Solution

Full Length Test – 01 (December 2017)

24-11-2017

Ans. 1: (a)

Solution: $\log = \frac{a}{b} + \log\frac{b}{a} = \log(a+b) \Rightarrow \log(a+b) = \log\left(\frac{a}{b}\times\frac{b}{a}\right) = \log 1$

So, $a+b=1$

Ans. 2: (c)

Solution: Area to be plastered $= 2$ (length + breath) × height + (length × breath)

$= 2(25+12)\times 16 + (25\times 12)m^2 = (444+300)m^2 = 744m^2$

$\therefore$ Cost of plastering = Rs. $\left(744\times\frac{75}{100}\right) = Rs.558$

Ans. 3: (b)

Solution: Time from $7\,\text{A.M}$ to $4:15\,\text{P.M}$ $=9$ hrs 15 minutes $=\frac{37}{4}$ hrs.

3 minutes 5 second of this clock $=3$ minutes of a correct clock.

$\Rightarrow \frac{37}{720}$ hrs of this clock $=\frac{1}{20}$ hrs of the correct clock.

$\Rightarrow \frac{37}{4}$ hrs of this clock $=\left(\frac{1}{20}\times\frac{720}{37}\times\frac{37}{4}\right)$ hrs of the correct clock

$=9$ hrs of the correct clock.

$\therefore$ The correct time is 9 hrs after $7\,\text{A.M}$ i.e. $4\,\text{P.M}$.

Ans. 4: (d)

Solution: We may have in the committee: (3 men and 2 women) or (4 men and 1 woman) or (5 men) only.

$\therefore$ Required number $=(7C_3\times 6C_2)+(7C_4\times 6C_1)+7C_5$ number of ways.

$=\left(\frac{7\times6\times5}{3\times2\times1}\times\frac{6\times5}{2\times1}\right)+(7C_3\times 6C_1)+(7C_2)$

$$=525+\left(\frac{7\times6\times5}{3\times2\times1}\times6\right)+\left(\frac{7\times6}{2\times1}\right)=525+210+21=756$$

Ans. 5: (b)

Solution: A's two day's work $=\left(\frac{1}{20}\times2\right)=\frac{1}{10}$,

$(A+B+C)$'s 1 day's work $=\left(\frac{1}{20}+\frac{1}{30}+\frac{1}{60}\right)=\frac{1}{10}$

Work done in 3 days $=\left(\frac{1}{10}+\frac{1}{10}\right)=\frac{1}{5}$.

Now, $\frac{1}{5}$ work is done in 3 days

$\therefore$ whole work will be done in $3\times5=15$ days.

Ans. 6: (c)

Solution: To reach the winning post A will have to cover a distance of $(500\,\text{m}-140\,\text{m})$, i.e. $360\,\text{m}$. While, A cover $3\,\text{m}$, B covers $4m$.

While A covers $360\,\text{m}$, B covers $\left(\frac{4}{3}\times360\right)\text{m}=480\,\text{m}$.

Thus, when A reaches the winning post, B covers $480\,\text{m}$ and therefore, remains $20\,\text{m}$ behind.

$\therefore$ A wins by $20\,\text{m}$.

Ans. 7: (d)

Solution: Average of 20 numbers $=0$

$\therefore$ Sum of 20 numbers $(0\times20)=0$

It is quite possible that 19 of these numbers may be positive and their sum is a then 20^{th} number is $(-a)$.

Ans. 8: (c)

Solution: Let the number of students in rooms A and B be x and y respectively.

Then, $x-10=y+10\Rightarrow x-y=20$ (i)

And, $x+20=2\times(y-20)\Rightarrow x-2y=60$ (ii)

Solving (i) and (ii), we get:

$x=100, y=80$

Hence, the number of students in room $A=100$.

Ans. 9: (a)

Solution: Let x be the number of people with both high blood pressure and high level of cholesterol. Hence, $(15-x)$ will be the number of people with high blood pressure only and $(25-x)$ will be the number of people with high level of cholesterol only. It is clear from the question that the total number of people with high blood pressure only, with high level of cholesterol only and with both is equal to 30.

Hence, $(15-x)+(25-x)+x=30$ or $x=10$

Ans. 10: (a)

Solution: This is just a counting question. We know that the sum of two sides of a triangle is greater than the third one.

Let us assume that the sides of triangle, $a\le b\le c$, when

$a=1$, possible triangle $1,7,7$

$a=2$, possible triangle $2,6,7$

$a=3$, possible triangles $3,6,6$ and $3,5,7$

$a=4$, possible triangles $4,4,7$ and $4,5,6$

$a=5$, possible triangle is $5,5,5$

There are total 7 triangles possible

Ans. 11: (d)

Solution: Let the speed of Deepti be $9S$ steps per second. Speed of Pavitra would be $16S$ steps per second. Let the speed of escalator be x steps per second.

Hence, $(9S+x)\times\frac{30}{9S}=(16S+x)\times\frac{40}{16S}$

On solving we get:

$x=12S$, and so, number of steps $=70$.

Ans. 12: (c)

Solution: As the numbers are co-prime, they contain only 1 as the common factor, also the given two products have the middle number in common.

So, middle number $= H.C.F$ of 551 and $1073 = 29$, first number $\frac{551}{29} = 19$, third number $= \left(\frac{1073}{29}\right) = 37$.

$\therefore$ Required sum $= (19 + 29 + 37) = 85$.

Ans. 13: (c)

Solution: Average of a, b and $c = 20$, so $a + b + c = 60$. As, $a \le b \le c$ and their median $= a + 11$, so, $b = a + 11$.

Theoretically, the least value of c is when $c = b$.

Therefore, $a + (a + 11) + (a + 11) = 60$, ($b$ and c are equal and b, the median, is $a + 11$)

or $3a = 38$ or $a = 12.66$ so $b = c = 12.66 + 11 = 23.66$.

However, we know that these numbers are all integers. Therefore, a, b and c can not take these values. So, the least value for c with this constraint is not likely to be when $c = b$.

Let us increment c by 1. Let $c = b + 1$.

In this scenario, $a + (a + 11) + (a + 12) = 60$ or $3a = 37$.

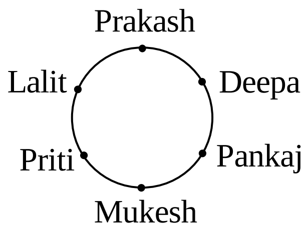
The value of the numbers is not an integer in this scenario as well. Let us increment c again by 1 i.e., $c = b + 2$.

Now, $a + (a + 11) + (a + 13) = 60$ or $3a = 36$ or $a = 12$. If $a = 12, b = 23$ and $c = 25$.

The least value for c, which satisfies all these conditions is 25.

Ans. 14: (d)

Solution:

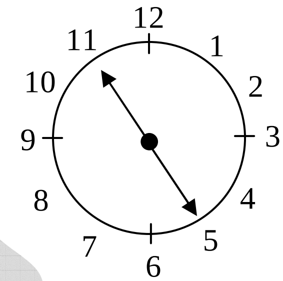


Hence, Lalit is sitting right to Prakash.

Ans. 15: (d)

Solution: At 4 'O clock, the hands of the watch are 20 minutes spaces apart. To be in opposite directions, they must be 30 minute spaces apart.

Minute hand must gain 50 minute spaces. 55 minute spaces are gained in 60 minute. 50 minute spaces are gained in $\frac{60}{55}\times 50 = 54\frac{6}{11}$ minute. Required time $=54\frac{6}{11}$ minute past 4.



