CLASSICAL MECHANICS SOLUTIONS

GATE- 2010

Q1. For the set of all Lorentz transformations with velocities along the x-axis consider the two statements given below:

P: If L is a Lorentz transformation then, L^{-1} is also a Lorentz transformation.

Q: If L_1 and L_2 are Lorentz transformations then, L_1L_2 is necessarily a Lorentz transformation.

Choose the correct option

(A) P is true and Q is false (B) Both P and Q are true

(C) Both P and Q are false (D) P is false and Q is true

Ans: (b)

Q2. A particle is placed in a region with the potential $V(x)=\frac{1}{2}kx^2-\frac{\lambda}{3}x^3$, where k, λ > 0.

Then,

(A) $x=0$ and $x=\frac{k}{\lambda}$ are points of stable equilibrium

(B) $x=0$ is a point of stable equilibrium and $x=\frac{k}{\lambda}$ is a point of unstable equilibrium

(C) $x=0$ and $x=\frac{k}{\lambda}$ are points of unstable equilibrium

(D) There are no points of stable or unstable equilibrium

Ans: (b)

Solution: $V=\frac{1}{2}kx^2-\frac{\lambda x^3}{3}\Rightarrow\frac{\partial V}{\partial x}=kx-\lambda x^2=0\Rightarrow x=0,x=\frac{k}{\lambda}$.

$$\Rightarrow\frac{\partial^2 V}{\partial x^2}=k-2\lambda x$$

$$\Rightarrow At\ x=0,\frac{\partial^2 V}{\partial x^2}=+ve\text{ (Stable) and }\Rightarrow At\ x=\frac{k}{\lambda},\frac{\partial^2 V}{\partial x^2}=-ve\text{ (unstable)}$$

Q3. A π^0 meson at rest decays into two photons, which move along the x-axis. They are both detected simultaneously after a time, $t = 10$s. In an inertial frame moving with a velocity $V = 0.6c$ in the direction of one of the photons, the time interval between the two detections is

(A) 15 s (B) 0 s (C) 10 s (D) 20 s

Ans: (a)

Solution: $t_1 = t_0\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} = 10\sqrt{\frac{1+0.6}{1-0.6}} = 10\times 2 = 20\sec$, $t_2 = t_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$

$= 10\sqrt{\frac{1-0.6}{1+0.6}} = 10\times\frac{1}{2} = 5\sec$

$\Rightarrow \mathrm{t}_1 - \mathrm{t}_2 = 15\sec$

Statement for Linked Answer Questions 4 and 5:

The Lagrangian for a simple pendulum is given by $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$

Q4. Hamilton's equations are then given by

(A) $\dot{p}_\theta = -mgl\sin\theta;\quad \dot{\theta} = \frac{p_\theta}{ml^2}$

(B) $\dot{p}_\theta = mgl\sin\theta;\quad \dot{\theta} = \frac{p_\theta}{ml^2}$

(C) $\dot{p}_\theta = -m\ddot{\theta};\quad \dot{\theta} = \frac{p_\theta}{m}$

(D) $\dot{p}_\theta = -\left(\frac{g}{l}\right)\theta;\quad \dot{\theta} = \frac{p_\theta}{ml}$

Ans: (b)

Solution: $H = \frac{P_\theta^2}{2ml^2} + mgl(1-\cos\theta) \Rightarrow \frac{\partial \mathrm{H}}{\partial\theta} = -\dot{\mathrm{P}}_\theta \Rightarrow \dot{\mathrm{P}}_\theta = \mathrm{mgl}\sin\theta;\ \frac{\partial H}{\partial P_\theta} = \dot{\theta} \Rightarrow \dot{\theta} = \frac{P_\theta}{ml^2}$ .Q5. The Poisson bracket between θ and $\dot{\theta}$ is

