D. U. (Ph.D. Entrance 2015)

Q1. Consider a gas of free electrons at a high temperature T. A uniform magnetic field B is applied and it is found that there is an excess of 0.1% of electrons polarized along the field compared to those polarized opposite. Then,

(a) $B = \frac{m_e k_B T}{\hbar e} \times 10^{-3}$

(b) $B = \frac{m_e k_B T}{2\hbar e} \times 10^{-3}$

(c) $B = \frac{m_e k_B T}{4\hbar e} \times 10^{-3}$

(d) $B = \frac{2m_e k_B T}{\hbar e} \times 10^{-3}$

Q2. According to the Thomas-Fermi model, the electron cloud in an atom can be described by a continuous charge distribution $p(x)$. An individual electron moves in the combined electrostatic potential $\phi(x)$ determined by this cloud and the nucleus of charge Ze.

Assume that the charge cloud adjusts itself locally until the electrons at Fermi sphere have zero energy, but a Fermi momentum p_F Then, the relation between $\phi(x)$ and $\rho(x)$ is

(a) $\rho(x) = \frac{e^{5/2}}{3\pi^2\hbar^3}\left[2m\phi(x)\right]^{3/2}$

(b) $\rho(x) = \frac{3e^{5/2}}{4\pi\hbar^3}\left[2m\phi(x)\right]^{3/2}$

(c) $\rho(x) = \frac{e^2}{3\pi^2\hbar^3}\left[2m\phi(x)\right]$

(d) $\rho(x) = \frac{e}{3\pi^2\hbar^3}\left[2m\phi(x)\right]$

Q3. Consider a black sphere of radius R at temperature T. which radiates to a distant environment which can be assumed to be at absolue zero. Now, surround the sphere with a heat shield in the form of a black shell whose temperature is determined by radiative equilibrium. What is the temperature of the shell and what is the effect of the shell on the total power radiated to the environment?

(a) $T_{\text{shell}} = \frac{T}{\sqrt{2}}$ and $P = \frac{P_{\text{original}}}{4}$

(b) $T_{\text{shell}} = \frac{T}{2}$ and $P = \frac{P_{\text{original}}}{4}$

(c) $T_{\text{shell}} = \frac{T}{2^{\frac{1}{4}}}$ and $P = \frac{P_{\text{original}}}{2}$

(d) $T_{\text{shell}} = T$ and $P = P_{\text{original}}$

Q4. The value of the integral

$$\int_C \frac{zdz}{z^2+1}$$

where C is the closed curve $|z+i| = 0.5$ is

(a) $+\pi i$ (b) 0 (c) $2\pi i$ (d) $-\pi i$

Q5. A long narrow uniform rod lies motion less on ice (assume the ice provides a frictionless surface). A small piece of wood slides without spinning on the ice towards the rod, hits one end and gets stuck to it. Consider the statements:

(i) Angular momentum of the wood about the center of mass of the rod.

(ii) Linear momentum of the rod-wood system

(iii) Angular momentum of the rod-wood system. . .

(iv) Mechanical energy of the rod-wood system.

Which of the above are conserved?

(a) (ii) and (iii) alone. (b) (ii), (iii) and (iv) alone.

(c) (ii) and (iv) alone. (d) All four

Q6. For a particle described by the Lagrangian

$$L = \frac{m}{2}\left(x^2 + y^2\right) + a\dot{x}t - \frac{k}{2}\left(x^2 + y^2\right) + ax$$

(a) energy is conserved but not angular momentum.

(b) both energy and angular momentum are conserved.

(c) angular momentum is conserved but not energy.

(d) neither energy nor angular momentum are conserved.

Q7. Anticommutator of two operators is de fined as $\{\hat{A}, \hat{B}\} = AB + BA$. Let $\hat{L}_x, \hat{L}_y$ and $\hat{L}_z$ be the components of the angular momentum operator. The commutator $\left[\hat{L}_x^2, \hat{L}_y^2\right]$ is equal to,

(a) $(i\hbar)^2\left\{\hat{L}_x, \hat{L}_y\right\}$ (b) $i\hbar \hat{L}_x \hat{L}_y \hat{L}_z$

(c) $i\hbar \hat{L}_z\left\{\hat{L}_x, \hat{L}_y\right\} + i\hbar\left\{\hat{L}_z, \hat{L}_x\right\}\hat{L}_y$ (d) $i\hbar \hat{L}_x\left\{\hat{L}_y, \hat{L}_x\right\} + i\hbar\left\{\hat{L}_y, \hat{L}_x\right\}\hat{L}_z$

Q8. A system of spin-half particles are placed in a uniform non-zero magnetic field. In the infinite temperature limit,

(a) all particles are in the higher energy state.

