<u>**JEST 2013**</u>

PPART A: THREE MARK QUESTIONS

Q1. In an observer's rest frame, a particle is moving towards the observer with an energy E and momentum P. If c denotes the velocity of light in vacuum, the energy of the particle in another frame moving in the same direction as particle with a constant velocity v is

(a) $\dfrac{(E+vp)}{\sqrt{1-(v/c)^2}}$ (b) $\dfrac{(E-vp)}{\sqrt{1-(v/c)^2}}$ (c) $\dfrac{(E+vp)}{\left[1-(v/c)^2\right]^2}$ (d) $\dfrac{(E-vp)}{\left[1-(v/c)^2\right]^2}$

Ans.: (a)

Solution: $t'=\dfrac{t+\dfrac{vx}{c^2}}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow \dfrac{x'}{c}=\dfrac{\dfrac{x}{c}+\dfrac{v}{c^2}x}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow x'=\dfrac{x+\dfrac{v}{c}x}{\sqrt{1-\dfrac{v^2}{c^2}}} \because x=ct,\quad x'=ct'$

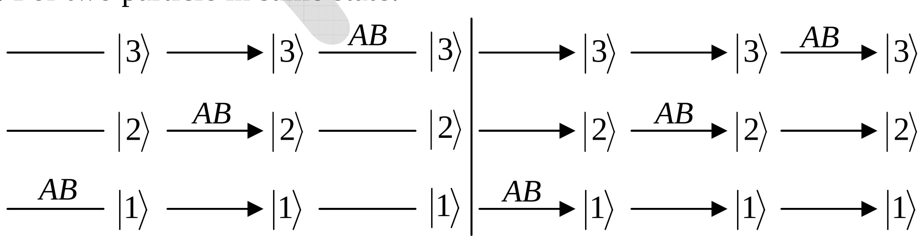
Now $x'=E',\quad x=E \Rightarrow E'=\dfrac{E+\dfrac{E}{c}v}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow E=mc^2,\quad E=Pc \Rightarrow P=\dfrac{E}{c} \Rightarrow E'=\dfrac{E+Pv}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Q2. Consider a system of two particles A and B. each particle can occupy one of three possible quantum states $|1\rangle, |2\rangle$ and $|3\rangle$. The ratio of the probability that the two particles are in the same state to the probability that the two particles are in different states is calculated for bosons and classical (Maxwell- Boltzman) particles. They are respectively

(a) 1,0 (b) 1/2,1 (c) 1,1/2 (d) 0,1/2

Ans.: (c)

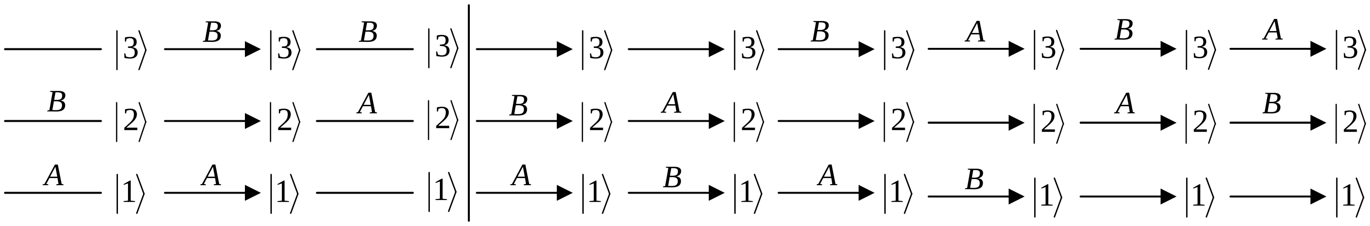
Solution: For two particle in same state:



Boson Classical (Maxwell - Boltzman)

Probability ratio: $\dfrac{1/3}{1/3}=1$

For two particle in different states



Boson Classical (Maxwell-Boltzmann)

Probability ratio: $\frac{1/3}{2/3}=\frac{1}{2}$

Q3. At ‘equilibrium there can not be any free charge inside a metal. However, if you forcibly put charge in the interior then it takes some finite time to ‘disappear’ i.e. move to the surface. If the conductivity, σ, of a metal is $10^6\ (\Omega m)^{-1}$ and the dielectric constant $\varepsilon_0 = 8.85\times 10^{-12}$ Farad/m, this time will be approximately:

(a) 10^{-5} sec (b) 10^{-11} sec (c) 10^{-9} sec (d) 10^{-17} sec

Ans.: (d)

Solution: Characteristic time: $\tau = \frac{\in}{\sigma} = \frac{8.85\times 10^{-12}}{10^6} = 8.85\times 10^{-18}$

Q4. The free fall time of a test mass on an object of mass M from a height $2R$ to R is

(a) $(\pi/2+1)\sqrt{\frac{R^3}{GM}}$ (b) $\sqrt{\frac{R^3}{GM}}$ (c) $(\pi/2)\sqrt{\frac{R^3}{GM}}$ (d) $\pi\sqrt{\frac{2R^3}{GM}}$

Ans.: (a)

Solution: Equation of motion $\frac{md^2r}{dt^2} = -\frac{GMm}{r^2} \Rightarrow \frac{d^2r}{dt^2} = -\frac{GM}{r^2} \Rightarrow \frac{d^2r}{dt^2} = -\frac{A}{r^2} \qquad \because GM = A$

$$v\frac{dv}{dt} = -\frac{A}{r^2}\frac{dr}{dt} \Rightarrow \frac{d}{dt}\left(\frac{v^2}{2}\right) = \frac{d}{dt}\left(\frac{A}{r}\right) \Rightarrow \frac{v^2}{2} = \frac{A}{r} + C$$

when $r = 2R,\ v = 0$

$$\frac{0}{2} = \frac{A}{2R} + C \Rightarrow C = -\frac{A}{2R} \Rightarrow \frac{v^2}{2} = \frac{A}{r} - \frac{A}{2R} \Rightarrow v = \sqrt{\frac{2A}{r} - \frac{2A}{2R}} \Rightarrow \frac{dr}{dt} = \frac{\sqrt{2A}}{\sqrt{2R}}\sqrt{\frac{2R-r}{r}}$$

$$\int_{2R}^{R}\frac{\sqrt{r}}{\sqrt{2R-r}}\,dr = -\sqrt{\frac{A}{R}}\int_0^t dt$$