Ans. 16: (a)

Solution: Join OE and OD. Internal angle of regular pentagon $=108^0$, $\angle EAB = \angle EDC = 108^0$

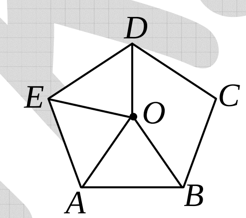
$\angle OAB = 60^0$, $\angle EAO = 48^0$, $AO = OB = AB$, as the triangle is equilateral, $AB = AE$, as this is a regular pentagon.

Triangle AEO is an isosceles $AO = EA$

Let $\angle AEO = \angle AOE = x$, in triangle AEO,

$\angle OAE + 2x = 180^0$ or $48^0 + 2x = 180^0$

or $2x = 132^0 \Rightarrow x = 66^0$



Ans. 17: (a)

Solution: Total number of balls $=(2+3+2)=7$

Let S be the sample space. Then,

$n(5)=$ Number of ways of drawing 2 balls out of 7

$$=7c_2 = \frac{(7\times 6)}{(2\times 1)} = 21$$

Let $E =$ event of drawing 2 balls, none of which is blue.

$\therefore$ Number of ways of drawing 2 balls out ot $2+3=5$ balls

$$=5c_2 = \frac{5\times 4}{2\times 1} = 10$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{10}{21}$$

Ans. 18: (c)

Solution: The first letters are in alphabetical order with a letter skipped in between each segment: C, E, G, I, K. The second and third letters are repeated and they are also in order with a skipped letter: M, O, Q, S, U.

Ans. 19: (d)

Solution: Let distance $= x$ km and usual rate $= y$ kmph.

Then, $\frac{x}{y} - \frac{x}{y+3} = \frac{40}{60}$

$\Rightarrow 2y(y+3) = 9x$ (i)

and $\frac{x}{y-2} - \frac{x}{y} = \frac{40}{60} \Rightarrow y(y-2) = 3x$ (ii)

On dividing (i) by (ii), we get $x = 40$

Ans. 20: (a)

Solution: Milk $= \frac{3}{5} \times 20 = 12$ liters

Water $= 8$ liters

If 10 liters of mixture are removed, amount of milk removed $= 6$ liters and amount of water removed $= 4$ liters.

Remaining milk $= 12 - 6 = 6$ liters,

Remaining water $= 8 - 4 = 4$ liters,

10 liters of pure milk are added. Therefore, total milk $= 6 + 10 = 16$ liters. The ratio of milk and water in the new mixture $= 16 : 4 = 4 : 1$. If the process is repeated one more time and 10 liters of mixture are removed, then amount of milk removed $= \frac{4}{5} \times 10 = 8$ liters.

Amount of water removed $= 2$ liters.

Remaining milk $= (16 - 8) = 8$ liters.

Remaining water $= (4 - 2) = 2$ liters.

The required ratio of milk and water in the final mixture obtained

$= (8 + 10) : 2 = 18 : 2 = 9 : 1$

Ans. 21: (b)

Solution: Taking Laplace transform of both sides of the given differential equation we obtain

$$\left[s^2Y(s)-sx(0)-x'(0)\right]+X(s)=e^{-s}$$

where $X(s)$, is the Laplace transform of $x(t)$.

Using the initial conditions we obtain

$$\Rightarrow \left(s^2+1\right)X(s)=1+e^{-s}$$

$$\Rightarrow X(s)=\frac{1+e^{-s}}{s^2+1}=\frac{1}{s^2+1}+e^{-s}\left(\frac{1}{s^2+1}\right)$$

Taking the inverse transform gives $x(t)=\sin t+u(t-1)\sin(t-1)$

Ans. 22: (d)

Solution: The given differential equation in a linear differential equation

$$\frac{dy}{dx}-\frac{2}{x+1}y=(x+1)^3 \text{ Integrating factor } =e^{-\int\frac{2}{x+1}dx}=e^{-2/n(x+1)}=\frac{1}{(x+1)^2}$$

Hence $\frac{1}{(x+1)^2}=\int(x+1)^3\frac{1}{(x+1)^2}dx+c$

$$\Rightarrow \frac{y}{(x+1)^2}=\int(x+1)^2dx+c\Rightarrow y=(x+1)^2\frac{(x+1)^3}{3}+c(x+1)^2$$

$$\Rightarrow y=\frac{(x+1)^5}{3}+c(x+1)$$

since $y(0)=\frac{1}{3}\Rightarrow c=0$

$$y=\frac{(x+1)^5}{3}\Rightarrow y(2)=81$$

Ans. 23: (d)

Solution: The given function can be written as $f(x)=\begin{cases}1+\frac{2x}{\pi},-\pi\le x\le 0\\ 1-\frac{2x}{\pi},0<x<\pi\end{cases}$

$f(x)$ can be written as $f(x)=1\mp\frac{2}{\pi}|x|, \quad -\pi\le x\le\pi$

For constant function (1) we have $a_0=1,\ a_n=0,\ b_n=0$

For $|x| - \pi\le x\le\pi$

$$a_0=\frac{1}{\pi}\int_0^{\pi}xdx=\frac{1}{\pi}\cdot\frac{\pi^2}{2}=\frac{\pi}{2}$$

$$a_n=\frac{2}{\pi}\int_0^{\pi}x\cos nxdx=\frac{2}{\pi}\left[\frac{x\sin nx}{n}+\frac{1}{n^2}\cos nx\right]_0^{\pi}\Rightarrow a_n=\frac{2}{\pi}-\left[\frac{1}{n^2}(\cos n\pi-1)\right]$$

For even n, $a_n=0$

For odd n, $\quad a_n\frac{2}{\pi}\cdot\frac{1}{n^2}(-2)=-\frac{4}{\pi n^2}$

Thus for the function $f(x)=1-\frac{2}{\pi}|x|$

We have $a_0=1-\frac{2}{\pi}\left(\frac{\pi}{2}\right)=0$

$a_n 0+0=$ for even n

$a_n=0-\frac{2}{\pi}\left(-\frac{4}{\pi n^2}\right)=\frac{8}{\pi^2n^2}$ for odd n.

Hence the Fourier series of $f(x)$ is $f(x)=\frac{8}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos nx$

Putting $x=0$ in this series we obtain $1=\frac{8}{\pi^2}\left(1+\frac{1}{3^2}+\frac{1}{5^2}+....\right)$

$$\Rightarrow 1+\frac{1}{3^2}+\frac{1}{5^2}+.....=\frac{\pi^2}{8}$$

Ans. 24: (c)

Solution: For calculation eigenvalues, we use

$$\begin{vmatrix}2-\lambda & 2 & 1\\ 1 & 3-\lambda & 1\\ 1 & 2 & 2-\lambda\end{vmatrix}=0$$

$$\Rightarrow (2-\lambda)\left[(3-\lambda)(2-\lambda)-2\right]-2\left[(2-\lambda)-1\right]+1\left[2-(3-\lambda)\right]=0$$

$$\Rightarrow (2-\lambda)\left(\lambda^2-5\lambda+6-2\right)-2(1-\lambda)+(-1+\lambda)=0$$

$$\Rightarrow (2-\lambda)\left(\lambda^2-5\lambda+4\right)-2+2\lambda-1+\lambda=0$$

$$\Rightarrow 2\lambda^2-10\lambda+8-\lambda^3+5\lambda^2-4\lambda+3\lambda-3=0$$

$$\Rightarrow -\lambda^3+7\lambda^2-11\lambda+5=0 \Rightarrow \lambda^3-7\lambda^2+11\lambda-5=0$$

Since the eigenvalues are equal, hence

$$\frac{d}{d\lambda}\left(\lambda-7\lambda^2+11\lambda-5\right)=3\lambda^2-14\lambda+11=0$$

$$\Rightarrow 3\lambda^2-3\lambda-11\lambda+11=0 \Rightarrow 3\lambda(\lambda-1)-11(\lambda-1)=0$$

Thus, $\lambda=\frac{11}{3}$ or $\lambda=1$

Only $\lambda=1$ satisfy the original equation.