(A) $\{\theta,\dot{\theta}\} = 1$

(B) $\{\theta,\dot{\theta}\} = \frac{1}{ml^2}$

(C) $\{\theta,\dot{\theta}\} = \frac{1}{m}$

(D) $\{\theta,\dot{\theta}\} = \frac{g}{l}$

Ans: (b)

$$\{\theta,\dot{\theta}\}=\left\{\theta,\frac{P_\theta}{ml^2}\right\} \text{ where } \dot{\theta}=\frac{P_\theta}{ml^2}. \Rightarrow \frac{1}{ml^2}\left(\frac{\partial\theta}{\partial\theta}\frac{\partial\theta}{\partial P_\theta}-\frac{\partial\theta}{\partial P_\theta}\frac{\partial P_\theta}{\partial\theta}\right)=1\cdot\frac{1}{ml^2}-0=\frac{1}{ml^2}.$$

GATE- 2011

Q6. A particle is moving under the action of a generalized potential $V(q,\dot{q})=\frac{1+\dot{q}}{q^2}$. The magnitude of the generalized force is

(A) $\frac{2(1+\dot{q})}{q^3}$ (B) $\frac{2(1-\dot{q})}{q^3}$ (C) $\frac{2}{q^3}$ (D) $\frac{\dot{q}}{q^3}$

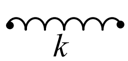
Ans: (c)

Solution: $$\frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}}\right)-\frac{\partial V}{\partial q}=F_q \Rightarrow F_q=\frac{2}{q^3}.$$

Q7. Two bodies of mass m and $2m$ are connected by a spring constant k. The frequency of the normal mode is

(A) $\sqrt{3k/2m}$ (B) $\sqrt{k/m}$ (C) $\sqrt{2k/3m}$ (D) $\sqrt{k/2m}$

Ans: (a)

Solution: m  $2m$ $$\omega=\sqrt{\frac{k}{\mu}}=\sqrt{\frac{k}{\frac{2m}{3}}}=\sqrt{\frac{3k}{2m}}$$ where reduce mass $\mu=\frac{2mm}{2m+m}=\frac{2m}{3}$.

Q8. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$$Q=q^\alpha\cos(\beta p),\ P=q^\alpha\sin(\beta p)$$

is canonical for

(A) $\alpha = 2, \beta = 1/2$ (B) $\alpha = 2, \beta = 2$ (C) $\alpha = 1, \beta = 1$ (D) $\alpha = 1/2, \beta = 2$

Ans: (d)

Solution: $$\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q}=1$$

$$\Rightarrow \alpha q^{\alpha-1}\cos(\beta p)\times q^\alpha\beta\cos(\beta p)-q^\alpha\beta(-\sin(\beta p))\times\alpha q^{\alpha-1}\sin(\beta p)=1$$

$$\alpha q^{2\alpha-1}\beta(\cos^2\beta p+\sin^2\beta p)=1\Rightarrow \alpha\beta q^{2\alpha-1}=1 \Rightarrow \alpha=\frac{1}{2},\beta=2.$$

Q9. Two particles each of rest mass m collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is

(A) $5m/4$ (B) $2m$ (C) $5m/2$ (D) $25\ m/8$

Ans: (c)

Solution: From conservation of energy

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}+\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=m_1c^2 \Rightarrow \frac{2mc^2}{\sqrt{1-\frac{v^2}{c^2}}}=m_1c^2$$

Since $v=0.6c \Rightarrow m_1=5m/2$

GATE- 2012

Q10. In a central force field, the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by,

$$\frac{1}{r}=\frac{m}{L^2}(1+\varepsilon\cos\theta)$$

where, ε is the eccentricity of the particle's motion. Which one of the following choice for ε gives rise to a parabolic trajectory?

(a) $\varepsilon=0$ (b) $\varepsilon=1$ (c) $0<\varepsilon<1$ (d) $\varepsilon>1$

Ans: (b)

Solution: $\frac{l}{r}=\frac{m}{l^2}(1+\varepsilon\cos\theta)$ for parabolic trajectory $\varepsilon=1$.

