JEST 2015

Part A: 3 Mark Questions

Q1. A circular loop of radius R, carries a uniform line charge density λ. The electric field, calculated at a distance z directly above the center of the loop, is maximum if z is equal to,

(a) $\frac{R}{\sqrt{3}}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{2}$ (d) $2R$

Q2. Consider two point charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum?

(a) μ (b) 1 (c) $\frac{1}{\mu}$ (d) $1+\mu$

Q3. Consider a harmonic oscillator in the state $|\psi\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^+} |0\rangle$, where $|0\rangle$ is the ground state, a^+ is the raising operator and α is a complex number. What is the probability that the harmonic oscillator is in the n-th eigenstate $|n\rangle$?

(a) $e^{-|\alpha^2|} \frac{|\alpha|^{2n}}{n!}$ (b) $e^{-\frac{|\alpha|^2}{2} \frac{|\alpha|^n}{\sqrt{n!}}}$

(c) $e^{-|\alpha|^2} \frac{|\alpha|^n}{n!}$ (d) $e^{-\frac{|\alpha|^2}{2}} \frac{|\alpha|^{2n}}{n!}$

Q4. The distance of a star from the Earth is 4.25 light years, as measured from the Earth. A space ship travels from Earth to the star at a constant velocity in 4.25 years, according to the clock on the ship. The speed of the space ship in units of the speed of light is,

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

Q5. Given an analytic function $f(z) = \phi(x,y) + i\psi(x,y)$, where $\phi(x,y) = x^2 + 4x - y^2 + 2y$. If C is a constant , which of the following relations is true?

(a) $\psi(x,y) = x^2 y + 4y + C$ (b) $\psi(x,y) = 2xy - 2x + C$

(c) $\psi(x.y) = 2xy + 4y - 2x + C$ (d) $\psi(x,y) = x^2 y - 2x + C$

Q6. For a system in thermal equilibrium with a heat bath at temperature T, which one of the following equalities is correct? ($\beta = \frac{1}{k_B T}$)

(a) $\frac{\partial}{\partial \beta}\langle E\rangle = \langle E\rangle^2 - \langle E^2\rangle$ (b) $\frac{\partial}{\partial \beta}\langle E\rangle = \langle E^2\rangle - \langle E\rangle^2$

(c) $\frac{\partial}{\partial \beta}\langle E\rangle = \langle E^2\rangle + \langle E\rangle^2$ (d) $\frac{\partial}{\partial \beta}\langle E\rangle = -\left(\langle E^2\rangle + \langle E\rangle^2\right)$

Q7. A classical particle with total energy E moves under the influence of a potential $V(x,y) = 3x^3 + 2x^2y + 2xy^2 + y^3$. The average potential energy, calculated over a long time is equal to,

(a) $\frac{2E}{3}$ (b) $\frac{E}{3}$ (c) $\frac{E}{5}$ (d) $\frac{2E}{5}$

Q8. If two ideal dice are rolled once, what is the probability of getting at least one ‘6’?

(a) $\frac{11}{36}$ (b) $\frac{1}{36}$ (c) $\frac{10}{36}$ (d) $\frac{5}{36}$

Q9. What is the maximum number of extrema of the function $f(x) = P_k(x)e^{-\left(\frac{x^4}{4}+\frac{x^2}{2}\right)}$ where $x \in (-\infty, \infty)$ and $P_k(x)$ is an arbitrary polynomial of degree k?

(a) $k+2$ (b) $k+6$ (c) $k+3$ (d) k

Q10, A chain of mass M and length L is suspended vertically with its lower end touching a weighing scale. The chain is released and falls freely onto the scale. Neglecting the size of the individual links, what is the reading of the scale when a length x of the chain has fallen?