(b) one third of the particles are in lower and two third in the higher energy state.

(c) all are in the lower energy state.

(d) half the particles are in the lower and half in the higher energy state.

Q9. A monochromatic source of light intensity I_e emitting with a frequency ν_e is moving away from an observer at a speed v. The observer detects the light to be of frequency ν_0. The intensity of light measured by the observer is

(a) $I_0 = I_e \frac{\nu_0}{\nu_e}$ (b) $I_0 = I_e \frac{\nu_e}{\nu_o}$

(c) $I_0 = I_e \frac{\nu_0^2}{\nu_e^2}$ (d) $I_o = I_e \frac{\nu_e^2}{\nu_o^2}$

Q10. According to the Debye theory for specific heat of solids, the solid is an

(a) isotropic elastic medium and the motion of the atoms is coupled

(b) anisotropic elastic medium and the motion of the atoms l's coupled

(c) isotropic elastic medium and the motion of the atoms is not coupled

(d) anisotropic elastic medium and the motion of the atoms is not coupled

Q11. A standard voltage divider bias circuit for *npn* silicon transistor is connected to a by $10\,V$ supply. The voltage dividing resistances at the base of a silicon transistor are $10\,k\Omega$ and $2.2\,k\Omega$. The resistances connected at the collector and the emitter terminals are $3.6\,k\Omega$ and $2k\Omega$, respectively. Assume that base current is 20 times less than the cur rent through voltage divider. What is the collector emitter voltage?

(a) $6.92\,V$ (b) $10\,V$ (c) $8.02\,V$ (d) $1.05\,V$

Q12. Two sinusoidal input waveform of peak to peak voltage of 1V and 4V are supplied to the two noninverting amplifiers (circuit 1 and circuit 2, respectively) using *OPAmp* 741 *C*, each has gain 10 and operating at $\pm 15\, V$. The shape of out put wave form is observed on channel 1 and channel 2 of *CRO*, respectively. The output wave forms on channel 1 and channel 2 of the *CRO* shall be respectively,

(a) both sinusoidal
(b) sinusoidal and square
(c) square and sinusoidal
(d) both saw toot

Q13. Taylor's series expansion of $\frac{e^z}{1+e^z}$ about $z=0$

(a) $\frac{1}{2}-\frac{z}{4}+\frac{z^3}{4^8}\dots$
(b) $\frac{1}{2}+\frac{z}{4}-\frac{z^3}{4^8}\dots$
(c) $\frac{1}{2}+\frac{z}{4}+\frac{z^3}{4^8}\dots$
(d) $-\frac{1}{2}+\frac{z}{4}-\frac{z^3}{4^8}\dots$

Q14. An unperturbed two level system has energy eigenvalues $2E_0$ and E_0 and eigenstates represented by $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, respectively. When the system is perturbed, its Hamiltonian is represented by

$$H=\begin{pmatrix}2E_0 & A\\ A^* & E_0\end{pmatrix},$$

where A is small as compared to E_0 order correction to $2E_0$ is

(a) $\frac{A^2}{E_0}$
(b) $-\frac{A^2}{E_0}$
(c) A
(d) 0

Q15. If the scattering amplitude $f(\theta)=4\sin\theta+i5\cos\theta$, the total cross-section σT is

(a) $\frac{20\pi}{k}$
(b) $\frac{5}{k^2}$
(c) $\frac{4}{k^2}$
(d) 0

Q16. For a one dimensional motion in a potential $V(x)=V_0\left(\frac{x}{x_0}\right)^6$ where V_0 and x_0 are constants, if the amplitude of oscillation is doubled, the time period of oscillation,

(a) remains same.

(b) becomes approximately one fourth for small oscillation and becomes four times for large oscillation.

(c) becomes one fourth.

(d) becomes four times

Q17. The phase space trajectory of a hamiltonian system is a plot of $p\ vS\ q$, where p and q are the momentum and coordinates, respectively. The phase space trajectory for one such system in the shape of the number '8'

(a) is not allowed as p is multivalued for certain values of q.

(b) is not admissible q is multivalued for certain values of p

(c) is admissible.