Q11. A particle of unit mass moves along the x-axis under the influence of a potential, $V(x)=x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x=2$. The time period of oscillation of the particle is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

Ans: (b)

$$V(x) = x(x-2)^2 \Rightarrow \frac{\partial V}{\partial x} = (x-2)^2 + 2x(x-2) = 0 \Rightarrow \mathrm{x} = 2, \mathrm{x} = \frac{2}{3}$$

$$\frac{\partial^2 V}{\partial x^2} = 2(x-2)^2 + 2(x-2) + 2x \Rightarrow \frac{\partial^2 V}{\partial x^2} = 2 \times 2 = 4$$

$$\Rightarrow \omega = \sqrt{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=2}} \Rightarrow \omega = \frac{2\pi}{T} = 2 \qquad \Rightarrow T = \pi .$$

Q12. A rod of proper length l_0 oriented parallel to the x-axis moves with speed $2c/3$ along the x-axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed $c/2$ with respect to the S-frame. The length of the rod as measured by the observer is

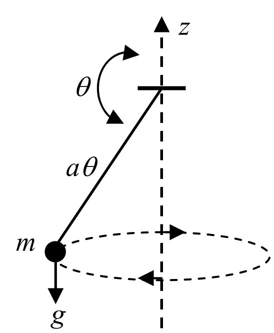
(a) $0.35l_0$ (b) $0.48l_0$ (c) $0.87l_0$ (d) $0.97l_0$

Ans: (d)

Solution: $l = l_0\sqrt{1-\frac{u^2{}_x}{c^2}} = 0.97\ l_0$

Q13. A particle of mass m is attached to fixed point O by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure. The Lagrangian of the particle is

$L(\theta,\phi) = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) - mga\cos\theta$ where θ and ϕ are the polar angles. The Hamiltonian of the particles is



(a) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) - mga\cos\theta$

(b) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

(c) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) - mga\cos\theta$ (d) $H = \frac{1}{2ma^2}\left(p_\theta^2 + p_\phi^2\right) + mga\cos\theta$

Ans: (b)

Solution: $H = P_\theta\dot{\theta} + P_\phi\dot{\phi} - L = P_\theta\dot{\theta} + P_\phi\dot{\phi} - \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) + mga\cos\theta$

$$\frac{\partial L}{\partial\dot{\theta}} = P_\theta \Rightarrow ma^2\dot{\theta} = P_\theta \Rightarrow \dot{\theta} = \frac{P_\theta}{ma^2} \text{ and } P_\phi = \frac{\partial L}{\partial\dot{\phi}} = ma^2\sin^2\theta\dot{\phi} \Rightarrow \dot{\phi} = \frac{P_\phi}{ma^2\sin^2\theta}$$

Put the value of $\dot{\theta}$ and $\dot{\phi}$

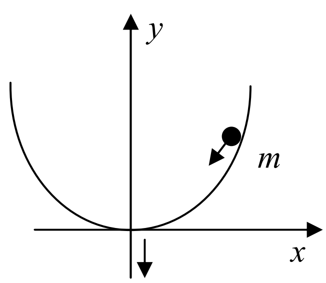
$$H = P_\theta \times \frac{P_\theta}{ma^2} + P_\phi \times \frac{P_\phi}{ma^2\sin^2\theta} - -\frac{1}{2}ma^2\left(\left(\frac{P_\theta}{ma^2}\right)^2 + \sin^2\theta\left(\frac{P_\phi}{ma^2\sin^2\theta}\right)^2\right) + mga\cos\theta$$

$$H = \frac{P_\theta^2}{ma^2} - \frac{P_\theta^2}{2ma^2} + \frac{P_\phi^2}{ma^2\sin^2\theta} - \frac{P_\phi^2}{2ma^2\sin^2\theta} + mga\cos\theta$$

$$H = \frac{1}{2ma^2}\left(P_\theta^2 + \frac{P_\theta^2}{\sin^2\theta}\right) + mga\cos\theta$$

Statement for Linked Answer Questions 14 and 15:

Q14. A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$ axis shown in the figure. Here a is a constant.



