IIT-JAM-2010(PHYSICS)

IMPORTAN NOTE FOR CANDIDTES

- **Attempt ALL 25 questions.**
- **Questions 1-15(objective questions) carry *six* marks each and questions 16-25(subjective questions) carry *twenty one* marks each.**

Q.1 A matrix is given by $M = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of the M are

(a) real and positive
(b) purely imaginary with modulus 1
(c) complex with modulus 1
(d) real and negative

Q.2 The value of θ at which the first-order peak in X-ray (λ =1.53 ´) diffraction corresponding to (111) plane of a simple cubic structure with the lattice constant, α = 2.65 ´, is approximately

(a) 15^o
(b) 30^o
(c) 45^o
(d) 60^o

Q.3 The magnetic field associated with the electric field vector $\vec{E} = E_0 \sin(kz - \omega t)\hat{j}$ is given by

(a) $\vec{B} = -\frac{E_0}{c}\sin(kz - \omega t)\hat{i}$
(b) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{i}$
(c) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{j}$
(d) $\vec{B} = \frac{E_0}{c}\sin(kz - \omega t)\hat{k}$

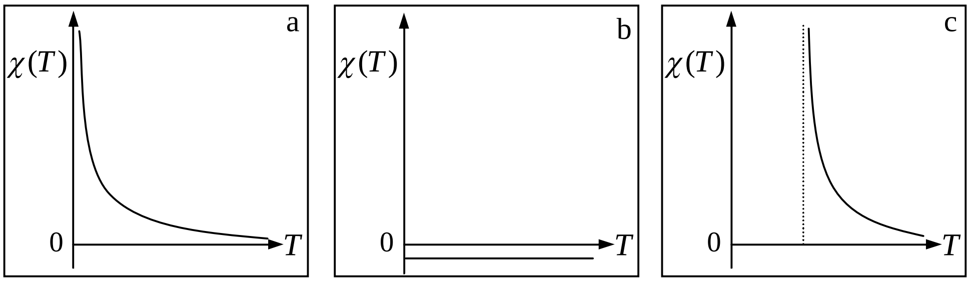
Q.4 Consider the following truth table

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

The logic expression for F is

(a) AB + BC + CA
(b) $\bar{A}B + A\bar{C} + \bar{B}C$
(c) $\overline{C\bar{A}}\,\bar{B} + A\bar{B}$
(d) $\bar{C}(A + \bar{B}) + A\bar{B}$

Q.5 The following are the plots of the temperature dependence of the magnetic susceptibility for three different samples.



The plots a, b and c correspond to

(a) ferromagnet, paramagnet and diamagnet, respectively

(b) paramagnet, diamagnet and ferromagnet, respectively

(c) ferromagnet, diamagnet and paramagnet, respectively

(d) diamagnet, paramagnet and ferromagnet, respectively

Q.6 The equation of a surface of revolution is $z = \pm\sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$

The unit normal to the surface at the point $A\left(\sqrt{\frac{2}{3}},0,1\right)$ is

(a) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$

(b) $\sqrt{\frac{3}{5}}\hat{i} - \frac{2}{\sqrt{10}}\hat{k}$

(c) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$

(d) $\sqrt{\frac{3}{10}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$

Q.7 A gas of molecules each having mass m is in thermal equilibrium at a temperature T. Let v_x, v_y, v_z be the Cartesian components of velocity, $\vec{v}$, of a molecules. The mean value of $\left(v_x - \alpha v_y + \beta v_z\right)^2$ is

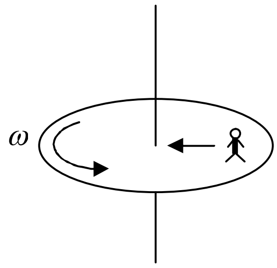
(a) $\left(1+\alpha^2+\beta^2\right)\frac{k_B T}{m}$

(b) $\left(1-\alpha^2+\beta^2\right)\frac{k_B T}{m}$

(c) $\left(\beta^2-\alpha^2\right)\frac{k_B T}{m}$

(d) $\left(\alpha^2+\beta^2\right)\frac{k_B T}{m}$

Q.8 A circular platform is rotating with a uniform angular speed ω counterclockwise about an axis passing through its centre and perpendicular to its plane as shown in the figure. A person of mass m walks radially inward with a uniform speed v on the platform. The magnitude and the direction of the Coriolis force (with respect to the direction along which the person walks) is