put $r = u^2,\ dr = 2udu$ when $r = 2R,\ r = R,\ u = \sqrt{2R},\ u = \sqrt{R}$

$$\int_{\sqrt{2R}}^{\sqrt{R}} \frac{u}{\sqrt{2R-u^2}} \times 2udu = -\sqrt{\frac{A}{R}} \int_0^t dt \Rightarrow -\sqrt{\frac{A}{R}} t = 2\int_{\sqrt{2R}}^{\sqrt{R}} \frac{u^2}{\sqrt{2R-u^2}} du$$

$$\Rightarrow -\sqrt{\frac{A}{R}} t = 2\left[-\frac{u}{2}\sqrt{2R-u^2} + \frac{2R}{2}\sin^{-1}\frac{u}{\sqrt{2R}} \right]_{\sqrt{2R}}^{\sqrt{R}}$$

$$\Rightarrow -\sqrt{\frac{A}{R}} t = 2\left[\frac{-\sqrt{R}}{2}\sqrt{2R-R} + \frac{2R}{2}\sin^{-1}\frac{\sqrt{R}}{\sqrt{2R}} + \frac{\sqrt{2R}}{2}\sqrt{2R-2R} - R\sin^{-1}\frac{\sqrt{2R}}{\sqrt{2R}} \right]$$

$$\Rightarrow -\sqrt{\frac{A}{R}} t = 2\left[\frac{-R}{2} + \frac{R\pi}{4} - \frac{R\pi}{2} \right] \Rightarrow t = \frac{R\sqrt{R}}{\sqrt{A}}\left(\frac{\pi}{2}+1\right) \Rightarrow t = \left(\frac{\pi}{2}+1\right)\sqrt{\frac{R^3}{GM}} \quad \because A = GM$$

Q5. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one?

(a) 0.99 (b) 0.925 (c) 0.75 (d) 0.01

Ans.: (c)

Q6. Under a Galilean transformation, the coordinates and momenta of any particle/ system transform as: $t' = t$, $\vec{r}' = \vec{r} + \vec{v}t$ and $\vec{p}' = \vec{p} + m\vec{v}$ where $\vec{v}$ is the velocity of the boosted frame with respect to the original frame. A unitary operator carrying out these transformations for a system having total mass M, total momentum $\vec{P}$ and centre of mass coordinate $\vec{X}$ is

(a) $e^{iM\,\vec{v}.\vec{X}/\hbar} e^{it\vec{v}.\vec{P}/\hbar}$

(b) $e^{iM\,\vec{v}.\vec{X}/\hbar} e^{-it\vec{v}.\vec{P}/\hbar} e^{-iMv^2 t/(2\hbar)}$

(c) $e^{iM\,\vec{v}.\vec{X}/\hbar} e^{it\vec{v}.\vec{P}/\hbar} e^{iMv^2 t/(2\hbar)}$

(d) $e^{it\vec{v}.\vec{P}/\hbar} e^{-iMv^2 t/(2\hbar)}$

Ans.: (b)

Q7. A particle of mass m is contained in a one-dimensional infinite well extending from $x = -L/2$ to $x = L/2$. The particle is in its ground state given by $\varphi_0(x) = \sqrt{2/L}\cos(\pi x/L)$. The walls of the box are moved suddenly to form a box extending from $x = -L$ to $x = L$. what is the probability that the particle will be in the ground state after this sudden expansion?

(a) $(8/3\pi)^2$ (b) 0 (c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

Ans.: (a)

Solution: Probability $\left|\langle\phi_0|\phi_1\rangle\right|^2, \phi_0 = \sqrt{\frac{2}{L}}\cos\frac{\pi x}{L},\ \phi_1\sqrt{\frac{2}{2L}}\cos\frac{\pi x}{2L}$

Since the wall of box are moved suddenly then

$$\text{Probability } = \left|\int_{-L/2}^{L/2}\sqrt{\frac{2}{L}}\cdot\sqrt{\frac{1}{L}}\frac{\cos\pi x}{L}\cdot\frac{\cos\pi x}{2L}dx\right|^2 = \left|\frac{\sqrt{2}}{L}\frac{1}{2}\int_{-L/2}^{L/2}\frac{2\cos\pi x}{L}\cdot\frac{\cos\pi x}{2L}dx\right|^2$$

$$\Rightarrow\left|\frac{\sqrt{2}}{L}\cdot\frac{1}{2}\int_{-L/2}^{L/2}\left[\cos\left(\frac{3\pi x}{2L}\right)+\cos\left(\frac{\pi x}{2L}\right)\right]dx\right|^2 \Rightarrow\left|\frac{\sqrt{2}}{L}\cdot\frac{1}{2}\left[\frac{2L}{3\pi}\sin\frac{3\pi x}{2L}+\frac{2L}{\pi}\sin\frac{\pi x}{2L}\right]_{-L/2}^{L/2}\right|^2$$

$$\Rightarrow\left|\frac{\sqrt{2}}{L}\cdot\frac{1}{2}\left[\frac{2L}{3\pi}\left(\sin\frac{3\pi}{4}+\sin\frac{3\pi}{4}\right)+\frac{2L}{\pi}\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{4}\right)\right]\right|^2$$

$$\Rightarrow\left|\frac{2}{3\pi}+\frac{2}{\pi}\right|^2 = \left|\frac{8}{3\pi}\right|^2$$

Q8. The electric fields outside ($r > R$) and inside ($r < R$) a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r<R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$, q being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is

(a) 1:3 (b) 9:16 (c) 3:8 (d) 5:6

Ans.: (d)

Solution: Electrostatic energy in spherical shell $w_{sp} = \frac{\epsilon_0}{2}\int_0^R\left|\vec{E}_1\right|^2 4\pi r^2 dr + \frac{\epsilon_0}{2}\int_R^\infty\left|\vec{E}_2\right|^2 4\pi r^2 dr$

$$\Rightarrow\frac{\epsilon_0}{2}\int_R^\infty\frac{q^2}{\left(4\pi\epsilon_0\right)^2 r^4}4\pi r^2 dr = \frac{q^2}{8\pi\epsilon_0}\left(-\frac{1}{r}\right)_R^\infty = \frac{q^2}{8\pi\epsilon_0}\frac{1}{R}$$

