Answer of Full Length Test-02

PART A

Q1:	Ans:	(d)	Q2:	Ans:	(d)	Q3:	Ans:	(c)
Q4:	Ans:	(c)	Q5:	Ans:	(c)	Q6:	Ans:	(c)
Q7:	Ans:	(b)	Q8:	Ans:	(a)	Q9:	Ans:	(a)
Q10:	Ans:	(b)	Q11:	Ans:	(d)	Q12:	Ans:	(a)
Q13:	Ans:	(d)	Q14:	Ans:	(b)	Q15:	Ans:	(c)
Q16:	Ans:	(c)	Q17:	Ans:	(b)	Q18:	Ans:	(b)
Q19:	Ans:	(c)	Q20:	Ans:	(c)			

PART B

Q21. Ans: (*a*)

Q22. Ans: (b)

Q23. Ans: (d)

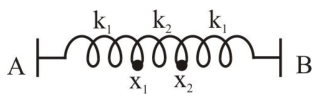
Q24. Ans: (*a*) $\hat{n} = \frac{\vec{\nabla}\phi}{\left|\vec{\nabla}\phi\right|} = \frac{2i+2j-k}{3}$

Q25. Solution: (d)

Q26. Ans (c):

$$T = \frac{1}{2}m(\dot{x}_1^2 + \dot{x}_2^2) \text{ and } V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2 + \frac{1}{2}kx_2^2$$

$$= \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2^2 + x_1^2 - 2x_1x_2) + \frac{1}{2}k_1x_2^2$$



$$= x\frac{1}{2}(k_1 + k_2)x_1^2 + \frac{1}{2}(k_1 + k_2)x_2^2 - \frac{1}{2}k\ 2x_1x_2$$

$$T = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \text{ and } V = \begin{pmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_1 + k_2 \end{pmatrix}$$

For Normal frequencies.

$$(V-\omega^2 T)=0$$

$$\begin{pmatrix} k_1+k_2-\omega^2 m & -k_2 \\ -k_2 & k_1+k_2-\omega^2 m \end{pmatrix}=0$$

$$(k_1+k_2-\omega^2 m)^2-k_2^2=0$$

$$(k_1+k_2-\omega^2 m+k_2)(k_1+k_2-\omega^2 m-k_2)=0$$

$$(k_1+2k_2-\omega^2 m)(k_1-\omega^2 m)=0$$

$$\omega_1=\sqrt{\frac{k_1}{m}} \quad \text{and} \quad \omega_2=\sqrt{\frac{k_1+2k_2}{m}}$$

Q27. Solution: (c)

$$\frac{dF}{dt}=[H,F]+\frac{\partial F}{\partial t} \qquad =\frac{P}{m}-\frac{P}{m}=0$$

$$\frac{dF}{dt}=0$$ F = constant of motion.

Q28. Solution: (c)

Q29. Solution: (b)

$$\langle H\rangle=\frac{1}{14}\times E_1+\frac{2}{7}\times E_2+\frac{9}{14}\times E_3=\frac{1}{14}\times -E_0+\frac{2}{7}\times\frac{-E_0}{4}+\frac{9}{14}\times\frac{-E_0}{9}$$

$$=-E_0\left(\frac{1}{14}+\frac{1}{14}+\frac{1}{14}\right)=\frac{-3E_0}{14}$$

Q30. Solution: (b)

$$[L_z \cos\phi]\psi$$

$$=L_z\cos\phi\psi-\cos\theta L_z\psi \quad =\psi L_z\cos\phi+\cos\phi L_z\psi-\cos\phi L_z\psi$$

$$\psi L_z\cos\phi \qquad \psi-i\hbar\frac{\partial}{\partial\phi}\cos\phi$$

$$\psi i\hbar\sin\phi \qquad =i\hbar\sin\phi$$

Q31. Ans: (c)

Solution: The possible energy for given system in $1\in, 3\in, 4\in$. (For FD statistics).

Q32. Ans: (a)

$E = -P \cdot E = -PE\cos\theta$

$$z \approx \int_0^{\pi} e^{PE\cos\theta/kT}\sin\theta d\theta \quad \Rightarrow \quad z = \frac{2kT}{PE}\sinh\left(\frac{PE}{kT}\right)$$

Q33. Solution: (b)

$$S = -\left(\frac{\partial G}{\partial T}\right)_p = \frac{5}{2}R - R\ln\left[\frac{ap}{(RT)^{5/2}}\right]$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right) = \frac{5}{2}R$$

Q34. Ans: (a)