(a) $\frac{Mgx}{L}$ (b) $\frac{2Mgx}{L}$ (c) $\frac{3Mgx}{L}$ (d) $\frac{4Mgx}{L}$

Q11. For non-interacting Fermions in $d-$dimensions, the density of states $D(E)$ varies as $E^{\left(\frac{d}{2}-1\right)}$. The Fermi energy E_F of an N particle system in $3-$, $2-$ and $1-$dimensions will scale respectively as,

(a) $N^2, N^{2/3}, N$ (b) $N, N^{2/3}, N^2$

(c) $N, N^2, N^{2/3}$ (d) $N^{2/3}, N, N^2$

Q12. A particle of mass m moves in $1-$dimensional potential $V(x)$, which vanishes at infinity. The exact ground state eigenfunction is $\psi(x) = A$ such(λx) where A and λ are constants. The ground state energy eigenvalue of this system is,

(a) $E = \frac{\hbar^2 \lambda^2}{m}$ (b) $E = -\frac{\hbar^2 \lambda^2}{m}$

(c) $E = -\frac{\hbar^2 \lambda^2}{2m}$ (d) $E = \frac{\hbar^2 \lambda^2}{2m}$

Q13. Consider a spin$-\frac{1}{2}$ particle characterized by the Hamiltonian $H = \omega S_z$. Under a perturbation $H' = gS_x$, the second order correction to the ground state energy is given by,

(a) $-\frac{g^2}{4\omega}$ (b) $\frac{g^2}{4\omega}$ (c) $-\frac{g^2}{2\omega}$ (d) $\frac{g^2}{2\omega}$

Q14. Given that ψ_1 and ψ_2 are eigenstates of a Hamiltonian with eigenvalues E_1 *and* E_2 respectively, what is the energy uncertainty in the state $(\psi_1 + \psi_2)$?

(a) $-\sqrt{E_1 E_2}$ (b) $\frac{1}{2}|E_1 - E_2|$

(c) $\frac{1}{2}(E_1 + E_2)$ (d) $\frac{1}{\sqrt{2}}|E_2 - E_1|$

Q15. An ideal gas is compressed adiabatically from an initial volume V to a final volume αV and a work W is done on the system in doing so. The final pressure of the gas will be $\left(\gamma = \frac{C_P}{C_V}\right)$

(a) $\frac{W}{V^{\gamma}}\frac{1-\gamma}{\alpha-\alpha^{\gamma}}$ (b) $\frac{W}{V^{\gamma}}\frac{\gamma-1}{\alpha-\alpha^{\gamma}}$

(c) $\frac{W}{V}\frac{1-\gamma}{\alpha-\alpha^{\gamma}}$ (d) $\frac{W}{V}\frac{\gamma-1}{\alpha-\alpha^{\gamma}}$

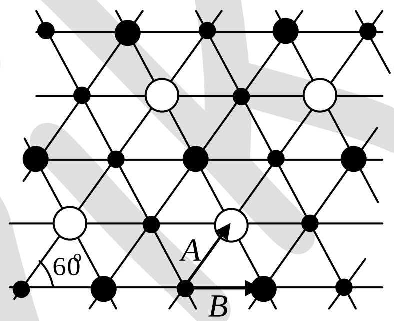
Q16. What is the area of the irreducible Brillouin zone of the crystal structure as given in the figure?

(a) $\frac{2\pi^2}{\sqrt{3}A^2}$

(b) $\frac{\sqrt{3}\pi^2}{2A^2}$

(c) $\frac{2\pi^2}{A^2}$

(d) $\frac{\pi^2}{\sqrt{3}A^2}$



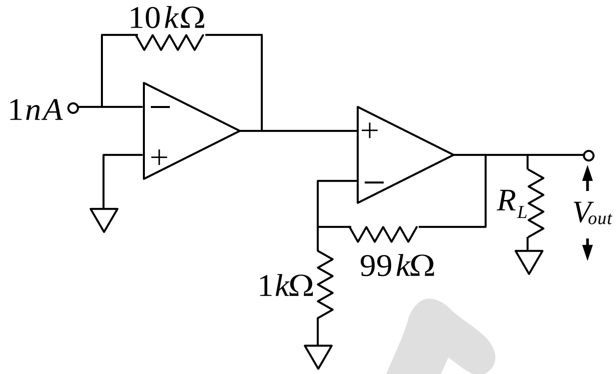
$|A| = |B| = A$

Q17. A particle in thermal equilibrium has only 3 possible states with energies $-\in,\ 0,\ \in$. If the system is maintained at a temperature $T >> \frac{\in}{k_B}$, then the average energy of the particle can be approximated to,

(a) $\frac{2\in^2}{3k_BT}$ (b) $\frac{-2\in^2}{3k_BT}$

(c) $\frac{-\in^2}{k_BT}$ (d) 0

Q18. What is the voltage at the output of the following operational amplifier circuit. [See in the figure]?