(d) is not admissible as the phase space trajectory cannot intersect itself.

Q18. For a particle whose dynamics is described by the Lagrangian

$$\frac{1}{2}m\dot{x}^2-\frac{1}{2}k(t)x^2$$

where $\left(\frac{dk}{dt}\right)\ll\sqrt{\frac{k^3}{m}}$ amplitude of oscillation A varies with frequency ω as

(a) $A^{\frac{3}{2}}\propto\omega$ (b) $\sqrt{A}\propto\omega$ (c) $A\propto\omega$ (d) $A^2\propto\omega$

Q19. Polarized monochromatic light is incident on a long thin slit parallel to the vertical plane and gets diffracted and falls on a screen. The distribution of brightness on the screen is

(a) along a long thin vertical line.

(b) along a long thin horizontal line.

(c) uniform in the image plane.

(d) in the form of circular diffraction pattern

Q20. Consider two independent random variables x_1 and x_2 that have a Gaussian distribution with the standard deviations σ_1 and σ_2 respectively. We define a new variable $x_3 = 3x_1 + 4x_2$. Then the distribution of the variable x_3 is a Gaussian with standard deviation σ given by

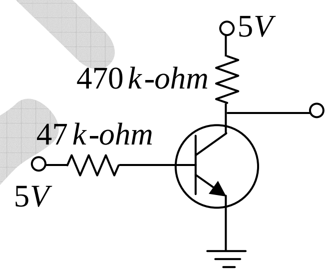
(a) $\sigma = \sqrt{9\sigma_1^2 + 16\sigma_2^2}$ (b) $\sigma = 3\sigma_1 + 4\sigma_2$

(c) $\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$ (d) $\frac{1}{\sigma} = \frac{1}{\sigma_1} + \frac{1}{\sigma_2}$

Q21. An experimentalist wants to determine the value of g by dropping a ball at rest from a b of height h and in sure the time the ball takes to reach the ground. Let $h = 10\ m$ and is measured to an accuracy of $0.01\,m$ How accurately should the experimentalist deter mine the time to measure g with at least 1% accuracy

(a) 10 ms (b) 1 ms (c) 0.1 S (d) 1 S

Q22. What is the approximate output for the following circuit for the silicon transistor when input voltage is $5\,V$?



(a) $5\,V$

(b) $0.5\,V$

(c) $0V$

(d) None of the above

Q23. A cylindrical jar contains molecular oxygen in gaseous form, maintained at temperature T. Average energy per oxygen molecule in the jar is:

(a) $\frac{1}{2}k_BT$ (b) $\frac{3}{2}k_BT$ (c) $\frac{5}{2}k_BT$ (d) $3k_BT$

Q24. Heat energy flows from a high temperature system to a low temperature system, because:

(a) higher temperature corresponds to higher potential energy.

(b) higher temperature corresponds to higher heat energy.

(c) higher temperature corresponds to higher chemical potential.

(d) that is how entropy of the combined system increases.

Q25. Find the first order probability of transition of a harmonic oscillator to go from its ground state $|0>$ to the first exited state $|1>$ for a time-dependent perturbation

$$H(t) = xe^{\frac{-t}{tau}}, \qquad t \geq 0, \tau > 0$$

for $t \to \infty$ late time and $\tau \to 0$. $P_{0\to1}^{(1)}$ is

(a) $\frac{1}{2m\hbar\omega^3}$ (b) 0 (c) 1 (d) ∞

Q26. If the electrostatic potential is given by

$$\phi = \phi_0\left(x^2 + y^2 + z^2\right)$$

where ϕ_0 is a constant then the charge density giving rise to the above, potential would be

(a) 0 (b) $-2\phi_0 \in_0$ (c) $-6\phi_0 \in_0$ (d) $\frac{-6\phi_0}{\in_0}$

Q27. A long cylinder kept along the z-axis carries a current density, $\vec{J} = J_0 r\hat{k}$, where J_0 is a constant and r is the radial distance from the axis of the cylinder. The magnetic induction $\vec{B}$ inside the conductor at a distance d from the axis of the cylinder is

(a) $\mu_0 J_0 \hat{\phi}$ (b) $\frac{-\mu_0 J_0 d}{2}\hat{\phi}$ (c) $\frac{-\mu_0 J_0 d^3}{4}\hat{\phi}$ (d) $\frac{\mu_0 J_0 d^2}{3}\hat{\phi}$