Thus, $\lambda=1$ is a twice repeated eigenvalue.

Third eigenvalue is $\lambda_3=(2+3+2)-2=5$

Eigenvalues of A^3+2I are

$$\lambda_1=1^3+2.1=3,\ \lambda_2=1^3+2.1=3 \text{ and } \lambda_3=5^3+2.1=127$$

Hence, the eigenvalues of B^{-1} are

$$\lambda_1=\frac{1}{3}, \lambda_2=\frac{1}{3}, \lambda_3=\frac{1}{127}$$

Hence, trace of $B^{-1}=\frac{1}{3}+\frac{1}{3}+\frac{1}{127}=\frac{254+3}{381}=\frac{257}{381}$

Ans. 25: (a)

Solution: $H=ax^2p+\frac{p^2}{2(1+2\beta x)}+\frac{1}{2}\omega^2x^2.$

$$\frac{\partial H}{\partial p}=\dot{x}=ax^2+\frac{p}{(1+2\beta x)} \Rightarrow p=\left(\dot{x}-ax^2\right)(1+2\beta x)\cdot$$

$$L=\dot{x}P-H=\dot{x}P-ax^2P-\frac{p^2}{(1+2\beta x)}-\frac{1}{2}\omega^2x^2\cdot$$

$$= \dot{x}\left(\dot{x}-\alpha x^2\right)(1+2\beta x)-\alpha x^2\left(\dot{x}-\alpha x^2\right)(1+2\beta x)-\frac{\left(\dot{x}-\alpha x^2\right)^2(1+2\beta x)^2}{2(1+2\beta x)}-\frac{1}{2}\omega^2 x^2$$

$$=(1+2\beta x)\left(\dot{x}-\alpha x^2\right)\left[\dot{x}-\alpha x^2-\frac{\left(\dot{x}-\alpha x^2\right)}{2}\right]-\frac{1}{2}\omega^2 x^2$$

$$=(1+2\beta x)\frac{\left(\dot{x}-\alpha x^2\right)^2}{2}-\frac{1}{2}\omega^2 x^2$$

Ans. 26: (a)

Solution: $x_2^1 - x_1^1 = 0$

$$t_2^1 - t_1^1 = 6\times10^{-6}$$

$$t_2 - t_1 = 9\times10^{-6}$$

$$\left(\frac{t_2^1+\frac{v}{c^2}x_2^1}{\sqrt{1-v^2/c^2}}\right)-\frac{\left(t_1^{'}+\frac{v}{c^2}x_1^{'}\right)}{\sqrt{1-v^2/c^2}}=9\times10^{-6}$$

$$\frac{t_2'-t_1'}{\sqrt{1-v^2/c^2}}=9\times10^{-6}\Rightarrow\frac{6\times10^{-6}}{\sqrt{1-v^2/c^2}}=9\times10^{-6}$$

$$v=\sqrt{\frac{5}{9}}c\Rightarrow\sqrt{1-\frac{v^2}{c^2}}=2/3\cdot$$

Ans. 27: (c)

Solution: (1) $\vec{r}(t)=A\cos\omega t\,\hat{i}+B\sin\omega t\,\hat{j}\Rightarrow x=A\cos\omega t,\ y=B\sin\omega t\Rightarrow\frac{x}{A}=\cos\omega t,\ \frac{y}{B}=\sin\omega t$

$$\Rightarrow\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$$

Since $A\neq B$, the orbit of the particle is an ellipse.

(2) $\frac{d\vec{r}}{dt}=-A\omega\sin\omega t\,\hat{i}+B\omega\cos\omega t\,\hat{j}$

Speed $=\left|\frac{d\vec{r}}{dt}\right|=\omega\sqrt{A^2\sin^2\omega t+B^2\cos^2\omega t}$. Speed is function of time, so it is not a constant.

(3) $\frac{d^2\vec{r}}{dt^2}=-A\omega^2\cos\omega t\,\hat{i}-B\omega^2\sin\omega t\,\hat{j}=-\omega^2\vec{r}$. Force act towards origin.

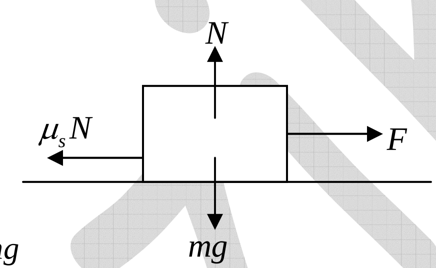
Ans. 28: (c)

Solution: For $n=1$ force is linear so angular frequency is constant

For $n=2,3,4..$ force is non linear so frequency will be dependent on energy as well amplitude.

Ans. 29: (b)

Solution: The forces acting on the block are shown in figure.



$f=\mu_s N$ and $N=mg$

$F=\mu_s Mg$

$$\mu_s=\frac{F}{mg}=\frac{15N}{(2\cdot5\,kg)(10\,m/s^2)}=0\cdot60.$$

$$10\,m=\frac{1}{2}a(5\,s)^2\Rightarrow a=\frac{20}{25}m/s^2=0\cdot8\,m/s^2$$

$f=\mu_k N=\mu_k Mg$

$$F-\mu_k Mg=Ma\Rightarrow\mu_k=\frac{F-Ma}{Mg}=\frac{15N-(2\cdot5\,kg)(0\cdot8\,m/s^2)}{(2\cdot5\,kg)(10\,m/s^2)}=0\cdot52.$$

$$\left(\frac{\mu_s}{\mu_k}\right)=\frac{0.6}{0.52}=1.15$$

Ans. 30: (d)

Solution: $r < R_1, \vec{E}_1 = 0$; $R_1 < r < R_2, \vec{E}_2 = \dfrac{\lambda}{2\pi \in_0 r}\hat{r}$; $r > R_2$, $\vec{E}_3 = 0$

$$W = \frac{\in_0}{2}\int_{\text{all space}} E^2 dz = \frac{\in_0}{2}\int_{R_1}^{R_2} \frac{\lambda^2}{4\pi^2 \in_0^2 r^2} \times 2\pi r l dr$$

$$\frac{W}{l} = \frac{\in_0}{2} \times \frac{\lambda^2}{2\pi \in_0^2}\int_{R_1}^{R_2} \frac{1}{r} dr = \frac{\lambda^2}{4\pi \in_0}\ln\left(\frac{R_2}{R_1}\right) = \frac{\lambda^2}{8\pi \in_0}\ln\left(R_2^2 / R_1^2\right)$$

Ans. 31: (c)

Solution: $B = \dfrac{\mu_0 I}{2}\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} + \dots\dots\dfrac{1}{r_n}\right)$

$r_1 = a$, $r_n = n r_{n-1}$

$r_1 = r_0 = a$, $r_2 = 2r_1 = 2a$, $r_3 = 3r_2 = 3.2a$, $r_4 = 4r_3 = 4.3.2a$

$$\Rightarrow B = \frac{\mu_0 I}{2a}\left(1 + \frac{1}{2} + \frac{1}{3.2} + \frac{1}{4.3.2} + \dots\dots\right)$$

$$B = \frac{\mu_0 I}{2a}\left(\sum_{n=1}^{N} \frac{1}{\lfloor n}\right)$$

Ans. 32: (b)

