

(d) $\{C_2,\ C_3\}=C_1$ and $\{C_3,\ C_1\}=-C_2$

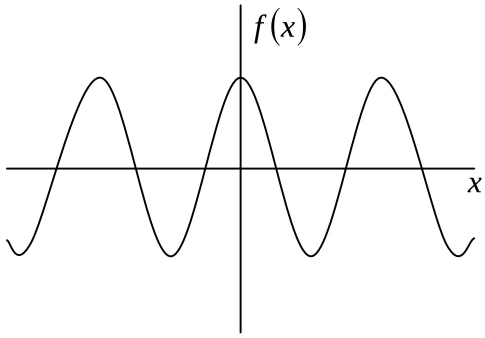
Q48. The recently-discovered Higgs boson at the LHC experiment has a decay mode into a photon and a Z boson. If the rest masses of the Higgs and Z boson are $125\ \text{GeV/c}^2$ and $90\ \text{GeV/c}^2$ respectively, and the decaying Higgs particle is at rest, the energy of the photon will approximately be

(a) $35\sqrt{3}$ GeV (b) 35 GeV (c) 30 GeV (d) 15 GeV

Q49. Let $y = \frac{1}{2}(x_1 + x_2) - \mu$, where x_1 and x_2 are independent and identically distributed Gaussian random variables of mean μ and standard deviation σ. Then $\langle y^4 \rangle / \sigma^4$ is

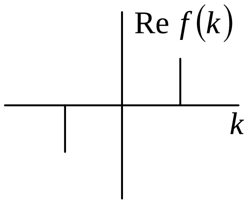
(a) 1 (b) 3/4 (c) 1/2 (d) 1/4

Q50. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below

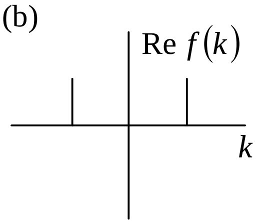


Which of the following graphs represents the real part of its Fourier transform?

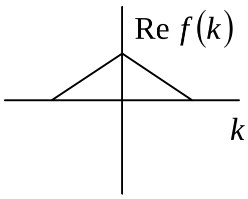
(a)



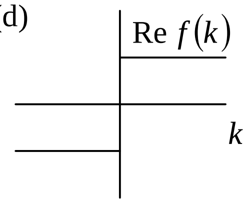
(b)



(c)



(d)



Q51. The matrices

$$A=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \qquad B=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

and $$C=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

satisfy the commutation relations

(a) $[A, B]=B+C, [B, C]=0, [C, A]=B+C$

(b) $[A, B]=C, [B, C]=A, [C, A]=B$

(c) $[A, B]=B, [B, C]=0, [C, A]=A$

(d) $[A, B]=C, [B, C]=0, [C, A]=B$

Q52. The time evolution of a one-dimensional dynamical system is described by

$$\frac{dx}{dt}=-(x+1)(x^2-b^2).$$

If this has one stable and two unstable fixed points, then the parameter b satisfies

(a) $0<b<1$ (b) $b>1$ (c) $b<-1$ (d) $b=2$

Q53. A canonical transformation relates the old coordinates (q, p) to the new ones (Q, P) by the relations $Q=q^2$ and $P=p/2q$. The corresponding time independent generating function is

(a) P/q^2 (b) q^2P (c) q^2/P (d) qP^2

Q54. The integral $\int_0^1 \sqrt{x}dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval $[0, 1]$ is divided into 4 equal parts, the correct result is

(a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638

Q55. The pressure of a nonrelativistic free Fermi gas in three-dimensions depends, at $T=0$, on the density of fermions n as

(a) $n^{5/3}$ (b) $n^{1/3}$ (c) $n^{2/3}$ (d) $n^{4/3}$

Q56. Consider an electron in b.c.c. lattice with lattice constant a. A single particle wavefunction that satisfies the Bloch theorem will have the form $f(\vec{r})\exp(i\vec{k}.\vec{r})$, with $f(\vec{r})$ being

(a) $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$

(b) $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

(c) $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$

(d) $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

Q57. A charge $(-e)$ is placed in vacuum at the point $(d,0,0)$, where $d>0$. The region $x\le 0$ is filled uniformly with a metal. The electric field at the point $\left(\frac{d}{2},0,0\right)$ is

(a) $-\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (b) $\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$

(c) $\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$ (d) $-\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$

Q58. A beam of light of frequency ω is reflected from a dielectric-metal interface at normal incidence. The refractive index of the dielectric medium is n and that of the metal is $n_2=n(1+i\rho)$. If the beam is polarised parallel to the interface, then the phase change experienced by the light upon reflection is

(a) $\tan(2/\rho)$ (b) $\tan^{-1}(1/\rho)$ (c) $\tan^{-1}(2/\rho)$ (d) $\tan^{-1}(2\rho)$

Q59. A permanently deformed even-even nucleus with $J^p=2^+$ has rotational energy 93 keV. The energy of the next excited state is

(a) 372 keV (b) 310 keV (c) 273 keV (d) 186 keV

Q60. A thin, infinitely long solenoid placed along the z- axis contains a magnetic flux ϕ. Which of the following vector potentials corresponds to the magnetic field at an arbitrary point (x, y, z)?