Q28. A particle executes motion in one dimension in a potential

$$V(x) = V_0\left(\frac{x^2}{x_0^2} - \frac{x^4}{x_0^4}\right)$$

where, V_0 and x_0 are constants. For small perturbations about the stable equilibrium, the particle oscillates with an angular frequency ω given by,

(a) $\frac{4V_0}{mx_0^2}$ (b) 0 (c) $\frac{2V_0}{mx_0^2}$ (d) $\frac{V_0}{mx_0^2}$

Q29. The magnitude of the electric field due a charge Q at a position $\vec{r}$ from it is $E=\frac{kQ}{r^2}$ where k is positive constant. In a Rutherford scattering experiment, the α-particles of mass m approach the gold nucleus from a large distance with a speed v_∞ and with an impact parameter ρ. During the scattering process, the minimum distance between the nucleus and the α-particle is $r_{\min}$. If Q_n and q_α denote the charge on the nucleus and the α-particle, respectively, the speed v_∞, is given by,

(a) $v_\infty^2=\frac{2kq_\alpha Q_n\rho}{m\left(r_{\min}^2-\rho^2\right)}$ (b) $v_\infty^2=\frac{2kq_\alpha Q_n r_{\min}}{m\left(r_{\min}^2-\rho^2\right)}$

(c) $v_\infty^2=\frac{2kq_\alpha Q_n r_{\min}}{m\left(r_{\min}-\rho\right)^2}$ (d) $v_\infty^2=\frac{2kq_\alpha Q_n\rho}{m\left(r_{\min}-\rho\right)^2}$

Q30. The magnitude of the electric field due to a charge Q at a position $\vec{r}$ is $E=\frac{kQ}{r^2}$, where K is positive constant A negative charge Q is placed at every vertex, of a cube except at one vertex, The side of the cube is of length l. The electric field at the center of the cube is of magnitude

a) $\frac{4KQ}{3l^2}$ away from the vertex without charge

(b) $\frac{4KQ}{3l^2}$ towards the vertex without charge

(c) $\frac{KQ}{3l^2}$ towards the vertex without charge.

(d) $\frac{KQ}{3l^2}$ away from the vertex without charge.

Q31. It is known that the root of the function $f(x)$ lies between 0 and 1. After 10 iterations of the bisection method, what will be the approximate precision of the root?

(a) Correct up to 2 places of decimal. (b) Correct up to 3 places of decimal.

(c) Correct up to 4 places o decimal (d) Correct up to 5 places of decimal

Q32. For a nucleus having homogeneous spherical charge distribution, what would be the nature of the corresponding nuclear Form Factor?

(a) Exponential (b) Gaussian

(c) Oscillating (d) Constant

Q33. A certain odd-parity state in shell model can hold up to a maximum of 16 nucleons What are its values of J and 1?

(a) $j=\frac{7}{2};1=4$ (b) $j=\frac{9}{2};1=5$

(c) $j=\frac{5}{2};1=3$ (d) $j=\frac{7}{2};1=3$

Q34. A biased coin has a 40% chance of turning up heads. In a series of ten tosses, what is the probability that you will see at least three heads?

(a) 0.62 (b) 0.48 (c) 0.96 (d) 0.36

Q35. A function $f(x)$ is defined in the interval $[-2,\quad 2]$ by

$$f(x)=\begin{cases}-1 & -2\le x<0\\ 2 & 0<x\le 2\end{cases}$$

$f(x)$ can also be written as a series

(a) $f(x)=\frac{1}{2}+\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{\left[1-(-1)^n\right]}{n}\sin\frac{n\pi x}{2}+\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{\left[1+(-1)^n\right]}{n}\cos\frac{n\pi x}{2}$

(b) $f(x)=\frac{1}{2}+\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{\left[1-(-1)^n\right]}{n}\sin\frac{n\pi x}{2}$

(c) $f(x)=\frac{1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left[1+(-1)^n\right]}{n}\sin\frac{n\pi x}{2}$

(d) $f(x)=\frac{-1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left[1+(-1)^n\right]}{n}\sin\frac{n\pi x}{2}\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left[1-(-1)^n\right]}{n}\cos\frac{n\pi x}{2}$

Q36. Consider an atom with Z protons and N neutrons in its nucleus Estimate the strength of the electric field E needed to pull the innermost electron out of the atom in a time comparable to that for the electron to go around the nucleus