Solution: $d = \dfrac{1}{\chi} = \dfrac{\lambda_0}{4\pi}$, $\dfrac{\in_I}{\in_R} = \sqrt{3} = \dfrac{\sigma}{\omega \in}$

$$\chi = \omega\sqrt{\frac{\in \mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega \in}\right)^2} - 1\right]^{1/2}$$

$$\chi = \omega\sqrt{\frac{\in \mu}{2}} = \frac{4\pi}{\lambda_0} \Rightarrow \sqrt{\in \mu} = \frac{\sqrt{2}}{\omega}\frac{4\pi}{\lambda_0}$$

$$K = \sqrt{k^2 + \chi^2} = \omega\left[\in \mu\sqrt{1 + \left(\frac{\sigma}{\omega \in}\right)^2}\right]^{1/2}$$

$$\frac{E_0}{B_0} = \frac{\omega}{K} = \frac{\omega}{\omega\left[\in \mu\sqrt{1 + \left(\frac{\sigma}{\omega \in}\right)^2}\right]^{1/2}} = \frac{1}{\sqrt{2 \in \mu}} = \frac{1}{\sqrt{2} \times \frac{\sqrt{2}}{\omega} \times \frac{4\pi}{\lambda_0}} = \frac{\lambda_0 \omega}{8\pi} = \frac{\lambda_0 \times 2\pi c / \lambda_0}{8\pi} = \frac{c}{4}$$

Ans. 33: (d)

Solution: Gauge Transformation $\vec{A}' = \vec{A} + \vec{\nabla}\lambda,\ \phi' = \phi - \frac{\partial \lambda}{\partial t}$

$$\vec{A}' - \vec{A} = -4ke^{-at} r\hat{r} = \vec{\nabla}\lambda = \frac{\partial \lambda}{\partial r}\hat{r}$$

$$\Rightarrow \lambda = -2ke^{-at} r^2 \Rightarrow \frac{\partial \lambda}{\partial t} = 2kae^{-at} r^2$$

$$\Rightarrow \phi' - \phi = -\frac{\partial \lambda}{\partial t} = -2kae^{-at} r^2$$

Ans. 34: (d)

Solution: $I = I_0 \cos^2\theta$ (Malus Law)

$$\Rightarrow I_1 = \frac{I_0}{2}, \quad I_2 = \frac{I_0}{2}\cos^2\theta, \quad I_3 = \frac{I_0}{2}\cos^2\theta \times \cos^2(90-\theta) = \frac{I_0}{8}\sin^2 2\theta.$$

$$\Rightarrow I_3 = \frac{I_0}{8}\sin^2 60 = \frac{I_0}{8}\frac{3}{4} = \frac{3I_0}{32} \approx \frac{I_0}{11}$$

Ans. 35: (d)

Solution: $I_2 = \frac{V}{R_2} = \frac{20}{1} = 20\mu A$

Ans. 36: (a)

Solution: $P = \sqrt{m\omega\hbar}\hat{P} = \sqrt{m\omega\hbar}\frac{(a - a^\dagger)}{\sqrt{2}i}$

$$P^2 = -\frac{m\omega\hbar}{2}\left(a^2 + a^{\dagger 2} - (2N+1)\right)$$

$$\langle P^2 \rangle = -\frac{m\omega\hbar}{2}\left(\langle a^2 \rangle + \langle a^{\dagger 2} \rangle - \langle 2N+1 \rangle\right)$$

For any state $|n\rangle$,

$\langle a^2 \rangle = 0, \langle a^{\dagger 2} \rangle = 0$ and $\langle 2N+1 \rangle = 2n+1$

$$\langle P^2\rangle=(2n+1)\frac{m\omega\hbar}{2} \text{ and } \langle P\rangle=0$$

$$P_{rms}=\sqrt{\langle P^2\rangle-\langle P\rangle^2} \Rightarrow P_{rms}=\sqrt{\frac{m\omega\hbar}{2}}\sqrt{2n+1}$$

Ans. 37: (c)

Solution: $T\propto e^{\frac{-\sqrt{2m(V-E)}}{\hbar^2}}$, where $E=\frac{V_0}{10}$

For potential A, $V=V_0$

$$T_A\propto e^{-\sqrt{\frac{2m}{\hbar^2}\left(V_0-\frac{V_0}{10}\right)}}$$

$$T_A\propto e^{-\sqrt{\frac{2m}{\hbar^2}\left(\frac{9}{10}V_0\right)}}\propto e^{-\sqrt{\frac{2m(0.9V_0)}{\hbar^2}}}$$

For Potential B, $V=4V_0$ and $E=\frac{V_0}{10}$

$$T_B\propto e^{-\sqrt{\frac{2m}{\hbar^2}\left(4V_0-\frac{V_0}{10}\right)}}$$

$$T_B\propto e^{-\sqrt{\frac{2m}{\hbar^2}\left(\frac{39V_0}{10}\right)}}\Rightarrow T_B\propto e^{-\sqrt{\frac{2m(3.99V_0)}{\hbar^2}}}$$

$$\frac{T_A}{T_B}=\frac{e^{-\sqrt{.9V_0}}}{e^{-\sqrt{3.9V_0}}}$$

$$\frac{T_A}{T_B}=\exp\left[\sqrt{3.9}-\sqrt{0.9}\right]$$

Ans. 38: (c)

Solution:, $|\phi_1\rangle=a|n\rangle$ and $|\phi_2\rangle=b|n\rangle+c|m\rangle$

$$\langle\phi_1|\phi_1\rangle=1 \quad\Rightarrow a=\pm1$$

$$\langle\phi_2|\phi_2\rangle=1 \quad\Rightarrow |b|^2+|c|^2=1$$

$$\langle\phi_1|\phi_2\rangle=0 \quad\Rightarrow ab=0 \text{ so } b=0$$

$$|c|^2=1,\quad c=\pm1$$

$a=\pm 1,\qquad b=0,\qquad c=\pm 1$

Ans. 39: (a)

Solution: $H'=bx$ put $x=r\sin\cos\phi$

$H'=br\sin\theta\cos\phi.$

$$E_1^1=\langle\psi_1|H'|\psi_1\rangle,|\psi_1\rangle=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}=\int\psi_1^* H'\psi_1 r^2\sin\theta\, dr\, d\theta\, d\phi$$

$$\frac{b}{\pi a_0^3}\int_0^\infty re^{-\frac{2r}{a_0}}r^2 dr\int_0^\pi \sin^2\theta d\theta\int_0^{2\pi}\cos\phi d\phi=0$$

Ans. 40: (c)

Ans. 41: (c)

Solution: $I_B=\dfrac{V_{CC}-V_{BE}}{R_B+\beta(R_C+R_E)}=\dfrac{12-0}{150+100(3+3)}=0.016\, mA\Rightarrow I_C=\beta I_B=1.6\, mA$

Ans. 42: (c)

Solution: $G=U-TS+PV$

$$dG=dU-TdS-SdT+PdV+VdP=TdS-PdV-TdS-SdT+PdV+VdP$$

$$dG=-SdT+VdP$$

$$\left(\frac{\partial G}{\partial T}\right)_{N,P}=-S \text{ and } \left.\frac{\partial G}{\partial P}\right|_{N,T}=V$$

Ans. 43: (c)

Solution: $V_1+V_2=2V$

$V_2=2V-V_1$

$2P_0V^\gamma=PV_1^\gamma$

$P_0V^\gamma=PV_2^\gamma$

$P_0V^\gamma=P(2V-V_1)^\gamma$

From (i) and (ii), $2=\left(\dfrac{V_1}{2V-V_1}\right)^\gamma$

$$2^{1/\gamma}=\frac{V_1}{2V-V_1}\Rightarrow\frac{1}{2^{1/\gamma}}=\frac{2V-V_1}{V_1}=\frac{2V}{V_1}-1$$

$$V_1=\frac{2V}{\left(1+1/2^{1/\gamma}\right)}$$

put the value of V_1 in (i)

$$2P_0V^\gamma=P\left(\frac{2V}{1+1/2^{1/\gamma}}\right)^\gamma$$

$$P=\frac{2P_0}{2^\gamma}\left(1+1/2^{1/\gamma}\right)^\gamma=\frac{P_0}{2^{\gamma-1}}\left(1+2^{1/\gamma}\right)^\gamma$$