Electrostatic energy in solid sphere $w_s = \frac{\epsilon_0}{2}\int_0^R\left|E_1\right|^2 4\pi r^2 dr + \frac{\epsilon_0}{2}\int_R^\infty\left|E_2\right|^2 4\pi r^2 dr$

$$\Rightarrow\frac{q^2}{8\pi\epsilon_0}\times\frac{1}{R^6}\left[\frac{r^5}{5}\right]_0^R + \frac{q^2}{8\pi\epsilon_0}\left[-\frac{1}{r}\right]_R^\infty$$

$$w_s = \frac{q^2}{5\times 8\pi \in_0}\cdot\frac{1}{R} + \frac{q^2}{8\pi \in_0 R} = \frac{6q^2}{40\pi \in_0 R}$$

Now $\frac{W_{spherical}}{W_{sphere}} = \frac{\frac{q^2}{8\pi \in_0}}{\frac{6q^2}{40\pi \in_0 R}} = \frac{5}{6}$

Q9. A quantum mechanical particle in a harmonic oscillator potential has the initial wave function $\psi_0(x)+\psi_1(x)$, where ψ_0 and ψ_1 are the real wavefunctions in the ground and first excited state of the harmonic oscillator Hamiltonian. For convenience we take $m=\hbar=\omega=1$ for the oscillator. What is the probability density of finding the particle at x at time $t=\pi$?

(a) $(\psi_1(x)-\psi_0(x))^2$

(b) $(\psi_1(x))^2-(\psi_0(x))^2$

(c) $(\psi_1(x)+\psi_0(x))^2$

(d) $(\psi_1(x))^2+(\psi_0(x))^2$

Ans.: (a)

Solution: $\psi(x)=\psi_0(x)+\psi_1(x)$

$$\psi(x,t)=\psi_0(x)e^{-i\frac{E_0 t}{\hbar}}+\psi_1(x)e^{-i\frac{E_1 t}{\hbar}}$$

Now probability density at time t

$$|\psi(x,t)|^2=\psi^*(x,t)\psi(x,t)=|\psi_0(x)|^2+|\psi_1(x)|^2+2\,\mathrm{Re}\,\psi_0^*(x)\psi_1(x)\cos(E_1-E_0)\frac{t}{\hbar}$$

putting $t=\pi$

$$|\psi(x,t)|^2=|\psi_0(x)|^2+|\psi_1(x)|^2+2\,\mathrm{Re}\,\psi_0^*(x)\psi_1(x)\cos\pi \qquad \because E_1-E_0=\hbar\omega=1$$

$$|\psi(x,t)|^2=|\psi_0(x)|^2+|\psi_1(x)|^2-2\,\mathrm{Re}\,\psi_0^*(x)\psi_1(x)=\left[\psi_1(x)-\psi_0(x)\right]^2$$

Q10. A spherical planet of radius R has a uniform density ρ and does not rotate. If the planet is made up of some liquid, the pressure at point r from the center is

(a) $\frac{4\pi\rho^2 G}{3}(R^2-r^2)$

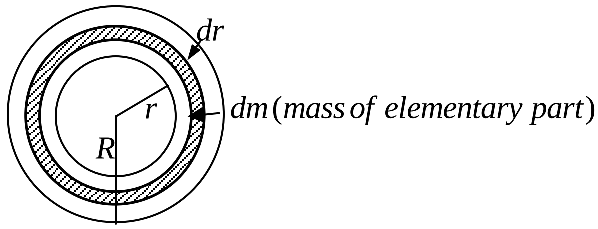
(b) $\frac{4\pi\rho G}{3}(R^2-r^2)$

(c) $\frac{2\pi\rho^2 G}{3}(R^2-r^2)$

(d) $\frac{\rho G}{2}(R^2-r^2)$

Ans.: (c)

Solution: Pressure $dp = \frac{dm \cdot g}{A} \Rightarrow dp = \frac{dm \cdot g}{4\pi r^2} \Rightarrow dp = \frac{\rho \cdot 4\pi r^2 drGM \frac{r}{R^3}}{4\pi r^2}$



$$\Rightarrow dp = \frac{\rho \cdot 4\pi r^2 drG \cdot \rho \cdot \frac{4\pi}{3} R^3 \frac{r}{R^3}}{4\pi r^2} \Rightarrow dp = \frac{4\pi}{3}\rho^2 Grdr$$

$$\int dp = \int_r^R \frac{4\pi}{3}\rho^2 Grdr \Rightarrow p = \frac{4\pi}{3}\rho^2 G\left(\frac{r^2}{2}\right)_r^R \Rightarrow p = \frac{4\pi}{3}\rho^2 G\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

$$\Rightarrow p = \frac{4\pi}{3}\frac{\rho^2 G}{2}\left(R^2 - r^2\right) \Rightarrow p = \frac{2\pi}{3}\rho^2 G\left(R^2 - r^2\right)$$

Q11. Compute $\lim_{z \to 0} \frac{\text{Re}\left(z^2\right) + \text{Im}\left(z^2\right)}{z^2}$

(a) The limit does not exist. (b) 1

(c) $-i$ (d) -1

Ans.: (a)

Solution: $\lim_{z \to 0} \frac{\text{Re}\left(z^2\right) + \text{Im}\left(z^2\right)}{z^2} = \lim_{z \to 0} \frac{x^2 - y^2 + 2xy}{x^2 - y^2 + 2ixy} \Rightarrow \lim_{\substack{y=0 \\ x \to 0}} \frac{x^2 - y^2 + 2xy}{x^2 - y^2 + 2ixy} = 1$

$$\lim_{\substack{x=0 \\ y \to 0}} \frac{x^2 - y^2 + 2xy}{x^2 - y^2 + 2ixy} = -1$$

Q12. A particle of mass *m* is thrown upward with velocity *v* and there is retarding air resistance proportional to the square of the velocity with proportionality constant *k*. If the particle attains a maximum height after time *t*, and *g* is the gravitational acceleration, what is the velocity?