Q35. Ans: (c)

$$F = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3Q}{(1)^2} + \frac{Q}{(1)^2} + \frac{3Q}{(2)^2}\right] = \frac{19Q^2}{16\pi\varepsilon_0}$$

Q36. Ans: (d)

Q: 37. Ans: (c)

Solution: $\omega = 10^{15}, k = \sqrt{2}\times 10^7 \Rightarrow v = \frac{\omega}{k} = \frac{10^8}{\sqrt{2}} \qquad \Rightarrow n = \frac{c}{v} = 3\times\sqrt{2} = 4.2$

Q38. Ans: (*d*) Q39. Ans: (c) Q40. Ans: (b)

Q41: Ans: (d) Q42: Ans: (c)

Q43: Ans: (d)

$$L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - mgZ \qquad Z = \frac{1}{2}a\left(x^2 + y^2\right)$$

Which has cylindrical symmetries $x = r\cos\phi$, $y = r\sin\phi$, z=z $\quad Z = \frac{1}{2}a\left(r^2\right)$

$\dot{x} = \dot{r}\cos\phi - r\sin\phi\dot{\phi}$ $\dot{y} = \dot{r}\sin\phi + r\cos\phi\dot{\phi}$ $\dot{Z} = a\left(r\dot{r}\right)$ so $L = \frac{1}{2}m\left[\left(1 + a^2r^2\right)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2\right]$

Q44: Ans: (a)

Solution:

$$\mu(T) \approx E_F\left[1-\frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2\right]$$

Q45: Ans : (c)

PART C

Q46. Ans: (c)

Solution: Separation of variables with $u(x,t)=\phi(x)G(t)$ gives

$$\frac{dG}{dt}=-(\lambda k+1)G \quad \text{and} \quad \frac{d^2\phi}{dx^2}=-\lambda\phi.$$

The boundary conditions gives

$$G(t)\phi(0)=0$$

$$G(t)\frac{d\phi}{dx}(L)+G(t)\phi(L)=G(t)\left[\frac{d\phi}{dx}(L)+\phi(L)\right]=0$$

So for non- trivial solutions

$$\phi(0) \text{ and } \frac{d\phi}{dx}(L)+\phi(L)=0.$$

Q47. Ans: (a)

Q48. Ans: (d) Degeneracy is 2S+1 =3

Q49. Ans: (*a*)

$$|\psi_n'\rangle=\sum_{m\neq n}\frac{\langle\phi_m|w|\phi_n\rangle}{E_n^0-E_m^0}|\phi_m\rangle = \frac{qE}{\hbar\omega}\sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{n}\,|n-1\rangle-\sqrt{n+1}\,|n+1\rangle\right]$$

Where $W=qE_x=qE\sqrt{\frac{\hbar}{2m\omega}}(a^{-1}+a)$

$$\langle m|X|n\rangle=\frac{\hbar}{\sqrt{2m\omega}}\left[\sqrt{n}\delta_{m,m-1}+\sqrt{n+1}\delta_{m,m+1}\right]$$

Q50. Ans: (b)

Q51. Ans: (*a*)

Let the frequency of the photon be v and the momentum of the atom in the excited state be p.

The conservation laws of the energy and momentum give

$$Mc^2 + h\nu = \left[(M+\Delta)^2 + p^2c^2\right]^{1/2} \qquad \text{Conservation of Energy}$$

$$\frac{h\nu}{c} = p \qquad \text{Conservation of Momentum}$$

And hence,

$$\nu = \frac{\Delta c^2}{h}\left(1+\frac{\Delta}{2M}\right)$$

Q52. Ans: (c)

$C_V \propto T^{n/s}$, here n is the dimensionality

Q53. Ans: (b)

Q54. Ans: (d)

Hints: E_1 =3.27MeV, E_2 =4.03Mev, E_3 =17.59MeV

Q55. Ans: (c)

Q56. Ans: (d)

Hints:

(a) $u_t = (pu_x)_x$

$pu_{xx} + 0u_{xt} p_x u_x - u_t = 0$

Since $0^2 - p0 = 0$, the equation is parabolic

(b) $u_{tt} = c^2 u_{xx} - \gamma u$

$u_{tt} + 0u_{tx} - c^2 u_{xx} + \gamma u = 0$

Since $0^2 - (1)(-c^2) > 0$, the equation is hyperbolic.

(c) $(qu_x)_x + (qu_t)_t = 0$

$qu_{xx} + 0u_{xt} + qu_{tt} + q_x u_x + q_t u_t = 0$

Since $0^2 - qq < 0$, the equation is elliptic

$u_{tt} - u_{xx} = 0$ is wave equation which is 'hyperbolic'.