Ans. 44: (a)

Solution: For 3 dimensional system. $P\propto T^4$

$$\frac{P_2}{P_1}=\left(\frac{T}{2T}\right)^4\Rightarrow P_1\left(\frac{1}{2}\right)^4=P_2\Rightarrow P_2=\frac{1}{16}P_1$$

Ans. 45: (c)

Solution: The relative percentage uncertainty in the measurement of force is

$$\sigma_F^2=\left(\frac{\partial F}{\partial V}\right)^2\sigma_V^2$$

$\Rightarrow\sigma_F=\left(\frac{\partial F}{\partial V}\right)\sigma_V$ where σ_V is the uncertainty in the measurement of volume.

$$\because F=V^{1/3}\Rightarrow\frac{\partial F}{\partial V}=\frac{1}{3}V^{-2/3}$$

$$\therefore\ \sigma_F=\frac{1}{3V^{2/3}}\times\sigma_V=\frac{1}{3(30)^{2/3}}\times19.4=\frac{1}{3\times(900)^{1/3}}\times19.4=\frac{1}{3\times9.7}\times2\times9.7\Rightarrow\sigma_F=0.66$$

Ans. 46: (a)

Solution: $\int_{-\infty}^{+\infty}\frac{x}{\left(x^2+1\right)\left(x^2+4\right)}dx$

$$\oint_C\frac{zdz}{\left(z^2+1\right)\left(z^2+4\right)}=\int_{-\infty}^{+\infty}\frac{xdx}{\left(x^2+1\right)\left(x^2+4\right)}+\int_\Gamma\frac{zdz}{\left(z^2+1\right)\left(z^2+4\right)}\left[\int_\Gamma\frac{zdz}{\left(z^2+1\right)\left(z^2+4\right)}=0\right]$$

Poles are $z^2+1=0 \Rightarrow z^2=-1 \Rightarrow z=\pm\sqrt{-1}$

$\therefore z=\pm i$

$z^2+4=0 \Rightarrow z^2=-4 \Rightarrow z=\pm\sqrt{-4}$

$\therefore z=\pm 2i$

$z=i$ and $2i$ are inside c

$$\text{Res}(i)=\lim_{z\to i}\frac{(z-i)z}{(z-i)(z+i)(z^2+4)}=\frac{i}{2i\times 3}=\frac{1}{6}$$

$$\text{Res}(2i)=\lim_{z\to 2i}\frac{(z-2i)z}{(z^2+i)(z-2i)(z+2i)}=-\frac{1}{6}$$

Sum of residues $=0$. Hence the value of integral is zero.

Ans. 47: (b)

Solution: $f(T)=\begin{cases}-1 & -1<x<0\\ 1 & 0<x<1\\ 0 & \text{otherwise}\end{cases}$

$$F[f(T)]=\frac{1}{\sqrt{2\pi}}\left[\int_{-1}^{0}-e^{-i\omega x}dx+\int_{0}^{1}e^{-i\omega x}dx\right]$$

$$=\frac{1}{\sqrt{2\pi}}\left[-\left(\frac{e^{-i\omega x}}{-i\omega}\right)_{-1}^{0}+\left(\frac{e^{-i\omega x}}{-i\omega}\right)_{0}^{1}\right]=\frac{1}{\sqrt{2\pi}}\left[\frac{-1+e^{i\omega}}{-i\omega}+\frac{\cdot e^{-i\omega}-1}{-i\omega}\right]$$

$$=\frac{1}{\sqrt{2\pi}}\left[\frac{2\cos\omega-2}{-i\omega}\right]=\frac{i}{\omega}\sqrt{\frac{2}{\pi}}(\cos\omega-1)$$

Ans. 48: (a)

Solution: $E^2=p^2c^2+m_0^2c^4$.

For constant force $\frac{dP}{dt}=F \Rightarrow P=Ft \Rightarrow t=\frac{P}{F}$

$$\frac{mu}{\sqrt{1-u^2/c^2}}=Ft \Rightarrow u=\frac{(F/m)t}{\sqrt{1+\left(\frac{Ft}{mc}\right)^2}}.$$

$$\frac{dx}{dt}=\frac{\left(\frac{F}{m}\right)t.}{\sqrt{1+\left(\frac{Ft}{mc}\right)^2}}$$

$$x=\frac{F}{m}\int_0^t\frac{tdt}{\sqrt{1+\left(\frac{Ft}{mc}\right)^2}}=\frac{mc^2}{F}\left[\sqrt{1+\left(\frac{Ft}{mc}\right)^2}-1\right]$$

$$\left(\frac{Fx}{mc^2}+1\right)^2=\left(1+\frac{P^2}{m^2c^4}\right)$$

$$P^2=F^2x^2+2mc^2Fx=\left(Fx+mc^2\right)^2-m^2c^4$$

$\left(Fx+mc^2\right)^2-P^2=m^2c^4$ which is equation of hyperbola.

Ans. 49: (d)

Solution: The mass of rod is $m=\int_0^L\lambda_0\left(1+\frac{x^2}{L^2}\right)dx=\frac{4\lambda_0L}{3}$ so (a) is wrong

The centre of gravity of the rod is located at $x_{cm}=\frac{\int_0^L x\lambda_0\left(1+\frac{x^2}{L^2}\right)dx}{\int_0^L\lambda_0\left(1+\frac{x^2}{L^2}\right)dx}=\frac{9L}{16}$

Force equation $T_1+T_2=\frac{4\lambda_0Lg}{3}$

and torque equation $T_1\times\frac{9L}{16}=T_2\times\left(L-\frac{9L}{16}\right)\Rightarrow T_1\times\frac{9L}{16}=T_2\times\frac{7L}{16}\Rightarrow T_2=T_1\times\frac{9}{7}$

Putting $T_2=\frac{9}{7}T_1$ in equation, it gives $T_1+T_2=\frac{4\lambda_0Lg}{3}$

$$\frac{16}{7}T_1=\frac{4\lambda_0Lg}{3}\Rightarrow T_1=\frac{7\lambda_0Lg}{12}$$

$$T_2=\frac{9}{7}\times T_1=\frac{9}{7}\times\frac{7\lambda_0Lg}{12}=\frac{9\lambda_0Lg}{12}$$

Ans. 50: (d)

Solution: (a). $0_1 = \bar{X}Y, 0_2 = XY, 0_3 = 0$

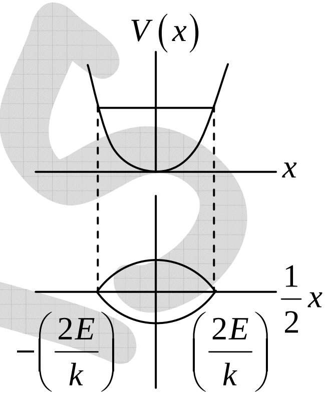
(b). $0_1 = \bar{X}Y, 0_2 = XY, 0_3 = Y$

(c). $0_1 = \bar{X}\bar{Y}, 0_2 = X + \bar{Y}, 0_3 = \bar{X}\bar{Y}$

(d). $0_1 = \bar{X}Y, 0_2 = X\bar{Y}, 0_3 = \bar{X}Y + X\bar{Y} = \overline{X \oplus Y}$ (equality comparator)

Ans. 51: (b)

Solution: $F(x) = -knx^{2n-1} \Rightarrow -\frac{dV}{dx} = -knx^{2n-1} \Rightarrow V = \frac{kx^{2n}}{2} + c$



It is given as $V = 0$ as $x = 0$) so $c = 0$

$$H = \frac{p^2}{2m} + \frac{kx^{2n}}{2}$$

$$\omega = \frac{\partial H}{\partial J} \Rightarrow T = \frac{\partial J}{\partial H} \quad \text{or}$$