The Lagrangian for this particle is given by

(a) $L = \frac{1}{2}m\dot{x}^2 - mgax^2$

(b) $L = \frac{1}{2}m\left(1 + 4a^2x^2\right)\dot{x}^2 - mgax^2$

(c) $L = \frac{1}{2}m\dot{x}^2 + mgax^2$

(d) $L = \frac{1}{2}m\left(1 + 4a^2x^2\right)\dot{x}^2 + mgax^2$

Ans: (d)

Solution: Equation of constrain is given by $y = ax^2$, K.E $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right)$

$\dot{y} = 2ax\dot{x} \Rightarrow T = \frac{1}{2}m\left(\dot{x}^2 + 4ax^2\dot{x}^2\right) = \frac{1}{2}m\dot{x}^2\left(1 + 4ax^2\right)$

$V = -mgy = -mgax^2$. Since particle is moving downward direction so potential V is negative.

$\because L = T - V \Rightarrow L = \frac{1}{2}m\left(1 + 4a^2x^2\right)\dot{x}^2 + mgax^2$

Q15. The Lagrange's equation of motion of the particle for above question is given by

(a) $\ddot{x} = 2gax$

(b) $m\left(1 + 4a^2x^2\right)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$

(c) $m\left(1 + 4a^2x^2\right)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$

(d) $\ddot{x} = -2gax$

Ans: (c)

Solution: $\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx} \Rightarrow m(1 + 4a^2x^2)\ddot{x} = 4ma^2x\dot{x}^2 + 2mgax$

GATE- 2013

Q16. In the most general case, which one of the following quantities is NOT a second order tensor?

(a) Stress

(b) Strain

(c) Moment of inertia

(d) Pressure

Ans: (b)

Solution: Strain is not a tensor.

Q17. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is

(a) c

(b) $-c$

(c) $0.15c$

(d) $-0.15c$

Ans: (b)

Q18. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

(a) $p_q = 2\beta q$ and it is a conserved quantity.

(b) $p_q = 2\beta q$ and it is not a conserved quantity.

(c) $p_q = 2\alpha\dot{q}$ and it is a conserved quantity.

(d) $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

Ans: (d)

Solution: $\frac{\partial L}{\partial \dot{q}} = p_q$ but $\frac{\partial L}{\partial q} \neq 0$

Q19. The relativistic form of Newton's second law of motion is

(a) $F = \frac{mc}{\sqrt{c^2 - v^2}}\frac{dv}{dt}$ (b) $F = \frac{m\sqrt{c^2 - v^2}}{c}\frac{dv}{dt}$

(c) $F = \frac{mc^2}{c^2 - v^2}\frac{dv}{dt}$ (d) $F = m\frac{c^2 - v^2}{c^2}\frac{dv}{dt}$

Ans:

Solution: $P = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow F = \frac{dP}{dt} = m\frac{dv}{dt}\cdot\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + mv\left(-\frac{1}{2}\right)\cdot\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}\cdot\frac{-2v}{c^2}\frac{dv}{dt}$

$$\Rightarrow F = m\frac{dv}{dt}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(1 + \frac{1}{2}\frac{v^2/c^2}{\left(1 - \frac{v^2}{c^2}\right)}\right) = m\frac{dv}{dt}\left(\frac{1 - v^2/2c^2}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}\right)$$

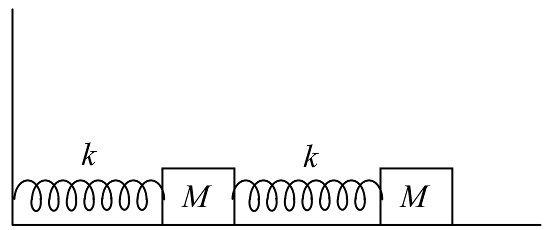
Q20. Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,