(a) $\sqrt{\frac{k}{g}} \tan\left(\sqrt{\frac{g}{k}} t\right)$ (b) $\sqrt{gk} \tan\left(\sqrt{\frac{g}{k}} t\right)$

(c) $\sqrt{\frac{g}{k}}\tan\left(\sqrt{gk}t\right)$ (d) $\sqrt{gk}\tan\left(\sqrt{gk}t\right)$

Ans.: (c)

Solution: Equation of motion $\frac{mdv}{dt} = mg + kv^2 \Rightarrow \frac{dv}{dt} = g + \frac{k}{m}v^2 \Rightarrow \frac{dv}{g + \frac{k}{m}v^2} = dt$

$$\Rightarrow \int \frac{dv}{g + \frac{k}{m}v^2} = \int dt \Rightarrow \int \frac{dv}{\frac{k}{m}\left(\frac{gm}{k} + v^2\right)} = \int dt \Rightarrow \frac{m}{k} \times \frac{1}{\sqrt{\frac{gm}{k}}}\tan^{-1}\frac{v}{\sqrt{\frac{gm}{k}}} = t$$

$$\Rightarrow \tan^{-1}\frac{v}{\sqrt{\frac{gm}{k}}} = \sqrt{\frac{gk}{m}}t \Rightarrow v = \sqrt{\frac{gm}{k}}\tan\sqrt{\frac{gk}{m}}t$$

Q13. Consider a uniform distribution of particles with volume density n in a box. The particles have an isotropic velocity distribution with constant magnitude v. The rate at which the particles will be emitted from a hole of area A on one side of this box is

(a) nvA (b) $nvA/2$ (c) $nvA/4$ (d) none of the above

Ans.: (c)

Q14. For a diatomic ideal gas near room temperature, what fraction of the heat supplied is available for external work if the gas is expanded at constant pressure?

(a) 1/7 (b) 5/7 (c) 3/4 (d) 2/7

Ans.: (d)

Solution: It is isobaric process (constant pressure)

Then $\delta\theta = nC_p\Delta T \Rightarrow \Delta W = nR\Delta T$

In this process $\delta\theta$ is heat exchange during process.

Function of heat supplied

$$= \frac{\delta W}{\Delta Q} = \frac{nR\Delta T}{nC_p\Delta T} = \frac{R}{R\frac{\gamma}{\gamma - 1}} = \frac{\gamma - 1}{\gamma} = 1 - \frac{1}{\gamma}$$

$$\Rightarrow 1 - \frac{1}{\left(1 + \frac{2}{f}\right)} \qquad \gamma = \frac{C_p}{C_V} \Rightarrow C_p = \frac{\gamma R}{\gamma - 1}$$

$$\Rightarrow 1-\frac{f}{f+2} \qquad f = \text{ degree of freedom, for diatomic molecule } f=5$$

$$\Rightarrow 1-\frac{5}{5+2} \Rightarrow \frac{2}{7}$$

Q15. A flat surface is covered with non-overlapping disks of same size. What is the largest fraction of the area that can be covered?

(a) $\frac{3}{\pi}$ (b) $\frac{5\pi}{6}$ (c) $\frac{6}{7}$ (d) $\frac{\pi}{2\sqrt{3}}$

Ans.: (d)

Solution: $n_{eff}=\frac{1}{3}n_C+\frac{1}{2}n_f+1\times n_i=\frac{1}{3}\times 6+1=3 \qquad a=2r$

Now largest fraction of area i.e. packing fraction $=\frac{n_{eff}\times A}{6\times\frac{\sqrt{3}}{4}\times a^2}=\frac{3\times\pi r^2}{6\times\frac{\sqrt{3}}{4}\times(2r)^2}=\frac{\pi}{2\sqrt{3}}$

Q16. If J_x, J_y, J_z are angular momentum operators, the eigenvalues of the operator $\left(J_x+j_y\right)/\hbar$ are

(a) real and discrete with rational spacing

(b) real and discrete with irrational spacing

(c) real and continuous

(d) not all real

Ans.: (b)

Solution: $J_x=\frac{1}{2}\left(J_++J_-\right),\quad J_y=\frac{i}{2}\left(J_--J_+\right)\Rightarrow J_+=\hbar\begin{bmatrix}0&1\\0&0\end{bmatrix},\quad J_-=\hbar\begin{bmatrix}0&0\\1&0\end{bmatrix}$

$$J_x=\frac{\hbar}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad J_y=\frac{i\hbar}{2}\begin{bmatrix}0&-1\\1&0\end{bmatrix}\Rightarrow\frac{J_x+J_y}{\hbar}=\frac{1}{2}\begin{bmatrix}0&1-i\\1+i&0\end{bmatrix}$$

eigen value $\frac{1}{2}\begin{pmatrix}-\lambda&1-i\\1+i&-\lambda\end{pmatrix}\Rightarrow\lambda^2-2=0\Rightarrow\lambda=\pm\sqrt{2}$

Q17. A metal suffers a structural phase transition from face-centered cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbor distances d_{fc} and d_{sc} for the FCC and the SC structures respectively are in the ratio (d_{fc}/d_{sc}) [Given $2^{1/3}=1.26$]

(a) 1.029 (b) 1.122 (c) 1.374 (d) 1.130

Ans. 17: ()

Solution: Nearest neighbour in $SC \rightarrow a \rightarrow C.N = 6$

Nearest neighbour in $FCC \rightarrow \frac{a}{\sqrt{2}} \rightarrow C.N = 12$

$$\frac{dFCC}{dSC} = \frac{\frac{a}{\sqrt{2}}}{a} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.707$$

Q18. If, in a Kepler potential, the pericentre distance of particle in a parabolic orbit is r_p while the radius of the circular orbit with the same angular momentum is r_c, then

(a) $r_c = 2r_p$ (b) $r_c = r_p$ (c) $2r_c = r_p$ (d) $r_c = \sqrt{2}r_p$

Ans.: (a)

Solution: Ionic equation $\frac{l}{r} = 1 + e\cos\theta$ for parabola $e = 1$ for circle, $e = 0$, $\theta = 0$

$$\frac{l}{r_p} = 1 + e \ , \quad \frac{l}{r_C} = 1, \quad l = 2r_p \ , \quad l = r_C \Rightarrow 2r_p = r_C$$

Q19. A K meson (with a rest mass of 494 MeV) at rest decays into a muon (with a rest mass of 106 MeV) and a neutrino. The energy of the neutrino, which can be massless, is approximately

(a) 120 MeV (b) 236 MeV (c) 300 MeV (d) 388 MeV

Ans.: (b)

Solution: $k \rightarrow \mu + \nu, \quad E_\nu = \frac{\left(m_k^2 - m_\mu^2\right)c^2}{2m_k} \Rightarrow \frac{\left(\frac{494}{c^2} \times \frac{494}{c^2} - \frac{106}{c^2} \times \frac{106}{c^2}\right)c^2}{2 \times \frac{494}{c^2}}$

$$\Rightarrow 244036 - \frac{11236}{988} = 235.6275 \approx 236\,\text{MeV}$$

Q20. The vector field $xz\hat{i} + y\hat{j}$ in cylindrical polar coordinates is

(a) $\rho\left(z\cos^2\phi + \sin^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 - z)\hat{e}_\phi$