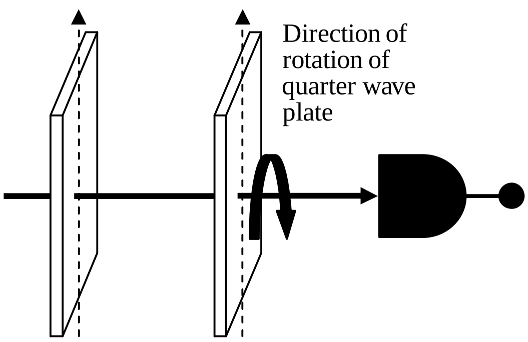
(a) $2m\omega v$ towards his left

(b) $2m\omega v$ towards his front

(c) $2m\omega v$ towards his right

(d) $2m\omega v$ towards his back

Q.9 A quarter-wave plate is placed in between a polarizer and a photo-director. When the optic axis of the quarter-wave plate is kept initially parallel to the pass axis of the polarizer and perpendicular to the direction of light propagation. The intensity of light passing through the quarter-wave plate is measured to be I_0 (see figure). If the quarter wave plate is now rotated by 45^0 about an axis parallel to the light propagation, what would be the intensity of the emergent light measured by the photo-director?



(a) $\dfrac{I_o}{\sqrt{2}}$

(b) $\dfrac{I_0}{2}$

(c) $\dfrac{I_0}{2\sqrt{2}}$

(d) I_0

Q.10 A particle of mass *m*, moving with a velocity $\vec{v} = v_0(\hat{i} + \hat{j})$, collides elastically with another particle of mass 2*m* which is at rest initially. Here, v_0 is a constant. Which of the following statements is correct?

(a) The direction along which the centre of mass moves before collision is $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

(b) The speed of the particle of mass *m* before collision in the center of mass frame is $\sqrt{2}v_0$.

(c) After collision the speed of the particle with mass *2m* in the centre of mass frame is $\frac{\sqrt{2}}{3}v_0$.

(d) The speed of the particle of mass 2*m* before collision in the center of mass frame is $\sqrt{2}v_0$.

Q.11 A trapped air bubble of volume V_0 is released from a depth *h* measured from the water surface in a large water tank. The volume of the bubble grows to 2 V_0 as it reaches just below the surface. The temperature of the water and the pressure above the surface of water (10^5 N/m^2) remain constant throughout the process. If the density of water is 1000 kg/ m^3 and the acceleration due to gravity is 10m/ s^2, then the depth *h* is

(a) 1 m (b) 10 m (c) 50 m (d) 100 m

Q.12 A particle of mass *m* is confined in a two-dimensional infinite square well potential of side *a*. The eigen-energy of the particle in a given state is $E = \frac{25\pi^2\hbar^2}{ma^2}$. The state is

(a) 4-fold degenerate (b) 3-fold degenerate

(c) 2-fold degenerate (d) Non-degenerate

Q.13 For a wave in a medium the angular frequency ω and the wave vector $\vec{k}$ are related by, $\omega^2 = \left(\omega_0^2 + c^2k^2\right)$ where ω_0 and c are constants. The product of group and phase velocities, i.e $v_g.v_p$ is

(a) 0.25 c^2 (b) 0.4 c^2 (c) 0.5 c^2 (d) c^2

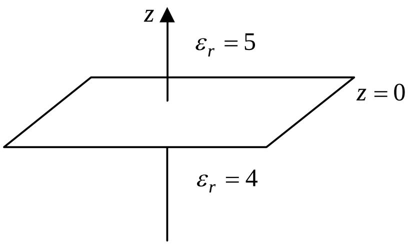
Q.14 Three identical non-interacting particles, each of spin ½ and mass *m*, are moving in a one-dimensional infinite potential well given by,

$$V(x)=\begin{cases}0 & \text{for } 0<x<a \\ \infty & \text{for } x\le 0 \text{ and } x\ge a\end{cases}$$

The energy of the lowest energy state of the system is

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{ma^2}$ (d) $\frac{5\pi^2\hbar^2}{2ma^2}$

Q.15. Assume that $z=0$ plane is the interface between two linear and homogenous dielectrics (see figure). The relative permittivities are $\varepsilon_r=5$ for $z>0$ and $\varepsilon_r=4$ for $z<0$. The electric field in the region $z>0$ is $\vec{E}_1=\left(3\hat{i}-5\hat{j}+4\hat{k}\right)kV/m$. If there are no free charges on the interface, the electric field in the region $z<0$ is given by