(a) $1V$

(b) $1mV$

(c) $1\mu V$

(d) $1nV$

Q19. The energy difference between the $3p$ and $3s$ levels in Na is $2.1\,eV$. Spin-orbit coupling splits the $3p$ level, resulting in two emission lines differing by $6\,\mathring{A}$. The splitting of the 3p level is approximately,

(a) $2\ eV$ (b) $0.2eV$ (c) $0.02eV$ (d) $2meV$

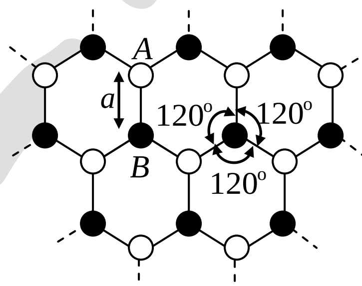
Q20. For a 2- dimensional honeycomb lattice as shown in the figure 3, the first Bragg spot occurs for the grazing angle θ_1 while sweeping the angle from 0°. The next Bragg spot is obtained at θ_2 given by

(a) $\sin^{-1}(3\sin\theta_1)$

(b) $\sin^{-1}\left(\frac{3}{2}\sin\theta_1\right)$

(c) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta_1\right)$

(d) $\sin^{-1}\left(\sqrt{3}\sin\theta_1\right)$



Q21. A spherical shell of inner and outer radii a and b, respectively, is made of a dielectric material with frozen polarization $\vec{P}(r)=\frac{k}{r}\hat{r}$,where k is a constant and r is the distance from the its centre. The electric field in the region $a<r<b$ is,

(a) $\vec{E}=\frac{k}{\epsilon_0 r}\hat{r}$ (b) $\vec{E}=-\frac{k}{\epsilon_0 r}\hat{r}$

(c) $\vec{E}=0$ (d) $\vec{E}=\frac{k}{\epsilon_0 r^2}\hat{r}$

Q22. The electrostatic potential due to a charge distribution is given by $V(r) = A\frac{e^{-\lambda r}}{r}$ where A and λ are constants The total charge enclosed within a sphere of radius $\frac{1}{\lambda}$, with its origin at $r = 0$ is given by,

(a) $\frac{8\pi \in_0 A}{e}$ (b) $\frac{4\pi \in_0 A}{e}$ (c) $\frac{\pi \in_0 A}{e}$ (d) 0

Q23. A bike stuntman rides inside a well of frictionless surface given by $z = a(x^2 + y^2)$,under the action of gravity acting in the negative z direction. $\vec{g} = -g\hat{z}$ What speed should he maintain to be able to ride at a constant height z_0 without falling down?

(a) $\sqrt{gz_0}$

(b) $\sqrt{3gz_0}$

(c) $\sqrt{2gz_0}$

(d) The biker will not be able to maintain a constant height, irrespective of speed.

Q24. A particle of mass m is confined in a potential well given by $V(x) = 0$ for $\frac{-L}{2} < x < \frac{L}{2}$ L/2 and $V(x) = \infty$ elsewhere. A perturbing potential $H'(x) = ax$ has been applied to the system. Let the first and second order corrections to the ground state be $E_0^{(1)}$ and $E_0^{(2)}$, respectively. Which one of the following statements is correct?

(a) $E_0^{(1)} < 0$ and $E_0^{(2)} > 0$ (b) $E_0^{(1)} = 0$ and $E_0^{(2)} > 0$

(c) $E_0^{(1)} > 0$ and $\mathrm{E}_0^{(2)} < 0$ (d) $E_0^{(1)} = 0$ and $\mathrm{E}_0^{(2)} < 0$

Q25. The Bernoulli polynominals $B_n(s)$are defined by, $\frac{xe^{xs}}{e^x - 1} = \sum B_n(s)\frac{x^n}{n!}$. Which one of the following relations is true?

(a) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)\frac{x^n}{(n+1)!}$ (b) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)(-1)^n \frac{x^n}{(n+1)!}$

(c) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(-s)(-1)^n \frac{x^n}{n!}$ (d) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)(-1)^n \frac{x^n}{n!}$

Q26. The skin depth of a metal is dependent on the conductivity (σ) of the metal and the angular frequency ω of the incident field. For a metal of high conductivity, which of the following relations is correct? (Assume that $\sigma >> \in \omega$, where $\in$ is the electrical permittivity of the medium.)