(a) $\sqrt{\frac{3+\sqrt{2}}{2}}\sqrt{\frac{k}{M}}$

(b) $\sqrt{\frac{3+\sqrt{3}}{2}}\sqrt{\frac{k}{M}}$

(c) $\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{M}}$

(d) $\sqrt{\frac{3+\sqrt{6}}{2}}\sqrt{\frac{k}{M}}$



Ans: (c)

Solution: $T=\frac{1}{2}m\dot{x}_1^2+\frac{1}{2}m\dot{x}_2^2$,

$$V=\frac{1}{2}kx_1^2+\frac{1}{2}k\left(x_2-x_1\right)^2=\frac{1}{2}kx_1^2+\frac{1}{2}k\left(x_2^2+x_1^2-2x_2x_1\right)=\frac{1}{2}k\left(2x^2+x^2-2x_2x_1\right)$$

$$T=\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix};\ V=\begin{pmatrix} 2k & -k \\ -k & k \end{pmatrix}$$

$$\begin{vmatrix} 2k-\omega^2 m & -k \\ -k & k-\omega^2 m \end{vmatrix}=0\Rightarrow\left(2k-\omega^2 m\right)\left(k-\omega^2 m\right)-k^2=0\Rightarrow\omega=\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{m}}$$

GATE- 2014

Q21. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is $5\times10^{-8}\,s$, then the proper half-life is ______________ $\times10^{-8}\,s.\left(c=3\times10^{8}\,m/s\right)$

Ans: 2.18

Solution: $t=\dfrac{t_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$, $t_0=t\times\sqrt{1-\dfrac{v^2}{c^2}}$ $=$ $t_0=5\times10^{-8}\times\sqrt{.19}$ $\Rightarrow 2.18\times10^{-8}s$

Q22. Two masses m and $3m$ are attached to the two ends of a massless spring with force constant K. If $m=100g$ and $K=0.3N/m$, then the natural angular frequency of oscillation is ________ Hz.

Ans: 0.318

Solution: $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{\mu}}$ $\mu=\dfrac{m_1.m_2}{m_1+m_2}=\dfrac{3m.m}{4m}=\dfrac{3m}{4}$ $\omega=\sqrt{\dfrac{4k}{3m}}=2=0.318\ Hz$

Q23. The Hamilton's canonical equation of motion in terms of Poisson Brackets are

(a) $\dot{q}=\{q,H\};\dot{p}=\{p,H\}$ (b) $\dot{q}=\{H,q\};\dot{p}=\{H,p\}$

(c) $\dot{q}=\{H,p\};\dot{p}=\{H,p\}$ (d) $\dot{q}=\{p,H\};\dot{p}=\{q,H\}$

Ans: (a)

Solution: $\dfrac{df}{dt}=\dfrac{\partial f}{\partial q}.\dfrac{\partial q}{\partial t}+\dfrac{\partial f}{\partial p}.\dfrac{\partial p}{\partial t}+\dfrac{\partial f}{\partial t}$

$$\frac{df}{dt}=\frac{\partial f}{\partial q}.\frac{\partial H}{\partial p}-\frac{\partial f}{\partial p}.\frac{\partial H}{\partial q}+\frac{\partial f}{\partial t}\Rightarrow\frac{df}{dt}=\{f,H\}+\frac{\partial f}{\partial t}$$

$$\frac{dq}{dt}=\{q,H\}\text{ and }\frac{dp}{dt}=\{p,H\}$$

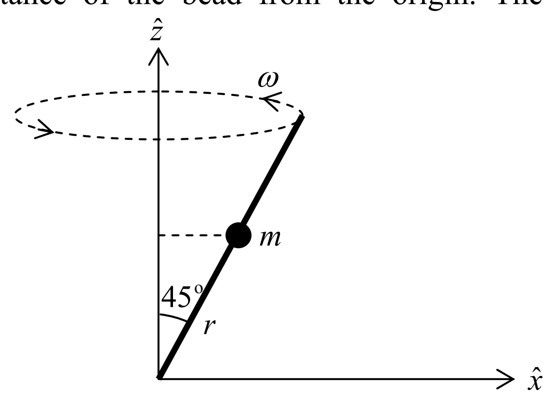
Q24. A bead of mass m can slide without friction along a mass less rod kept at 45^o with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant r is the distance of the bead from the origin. The momentum conjugate to r is