Time Period $T = \frac{\partial J}{\partial E}$.

where J is Action variable

$$J = \oint P dx$$

$$J = 4\int_0^{\infty} \sqrt{2m\left(E - V(x)\right)dx}$$

$$\alpha = \left(\frac{2E}{k}\right)^{1/2n}$$

$$J = 4\int_0^{\left(\frac{2E}{k}\right)} \sqrt{2m\left(E - \frac{1}{2}kx^{2n}\right)}dn = 4\sqrt{2mE}\int_0^{\left(\frac{2E}{k}\right)^{1/2n}} \sqrt{1 - \frac{k}{2E}x^{2n}dx}$$

Don't try to solve integration rather try to make E independent.

$$= 4\sqrt{2mE}\left(\frac{2E}{k}\right)^{1/2n}\int_0^1 \sqrt{1 - t^{2n}}dt \qquad \text{Put } \left(\frac{k}{2E}\right)^{\frac{1}{2n}} x = t$$

$$= 4\sqrt{2mE}\left(\frac{2E}{k}\right)^{1/2n}\int_0^1 \sqrt{1 - t^{2n}}dt \qquad dx = \left(\frac{2E}{k}\right)^{\frac{1}{2n}} dt$$

$$J = c4\sqrt{2m}\frac{1}{(k)^{1/2n}}E^{\left(\frac{1+n}{2n}\right)} \text{ where } c = \int_0^1 \sqrt{1-t^{2n}}\,dt$$

$$T \propto \frac{\partial J}{\partial E} \propto (k)^{\frac{-1}{2n}} E^{\left(\frac{1+n}{2n}-1\right)} \propto (k)^{\frac{-1}{2n}} E^{\frac{1-n}{2n}}$$

Ans. 52: (c)

Solution: $\frac{V_o}{V_{in}} = -\frac{z_F}{z_i} = -\frac{R_F \parallel X_{C_F}}{R_i + X_{C_i}} = -\frac{R_F \times \frac{1}{J\omega c_F} / R_F + \frac{1}{J\omega c_F}}{\left(R_i + \frac{1}{J\omega c_i}\right)}$

$$\frac{V_o}{V_{in}} = \frac{-R_F/\left(J\omega c_F R_F + 1\right)}{\left(j\omega c_i R_i + 1\right)/j\omega c_i} = \frac{-R_F}{\left(j\omega c_F R_F + 1\right)} \times \frac{j\omega c_i}{\left(1 + j\omega R_i c_i\right)}$$

$$\Rightarrow \left|\frac{V_o}{V_{in}}\right| = \frac{\omega c_i R_F}{\sqrt{1+\left(\omega c_F R_F\right)^2}\sqrt{1+\left(\omega R_i c_i\right)^2}}, \omega = 2\pi f$$

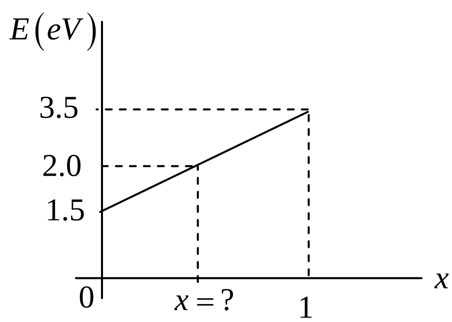
$$\Rightarrow \left|\frac{V_0}{V_{\text{In}}}\right| = \frac{\left(2\pi\times16\times10^3\right)\left(10\times10^{-9}\right)\left(400\times10^3\right)}{\sqrt{1+4\pi^2\left(16\times10^3\right)^2\left(400\times10^3\right)^2\left(50\times10^{-12}\right)^2}\sqrt{1+4\pi^2\left(16\times10^3\right)^2\left(20\times10^3\right)^2\left(10\times10^{-9}\right)^2}}$$

$$\Rightarrow \left|\frac{V_0}{V_{\text{In}}}\right| = \frac{128\pi}{2.12\times20.12} \approx 9.4$$

Ans. 53: (d)

Solution: $E_{g_{GaN}} = 3.5eV$ and $E_{g_{InN}} = 1.5eV$

Band Gap energy of $Ga_x In_{1-x} N$ is $E \propto x$.



For blue light of wavelength $400nm$, the band gap energy is $= \frac{hc}{\lambda} = \frac{1200 \;\; eV.nm}{600nm} = 2.0eV$.

Thus equating slopes we get; $\left(\frac{3.5-1.5}{1-0}\right) = \left(\frac{2.0-1.5}{x-0}\right) \Rightarrow 2x = 0.5 \Rightarrow x = 0.25$

Ans. 54: (b)

Solution: $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$ so $p_3 = 1 - \left(\frac{1}{2} + \frac{1}{3}\right) = \frac{1}{6}$

$$P_1 = \frac{1}{2},\ P_2 = 1/3 \text{ and } P_3 = 1/6 \cdot$$

$$S = -k_B\left(\frac{1}{2}\ln 1/2 + 1/3\ln 1/3 + 1/6\ln 1/6\right) \cdot$$

$$S = -k_b\left(\frac{1}{2}(\ln 1 - \ln 2) + \frac{1}{3}(\ln 1 - \ln 3) + \frac{1}{6}(\ln 1 - \ln 6)\right)$$

$$S = k_B\left[\frac{1}{2}\ln 2 + \frac{1}{3}\ln 3 + \frac{1}{6}\ln 2 + \frac{1}{6}\ln 3\right]$$

$$S = k_B\left[\frac{1}{2}\ln 2 + \frac{1}{6}\ln 2 + \frac{1}{3}\ln 3 + \frac{1}{6}\ln 3\right]$$

$$= k_B\left[\frac{3\ln 2 + \ln 2}{6} + \frac{2\ln 3 + \ln 3}{6}\right] = k_B\left(\frac{4\ln 2}{6} + \frac{3\ln 3}{6}\right)$$

$$S = k_B\left[\frac{2}{3}\ln 2 + \frac{1}{2}\ln 3\right]$$

Ans. 55: (b)

Solution: $E \propto p^s$, where p is momentum

$P = \frac{s}{3}\left(\frac{E}{V}\right)$, where P is pressure

$\frac{P}{E} \propto \frac{s}{3}$.

In problem, $E \propto k^2$, so, $s = 2$

pressure $P = \frac{2}{3}\left(\frac{E}{V}\right)$ at fixed T.

Ans. 56: (b)

Solution: $H = -JS_0(S_1 + S_2) = -J(S_0S_1 + S_0S_2)$ $\quad S_0 = \pm 1\ \ S_1 = \pm S_2 = \pm 1$

S_0	S_1	S_2	E
1	1	1	$-2J$
-1	1	1	$2J$

1	−1	1	0
−1	−1	1	0
1	1	−1	0
−1	1	−1	0
1	−1	−1	$2J$
−1	−1	−1	$-2J$

$E_1 = -2J \quad g_1 = 2$

$E_2 = 2J \quad g_2 = 2$

$E_3 = 0 \quad g_3 = 4$

$$Z = 4 + 2\exp-(2\beta J) + 2\exp 2\beta J \Rightarrow 4(1+\cosh 2\beta J)$$

$$F = -kT\ln Z = -kT\ln\left(4(1+\cosh 2\beta J)\right) = -2kT\ln 2 - kT\ln(1+\cosh 2\beta J)$$

Ans. 57: (a)