Q57. Ans: (c)

$$H = \frac{P_\theta^2}{2mR^2} \quad \Rightarrow \quad \frac{J_\theta}{2\pi} = P_\theta$$

$$H = E_\theta = \frac{J_\theta}{4\pi^2 2mR^2} = \frac{J_\theta^2}{8m\pi^2 R^2} \quad \Rightarrow \quad \nu_\theta = \frac{\partial E_\theta}{\partial \theta} = \frac{J_\theta}{4\pi mR^2}$$

Q58. Ans: (c)

Q59. Ans: (c)

Solution: (a) Forbidden as it violates $\Delta J = 0, \pm 1$ $(0 \not\leftrightarrow 0)$

(b) Forbidden as it violates $\Delta J = 0, \pm 1$ (Here $\Delta J = 2$)

(c) Forbidden as it violates $\Delta L = 0, \pm 1$ (Here $\Delta L = 2$)

Q60. Ans: (b)

Q61. Ans: c

Q62. Ans: (b)

$$PV = \frac{s}{n}U$$

Q63. Ans: (d)

Q64. Ans: (c)

Q65. Ans -: (b)

Solution: If the electron gas confined in one dimension rod of length L, then number of possible states in between k and k + dk

$$g(k)dk = \frac{L}{\pi}dk \qquad \text{Since,} \quad E = \frac{\hbar^2 k^2}{2m} \quad \Rightarrow \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\therefore \qquad dk = \left(\frac{2m}{\hbar^2}\right)^{1/2} \cdot \frac{1}{2} E^{-1/2} dE$$

$$\therefore \qquad g(E)dE = \frac{L}{\pi} \times \left(\frac{2m}{\hbar^2}\right)^{1/2} \frac{1}{2} E^{-1/2} dE$$

Since, electrons have spin ±1/2. So multiply above equation with 2.

$$g(E)dE = \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} E^{-1/2} dE$$

At $T = 0^0$ K. The total number of electron between E = 0 and E_F is

$$N=\int_0^{E_F}F(E)g(E)dE \Rightarrow N=\frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_0^{E_F}E^{-1/2}dE$$ (Since F(E) = 1 at T=0K)

$$=\frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{E_F^{1/2}}{1/2} = \frac{2L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}E_F^{1/2}$$

$$\therefore \quad E_F=\frac{\hbar^2\pi^2}{2m}\left(\frac{N}{2L}\right)^2=\frac{\hbar^2}{2m}\left(\frac{\pi N}{2L}\right)^2$$

Q: 66. Ans: (b)

Q: 67. (A) Ans: (a) $\quad Q_{enc}=\varepsilon_0\oint_S \vec{E}.d\vec{a}=4\pi\varepsilon_0\alpha$

(B) Ans: (a)

Q68. (A) Ans: (c)

Hints: (i) Forbidden; muon and electron lepton numbers are not conserved.

(ii) Forbidden; baryon number not conserved.

(iii) Allowed; by weak interaction as $|\Delta S|=1$, but not by strong because strangeness is not conserved.

(B) Ans: (c)

Hints:

$$\pi^- + p \to x_1 + \bar{K}^0$$
$$B: 0+1 \to x_1 + 0$$
$$S: 0+0 \to x_1 + (-1)$$
$$p+p \to x_2 + K^- + p$$
$$B: 1+1 \to x_1 + 0 + 1$$
$$S: 0+0 \to x_1 + (-1) + 0$$
$$K^+ + n \to x_3 + \pi^0 + \pi^0$$
$$B: 0+1 \to x_1 + 0 + 0$$
$$S: 1+0 \to x_1 + 0 + 0$$

Q69. Ans: (d)

Hints: $V_c = \frac{1}{4\pi\varepsilon_0}\frac{ZZ'e^2}{R+R'}$, where Z and Z' are the atomic numbers of the two nuclei and R and R' are their effective radii. Classically, the distance of closest approach is $R+R'$.

Q70. (A) Ans: (c)
(B) Ans: (b)

There are 12 members in the lepton family: $e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau,$ and their antiparticles. Out of these only 10 are discovered so far. ν_τ and its antiparticle are theoritically predicted but not discovered yet.

Q71: Ans: (d)

Q72: Ans: (c)

Q73: Ans: (b)

Q74: Ans: (*a*)

$$f_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2},\ \lambda_{mn} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$So,\ \lambda_{10} = 14.4\ cm, \quad \lambda_{01} = 6.8\ cm, \quad \lambda_{11} = 6.15\ cm, \quad \lambda_{20} = 7.21\ cm$

$\lambda < \lambda_{10}$. *Hence* TE_{10} mode *can be used*.

Q75: Ans: (a)