(a) $m\dot{r}$

(b) $\dfrac{1}{\sqrt{2}}m\dot{r}$

(c) $\dfrac{1}{2}m\dot{r}$

(d) $\sqrt{2}m\dot{r}$



Ans: (a)

Solution: $L=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2)-mgr\cos\theta$

equation of constrain is $\theta=\frac{\pi}{4}$ and it is given $\dot{\phi}=\omega$

$$L=\frac{1}{2}m(\dot{r}^2+\frac{1}{2}r^2\omega^2)-\frac{1}{\sqrt{2}}mgr$$

the momentum conjugate to r is $p_r=\frac{\partial L}{\partial \dot{r}}$ = $p_r=m\dot{r}$

Q25. A particle of mass m is in a potential given by

$$V(r)=-\frac{a}{r}+\frac{ar_0^2}{3r^3}$$

when a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

Ans: (a)

Solution: $V(r)=-\frac{a}{r}+\frac{ar_0^2}{3r^3}$ for equilibrium $\frac{\partial V}{\partial r}=\frac{a}{r^2}-\frac{3ar_0^2}{3r^4}=0$ $r=\pm r_0$

$$\frac{\partial^2V}{\partial r^2}=-\frac{2a}{r^3}+\frac{4ar_0^2}{r^5}\bigg|_{r_0}=-\frac{2a}{r_0^3}+\frac{4ar_0^2}{r_0^5}=\frac{2a}{r_0^3}$$

$$\omega=\sqrt{\frac{\left.\frac{\partial^2V}{\partial r^2}\right|_{r_0}}{m}}\Rightarrow T=2\pi\sqrt{\frac{mr_0^3}{2a}}$$

Q26. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r)=-\frac{k}{r}$, where k is a positive constant. The orbit angular momentum of the planet is

(a) $2r_0km$ (b) $\sqrt{2r_0km}$ (c) r_0km (d) $\sqrt{r_0km}$

Ans: (d)

Solution: $V_{effctive} = \frac{J^2}{2mr^2} - \frac{k}{r} \Rightarrow \frac{dV_{effect}}{dr} = -\frac{J^2}{mr^3} + \frac{k}{r^2} = 0$ at $r = r_0$

so $J = \sqrt{r_0 km}$

Q27. Given that the linear transformation of a generalized coordinate q and the corresponding momentum p,

$$Q = q + 4ap$$
$$P = q + 2p$$

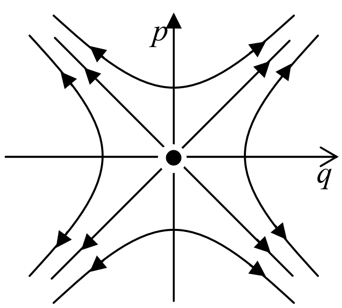
is canonical, the value of the constant a is ________________

Ans: 0.5

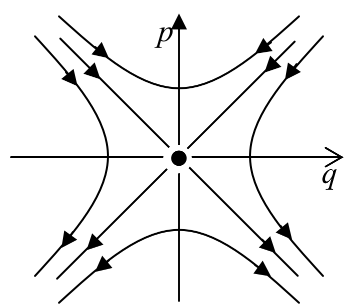
Solution: $\frac{\partial Q}{\partial q} \cdot \frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p} \cdot \frac{\partial P}{\partial q} = 0 \Rightarrow 1.2 - 4a.1 = 0 \Rightarrow a = 0.5$

Q28. The Hamiltonian of particle of mass m is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$.which one of the following figure describes the motion of the particle in phase space?