(b) $\rho\left(z\cos^2\phi + \sin^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 + z)\hat{e}_\phi$

(c) $\rho\left(z\sin^2\phi + \cos^2\phi\right)\hat{e}_\rho + \rho\sin\phi\cos\phi(1 + z)\hat{e}_\phi$

(d) $\rho\left(z\sin^2\phi+\cos^2\phi\right)\hat{e}_\rho+\rho\sin\phi\cos\phi(1-z)\hat{e}_\phi$

Ans.: (a)

Solution: $\vec{A}=xz\hat{i}+y\hat{j}$

$A_x=xz,\quad A_y=y,\quad A_z=0$

$A_\rho=\vec{A}\cdot\hat{e}_\rho=A_x\left(\hat{x}\cdot\hat{e}_\rho\right)+A_y\left(\hat{y}\cdot\hat{e}_\rho\right)+A_z\left(\hat{z}\cdot\hat{e}_\rho\right)$

$=\rho\cos\phi z\cdot\cos\phi+\rho\sin\phi\cdot\sin\phi+0\Rightarrow A_\rho=\left(\rho\cos\phi^2 z+\rho\sin^2\phi\right)\hat{e}_\rho$

$A_\phi=\vec{A}\cdot\hat{e}_\phi=A_x\left(\hat{x}\cdot\hat{e}_\phi\right)+A_y\left(\hat{y}\cdot\hat{e}_\phi\right)+A_z\left(\hat{z}\cdot\hat{e}_\phi\right)$

$=\rho\cos\phi\left(-\sin\phi\right)z+\rho\sin\phi\cdot\cos\phi$

$A_\phi=\rho\cos\phi\cdot\sin\phi\left(1-z\right)\hat{e}_\phi$

$\vec{A}=A_\rho\hat{e}_\rho+A_\phi\hat{e}_\phi+A_z\hat{e}_z=\rho\left(\cos^2\phi z+\sin^2\phi\right)\hat{e}_\rho+\rho\cos\phi\sin\phi\left(1-z\right)\hat{e}_\phi$

Q21. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to

(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19

Ans.: (d)

Q22. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x axis. The probability that they meet after n steps is

(a) $\frac{1}{4^n}\frac{2n!}{n!^2}$ (b) $\frac{1}{2^n}\frac{2n!}{n!^2}$ (c) $\frac{1}{2^n}2n!$ (d) $\frac{1}{4^n}n!$

Ans.: (a)

Solution: Into probability of taking $'r'$ steps out of N steps

$$=N_{C_r}\left(\frac{1}{2}\right)^r\left(\frac{1}{2}\right)^{N-r}$$

total steps $=N=n+n=2n$

for taking probability of n steps out of N

$$P=N_{C_n}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{N-n}=\frac{N!}{(N-n)!n!}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^{2n-n}=\frac{2n!}{n!n!}\left(\frac{1}{2}\right)^{2n}=\frac{2n!}{(n!)^2 4^n}$$

Q23. The equation describing the shape of curved mirror with the property that the light from a point source at the origin will be reflected in a beam of rays parallel to the x-axis is (with a as some constant)

(a) $y^2 = ax + a^2$ (b) $2y = x^2 + a^2$ (c) $y^2 = 2ax + a^2$ (d) $y^2 = ax^3 + 2a^2$

Ans.: (c)

Q24. A simple model of a helium-like atom with electron-electron interaction is replaced by Hooke's law force is described by Hamiltonian

$$\frac{-\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2\right)+\frac{1}{2}m\omega^2\left(r_1^2+r_2^2\right)-\frac{\lambda}{4}\ m\omega^2\left|\vec{r}_1-\vec{r}_2\right|^2.$$

What is the exact ground state energy?

(a) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{1+\lambda}\right)$ (b) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{\lambda}\right)$

(c) $E=\frac{3}{2}\hbar\omega\sqrt{1-\lambda}$ (d) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{1-\lambda}\right)$

Ans.: (b)

Q25. Consider the state $\begin{pmatrix}1/2\\1/2\\1/\sqrt{2}\end{pmatrix}$ corresponding to the angular momentum $l = 1$in the L_z basis of states with $m = +1, 0, -1$. If L_z^2 is measured in this state yielding a result 1, what is the state after the measurement?

(a) $\begin{pmatrix}1\\0\\0\end{pmatrix}$ (b) $\begin{pmatrix}1/\sqrt{3}\\0\\\sqrt{2/3}\end{pmatrix}$ (c) $\begin{pmatrix}0\\0\\1\end{pmatrix}$ (d) $\begin{pmatrix}1/\sqrt{2}\\0\\1/\sqrt{2}\end{pmatrix}$

Ans.: (d)

Solution: $L_z=\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$, $L_z^2=\begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}$, eigenvector $\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}$

Corresponding eigenvalue 1, 0, 1

Now state after measurement yielding $1\Rightarrow\left|\phi_1\right\rangle+\left|\phi_3\right\rangle=\begin{pmatrix}1\\0\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix}$

PART B: ONE MARK QUESTIONS

Q26. A thin uniform ring carrying charge Q and mass M rotates about its axis. What is the gyromagnetic ratio (defined as ratio of magnetic dipole moment to the angular momentum) of this ring?

(a) $Q/(2\pi M)$ (b) Q/M (c) $Q/(2M)$ (d) $Q/(\pi M)$

Ans.: (c)

Solution: Magnetic dipole moment $M' = IA = \frac{Q}{T}\pi r^2 \Rightarrow \frac{Q}{2\pi T} \times 2\pi \times \pi r^2 = \frac{Q\omega r^2}{2}$

Angular momentum $J = Mr^2\omega \Rightarrow \frac{M'}{J} = \frac{Q}{2M}$

Q27. The electric and magnetic field caused by an accelerated charged particle are found to scale as $E \propto r^{-n}$ and $B \propto r^{-m}$ at large distances. What are the value of n and m?