(a) $d \propto \sqrt{\frac{\sigma}{\omega}}$ (b) $d \propto \sqrt{\frac{1}{\sigma\omega}}$

(c) $d \propto \sqrt{\sigma\omega}$ (d) $d \propto \sqrt{\frac{\omega}{\sigma}}$

Q27. The blackbody at a temperature of $6000\,K$ emits a radiation whose intensity spectrum peaks at $600\,nm$. If the temperature is reduced to $300K$, the spectrum will peak at,

(a) $120\mu m$ (b) $12\,\mu m$ (c) $12\,mm$ (d) $120\,mm$

Q28. The wavelength of red helium-neon laser in air is $6328\,\mathring{A}$. What happens to its frequency in glass that has a refractive index of 1.50?

(a) Increases by a factor of 3

(b) Decreases by a factor of 1.5

(c) Remains the same

(d) Decreases by a factor of 0.5

Q29. Which of the following excited states of a hydrogen atom has the highest lifetime?

(a) $2p$ (b) $2s$ (c) $3s$ (d) $3p$

Q30. The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. Which of the following statements is true?

(a) This is a free particle

(b) The particle is experiencing velocity dependent damping

(c) The particle is executing simple harmonic motion

(d) The particle is under constant acceleration.

Q31. A particle moving under the influence of a potential $V(r)=\frac{kr^2}{2}$ has a wavefunction $\psi(r,t)$. If the wavefunction changes to $\psi(\alpha r,t)$, the ratio of the average final kinetic energy to the initial kinetic energy will be,

(a) $\frac{1}{\alpha^2}$ (b) α (c) $\frac{1}{\alpha}$ (d) α^2

Q32. How is your weight affected if the Earth suddenly doubles in radius, mass remaining the same?

(a) Increases by a factor of 4

(b) Increases by a factor of 2

(c) Decreases by a factor of 4

(d) Decreases by a factor of 2

Q33. The approximate force exerted on a perfectly reflecting mirror by an incident laser beam of power $10\,mW$ at normal incidence is

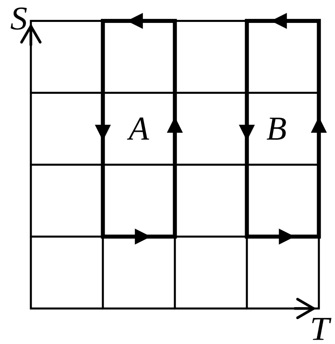
(a) $10^{-13}\,N$ (b) $10^{-11}\,N$ (c) $10^{-9}\,N$ (d) $10^{-15}\,N$

Q34. Which of the following statements is true for the energies of the terms of the carbon atom in the ground state electronic configuration $1s^2 2s^2 2p^2$?

(a) $^3P<^1D<^1S$ (b) $^3P<^1S<^1D$

(c) $^3P<^1F<^1S$ (d) $^3P<^1F<^1D$

Q35. The entropy-temperature diagram of two Carnot engines, A and B, are shown in the figure 4. The efficiencies of the engines are η_A and η_B respectively. Which one of the following equalities is correct?



(a) $\eta_A=\frac{\eta_B}{2}$

(b) $\eta_A=\eta_B$

(c) $\eta_A=3\eta_B$

(d) $\eta_A=2\eta_B$

Q36. The reference voltage of an analog to digital converter is $1\,V$. The smallest voltage step that the converter can record using a 12-bit converter is,

(a) $0.24V$ (b) $0.24mV$ (c) $0.24\mu V$ (d) $0.24nV$

Q37. A spring of force constant k is stretched by x. It takes twice as much work to stretch a second spring by $\frac{x}{2}$. The force constant of the second spring is,

(a) k (b) $2k$ (c) $4k$ (d) $8k$

Q38. Which of the following expressions represents an electric field due to a time varying magnetic field?