Solution: Total power radiated $P = \dfrac{\mu_0 q^2 a^2}{6\pi c}$

Total energy radiated in time t is $E = P\cdot t = \dfrac{\mu_0 e^2 a^2}{6\pi c}\cdot t = \dfrac{\mu_0 e^2 a^2}{6\pi c}\times\dfrac{u}{2a}$

$$\left[\because v = u - at \Rightarrow \frac{u}{2} = u - at \Rightarrow t = \frac{u}{2a}\right]$$

$$\Rightarrow E = \frac{\mu_0 e^2 au}{12\pi c}$$

$$\left[\therefore s = ut - \frac{1}{2}at^2 = u\times\frac{u}{2a} - \frac{1}{2}a\times\frac{u^2}{4a^2} = \frac{u^2}{2a} - \frac{u^2}{8a} = \frac{3u^2}{8a} \Rightarrow a = \frac{3u^2}{8s}\right]$$

$$\Rightarrow E = \frac{\mu_0 e^2 au}{12\pi c} = \frac{\mu_0 e^2 u}{12\pi c}\times\frac{3u^2}{8s} = \frac{3\mu_0 e^2 u^3}{96\pi cs}$$

Ans. 58: (a)

Solution: According to multipole expansion $Q_{mono} = -\dfrac{q}{2} + q - \dfrac{q}{2} + q - q = 0$

$$\vec{p} = q(a\hat{x} + a\hat{y}) - \frac{q}{2}(a\hat{x} + a\hat{y}) - q(a\hat{x} - a\hat{y}) + q(-a\hat{x} - a\hat{y}) - \frac{q}{2}(-a\hat{x} + a\hat{y}) + 0 = 0$$

Thus $V \propto \frac{1}{r^3} \Rightarrow E \propto \frac{1}{r^4} \Rightarrow \frac{E(2r)}{E(r)} = \frac{1}{16}$.

Ans. 59: (a)

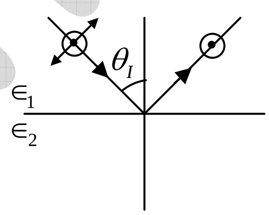
Solution: $I = I_0 e^{-2\kappa z} \Rightarrow z = \frac{1}{2\kappa} \ln\left(\frac{I_0}{I}\right)$

where $\frac{I_0}{I} = 100,\ \kappa = \sqrt{\frac{\sigma\mu\omega}{2}} = \sqrt{\frac{1}{2} \times \frac{1}{8\pi} \times 10^6 \times 4\pi \times 10^{-7} \times 10^7} = 500$

$\Rightarrow z = \frac{1}{2 \times 500} \ln(100) = 4.60\, mm$

Ans. 60: (d)

Solution: $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$



$\theta_B = \tan^{-1}\left(\frac{\sqrt{\in_x}}{\sqrt{\in_1}}\right) = \tan^{-1}\left(\sqrt{3}\right)$

$\Rightarrow \theta_B = 60^0$ (hence reflected light is plane polarized perpendicular to plane of incidence))

Ans. 61: (c)

Solution: $V(r) = \sum_i V_0 a^3 \delta^3(\vec{r} - \vec{r}_i)$

$$= \sum_i V_0 a^3 \delta(x - x_i)\delta(y - y_i)\delta(z - z_i)$$

where x_i, y_i, z_i are co-ordinates of 8 corners of the cube whose center is at the origin.

$$f(\theta) = -\frac{m}{2\pi\hbar^2}\int V(r) d^3 r$$

$$= \frac{-m}{2\pi\hbar^2} V_0 a^3 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sum_{i=1}^{8} \delta(x - x_i)\delta(y - y_i)\delta(z - z_i) dxdydz$$

$$= \frac{-m}{2\pi\hbar^2} V_0 a^3 [1+1+1+1+1+1+1+1]$$

$$= \frac{-8mV_0 a^3}{2\pi\hbar^2} = \frac{-4mV_0 a^3}{\pi\hbar^2}$$

total scattering cross section $\sigma = \int |f(\theta)|^2 \sin\theta d\theta d\phi.$

Differential scattering cross section $D(\theta) = |f(\theta)|^2 = \frac{16m^2 V_0^2 a^6}{\pi^2\hbar^4}$

Ans. 62: (d)

Solution: $\psi(x) = \begin{cases} A(a^2 - x^2), & -a < x < a \\ 0 \quad , & \text{otherwise} \end{cases}$

For normalization

$$\int \psi^* \psi \, dx = 1$$

$$A^2 = \frac{15}{16a^5} \Rightarrow A = \sqrt{\frac{15}{16a^5}}$$

$$\langle T \rangle = \frac{-\hbar^2}{2m} \int_{-a}^{a} \psi^* \frac{\partial^2}{\partial x^2} \psi \, dx = \frac{-\hbar^2}{2m} \frac{15}{16a^5} \cdot (-2)(2) \int_0^a (a^2 - x^2) dx$$

$$\langle T \rangle = \frac{5\hbar^2}{4ma^2}$$

$\langle V \rangle = \int_{-a}^{a} \psi^* V \psi \, dx$, where $V(x) = \frac{1}{2} m\omega^2 x^2$

$$= \frac{1}{2} m\omega^2 \frac{15}{16a^5} 2\int_0^a x^2 (a^2 - x^2)^2 \, dx.$$

$$\langle V \rangle = \frac{m\omega^2 a^2}{14}$$

$$E = T + V = \frac{5\hbar^2}{4ma^2} + \frac{m\omega^2 a^2}{14}$$

$$\frac{dE}{da} = 0 \Rightarrow \frac{5 \times (-2)\hbar^2}{4ma^3} + \frac{m\omega^2 a}{7} = 0 \Rightarrow a^4 = \frac{35}{2}\left(\frac{\hbar^2}{m^2\omega^2}\right).$$

$$a^2 = \left(\frac{35}{2}\right)^{1/2}\left(\frac{\hbar}{m\omega}\right).$$

$$E = \frac{5}{4}\times\frac{\hbar^2}{m}.\frac{m\omega}{\hbar}\sqrt{\frac{2}{35}} + \frac{m\omega^2}{14}\sqrt{\frac{35}{2}}\frac{\hbar}{m\omega}.$$

$$= \frac{\hbar\omega}{2}\left(\frac{5}{2}\sqrt{\frac{2}{35}} + \frac{1}{7}\sqrt{\frac{35}{2}}\right) = \frac{\hbar\omega}{2}\left(\sqrt{\frac{5}{14}} + \sqrt{\frac{5}{14}}\right) = \frac{\hbar\omega}{2}\sqrt{\frac{5\times 4}{14}} = \frac{\hbar\omega}{2}\sqrt{\frac{10}{7}}$$

Ans. 63: (a)

Solution: In Poisson's statistics the probability of finding the value n is given by $P(n) = \frac{\mu^n}{n!}e^{-\mu}$

The mean of Poisson's statistics is μ. From the question

$$P(0) = 10^{-6}$$

$$\Rightarrow 10^{-6} = \frac{\mu^0}{0!}e^{-\mu} \Rightarrow e^{-\mu} = 10^{-6}$$

Talking Log of both sides

$$-\mu = -6\ln 10 \Rightarrow \mu = 6\ln 10$$

Hence the expectation value of n is $\mu = 6\times 2.30 = 13.8 \approx 14$

variance For Poisson distribution is same as expectation value so it is also $13.8 \approx 14$

Ans. 64: (a)

Solution: $\frac{dL_z}{dt} = \frac{1}{i\hbar}[L_z, H] = \frac{1}{i\hbar}\left[L_z, c\vec{\alpha}.\vec{p} + \beta mc^2\right]$

$$i\hbar\frac{dL_z}{dt} = c\left[xp_y, \alpha_x p_x\right] - c\left[yp_x, \alpha_y p_y\right] = \frac{i\hbar c}{i\hbar}\left[p_y\alpha_x - p_x\alpha_y\right] = c\left[p_y\alpha_x - p_x\alpha_y\right]$$

Ans. 65: (a)