(a) $n = 1, m = 2$ (b) $n = 2, m = 1$ (c) $n = 1, m = 1$ (d) $n = 2, m = 2$

Ans.: (c)

Solution: For large distance $F = \frac{qa\sin\theta}{r}$, $B = \frac{qa\sin\theta}{r} \Rightarrow E \propto \frac{1}{r}$, $B \propto \frac{1}{r}$

So $m = n = 1$

Q28. Consider the differential equation

$$\frac{dG(x)}{dx} + kG(x) = \delta(x),$$

where k is a constant. Which of the following statements is true?

(a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$

(b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not.

(c) $G(x)$ is discontinuous at $x = 0$

(d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depend on the value of k.

Ans.: (c)

Q29. A metal bullet comes to rest after hitting its target with a velocity of 80m/s. If 50% of the heat generated remains in the bullet, what is the increase in its temperature? (The specific heat of the bullet = 160 Joule per Kg per degree C)

(a) 14° C (b) 12.5° C (c) 10° C (d) 8.2° C

Ans.: (c)

Solution: Conservation of momentum $\frac{1}{2}mv^2 \times 50 \quad \% = mc\Delta T \Rightarrow \frac{1}{2}80\times 80 = 160 \quad \Delta T$

$$\Rightarrow \Delta T = \frac{80\times 80}{4}\times\frac{1}{160} = 10^0 C$$

Q30. What are the eigenvalues of the operator $H = \vec{\sigma}\cdot\vec{a}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{a}$ is a vector?

(a) $a_x + a_y$ and a_z (b) $a_x + a_z \pm i a_y$ (c) $\pm\left(a_x + a_y + a_z\right)$ (d) $\pm|\vec{a}|$

Ans.: (d)

Solution: $H = \vec{\sigma}\cdot\vec{a} = \left(\sigma_x . a_x + \sigma_y . a_y + \sigma_z . a_z\right)$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} a_x + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} a_y + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} a_z$$

$$\Rightarrow \begin{pmatrix} a_z & \left(a_x - i a_y\right) \\ \left(a_x + i a_y\right) & -a_z \end{pmatrix} \Rightarrow \begin{pmatrix} \left(a_z - \lambda\right) & \left(a_x - i a_y\right) \\ \left(a_x + i a_y\right) & -\left(a_z + \lambda\right) \end{pmatrix}$$

$$\Rightarrow -\left(a_z - \lambda\right)\left(a_z + \lambda\right) - \left(a_x - i a_y\right)\left(a_x + i a_y\right)$$

$$-a_z^2 + \lambda^2 - a_x^2 - a_y^2 = 0$$

$$\lambda^2 = a_x^2 + a_y^2 + a_z^2$$

$$\Rightarrow \lambda = \pm|\,\bar{a}\,|$$

Q31. The hermitian conjugate of the operator $\left(\frac{-\partial}{\partial x}\right)$ is

(a) $\frac{\partial}{\partial x}$ (b) $-\frac{\partial}{\partial x}$ (c) $i\frac{\partial}{\partial x}$ (d) $-i\frac{\partial}{\partial x}$

Ans.: (a)

Solution: $\Rightarrow \left(\psi^*(x) - \frac{\partial}{\partial x}\psi(x)\right)^\dagger = \left(\frac{-\partial\psi^*(x)}{\partial x}\psi(x)\right)$

$$\Rightarrow \int_{-\infty}^{\infty}\psi^*(x)\left[-\frac{\partial}{\partial x}\psi(x)\right]dx - \psi^*(x)\psi(x)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} - \frac{\partial\psi^*(x)}{\partial x}\psi(x)\,dx$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{\partial\psi^*(x)}{\partial x}\psi(x)\,dx$$

Q32. If the expectation value of the momentum is $\langle p\rangle$ for the wavefunction $\psi(x)$, then the expectation value of momentum for the wavefunction $e^{ikx/\hbar}\psi(x)$ is

(a) k (b) $\langle p\rangle - k$ (c) $\langle p\rangle + k$ (d) $\langle p\rangle$

Ans.: (c)

Solution: $\int_{-\infty}^{\infty}\psi^*(x)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi(x)\,dx = \langle p\rangle$

Now

$$\int_{-\infty}^{\infty} e^{-\frac{ikx}{\hbar}}\psi^*(x)\left(-i\hbar\frac{\partial}{\partial x}\right)e^{\frac{ikx}{\hbar}}\psi(x)\,dx \Rightarrow \int_{-\infty}^{\infty} e^{\frac{-ikx}{\hbar}}\psi^*(x)(-i\hbar)\left[e^{\frac{ikx}{\hbar}}\frac{\partial}{\partial x}\psi(x)+\frac{ik}{\hbar}e^{\frac{ikx}{\hbar}}\psi(x)\right]$$

$$\Rightarrow \int_{-\infty}^{\infty} e^{-\frac{ikx}{\hbar}}\psi^*(x)\left(-i\hbar\frac{\partial}{\partial x}\psi(x)\right)e^{\frac{ikx}{\hbar}} + \int_{-\infty}^{\infty} -i\hbar.\frac{ik}{\hbar}e^{\frac{-ikx}{\hbar}}\psi^*(x)\psi(x)\,dx$$

$$\Rightarrow \int_{-\infty}^{\infty}\psi^*(x)\left[-i\hbar\frac{\partial}{\partial x}\psi(x)\right] + k\int_{-\infty}^{\infty}\psi^*(x)\psi(x) \Rightarrow \langle P\rangle + K$$

Q33. Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x) = \sqrt{2/L}\,\sin(n\pi x/L)$. What would be the ground state wave function $\psi(x_1, x_2)$ if both electrons are arranged to have the same spin state?