(a) $K(x\hat{x}+y\hat{y}+z\hat{z})$ (b) $K(x\hat{x}+y\hat{y}-z\hat{z})$

(c) $K(x\hat{x}-y\hat{y})$ (d) $K(y\hat{y}-x\hat{y}+2z\hat{z})$

Q39. In Millikan's oil drop experiment the electronic charge e could be written as $k\eta^{1.5}$,where κ is a function of all experimental parameters with negligible error. If the viscosity of air η is taken to be 0.4% lower than the actual value, what would be the error in the calculated value of e?

(a) 1.5% (b) 0.7% (c) 0.6% (d) 0.4%

Q40. Given the tight binding dispersion relation $E(k)=E_0+A\sin^2\left(\frac{ka}{2}\right)$, where E_0 and A are constants and a is the lattice parameter. What is the group velocity of an electron at the second Brillouin zone boundary?

(a) 0 (b) $\frac{a}{h}$ (c) $\frac{2a}{h}$ (d) $\frac{a}{2h}$

Q41. The total number of Na^+ and Cl^- ions per unit cell of $NaCl$ is,

(a) 2 (b) 4 (c) 6 (d) 8

Q42. if a Hamiltonian H is given as $H=|0\rangle\langle 0|-|1\rangle\langle 1|+i(|0\rangle\langle 1|-|1\rangle\langle 0|)$, where $|0\rangle$ and $|1\rangle$ are orthonormal states, the eigenvalues of H are

(a) ± 1 (b) $\pm i$ (c) $\pm\sqrt{2}$ (d) $\pm i\sqrt{2}$

Q43. The stable nucleus that has $\frac{1}{3}$ the radius of ^{189}Os nucleus is,

(a) Li (b) ^{16}O (c) ^{4}He (d) ^{14}N

Q44. A charged particle is released at time $t = 0$, from the origin in the presence of uniform static electric and magnetic fields given by $E = E_0\hat{y}$ and $B = B_0\hat{z}$ respectively. Which of the following statements is true for $t > 0$?

(a)The particle moves along the x-axis.

(b) The particle moves in a circular orbit.

(c) The particle moves in the (x, y) plane.

(d) particle moves in the (y, z) plane

Q45. Consider the differential equation $G'(x) + kG(x) = \delta(x)$; where k is a constant. Which following statements is true?

(a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$.

(b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not.

(c) $G(x)$ is discontinuous at $x = 0$.

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depends on the value of k.

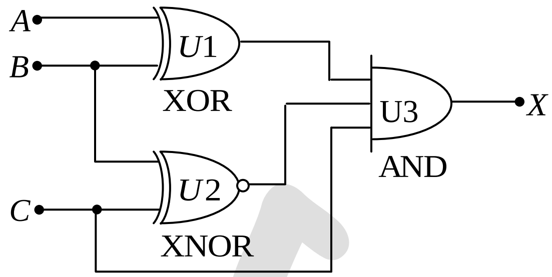
Q46. The sum $\sum_{m=1}^{99} \frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to

(a) 9 (b) $\sqrt{99}-1$ (c) $\frac{1}{\left(\sqrt{99}-1\right)}$ (d) 11

Q47. Let λ be the wavelength of incident light. The diffraction pattern of a circular aperture of dimension r_0 will transform from Fresnel to Fraunhofer regime if the screen distance z is,

(a) $z >> \frac{r_0^2}{\lambda}$ (b) $z >> \frac{\lambda^2}{r_0}$ (c) $z << \frac{\lambda^2}{r_0}$ (d) $z << \frac{r_0^2}{\lambda}$

Q48. For the logic circuit shown in figure 5, the required input condition (A, B, C) to make the output $(X) = 1$ is,



(a) 1,0,1

(b) 0,0,1

(c) 1,1,1

(d) 0,1,1

Q49. The reaction $e^{+} + e^{-} \rightarrow \gamma$ is forbidden because,

(a) lepton number is not conserved

(b) linear momentum is not conserved

(c) angular momentum is not conserved

(d) charge is not conserved

Q50. Electrons of mass m in a thin, long wire at a temperature T follow a one-dimensional Maxwellian velocity distribution. The most probable speed of these electrons is,

(a) $\sqrt{\left(\frac{kT}{2\pi m}\right)}$ (b) $\sqrt{\left(\frac{2kT}{m}\right)}$ (c) 0 (d) $\sqrt{\left(\frac{8kT}{\pi m}\right)}$.