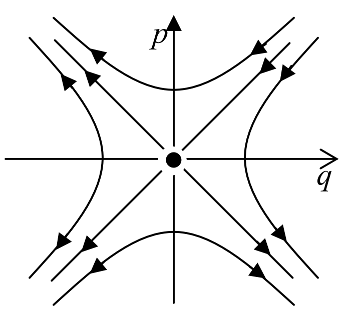
(a)



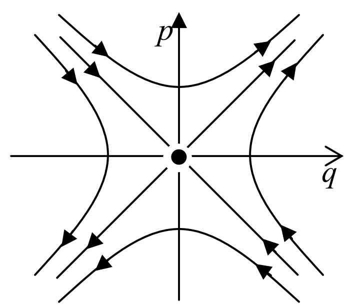
(b)



(c)



(d)



Ans: (d)

GATE- 2015

Q29. A satellite is moving in a circular orbit around the Earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

(a) $V = -2T; E = -T$ (b) $V = -T; E = 0$

(c) $V = -\frac{T}{2}; E = \frac{T}{2}$ (d) $V = \frac{-3T}{2}; E = \frac{-T}{2}$

Ans.: (a)

Solution: From Virial theorem $\langle T \rangle = \frac{n+1}{2}\langle V \rangle$ where $V \propto r^{n+1}$

$$\because V = \frac{-k}{r} \Rightarrow V \propto \frac{1}{r} \Rightarrow n = -2 \Rightarrow \langle V \rangle = -2\langle T \rangle$$

Q30. In an inertial frame S, two events A and B take place at $(ct_A = 0, \vec{r}_A = 0)$ and $(ct_B = 0, \vec{r}_B = 2\hat{y})$, respectively. The times at which these events take place in a frame S' moving with a velocity $0.6c\hat{y}$ with respect to S are given by

(a) $ct'_A = 0;\ ct'_B = -\frac{3}{2}$ (b) $ct'_A = 0;\ ct'_B = 0$

(c) $ct'_A = 0;\ ct'_B = \frac{3}{2}$ (d) $ct'_A = 0;\ ct'_B = \frac{1}{2}$

Ans.: (a)

Solution: Velocity of S' with respect to S is $v = .6c$

$$t'_A = \frac{t_A - \frac{v}{c^2}y}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{for event A } t_A = 0, y = 0 \text{ so } ct'_A = 0$$

$$t'_B = \frac{t_B - \frac{v}{c^2}y}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{for event B } t_B = 0, y = 2 \text{ so } ct'_B = -\frac{3}{2}$$

Q31. The Lagrangian for a particle of mass m at a position $\vec{r}$ moving with a velocity $\vec{v}$ is given by $L = \frac{m}{2}\vec{v}^2 + C\vec{r}.\vec{v} - V(r)$, where $V(r)$ is a potential and C is a constant. If $\vec{p}_c$ is the canonical momentum, then its Hamiltonian is given by

(a) $\frac{1}{2m}(\vec{p}_c + C\vec{r})^2 + V(r)$ (b) $\frac{1}{2m}(\vec{p}_c - C\vec{r})^2 + V(r)$

(c) $\frac{p_c^2}{2m} + V(r)$ (d) $\frac{1}{2m}p_c^2 + C^2r^2 + V(r)$

Ans.: (b)

Solution: $L = \frac{m}{2}\vec{v}^2 + C\vec{r}.\vec{v} - V(r)$ where $\vec{v} = \dot{r}$

$$H = \sum \dot{r}p_c - L = \dot{r}p_c - L \quad \text{where } L = \frac{m}{2}\dot{r}^2 + Cr.\dot{r} - V(r)$$

$$\Rightarrow \frac{\partial L}{\partial \dot{r}} = p_c = (m\dot{r} + Cr) \Rightarrow \dot{r} = \frac{p_c - Cr}{m}$$

$$\Rightarrow H = \left(\frac{p_c - Cr}{m}\right)p_c - \frac{m}{2}\left(\frac{p_c - Cr}{m}\right)^2 - cr\left(\frac{p_c - Cr}{m}\right) + V(r)$$

$$\Rightarrow H = \left(\frac{p_c - Cr}{m}\right)(p_c - Cr) - \frac{m}{2}\left(\frac{p_c - Cr}{m}\right)^2 + V(r)$$

$$\Rightarrow H = \frac{(p_c - Cr)^2}{m} - \frac{(p_c - Cr)^2}{2m} + V(r) \quad \Rightarrow H = \frac{1}{2m}(p_c - Cr)^2 + V(r)$$