(a) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(b) $\psi(x_1,x_2) = \frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(c) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2) = \frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

Ans.: (b)

Solution: Electrons are Fermions of spin $\frac{1}{2}$ and it wave functions are anti symmetric

Spin part is symmetric and space part will be anti symmetric (since total wave function is anti symmetric)

Then

$$=\frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right).\sin\left(\frac{2\pi x_2}{L}\right)-\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right).\sin\left(\frac{\pi x_2}{L}\right)\right]$$

Q34. What is the value of the following series?

$$\left(1-\frac{1}{2!}+\frac{1}{4!}-....\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}-...\right)^2$$

(a) 0 (b) e (c) e^2 (d) 1

Ans.: (d)

Solution: $\Rightarrow \cos\theta=1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}.....,\qquad \sin\theta=\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}.....$

$$\Rightarrow\left(1-\frac{1}{2!}+\frac{1}{4!}...\right)^2+\left(1-\frac{1}{3!}+\frac{1}{5!}\right)^2\Rightarrow\cos 1^2+\sin 1^2=1\qquad \because \sin^2\theta+\cos^2\theta=1$$

Q35. A light beam is propagating through a block of glass with index of refraction n. If the glass is moving at constant velocity v in the same direction as the beam, the velocity of the light in the glass block as measured by an observer in the laboratory is approximately

(a) $u=\frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$ (b) $u=\frac{c}{n}-v\left(1-\frac{1}{n^2}\right)$

(c) $u=\frac{c}{n}+v\left(1+\frac{1}{n^2}\right)$ (d) $u=\frac{c}{n}$

Ans.: (a)

Solution: now $u=\dfrac{v+\frac{c}{n}}{1+\frac{v\cdot c}{c^2\cdot n}}=\left(v+\frac{c}{n}\right)\left(1+\frac{v}{cn}\right)^{-1}=\left(v+\frac{c}{n}\right)\left(1-\frac{v}{cn}+\frac{v^2}{c^2n^2}\right)$

$$\Rightarrow v-\frac{v^2}{cn}+\frac{v^3}{c^2n^2}+\frac{c}{n}-\frac{v}{cn^2}+\frac{cv^2}{cn^3}\Rightarrow u=\frac{c}{n}+v\left(1-\frac{1}{n^2}\right)$$

Q36. If the distribution function of x is $f(x)=xe^{-x/\lambda}$ over the interval $0<x<\infty$, the mean value of x is

(a) λ (b) 2λ (c) $\lambda/2$ (d) 0

Ans.: (b)

Solution: $\because$ it is distribution function so $\langle x\rangle = \dfrac{\int_{-\infty}^{\infty} xf(x)dx}{\int_{-\infty}^{\infty} f(x)dx} = \dfrac{\int_0^{\infty} x.xe^{-\frac{x}{\lambda}}dx}{\int_0^{\infty} xe^{-\frac{x}{\lambda}}dx} \Rightarrow \dfrac{\int_0^{\infty} x^2 e^{\frac{-x}{\lambda}}dx}{\int_0^{\infty} xe^{\frac{-x}{\lambda}}dx} = 2\lambda$

Q37. Consider a particle with three possible spin states: s = 0 and ±1. There is a magnetic field *h* present and the energy for a spin state *s* is –*hs*. The system is at a temperature *T*. Which of the following statements is true about the entropy *S*(*T*)?

(a) *S*(*T*) = ln 3 at *T* = 0, and 3 at high *T*

(b) *S*(*T*) = ln 3 at *T* = 0, and zero at high *T*

(c) *S*(*T*) = 0 at *T* = 0, and 3 at high *T*

(d) *S*(*T*) = 0 at *T* = 0, and ln 3 at high *T*

Ans.: (d)

Solution: $S = k\ln\omega$ where $\omega =$ number of microstates

$S = k\ln 3$ at high T $\omega = 3$

and at $T = 0$ it is perfect ordered i.e. $S = 0$

Q38. The operator

$$\left(\frac{d}{dx} - x\right)\left(\frac{d}{dx} + x\right)$$

is equivalent to

(a) $\dfrac{d^2}{dx^2} - x^2$

(b) $\dfrac{d^2}{dx^2} - x^2 + 1$

(c) $\dfrac{d^2}{dx^2} - x\dfrac{d}{dx}x^2 + 1$

(d) $\dfrac{d^2}{dx^2} - 2x\dfrac{d}{dx} - x^2$

Ans.: (b)

Solution: $\Rightarrow \left(\dfrac{d}{dx} - x\right)\left(\dfrac{d}{dx} + x\right)f(x) \Rightarrow \left(\dfrac{d}{dx} - x\right)\left[\dfrac{d}{dx}f(x) + xf(x)\right]$

$$\Rightarrow \frac{d}{dx}\left[\frac{d}{dx}f(x) + xf(x)\right] - x\frac{d}{dx}f(x) - x^2 f(x)$$

$$\Rightarrow \frac{d^2}{dx^2}f(x) + f(x) + x\frac{df(x)}{dx} - x\frac{d}{dx}f(x) - x^2 f(x)$$

$$\Rightarrow \frac{d^2}{dx^2} f(x) - x^2 f(x) + f(x) = \left(\frac{d^2}{dx^2} - x^2 + 1\right) f(x)$$

Q39. Consider three situations of 4 particles in one dimensional box of width L with hard walls. In case (i), the particles are fermions, in case (ii) they are bosons, and in case (iii) they are classical. If the total ground state energy of the four particles in these three cases are E_F, E_B and E_{cl} respectively, which of the following is true?

(a) $E_F = E_B = E_{cl}$ (b) $E_F > E_B = E_{cl}$

(c) $E_F < E_B < E_{cl}$ (d) $E_F > E_B > E_{cl}$

Ans.: (b)

Solution: For fermions $\frac{\pi^2\hbar^2}{2ml^2} = \epsilon_0$

$$1\times \epsilon_0 + 1\times 4\epsilon_0 + 1\times 9\epsilon_0 + 1\times 16\epsilon_0 = 30\epsilon_0$$

$$For\ Boson = 4\times \epsilon_0, For\ Maxwell = 4\times \epsilon_0$$

$$E_F > E_B = E_{cl}$$

Q40. If a proton were ten times, the ground state energy of the electron in a hydrogen atom would be

(a) less

(b) more

(c) the same

(d) less, more or equal depending on the electron mass

Ans.: (b)

Solution: $E_n = \frac{-13.6}{n^2} \times \frac{0.99995\ m_e}{m_e} \Rightarrow -13.59932 \qquad \because \mu = 0.99995 m_e$

Q41. If $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$ then

(a) Both are impossible electrostatic fields

(b) Both are possible electrostatic fields

(c) Only $\vec{E}_1$ is a possible electrostatic field

(d) Only $\vec{E}_2$ is a possible electrostatic field

Ans.: (d)

Solution: For electrostatic field $\vec{\nabla}\times\vec{E} = 0$

$$\vec{\nabla}\times\vec{E}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ y^2 & 2xy+z^2 & 2yz \end{vmatrix}$$

$$(2z-2z)\hat{i}+0+(2y-2y)\hat{z}=0$$

$$\vec{\nabla}\times\vec{E}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xy & \partial yz & yxz \end{vmatrix} = (0-2y)\hat{i}+0+x\hat{j}\neq 0$$