(a) $\vec{E}_2=\left(\frac{3}{4}\hat{i}-\frac{5}{4}\hat{j}+\hat{k}\right)kV/m$

(b) $\vec{E}_2=\left(3\hat{i}-5\hat{j}+\hat{k}\right)kV/m$

(c) $\vec{E}_2=\left(3\hat{i}-5\hat{j}-5\hat{k}\right)kV/m$

(d) $\vec{E}_2=\left(3\hat{i}-5\hat{j}+5\hat{k}\right)kV/m$

SUBJECTIVE QUESTIONS

Q.16 A particle of mass 2/3 *kg* is subjected to a potential energy function

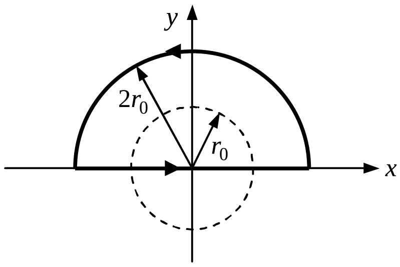
$V(x) = (3x^2 - 2x^3)\ J$, where $x \geq 0$ and expressed in meters.

(a) Sketch $V(x)$ with respect to x in the range 0 to 2 m. Mark the positions of all the maxima and minima. What is the maximum value of the potential energy? **(12)**

(b) Supposing the particle is released at $x = 4/3\ m$, find its velocity when it reaches $x = 3/2$ m. **(9)**

Q.17 A vector field is given by,

$$\vec{F}(\vec{r}) = \begin{cases} a(x\hat{j} - y\hat{i}) & \text{for } (x^2+y^2) \leq r_0^2 \text{ (region - I)} \\ ar_0^2\left(\dfrac{x\hat{j} - y\hat{i}}{x^2+y^2}\right) & \text{for } (x^2+y^2) > r_0^2 \text{ (region - II)} \end{cases}$$

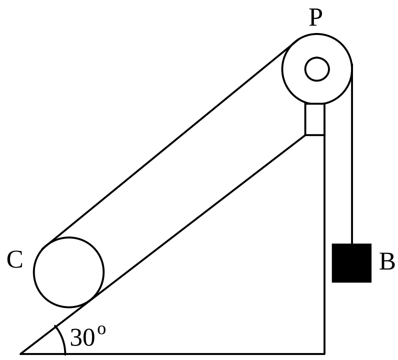


Here a and r_0 are two constants.

(a) Find the curl of this field in both the regions. **(9)**

(b) Find the line integral $\oint \vec{F} \cdot d\vec{l}$ along the closed semicircular path of radius $2r_0$ as shown in the figure below. **(12)**

Q.18 A solid cylinder C of mass 10 *kg* rolls without slipping in an inclined plane which offers friction. The angle of the inclined plane with the horizontal is 30^o as shown in the figure below. A massless inextensible string is wrapped around the cylinder and it passes over a frictionless pulley, P. The other end of the string holds a block B of mass 2 *kg* as shown in the figure. Take g = 10 ms^2.



(a) Draw the free body diagram of the cylinder and the block. If the block B moves down by 0.01 m, how much distance does C move along the incline? **(9)**

(b) If the block B moves with an acceleration of 0.05 g upward, find the magnitude of the frictional force. **(12)**

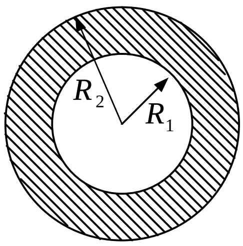
Q.19

(a) A photon of initial momentum p_0 collides with an electron of rest mass m_0 moving with relativistic momentum P and energy E. The change in wavelength of the photon after scattering by an angle θ is given by $\Delta\lambda = 2c\lambda_0 \frac{p_0 + P}{E - cP} \sin^2 \frac{\theta}{2}$, where c is the speed of light and λ_0 is the wavelength of the incident photon before scattering. What will be the value of $\Delta\lambda$ when the electron is moving in a direction opposite to that of the incident photon with momentum P and energy E? Show that the value of $\Delta\lambda$ becomes independent of the wavelength of the incident photon when the electron is at rest before collision.

(12)

(b) In a Compton experiment, the ultraviolet light of wavelength 2000 ´ is scattered from an electron at rest. What should be the minimum resolving power of an optical instrument to measure the Compton shift, if the observation is made at 90° with respect to the direction of the incident light? **(9)**

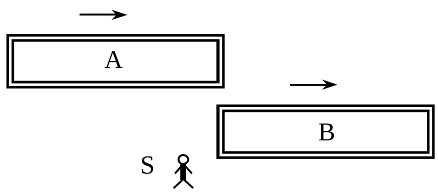
Q.20 A conducting spherical shell of radius R_1 carries a total charge Q. A spherical layer of a linear, homogeneous and isotropic dielectric of dielectric constant K and outer radius R_2 $(>R_1)$ covers the shell as shown in figure.



(a) Find the electric field and the polarization vector $\vec{P}$ inside the dielectric. From this $\vec{P}$, calculate the surface bound charge density, σ_b, on the outer surface of the the dielectric layer and the volume bound charge density ρ_b, inside the dielectric. **(12)**

(b) Calculate the electrostatic energy stored in the region $R_1 \le r \le R_2$. **(9)**

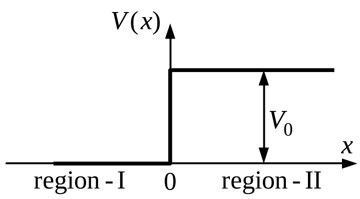
Q.21 Two spaceships A and B of proper length 50 m each are moving with relativistic speeds $0.8c$ and $0.6c$, respectively, in the same direction with respect to a stationary observer S. Here c is the speed of light. At $t = 0$, the spaceship A is just behind the spaceship B as shown below.



(a) Find the time taken by the spaceship A to completely overtake the spaceship B (that is the back of A is in line with the front of B) as seen by the observer S. **(9)**

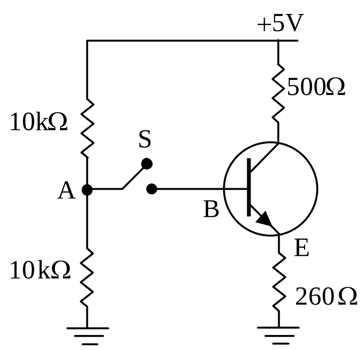
(b) Find the time taken by the spaceship A to completely overtake the spaceship B as seen by an observer in the spaceship B. **(12)**

Q.22 A free particles of mass m with energy $V_0/2$ is incident from left on a step potential of height V_0 as shown in the figure below.



Writing down the time independent Schrödinger equation in both the regions, obtain the corresponding general solutions. Apply the boundary conditions to find the wave functions in both the regions. Determine the reflection coefficient R. What is the transmission coefficient T? **(21)**

Q.23 For the transistor shown in the figure, the dc current gain $\beta_{dc} = 50$ and $V_{BE} = 0.7\ V$. The switch S is initially open.



(a) Calculate the voltage at point A. If the switch S is now closed, what would be the voltage at point A? **(12)**

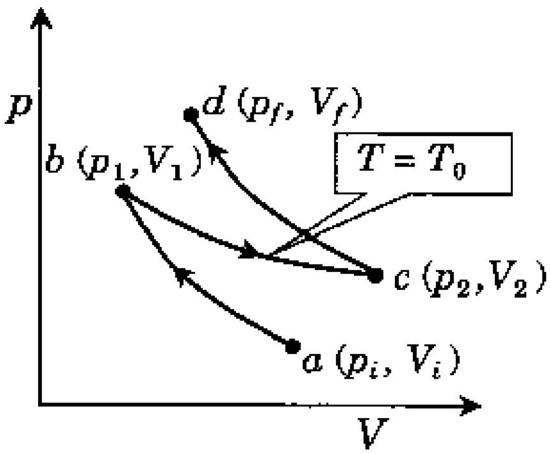
(b) Draw the dc load line and find the Q-point of the circuit with the switch S remaining closed. **(9)**

Q.24 Consider a capacitor placed in free space, consisting of two concentric circular parallel plates of radii r. The separation z between the plates oscillates with a constant frequency ω, i.e. $z(t) = z_0 + z_1 \cos \omega t$. Here z_0 and $z_1 (< z_0)$ are constants. The separation $z(t) (<< r)$ is varied in such a way that the voltage V_0 across the capacitor remains constant.

(a) Calculate the displacement current density and the displacement current between the plates through a concentric circle of radius $r/2$. **(12)**

(b) Calculate the magnetic field vector ($\vec{H}$) between the plates at a distance $r/2$ from the axis of the capacitor. **(9)**

Q.25 One mole of an ideal monatomic gas in an initial state a with pressure, p_i and volume V_i is to be taken to a final state d with $p_f = V_i/B$ through the path a → b → c → d as shown in the figure below for a particular value of B (> 2). Here a → b and c → d are adiabatic paths while b → c is an isotherm with temperature T_0. States b and c correspond to (p_1, V_1) and ((p_2, V_2), respectively.



Find the ratio $\frac{V_2}{V_1}$ and the total work done by the gas in terms of p_i, V_i, T_0 and B.

(21)