(a) $E \geq 2.6\times10^{9} Z^{2} \dfrac{V}{cm}$ (b) $E \geq 1.3\times10^{10} Z^{3} \dfrac{V}{cm}$

(c) $E \geq 2.6\times10^{9} Z^{3} \dfrac{V}{cm}$ (d) $E \geq 1.3\times10^{10} Z^{2} \dfrac{V}{cm}$

Q37. In a non-conducting medium characterized by $\in=\in_0, \mu=\mu_0$ and conductivity $\sigma=0$, the electric field (in $\dfrac{V}{m}$) is given by

$$\vec{E} = 20\sin\left(10^{8}t - kz\right)\hat{j}$$

The magnetic field $\vec{H}$ (in A/m) is given by

(a) $\dfrac{20k}{10^{8}\mu_0}\sin\left(10^{8}t-kz\right)\hat{j}$ (b) $20k\cos\left(10^{8}t-kz\right)\hat{j}$

(c) $-20k\cos\left(10^{8}t-kz\right)\hat{j}$ (d) $-\dfrac{20k}{10^{8}\mu_0}\sin\left(10^{8}t-kz\right)\hat{i}$

Q38. If the binary operation * is defined as $a*b=a+b-2$, what will the unit element be?

(a) 0 (b) 3 (c) 2 (d) 1

Q39. The solution for the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

is

(a) $(a+bx)\ln x$ (b) $(a+bx)e^{x}$ (c) $\left(a+bx^2\right)e^{-x^2}$ (d) $\left(a+bx^2\right)e^{x}$

Q40. In a certain co-ordinates system $S, F^{\mu\nu}F_{\mu\nu} = 0$ where $F^{\mu\nu}$ is the electromagnetic field tensor. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ are at an angle of 45 degrees with each other. In another frame S' (which is related to S by a Lorentz transformation), the electric field $\vec{E}'$ and magnetic field $\vec{B}'$ make an angle θ' with each other. Then,

(a) $\cos(\theta') = \frac{1}{\sqrt{2}}\frac{E^2}{E'^2}$ (b) $\sin(\theta') = \frac{1}{\sqrt{2}}\frac{E^2}{E'^2}$

(c) $\cos(\theta') = \frac{1}{\sqrt{2}}\frac{E'^2}{E^2}$ (d) $\cos(\theta') = \frac{1}{\sqrt{2}}\frac{E^2}{E'^2}$

(Option (a) and (d) are same)

Q41. A beam of polarized spin - $1/2$ particles are passed through a constant magnetic field. The two possible states of the particles, after passing through the set up are denoted by $|1\rangle$ and $|2\rangle$ and the ratio of the number of particles in these two states is N_1 and N_2 respectively. If $N_1 : N_2 = 3:1$, the incoming state can be represented by the $|\ \rangle$ given by

(a) $|\ \rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$ (b) $|\ \rangle = \frac{\sqrt{3}}{4}|1\rangle + \frac{1}{4}|2\rangle$

(c) $|\ \rangle = \frac{3}{4}|1\rangle + \frac{1}{4}|2\rangle$ (d) $|\ \rangle = \frac{1}{2}|1\rangle + \frac{1}{2}|2\rangle$

Q42. Which of the following is an admissible wave-function in one dimension in the range $-\infty < x < +\infty$? (Here A and a are a constant)

(a) $A\left(x^2 + x_0^2\right)^{-1/4}$ (b) $Ae^{-(x/x_0)^3}$ (c) $A\left(x^2 + x_0^2\right)^{-1}$ (d) $Ae^{(x/x_0)^3}$

Q43. A hypothetical two-nucleon system is given by the configuration $J^P = 2^-$. In which of the following state the following would exist?

(a) admixture of ${}^3P_2$ and ${}^3D_2$ (b) admixture of ${}^3S_2$ and ${}^3F_2$

(c) admixture of ${}^3S_2$ and ${}^3P_2$ (d) admixture of ${}^3P_2$ and ${}^3F_2$

Q44. The enthalpy of fusion of ice is $6.02\,kJ/mol$. The fraction of hydrogen bonds that are broken when ice melts is (hydrogen bond energy $= 20.5\,kJ/mol$)

(a) 0.001 (b) 0.1 (c) 0.15 (d) 0.005

Q45. The Fermi Surface meets the zone boundary at

(a) 30^{o} (b) 0^{o} (c) 60^{o} (d) 90^{o}

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