Q42. ^{238}U decays with a half life of 4.51×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock sample analysis shows that the ratio of the numbers of atoms of ^{206}Pb to ^{238}U is 0.0058. Assuming that all the ^{206}Pb has been produced by the decay of ^{238}U and that all other half-lives in the chain are negligible, the age of the rock sample is

(a) 38×10^6 years (b) 48×10^6 years (c) 38×10^7 years (d) 48×10^7 years

Ans.: (a)

Solution: $t=\dfrac{1}{\lambda_u}\ln\left(\dfrac{N_{pb}+N_u}{N_u}\right)$

Q43. The period of a simple pendulum inside a stationary lift is T. If the lift accelerates downwards with an acceleration g / 4, the period of the pendulum will be

(a) T (b) $T/4$ (c) $2T/\sqrt{3}$ (d) $2T/\sqrt{5}$

Ans.: (c)

Solution: $T=2\pi\sqrt{\dfrac{l}{g}}\Rightarrow$ lift accelerates down wards then

$$T=2\pi\sqrt{\frac{l}{g-g'}}\Rightarrow T=2\pi\sqrt{\frac{l}{g-\frac{g}{4}}}=2\pi\sqrt{\frac{4l}{3g}}\Rightarrow 2\pi\times 2\sqrt{\frac{l}{3g}}$$

$$T'=\frac{2T}{\sqrt{3}}$$

Q44. The velocity of a particle at which the kinetic energy is equal to its rest energy is (in terms of c, the speed of light in vacuum)

(a) $\sqrt{3}c/2$ (b) $3c/4$ (c) $\sqrt{3/5}c$ (d) $c/\sqrt{2}$

Ans.: (a)

Solution: $K.E = mc^2 - m_0c^2$, rest mass energy $= m_0c^2$

$K.E. =$ rest mass energy

$$mc^2 - m_0c^2 = m_0c^2$$

$$mc^2 = 2m_0c^2$$

$$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}c^2 = 2m_0c^2 \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2 \Rightarrow 4\left(1-\frac{v^2}{c^2}\right) = 1 \Rightarrow 4\frac{v^2}{c^2} = 3 \Rightarrow v = \frac{\sqrt{3}}{2}c$$

Q45. If the Poisson bracket $\{x, p\} = -1$, then the Poisson bracket $\{x^2 + p, p\}$ is

(a) $-2x$ (b) $2x$ (c) 1 (d) -1

Ans.: (a)

Solution: $\{x^2 + p, p\} = \{x^2, p\} + \{p, p\} \Rightarrow x\{x, p\} + \{x, p\}x + 0 \Rightarrow x(-1) + (-1)x \Rightarrow -2x$

Q46. The coordinate transformation

$$x' = 0.8x + 0.6y, \qquad y' = 0.6x - 0.8y$$

represents

(a) a translation (b) a proper rotation

(c) a reflection (d) none of the above

Ans.: (b)

Q47. The binding energy of the k-shell electron in a Uranium atom ($Z = 92$, $A = 238$) will be modified due to (i) screening caused by other electrons and (ii) the finite extent of the nucleus as follows:

(a) increases due to (i), remains unchanged due to (ii)

(b) decreases due to (i), decreases due to (ii)

(c) increases due to (i), increases due to (ii)

(d) decreases due to (i), remains unchanged due to (ii)

Ans.: (b)

Q48. A small mass M hangs from a thin string and can swing like a pendulum. It is attached above the window of a car. When the car is at rest, the string hangs vertically. The angle made by the string with the vertical when the car has a constant acceleration a = 1.2 m/s^2 is approximately

(a) 1° (b) 7° (c) 15° (d) 90°

Ans.: (b)

Solution: $T\sin\theta = ma$, $T\cos\theta = mg$, $\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\frac{a}{g} = \tan^{-1}\left(\frac{1.2}{9.8}\right) = 6.98^0 \approx 7^0$

Q49. A charge q is placed at the centre of an otherwise neutral dielectric sphere of radius a and relative permittivity ε_r. We denote the expression $q/4\pi\varepsilon_0 r^2$ by $E(r)$. Which of the following statements is false?

(a) The electric field inside the sphere, $r < a$, is given by $E(r)/\varepsilon_r$

(b) The field outside the sphere, $r > a$, is given by $E(r)$

(c) The total charge inside a sphere of radius $r > a$ is given by q.

(d) The total charge inside a sphere of radius $r < a$ is given by q.

Ans.: (d)

Solution: $\vec{E}$ $\quad r > a$

$$\Rightarrow \oint \vec{E}.d\vec{a} = Qenc \Rightarrow |\vec{E}| \times 4\pi \quad r^2 = q \Rightarrow |\vec{E}| = \frac{q}{4\pi \in_0 \quad r^2}\hat{r} \quad r > a$$

Q50. An electromagnetic wave of frequency ω travels in the x-direction through vacuum. It is polarized in the y-direction and the amplitude of the electric field is E_0. With $k = \omega/c$ where c is the speed of light in vacuum, the electric and the magnetic fields are then conventionally given by

(a) $\vec{E} = E_0\cos(ky - \omega t)\hat{x}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{z}$

(b) $\vec{E} = E_0\cos(kx - \omega t)\hat{y}$ and $\vec{B} = \frac{E_0}{c}\cos(kx - \omega t)\hat{z}$

(c) $\vec{E} = E_0\cos(kx - \omega t)\hat{z}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{y}$

(d) $\vec{E} = E_0\cos(kx - \omega t)\hat{x}$ and $\vec{B} = \frac{E_0}{c}\cos(ky - \omega t)\hat{y}$

Ans.: (b)

Solution: $\vec{E} = E_0\cos(kx - \omega t)\hat{y}$

$$\vec{B} = \frac{1}{c}\left(\hat{k} \times \vec{E}\right) \Rightarrow \vec{B} = \frac{1}{c}\left[\hat{x} \times E_0 \cos\left(kx - \omega t\right)\hat{y}\right]$$

$$\Rightarrow \vec{B} = \frac{E_0}{c}\cos\left(kx - \omega t\right)\left(\hat{x} \times \hat{y}\right) \Rightarrow \vec{B} = \frac{E_0}{c}\cos\left(kx - \omega t\right)\left(\hat{z}\right)$$