Solution: $E = \frac{p_x^2}{2m} + V_0 - \lambda x^2$ The tunneling point is $x = 0$ and $x = \left(\frac{V_0 - E}{\lambda}\right)^{\frac{1}{2}}$

$$\gamma = \frac{1}{\hbar}\int_0^{\left(\frac{V_0-E}{\lambda}\right)^{\frac{1}{2}}}\sqrt{2m\left(V_0 - \lambda x^2 - E\right)}dx = \frac{1}{\hbar}\sqrt{2mE}\int_0^{\sqrt{\frac{V_0-E}{\lambda}}}\left(\frac{V_0}{E} - \frac{\lambda x^2}{E} - 1\right)^{\frac{1}{2}}dx$$

put $\frac{V_0}{E}-1=a^2$

$$\sqrt{\frac{\lambda}{E}}x=a\sin\theta \Rightarrow \sqrt{\frac{\lambda}{E}}dx=a\cos\theta d\theta$$

If $x=0, \quad \theta=0$ and $x=\left(\frac{V_0-E}{\lambda}\right)^{\frac{1}{2}}, \theta=\frac{\pi}{2}$

$$\gamma=\frac{\sqrt{2mE}}{\hbar}\int_0^{\frac{\pi}{2}}\sqrt{a^2-a^2\sin^2\theta}\,a\cos\theta\sqrt{\frac{E}{\lambda}}d\theta=\frac{\sqrt{2mE}}{\hbar}a^2\sqrt{\frac{E}{\lambda}}\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2\theta}\cos\theta d\theta$$

$$=\frac{\sqrt{2mE}}{\hbar}\left(\frac{V_0}{E}-1\right)\sqrt{\frac{E}{\lambda}}\int_0^{\frac{\pi}{2}}\cos^2\theta d\theta=\frac{\sqrt{2mE}}{\hbar}\left(\frac{V_0}{E}-1\right)\sqrt{\frac{E}{\lambda}}\times\frac{1}{2}\times\frac{\pi}{2}$$

$$T\propto\exp\left[\frac{-\pi}{2\hbar}\sqrt{\frac{2m}{\lambda}}(V_0-E)\right]$$

$T\propto\exp\left[-c(V_0-E)\right]$. So $\alpha=1$

Ans. 66: (c)

Solution: For $6p^2$: $s_1=\frac{1}{2}, s_2=\frac{1}{2}\Rightarrow S=0,1$

$l_1=1, l_2=1\Rightarrow L=0,1,2$

The total spin quantum number S is determined by Pauli's exclusion principle for electrons in the same subshell which requires $L+S=$ even

$S=0,\ L=0,\ J=0\Rightarrow\ ^1S_0$

$S=0,\ L=2,\ J=2\Rightarrow\ ^1D_2$

$S=1,\ L=1,\ J=0,1,2\Rightarrow\ ^3P_{0,1,2}$

Therefore, the spectroscopic terms for configuration are

$^1S_0,\ ^3P_{0,1,2},\ ^1D_2$

In magnetic field each level with quantum number J splits into $2J+1$ component with different M_J. For the $6p^2$ levels listed above the total number of sublevels is $1+1+3+5+5=15$

Ans. 67: (c)

Solution: $\Delta\lambda = g\lambda^2 \frac{\mu_B B}{hc}$

where $\lambda = 1849\overset{0}{\text{A}} = 1849\times10^{10}\ m$

$hc = 4\pi\times10^{-7}\,eV\text{-}m$

$\mu_B B = 5.78\times10^{-5}\,eVT^{-1}$

$B = 0.1\,T$

$\therefore \Delta\lambda = 1\times\left(1.849\times10^{-7}\right)^2\times\frac{5.78\times10^{-5}\times0.1}{4\pi\times10^{-7}} = 1.572\times10^{-13}\,m = 0.0016\overset{0}{\text{A}}$

Ans. 68: (b)

Solution: Energy carried by the $\alpha-$ particle is

$$KE_\alpha = \left(\frac{A-4}{A}\right)Q = \frac{228}{232}Q = \frac{57}{58}Q = \frac{57}{58}\times5.2\,MeV = 5.1\,MeV$$

Ans. 69: (d)

Solution: Range for nuclear force between nucleon will be $R = c\Delta t = \frac{\hbar c}{mc^2}$ and $\hbar c = 199\,MeVfm$

$\Rightarrow R = \frac{199\,MeVfm}{1106\frac{MeV}{c^2}\times c^2} \approx 0.18\,fm$

Ans. 70: (c)

Solution: According to Fermi Selection Rule:

$\Delta I = 0, \qquad Parity = No\ Change$

According to Gammow-Teller Selection Rule:

$\Delta I = 0,\pm1, \qquad Parity = No\ Change$

In the β decay process, the transition $2^+ \to 3^+$,

$\Delta I = \pm1, \qquad Parity = No\ Change$.

Ans. 71: (c)

Solution: $n = \frac{N}{V} = \frac{4}{a^3} = \frac{4}{(4.08K)^3} = 5.9\times10^{28}\, m^{-3}$

$$E_f = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} = 5.5\,eV$$

Heat capacity at temperature T is

$$C_{el} = \frac{\pi^2}{2}k_B \times \frac{T}{T_F},\ \text{where } T_F = \frac{E_F}{k_B} = \frac{5.5\,eV}{8.62\times10^{-5}\,eVk^{-1}} = 6.37\times10^4\, k$$

$$C_{el} = \frac{\pi^2}{2}\times 8.62\, m^{-5} \times \frac{298}{6.37\times10^4} \cong 2\times10^{-6}\, eVk^{-1}$$

Ans. 72: (d)

Solution: The cut off frequency of acoustic branch for $\beta_2 > \beta_2$ is

$$\omega_2 = \sqrt{\frac{2\beta_1}{m}} = \sqrt{\frac{2c}{m}}$$

Ans. 73: (a)

Solution: In two dimensions, the area of the Fermi sphere is πk_F^2. The area of k-point is $\frac{2\pi}{a_2}\times\frac{2\pi}{a_1}$. Therefore, taking into account spin degeneracy, we have

$$N = 2\frac{\pi k_F^2}{\left(\frac{2\pi}{a_1}\right)\left(\frac{2\pi}{a_2}\right)}$$

which results in

$$k_F = \left(\frac{2\pi N}{a_1 a_2}\right)^{1/2} = \left(\frac{2\pi\times1}{2\overset{0}{\mathrm{A}}\times4\overset{0}{\mathrm{A}}}\right)^{1/2} = \frac{\sqrt{\pi}}{2}\left(\overset{0}{\mathrm{A}}\right)^{-1} = 0.89\left(\overset{0}{\mathrm{A}}\right)^{-1}$$

Ans. 74: (c)

Solution: $P_2(x)=\frac{1}{2}(3x^2-1)\Rightarrow P_2(-1)=\frac{1}{2}\left[3(-1)^2-1\right]=\frac{1}{2}[3-1]=1$

Ans. 75: (a)

Solution: Taylor's series for $y(x)$ is given by

$$y(x)=1+xy_0'+\frac{x^2}{2}y_0''+\frac{x^3}{6}y_0'''+\frac{x^4}{24}y_0^{iv}.........$$

$$y''(x)=xy'(x)+y(x)\Rightarrow y''(0)=xy'(0)+y(0)=1$$

$$y'''(x)=xy''(x)+2y'(x)\Rightarrow y'''(0)=xy''(x)+2y'(0)=0$$

$$y^{iv}(x)=xy'''+3y''\Rightarrow y^{iv}(0)=xy'''(0)+3y''(0)=3$$

$$y(x)=1+\frac{x^2}{2}+\frac{x^4}{24}(3).........\Rightarrow y(0.1)=1.0050$$