(a) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2}, 0\right)$

(b) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2+z^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2+z^2}, 0\right)$

(c) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, 0\right)$

(d) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x}{x^2+y^2}, \frac{\phi}{2\pi}\frac{y}{x^2+y^2}, 0\right)$

Q61. The ground state eigenfunction for the potential $V(x) = -\delta(x)$ where $\delta(x)$ is the delta function, is given by $\psi(x) = Ae^{-\alpha|x|}$, where A and $\alpha > 0$ are constants. If a perturbation $H' = bx^2$ is applied, the first order correction to the energy of the ground state will be

(a) $\frac{b}{\sqrt{2}\alpha^2}$ (b) $\frac{b}{\alpha^2}$ (c) $\frac{2b}{\alpha^2}$ (d) $\frac{b}{2\alpha^2}$

Q62. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to

(a) 0.60 (b) 0.90 (c) 0.16 (d) 0.32

Q63. A particle in the infinite square well $V(x) = \begin{cases} 0 & 0 < x < a \\ \infty & \text{otherwise} \end{cases}$

is prepared in a state with the wavefunction $\psi(x) = \begin{cases} A\sin^3\left(\frac{\pi x}{a}\right) & 0 < x < a \\ 0 & \text{otherwise} \end{cases}$

The expectation value of the energy of the particle is

(a) $\frac{5\hbar^2\pi^2}{2ma^2}$ (b) $\frac{9\hbar^2\pi^2}{2ma^2}$ (c) $\frac{9\hbar^2\pi^2}{10ma^2}$ (d) $\frac{\hbar^2\pi^2}{2ma^2}$

Q64. The scattering amplitude $f(\theta)$ for the potential $V(r)=\beta e^{-\mu r}$, where β and μ are positive constants, is given, in the Born approximation, by

(in the following $b=2k\sin\frac{\theta}{2}$ and $E=\frac{\hbar^2k^2}{2m}$)

(a) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^2}$ (b) $-\frac{4m\beta\mu}{\hbar^2 b^2\left(b^2+\mu^2\right)}$

(c) $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$ (d) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^3}$

Q65. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?

(a) 3/8 (b) 5/16 (c) 1/4 (d) 1/16

Q66. The average local internal magnetic field acting on an Ising spin is $H_{\text{int}}=\alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_c, the magnetic susceptibility at zero external field is proportional to (k_B is the Boltzmann constant)

(a) $k_BT-\alpha$ (b) $\left(k_BT+\alpha\right)^{-1}$ (c) $\left(k_BT-\alpha\right)^{-1}$ (d) $\tanh\left(k_BT+\alpha\right)$

Q67. An electromagnetically-shielded room is designed so that at a frequency $\omega=10^7$ rad/s the intensity of the external radiation that penetrates the room is 1% of the incident radiation. If $\sigma=\frac{1}{2\pi}\times10^6(\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that $\ln 10=2.3$)

(a) 4.60 mm (b) 2.30 mm (c) 0.23 mm (d) 0.46 mm

Q68. The following data is obtained in an experiment that measures the viscosity η as a function of molecular weight M for a set of polymers,

$M(Da)$	$\eta(kPa-s)$
990	0.28 ± 0.03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is

(a) $\eta \sim M^{4/9}$ (b) $\eta \sim M^{3/2}$ (c) $\eta \sim M^{2}$ (d) $\eta \sim M^{3}$

Q69. In a classical model, a scalar (spin-0) meson consists of a quark and an antiquark bound by a potential

$$V(r) = ar + \frac{b}{r}$$

where $a = 200\,\text{MeV fm}^{-1}$ and $b = 100$ MeV fm. If the masses of the quark and antiquark are negligible, the mass of the meson can be estimated as approximately

(a) 141 MeV/c^2 (b) 283 MeV/c^2 (c) 353 MeV/c^2 (d) 425 MeV/c^2

Q70. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is

(a) 10 (b) 3 (c) 6 (d) 9

Q71. A double slit interference experiment uses a laser emitting light of two adjacent frequencies v_1 and v_2 $(v_1 < v_2)$. The minimum path difference between the interfering beams for which the interference pattern disappears is

(a) $\frac{c}{v_2 + v_1}$ (b) $\frac{c}{v_2 - v_1}$ (c) $\frac{c}{2(v_2 - v_1)}$ (d) $\frac{c}{2(v_2 + v_1)}$

Q72. The van der Waals' equation of state for a gas is given by

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

where P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by

(a) $\frac{a}{9b}$ (b) $\frac{a}{27b^2}$ (c) $\frac{8a}{27bR}$ (d) $3b$

Q73. How much does the total angular momentum quantum number J change in the transition of $Cr(3d^6)$ atom as it ionize to $Cr^{2+}(3d^4)$?

(a) Increases by 2 (b) Decreases by 2 (c) Decreases by 4 (d) Does not change

Q74. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation, by

$$\varepsilon(k)=-4\varepsilon_0\left[\cos\frac{k_x a}{2}\cos\frac{k_y a}{2}+\cos\frac{k_y a}{2}\cos\frac{k_z a}{2}\right]$$

where a is the lattice constant and ε_0 is a constant with the dimension of energy. The x-component of the velocity of the electron at $\left(\frac{\pi}{a},0,0\right)$ is

(a) $-2\varepsilon_0 a/\hbar$ (b) $2\varepsilon_0 a/\hbar$ (c) $-4\varepsilon_0 a/\hbar$ (d) $4\varepsilon_0 a/\hbar$

Q75. The function $\Phi(x,y,z,t)=\cos(z-vt)+\mathrm{Re}(\sin(x+iy))$ satisfies the equation

(a) $\frac{1}{v^2}\frac{\partial^2\Phi}{\partial t^2}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi$ (b) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi$

(c) $\left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$ (d) $\left(\frac{\partial^2}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\Phi=\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi$