Solution

IIT - JAM – 2018 (Full Length Test – 01)

12-01-2018

Ans.1: (a)

Solution: Mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Image is inverted, so u and v both are real and negative. Magnification is $1/2$, therefore

$$v=\frac{u}{2}.$$

Given, $u=-30\,cm$, $v=-15\,cm$

$$\therefore \frac{1}{f}=-\frac{1}{15}-\frac{1}{30}=\frac{-1}{10} \Rightarrow f=-10\,cm$$

Ans. 2: (c)

Solution: $E_F \propto (n)^{2/3}$

If n increases 27 times, then $E_F \propto (27n)^{2/3} \Rightarrow E_F \propto 9(n)^{2/3}$

Ans. 3: (c)

Solution: $\because \quad I_R = I_C \Rightarrow \frac{-V-0}{R}=C\frac{d(0-V_0)}{dt} \Rightarrow \frac{dV_0}{dt}=+\frac{V}{RC} \Rightarrow V_0=+\frac{V}{RC}t+c$

Ans. 4: (c)

Solution: $\rho_f LWH_g - \rho LWH_g = 0$

which gives us $h=\frac{\rho}{\rho_f}H=\frac{800\,kg/m^3}{1200\,kg/m^3}(6.0\,cm)=4.0\,cm$

Ans. 5: (d)

Solution: We know that the sum of n th roots of unity is zero.

$$\Rightarrow 1+e^{\frac{2\pi i}{5}}+e^{\frac{4\pi i}{5}}+e^{\frac{6\pi i}{5}}+e^{\frac{8\pi i}{5}}=0$$

$$\Rightarrow e^{\frac{2\pi i}{5}}+e^{\frac{4\pi i}{5}}+e^{\frac{6\pi i}{5}}+e^{\frac{8\pi i}{5}}=-1$$

Thus, $z_1=-1$

$$z_2 = 1 + \frac{i}{2} + \frac{i^2}{4} + \frac{i^3}{8} + \ldots..$$

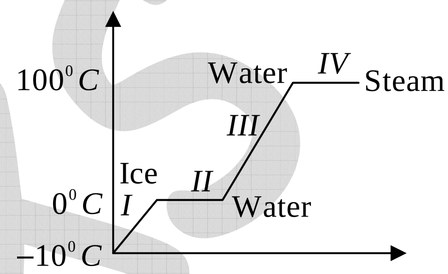
This is a geometric series with common ratio $\frac{i}{2}$.

Thus, $z_2 = \frac{1}{1-\frac{i}{2}} = \frac{2}{2-i}$

Thus, $\frac{z_1}{z_2} = -\frac{(2-i)}{2} = -1 + \frac{i}{2}$

Ans. 6: (a)

Solution: The change of ice at $-10^0 C$ into steam at $100^0 C$ occurs in four stages: it is represented by curve (a).



Ans. 7: (b)

Solution: Stopping potential is the negative potential which stops the emission of $(K.E)_{max}$ electrons when applied.

$\therefore$ Stopping potential = 4 volt

Ans. 8: (d)

Solution: Since in Fourier sine series

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x).\sin nx dx.$$

$$b_3 = \frac{2}{\pi}\int_0^{\pi} e^x \sin 3x dx$$

$$= \frac{2}{\pi}\left(\frac{e^x}{(1+9)}\right)\{\sin 3x - 3\cos 3x\}_0^{\pi}$$

$$= \frac{1}{5\pi}\left[3e^{\pi} + 3\right] = \frac{3}{5\pi}\left(e^{\pi} + 1\right)$$

Ans. 9: (d)

Solution: For a simple pendulum, $T = 2\pi\sqrt{\frac{l}{g}} \therefore \frac{T_2}{T_1} = \sqrt{\frac{g_1}{g_2}}$

Now, $g_1 = \frac{GM}{R^2}, g_2 = \frac{GM}{(2R)^2} = \frac{GM}{4R^2}$

$\therefore \frac{T_2}{T_1} = \sqrt{\frac{GM}{R} \times \frac{4R^2}{GM}} = \sqrt{\frac{4}{1}} = \frac{2}{1} \therefore \frac{T_2}{T_1} = \frac{2}{1}$

Ans. 10: (c)

Solution: De-Broglie wavelength, $\lambda = \frac{h}{p} \quad \therefore \frac{\lambda_1}{\lambda_2} = \frac{p_2}{P_1}$

Since momentum p is conserved in the decay process, $p_2 = p_1 \quad \therefore \frac{\lambda_1}{\lambda_2} = 1$

Ans. 11: (b)

Solution: The average energy of oscillation is

$E(t) = E_0 e^{-\omega_0 t/Q}$

When $E(t) = \frac{E_0}{e}$ for $t = \Gamma$

$\Rightarrow e^{\frac{\omega_0 \Gamma}{Q}} = e$ [Take log on both sides]

$\Rightarrow \frac{\omega_0 \Gamma}{Q} = 1 \Rightarrow \Gamma = \frac{Q}{\omega_0} = \frac{Q}{2\pi\nu_0}$

where $Q = 7000$

$\nu_0 = 400\, Hz$

$\therefore \quad \Gamma = \frac{7000}{2\pi(400)} = 2.8\sec$

Ans. 12: (b)

Solution: $d\vec{F} = I(d\vec{l} \times \vec{B}) = IdlB\sin 90^0 \Rightarrow dF = I(Rd\theta)B_0$ since $dl = Rd\theta$

$\Rightarrow \quad F = \int_{60^0}^{120^0} dF\sin\theta = IB_0R$

(Horizontal component cancels only perpendicular component add up).

Ans. 13: (a)

Solution: $\lambda_P = \frac{1}{\tau}; \lambda_Q = \frac{1}{2\tau}$

If $A = A_0 e^{-\lambda t} \Rightarrow R = -\lambda A_0 e^{-\lambda t} \Rightarrow \dfrac{R_P}{R_Q} = \dfrac{(A_0\lambda_P)e^{-\lambda_P t}}{A_0\lambda_Q e^{-\lambda_Q t}}$

At $t = 2\tau$; $\dfrac{R_P}{R_Q} = \dfrac{2}{e}$, then the value of n is 2.

Ans. 14: (a)

Solution: Wavelength of electrons

$$\lambda = \sqrt{\frac{150}{V}}\left(\overset{0}{\mathrm{A}}\right) = \sqrt{\frac{150}{80}} = 1.37\times10^{-10}\,m$$

and $d_{111} = \dfrac{a}{\sqrt{3}} = \dfrac{3.5\times10^{-10}}{\sqrt{3}} = 2.02\times10^{-10}\,m$

$$\therefore \theta_{111} = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{1.37\times10^{-10}}{2\times2.02\times10^{-10}}\right) = \sin^{-1}(0.34)$$

Ans. 15: (b)

Solution: Since, $y'' - 8y' + 16y = 32t$

Then auxilliary equation is

$m^2 - 8m + 16 = 0$

$\Rightarrow (m-4)^2 = 0 \Rightarrow m = 4, 4$

and $y_p = \dfrac{1}{(D^2 - 8D + 16)}\times 32t = \dfrac{32}{16}\left[1 + \dfrac{(D^2 - 8D)}{16}\right]^{-1} t = 2\left[1 - \dfrac{(D^2 - 8D)}{16}\right]t$

$= 2\left[t - \dfrac{1}{16}(0 - 8\times1)\right] = \left(2t + \dfrac{1}{2}\times 2\right) = (2t+1)$

Hence, $y(t) = (c_1 + c_2 t)e^{4t} + (2t+1)$

Now, $y(0) = 1 \Rightarrow 1 = c_1 + 1 \Rightarrow c_1 = 0$

Hence, $y(t) = c_2 t e^{4t} + (2t+1)$

$y'(t) = c_2\left[t.4e^{4t} + e^{4t}\times1\right] + 2$

$y'(0) = 2 \Rightarrow 2 = c_2 + 2 \Rightarrow c_2 = 0$

Hence, $y(t) = (2t+1)$

$$\Rightarrow y\left(\frac{1}{2}\right)=2$$

Ans. 16: (c)

Solution: The electric field for incident beam at $z=0$ is $E_x=\frac{E_0}{\sqrt{2}}\sin\omega t$ and $E_y=\frac{E_0}{\sqrt{2}}\cos\omega t$

Now, $\theta=\frac{(\mu_0-\mu_e)d\times 2\pi}{\lambda}=\frac{0.17195\times 0.00514\times 2\pi}{5893\times 10^{-7}}\cong 3\pi$

Thus, emergent beam will be

$$E_x=\frac{E_0}{\sqrt{2}}\sin(\omega t-3\pi)=-\frac{E_0}{\sqrt{2}}\sin\omega t$$

$$E_y=\frac{E_0}{\sqrt{2}}\cos\omega t$$

This represent a right circularly polarized. Thus, option (c) is correct.

Ans. 17:(c)

Solution: $\sigma=-\varepsilon_0\frac{\partial V}{\partial r}\bigg|_{r=R}=-\varepsilon_0\times -E_0\left[1+\frac{2R^3}{R^3}\right]\cos\theta=3\varepsilon_0 E_0\cos\theta$

Ans. 18: (c)

Solution: Since $n_i=n=\sqrt{N_cN_v}\exp\left(-\frac{E_g}{2kT}\right)\Rightarrow\frac{n_1}{n_2}=\exp\left[\frac{E_g}{2k}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right]$

$$\Rightarrow E_g=2k\frac{\ln\left(\frac{n_1}{n_2}\right)}{\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}=2\times\frac{1.38\times 10^{-23}}{1.6\times 10^{-19}}\left[\frac{\ln(3\times 10^{14})-\ln(1\times 10^{14})}{6\times 10^{-3}-2\times 10^{-3}}\right]eV\Rightarrow E_g=0.65\,eV$$

Ans. 19: (b)

Solution: The energy of a damped harmonic oscillator at an instant, t is given by $E=E_0e^{-t/\Gamma}$.

Since, $E=E_0/e=E_0e^{-1}\Rightarrow E_0e^{-1}=E_0e^{-t/\Gamma}$.

Where, $1=t/\Gamma\Rightarrow t=\Gamma$

And since, $Q=w_0\Gamma\Rightarrow\Gamma=\frac{Q}{w_0}$

Number of oscillation made by the oscillator in this time,

$$N=\frac{w_0}{2\pi}\times\Gamma=\frac{Q}{2\pi}=8\times10^4/2\pi=12740$$

Ans. 20: (c)

Solution: $\int_{-\infty}^{\infty} f(x)\frac{d}{dx}\text{sgn}(x)dx = f(x)\text{sgn}(x)\Big|_{-\infty}^{\infty}-\int_{-\infty}^{\infty}\frac{df}{dx}\text{sgn}(x)\,dx$

$$=2f(\infty)-\left[-\int_{-\infty}^{0}\frac{df}{dx}dx+\int_{0}^{\infty}\frac{df}{dx}dx\right]$$

$$=2f(\infty)-\left[\left(-f(0)+f(\infty)\right)+\left(f(\infty)-f(0)\right)\right]=2f(0)$$

$$=2f(0)=\int_{-\infty}^{\infty}f(x)2\delta(x)\,dx\Rightarrow\frac{d}{dx}\text{sgn}(x)=2\delta(x)$$

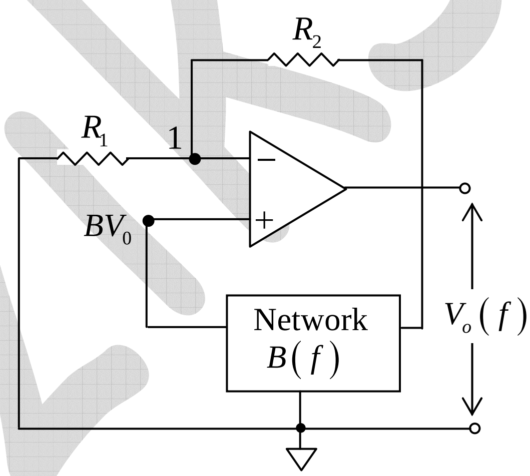
Ans. 21: (b)

Solution: Apply KCL at node 1

$$\frac{V_0-BV_0}{R_2}=\frac{BV_0-0}{R_1}$$

$$\frac{1}{R_2}=B\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$\Rightarrow B=\frac{R_1}{R_1+R_2}=\frac{1}{7}\Rightarrow R_2=6R_1$$



Ans. 22: (b)

Solution: $p:q:r=\frac{a}{h}:\frac{b}{k}:\frac{c}{l}=\frac{1.21}{2}:\frac{1.84}{3}:\frac{1.97}{-1}$

Now, $p:r=\frac{1.21/2}{1.97/-1}=-\frac{1.21}{2}\times\frac{1}{1.97}\Rightarrow\frac{p}{r}=-\frac{1.21}{2\times1.97}$

$$\Rightarrow r=-\frac{2\times1.97}{1.21}p=-\frac{2\times1.97}{1.21}\times1.21=-3.94\,\overset{0}{\text{A}}$$

Ans. 23: (b)

Solution: $\omega_r=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}=\sqrt{\frac{1}{2\times0.25}-\frac{4}{4}}=1.0\Rightarrow f_r=\frac{\omega_r}{2\pi}=0.16\,Hz$

Ans. 24: (c)

Solution: $\left(\frac{T\partial S}{\partial P}\right)_T=-T\left(\frac{\partial V}{\partial T}\right)_P\Rightarrow\left(\frac{\partial Q}{\partial P}\right)_T=-T\frac{V}{V}\left(\frac{\partial V}{\partial T}\right)_P=-T\alpha V$, where $\alpha=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$

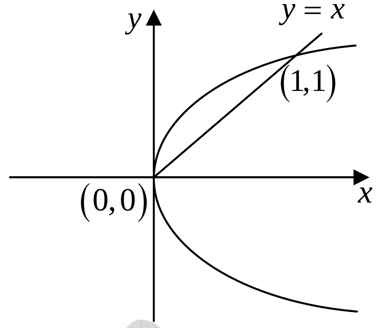
Ans. 25: (a) 0.05

Solution: $\because y = x$ (I)

And $y^2 = x$ (II)

From equations (I) and (II), $x = 0$ or $x = 1$

When $x = 0$, $y = 0$ and when $x = 1$, $y = 1$.



Hence $\int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} xy(x+y)\,dxdy = \int_{x=0}^{1}\left[\frac{x^2y^2}{2} + x\frac{y^3}{3}\right]_x^{\sqrt{x}} dx$

$$= \int_0^1 \left[\frac{1}{2}x^3 + \frac{1}{3}x^{7/2} - \frac{x^4}{2} - \frac{x^4}{3}\right] dx$$

$$= \int_0^1 \left[\frac{x^3}{2} + \frac{x^{5/2}}{3} - \frac{1}{6}\left(5x^4\right)\right] dx = \left(\frac{x^4}{8} + \frac{x^{1/2}}{3\times\frac{7}{2}} - \frac{5}{6}\cdot\frac{x^5}{5}\right)_0^1 = \left(\frac{1}{8} + \frac{2}{21} - \frac{1}{6}\right) = \frac{3}{56} = 0.053$$

Ans. 26: (d)

Solution: $Q_P = \Delta U + W = C_V \Delta T + P\Delta V$

$$= C_V\Delta T + R\Delta T = (C_V + R)\Delta T$$

$$\Rightarrow \frac{\Delta U}{Q_P} = \frac{C_V}{C_V + R} = \frac{C_V}{C_P} \Rightarrow \frac{C_V}{C_P} = \frac{1}{\gamma} = \frac{5}{7}$$

Ans. 27: (d)

Solution: $\psi(x,t) = \frac{1}{\sqrt{5}}\left[|\phi_1\rangle e^{-\frac{i}{\hbar}E_1 t} + 2|\phi_2\rangle e^{-\frac{i}{\hbar}E_2 t}\right]$. Now put $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$, $E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$ and

$$t = \frac{2ma^2}{\pi\hbar}$$

$$\psi(x,t) = \frac{1}{\sqrt{5}}\left[|\phi_1\rangle e^{-i\pi} + 2|\phi_2\rangle e^{-i4\pi}\right] = \frac{1}{\sqrt{5}}[2|\phi_2\rangle - |\phi_1\rangle]$$

Ans. 28: (a)

Solution: Mass per unit area of disc $= \frac{9M}{\pi R^2}$

$\therefore$ Mass of removed portion $= \frac{9M}{\pi R^2}\times\pi\left(\frac{R}{3}\right)^2 = M$

Let moment of inertia of removed portion $= I_1$

$\therefore \; I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^2$, by theorem parallel axis.

$\Rightarrow I_1 = \frac{MR^2}{2}$

Let $I_2 =$ Moment of inertia of the whole disc

$$I_2 = \frac{9MR^2}{2}$$

$\therefore$ Let $I =$ Moment of inertia of remaining disc

$\therefore I = I_2 - I_1$

or, $I = \frac{9MR^2}{2} - \frac{MR^2}{2} = \frac{8MR^2}{2} = 4MR^2$ or $I = 4MR^2$.

Ans. 29: (c)

Solution: From conservation of momentum

$$\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} + 0 = p$$

From conservation of energy

$$\frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} + m_0 c^2 = \sqrt{p^2c^2 + M^2c^4}$$

$$\Rightarrow \left(\frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} + m_0 c^2\right)^2 = p^2c^2 + M^2c^4 = \frac{m_0^2 v^2 c^2}{1-\frac{v^2}{c^2}} + M^2c^4$$

$$\Rightarrow \frac{m_0^2}{1-\frac{v^2}{c^2}} + m_0^2 + 2\frac{m_0^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0^2 v^2}{c^2 - v^2} + M^2$$

$$\Rightarrow \frac{m_0^2 c^2}{c^2 - v^2} + m_0^2 + 2\frac{m_0^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_0^2 v^2}{c^2 - v^2} + M^2$$

$$\Rightarrow \frac{m_0^2c^2}{c^2-v^2}-\frac{m_0^2v^2}{c^2-v^2}+m_0^2+2\frac{m_0^2}{\sqrt{1-\frac{v^2}{c^2}}}=M^2 \Rightarrow \frac{m_0^2\left(c^2-v^2\right)}{c^2-v^2}+m_0^2+2\frac{m_0^2}{\sqrt{1-\frac{v^2}{c^2}}}=M^2$$

$$2m_0^2+\frac{2m_0^2}{\sqrt{1-\frac{v^2}{c^2}}}=M^2 \Rightarrow M=m_0\sqrt{2\left(1+\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)}=m_0\sqrt{2(1+\gamma)}$$

Ans. 30: (a)

Solution: The frictional force provides the necessary centripetal force for circular motion. Linear acceleration $a = L\alpha$

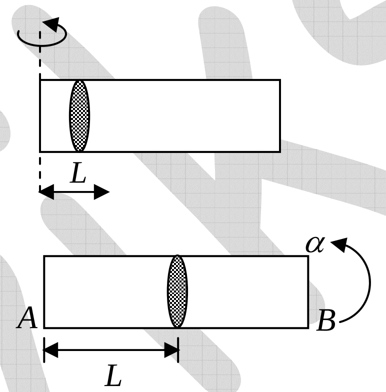
$mL\omega^2 = \mu(ma)$

$\Rightarrow mL\omega^2 = \mu mL\alpha$

$\Rightarrow \omega^2 = \mu\alpha$

$\Rightarrow (\alpha t)^2 = \mu\alpha$

$\Rightarrow \alpha t = \sqrt{\mu\alpha}$ or $t = \sqrt{\frac{\mu}{\alpha}}$.



Ans. 31: (a), (b) and (c)

Solution: For $x = A\sin\omega t$ and $y = a\sin(\omega t + t)$

The resultant motion is described as

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\cos\delta=\sin^2\delta$$

(a) At $\delta = \frac{\pi}{2}$, and $a = b$, we get

$x^2 + y^2 = a^2$, which is equation of circle

Also, $x = a\sin\omega t$ and $y = b\cos\omega t$

This gives clockwise motion of resultant amplitude. Option (a) is correct.

(b) At $\delta = \pi$, we get

$$y = \frac{b}{a}x$$

This is the equation of line. Thus, option (b) is correct.

(c) At $\delta = \frac{5\pi}{2}$, we get

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is equation of ellipse

Also, at $\delta = \frac{5\pi}{2}$

$x = a\sin\omega t$ and $y = b\sin(\omega t + t) = b\cos\omega t$

(d) At $\delta = \frac{9\pi}{2}$, we get

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which is equation of ellipse.

Thus, option (d) is not correct.

Ans. 32: (b), (c), (d)

Ans. 33: (a), (b) and (c)

Solution: $736_8 = 7\times 8^2 + 3\times 8^1 + 6\times 8^0 = 448 + 24 + 6 = 478_{10}$

$673_8 = 6\times 8^2 + 7\times 8^1 + 3\times 8^0 = 384 + 56 + 3 = 443_{10}$

$637_8 = 6\times 8^2 + 3\times 8^1 + 7\times 8^0 = 384 + 24 + 7 = 415_{10}$

$367_8 = 3\times 8^2 + 6\times 8^1 + 7\times 8^0 = 192 + 48 + 7 = 247_{10}$

Ans. 34: (c) , (d)

Solution: We have, $f(x) = \frac{|x-1|}{x^2} = \begin{cases} \frac{x-1}{x^2} &, x \ge 1 \\ \frac{1-x}{x^2} &, x < 1 \end{cases}$

Clearly, $f(x)$ is not differentiable at $x = 1$. So, by definition, $x = 1$ is a critical point.

For points other than $x = 1$, we have $f'(x) = \begin{cases} \frac{-x+2}{x^3} &, x > 1 \\ \frac{x-2}{x^3} &, x < 1 \end{cases}$

Clearly $f'(x) = 0$ at $x = 2$. So, $x = 2$ is also a critical point.

Ans. 35: (a) and (c)

Solution: $\rho_b = -\vec{\nabla}.\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \times kr^2\right) = -4kr$

$$\oint \vec{E}.d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc},\ Q_{enc} = \int \rho_b d\tau = \int_0^d (-4kr)(4\pi r^2 dr) = \frac{-16\pi k}{4}d^4 = -4\pi k d^4.$$

$$\int \vec{E}.d\vec{a} = \int |\vec{E}|\,da = |E|\int da = |E|\times 4\pi d^2 = -\frac{4\pi k d^4}{\varepsilon_0} \Rightarrow \vec{E} = -\frac{kd^2}{\varepsilon_0}\hat{r}$$

Ans. 36: (a) and (d)

Solution: The process is isobaric $P(V_2 - V_1) = 1.01\times 10^5(1.67 - 0.001) = 168.67kJ$

$Q = mL = 1\times 2256\, kg\, kJ/kg = 2256\, kJ$, where L is latent heat of vaporization.

$U = Q - W = 2087kJ$

Ans. 37: (b) (c) and (d)

Solution: The ground state wave function for symmetric potential is $\sqrt{\frac{2}{2a}}\cos\frac{\pi x}{2a}$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is

$$= \int_{-a/2}^{a/2}\sqrt{\frac{2}{2a}}\cdot\sqrt{\frac{2}{2a}}\cos\frac{\pi x}{2a}\cdot\cos\frac{\pi x}{2a}dx = \int_{-a/2}^{a/2}\frac{1}{a}\cos^2\frac{\pi x}{2a}dx = \frac{1}{a}\times\frac{1}{2}\left[\int_{-a/2}^{a/2}\left(1+\cos\frac{2\pi x}{2a}\right)dx\right]$$

$$= \frac{1}{2a}\left[x + \frac{a}{\pi}\sin\frac{\pi x}{a}\right]_{-a/2}^{a/2} = \frac{1}{2a}\left[\frac{a}{2}+\frac{a}{2}+\frac{a}{\pi}(1+1)\right] = \frac{1}{2a}\left[a + \frac{2a}{\pi}\right] = \left(\frac{1}{2}+\frac{1}{\pi}\right)$$

$$E_2 - E_1 = (4-1)\frac{\pi^2\hbar^2}{2m(2a)^2} = \frac{3\pi^2\hbar^2}{8ma^2}$$

Ans. 38: (a), (b) and (d)

Solution: $W = p\Delta V = 0$

$V = C \Rightarrow \rho = C$

Slope $= \frac{P}{T} = \frac{nR}{V} \propto n$ (R, V are constant)

Ans. 39: (a), (b) and (c)

Solution: (a) Given: $\vec{\tau} = \vec{A}\times\vec{L}$

$\vec{\tau} = \frac{d\vec{L}}{dt} \therefore \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$

By the rule of cross – product of vectors, $\frac{d\vec{L}}{dt}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{L}$. Hence option (a) correct

(b) For vector $\vec{L}$, the magnitude of L is constant but $\vec{L}$ is not constant, it changes.

$\therefore$ Let $\vec{L} = (a\cos\theta)\hat{i} + (a\sin\theta)\hat{j}$, where a is a constant. Differentiate it to obtain τ

$\therefore \vec{\tau} = -(a\sin\theta)\hat{i} + (a\cos\theta)\hat{j}$

$\vec{L}\cdot\vec{\tau} = -a^2\sin\theta\cos\theta + a^2\sin\theta\cos\theta$ or $\vec{L}\cdot\vec{\tau} = 0$ or $\vec{L}$ is a perpendicular to τ

A is a constant vector and it is always $\perp$ to τ

Let $\vec{A} = A\hat{K}$

$\vec{L}\cdot\vec{A} = \left[(a\cos\theta)\hat{i} + (a\sin\theta)\hat{j}\right]\cdot\left[A\hat{k}\right]$ or $\vec{L}\cdot\vec{A} = 0$

$\therefore \vec{L}$ is perpendicular to $\vec{A}$

$\therefore$ Component of $\vec{L}$ along $\vec{A}$ is zero. ($\therefore L\cos 90^0 = 0$)

$\therefore$ Component of $\vec{L}$ along $\vec{A}$ does not change with time. Hence option (b) is correct.

(c) By the rule of dot product of vectors, $\vec{L}\cdot\vec{L} = L^2$. Differentiate it $w.r.t$, time

$\therefore \vec{L}\cdot\frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt}\cdot\vec{L} = 2L\frac{dL}{dt}$, where $L = |\vec{L}|$ or, $2\vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{d\vec{L}}{dt}$

Since, $\vec{L}$ is perpendicular to $\frac{d\vec{L}}{dt}$, therefore their dot product is zero.

$\therefore 0 = 2L\frac{dL}{dt}$, this is possible if L is a constant.

$\therefore$ Magnitude of $\vec{L} =$ constant

or, Magnitude of $\vec{L}$ does not change with time. Hence option (c) is correct.

(d) $\vec{L}$ changes with time on account of change in its direction. Magnitude of $\vec{L}$ does not change with time, as shown option (c).

Hence option (a), (b) and (c) are correct. Option (d) is not correct.

Ans. 40: (a), (c) and (d)

Solution: (a) $r_n \propto n^2$. Option (a) is correct

(b) Total energy of electron is

$$T.E = \frac{-13.6Z^2}{n^2}$$

Option (b) is not correct

(c) Angular momentum of electron $= \frac{nh}{2\pi}$

Option (c) is correct

(d) Potential energy of electron $= \left(\frac{-27.2}{n^2}\right)eV$ for hydrogen atom.

Kinetic energy of electrons $= \left(\frac{13.6}{n^2}\right)eV$

$$\therefore |P.E.| = 2\times|K.E.|$$

$$\therefore |P.E.| = \frac{27.2}{n^2}$$

The option (d) is correct

Ans. 41: 1450

Solution: Resulting power is

$$R = \frac{\lambda}{\Delta\lambda} = nN$$

$$\Rightarrow N = \frac{1}{n}\frac{\lambda}{\Delta\lambda} = \frac{1}{2}\times\frac{5800}{2} = 1450$$

Ans. 42: 60

Solution: $I = \frac{P}{A} \Rightarrow P = IA = \frac{1}{2}\varepsilon_0 c E_0^2 \times \pi r^2 = 60\ Watt$.

Ans. 43: -4

Solution: $\because w = x^3 - y^3 - 2xy + 6$

$$\frac{\partial w}{\partial x} = 3x^2 - 2y \text{ and } \frac{\partial w}{\partial y} = -3y^2 - 2x$$

and, $\frac{\partial^2 \omega}{\partial x^2} = 6x$, $\frac{\partial^2 w}{\partial y^2} = -6y$

$\because \frac{\partial w}{\partial x} = 0 \Rightarrow x^2 = \frac{2y}{3}$ (I)

$\frac{\partial w}{\partial y} = 0 \Rightarrow 3y^2 = -2x$ (II)

from (I) and (II) we have

$$3.\left(\frac{3x^2}{2}\right)^2 = -2x$$

$$\Rightarrow \frac{27}{4\times 2}x^3 = -1 \Rightarrow x = \left(\frac{2}{3}\right)(-1)^{1/3}$$

and $y = \frac{3}{2}.x^2 = \frac{3}{2}\times\left(\frac{4}{9}\right)(-1)^{2/3} = \frac{2}{3}$

Hence, $\frac{\partial^2 w}{\partial x^2} = 6x = 6\times\frac{2}{3}(-1)^{1/3} = 4(-1)^{1/3} = -4$

And $\frac{\partial^2 w}{\partial y^2} = 6y = -6.\frac{2}{3} = -4$

Ans. 44: 0.387

Solution: $f = \frac{1}{2\pi}\sqrt{\frac{g+a}{l}}$

Where, $g = 9\cdot 8\, m/\sec^2, a = 2m/\sec^2$ and $l = 2m$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{11\cdot 8}{2}} = \frac{1}{2\pi}\sqrt{5\cdot 9} = 0\cdot 3866$$

Ans. 45: 66.8

Solution: $v_C = V\exp(-t/RC) = 110e^{\left(-\frac{1}{2000\times 10^{-3}}\right)} = 110e^{(-0.5)} = 110(0.607) = 66.8V$

Ans. 46: 5000

Solution: $R_{L_{\max}} = \frac{15}{I_{L_{\min}}}$ where, $I_{L_{\min}} = I_R - I_{ZM} = \frac{50-15}{1} - 32 = 3\ mA$

$$\Rightarrow R_{L_{\max}} = \frac{15}{3} = 5k\Omega = 5000\ \Omega.$$

Ans. 47: 0

Solution: Energy of Fermion, $E_F = 3\times 1\varepsilon_0$, ground state is doubly degenerate so all particle will be in ground state.

Energy of boson, $E_B = 3\times 1\varepsilon_0 = 3\varepsilon_0 \Rightarrow E_F - E_B = 3\varepsilon_0 - 3\varepsilon_0 = 0$

Ans. 48: 3.1

Solution: $P\left(\frac{3\hbar\omega}{2}\right) = \frac{1}{5},\ P\left(\frac{7\hbar\omega}{2}\right) = \frac{4}{5} \Rightarrow \langle E\rangle = \frac{3\hbar\omega}{2}\times\frac{1}{5} + \frac{7\hbar\omega}{2}\times\frac{4}{5} = \frac{31\hbar\omega}{10} = 3.1\hbar\omega$

Ans. 49: 4.16

Solution: $W =$ area of closed curve $ABC = \frac{1}{2}(200-100)\times 10^3 \times (700-500)\times 10^{-6}\, J = 10\, J$

$$J = \frac{W}{Q} = \frac{10}{2.4} = 4.16\, J/cal$$

Ans. 50: 1.94

Solution: For sphere $I = \frac{2}{5}MR^2$

For the recast disc, $I = \frac{Mr^2}{2} + Mr^2$ (by parallel axis theorem)

$$\therefore \frac{2}{5}MR^2 = \frac{3}{2}Mr^2 \text{ or } r = \frac{2}{\sqrt{15}}R \Rightarrow R = \frac{\sqrt{15}}{2}r = 1.94r$$

Ans. 51: 106

Solution: We suppose these are the lowest resonances of the enclosed air column

For one piece, $\lambda = \frac{v}{f} = \frac{343\, m/s}{256\, s^{-1}} = 1\cdot 34\, m$

Length, $d_1 = 0\cdot 67\, m$

For the other piece, $\lambda = \frac{v}{f} = \frac{343\, m/s}{440\, s^{-1}} = 0\cdot 78\, m$

Length, $d_2 = 0\cdot 39\, m$

$\therefore$ Original length, $d_1 + d_2 = 1\cdot 06\, m = 106\, cm$.

Ans. 52: 255

Solution: $I = \oint_{r=a} \vec{H}.d\vec{l} = \int_0^{2\pi} \frac{10^4}{a}\left[\frac{4a^2}{\pi^2}\sin\frac{\pi}{2} - \frac{2a^2}{\pi}\cos\frac{\pi}{2}\right] a\, d\phi$

$$I = 10^4 \times \frac{4a^2}{\pi^2} \times 2\pi = \frac{8 \times 10^4 \times a^2}{\pi} = \frac{800}{\pi} \approx 255\,A$$

Ans. 53: 22.67

Solution: Let the specific heats of liquid A, B and C be respectively C_A, C_B and C_C. When A and B are mixed, equilibrium of the mixture requires that

$$MC_A(16-12) = MC_B(18-16)$$

$$\Rightarrow C_B = 2C_A$$

When B and C are mixed

$$MC_B(23-18) = MC_C(28-23)$$

or, $C_C = C_B = 2C_A$

When A and C are mixed, let the equilibrium temperature be T.

$$MC_A(T-12) = MC_C(28-T) = 2MC_C(28-T)$$

$$\Rightarrow T = 22.67^\circ C.$$

Ans. 54: 120

Solution: R_E is "shorted out" by C_E for the ac analysis. Therefore

$$Z_i = R_B \,||\, \beta r_e = 470k\Omega \,||\, (120)6\Omega \approx 717\Omega, \quad Z_o = R_C = 2.2\,k\Omega$$

$$A_v = -\frac{R_C}{r_e} = -\frac{2.2\,k\Omega}{5.99\,\Omega} \approx -367$$

$$A_i = -A_v \frac{Z_i}{R_L} = -(-367)\frac{717\,\Omega}{2.2\,k\Omega} \approx 120$$

Ans. 55: 2.5

Solution: The whole system of blocks, wires and support have an upward acceleration of $0.2\,m/s^2$.

(i) Tension at midpoint of lower wire:

Let $T_1 =$ Tension

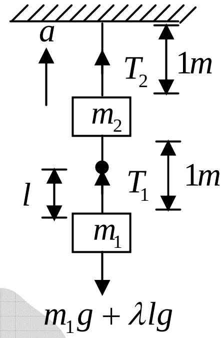
$\lambda =$ Mass of unit length of wire $= 0.2\,kg/m$.

$l =$ Half-length $= 0.5\,m$

$$\therefore T_1 - (m_1 + \lambda l)g = (m_1 + \lambda l)a$$

$$T_1 = (m_1 + \lambda l)(a+g) = [1.9 + (0.2\times 0.5)](0.2+9.8) = 2\times 10 \Rightarrow T_1 = 20N$$

Tension at mid-point of upper wire:



Let $T_2 =$ Tension

$$\therefore T_2 = [m_1 + (\lambda \times 2l) + m_2]a + [m_2 g + \lambda \times 2\lg + m_1 g]$$

or, $T_2 = [m_1 + (\lambda\times 2l) + m_2](a+g)$

$$= [1.9 + (0.2\times 1) + 2.9][0.2+9.8] = 5\times 10 \Rightarrow T_2 = 50N.$$

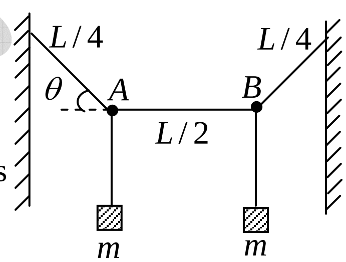
$$\frac{T_2}{T_1} = 2.5$$

Ans. 56: 2.26

Solution: From the free body diagram of point A

$$\sum f_y = 0 \Rightarrow T_1 \sin\theta = mg \text{ and } \sum f_x = 0 \Rightarrow T_1\cos\theta = T$$

Combining these equations to eliminate T_1 gives the tension in the string connecting points A and B as,

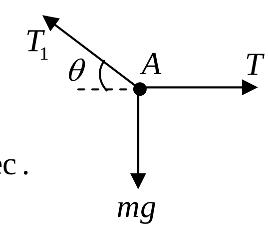


$$T = \frac{Mg}{\tan\theta}$$

The speed of the transverse waves in this segment of string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg/\tan\theta}{m/L}} = \sqrt{\frac{MgL}{m\tan\theta}}$$

and time for a pulse to travel from A to B is,



$$t = \frac{L/2}{v} = \sqrt{\frac{ML\tan\theta}{4mg}} = \sqrt{\frac{2\times 10^{-3}\times 0\cdot 1\times 1}{4\times 1\times 9\cdot 8}} = 2\cdot 26\times 10^{-3}\text{ sec}.$$

Ans. 57: 1.44

Solution: $E = \frac{q^2B^2R^2}{2m_p} \Rightarrow 1.6\times 10^{-13} = \frac{(1.6\times 10^{-19})^2 B^2 (0.1)^2}{2(1.67\times 10^{-27})} \Rightarrow B^2 = \frac{1.6\times 10^{-13}\times 2(1.67\times 10^{-27})}{(1.6\times 10^{-19})^2(0.1)^2}$

$$\Rightarrow B^2 = \frac{10^{-13}\times 2(1.67\times 10^{-27})}{(1.6\times 10^{-38})(0.01)} = \frac{3.34\times 10^{-40}}{1.6\times 10^{-40}} = 2.08 \Rightarrow B = \sqrt{2.08}\,Tesla = 1.44 Tesla$$

Ans. 58: 4

Solution: In cylindrical coordinates $\vec{\nabla}\cdot\vec{A}=\frac{1}{r}\frac{\partial}{\partial r}(rA_r)+\frac{1}{r}\frac{\partial A_\phi}{\partial\phi}+\frac{\partial A_z}{\partial z}$

$$\because A_r=2r\cos^2\phi,\quad A_\phi=3r^2\sin z\ ,A_z=4z\sin^2\phi$$

$$\vec{\nabla}\cdot\vec{A}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\times 2r\cos^2\phi\right)+\frac{1}{r}\frac{\partial\left(3r^2\sin z\right)}{\partial\phi}+\frac{\partial\left(4z\sin^2\phi\right)}{\partial z}$$

$$\Rightarrow\vec{\nabla}\cdot\vec{A}=\frac{1}{r}4r\cos^2\phi+0+4\sin^2\phi=4\left(\cos^2\phi+\sin^2\phi\right)=4$$

Ans. 59: 4

Solution: Since $A^TA=I$

$$\Rightarrow\begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}\begin{bmatrix}a&b&c\\b&c&a\\c&a&b\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}a^2+b^2+c^2&ab+bc+ca&ab+bc+ca\\ab+bc+ca&a^2+b^2+c^2&ab+bc+ca\\ab+bc+ca&ab+bc+ca&a^2+b^2+c^2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Therefore, $a^2+b^2+c^2=1$ and $ab+bc+ca=0$

We know, $a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$

$$\Rightarrow a^3+b^3+c^3=(a+b+c)(1-0)+3$$

$$\Rightarrow a^3+b^3+c^3=(a+b+c)+3$$

Now $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)=1$

$$\Rightarrow(a+b+c)=\pm 1$$

Thus the largest possible value of $a^3+b^3+c^3$ is $1+3=4$.

Ans. 60: 6

Solution: $E=\left(1^2+2^2+3^2\right)\dfrac{\pi^2\hbar^2}{2ma^2}=\dfrac{14\pi^2\hbar^2}{2ma^2}$ have 6 fold degeneracy

n_x, n_y, n_z
1, 2, 3
1, 3, 2
2, 1, 3
2, 3,1
3, 1, 2
3, 2,1