Q32. The Hamiltonian for a system of two particles of masses m_1 and m_2 at $\vec{r}_1$ and $\vec{r}_2$ having velocities $\vec{v}_1$ and $\vec{v}_2$ is given by $H = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{C}{(\vec{r}_1 - \vec{r}_2)^2}\hat{z}\cdot(\vec{r}_1 \times \vec{r}_2)$, wrong where C is constant. Which one of the following statements is correct?

(a) The total energy and total momentum are conserved

(b) Only the total energy is conserved

(c) The total energy and the z - component of the total angular momentum are conserved

(d) The total energy and total angular momentum are conserved

Ans.: (c)

Solution: Lagrangian is not function of time so energy is conserve and component of $(\vec{r}_1 \times \vec{r}_2)$ are Only in $\hat{z}$ direction means potential is symmetric under ϕ so L_z is conserve.

Q33. A particle of mass $0.01\,kg$ falls freely in the earth's gravitational field with an initial velocity $(0) = 10ms^{-1}$. If the air exerts a frictional force of the form, $f = -kv$, then for $k = 0.05\,Nm^{-1}s$, the velocity (in ms^{-1}) at time $t = 0.2\,s$ is _________ (upto two decimal places). (use $g = 10\,ms^{-2}$ and $e = 2.72$)

Ans.: Data given is incorrect

Solution: $m\frac{dv}{dt} = mg - kv \Rightarrow \frac{dv}{dt} = g - \frac{k}{m}v \Rightarrow \frac{dv}{g - \frac{k}{m}v} = dt \Rightarrow \int_{10}^{u} \frac{dv}{g - \frac{k}{m}v} = \int_0^{0.2} dt$

$$\Rightarrow -\frac{m}{k}\left[\ln\left[g - \frac{k}{m}v\right]\right]_{10}^{u} = [t]_0^{0.2} \Rightarrow -\frac{m}{k}\left\{\left[\ln\left(g - \frac{k}{m}u\right)\right] - \ln\left(g - \frac{10k}{m}\right)\right\} = 0.2$$

$$\Rightarrow -\frac{m}{k}\left\{\ln\left(10 - \frac{0.05}{0.01}u\right) - \ln\left(10 - 10 \times \frac{.05}{.01}\right)\right\} = 0.2$$

$$\Rightarrow -\frac{m}{k}\{\ln(10 - 5u) - \ln(-40)\} = 0.2$$

$\because \ln(-40)$ can not be defined. So given data are not correct.

Q34. Consider the motion of the Sun with respect to the rotation of the Earth about its axis. If $\vec{F}_c$ and $\vec{F}_{Co}$ denote the centrifugal and the Coriolis forces, respectively, acting on the Sun, then

(a) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = \vec{F}_c$

(b) $\vec{F}_c$ is radially inward and $\vec{F}_{Co} = -2\vec{F}_c$

(c) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = -2\vec{F}_c$

(d) $\vec{F}_c$ is radially outward and $\vec{F}_{Co} = 2\vec{F}_c$

Ans.: (b)

Q35. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{3}{10}M$ which move along the z axis. Their velocities are given by

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z}$; $\vec{v}_2 = (-0.8c)\hat{z}$

Ans.: (b)

Solution: $M \to \frac{3}{10}M + \frac{3}{10}M$

From momentum conservation

$$0 = \vec{P}_1 + \vec{P}_2 \Rightarrow \vec{P}_1 = -\vec{P}_2 \Rightarrow |P_1| = |P_2|$$

From energy conservation $E = E_1 + E_2$

$$\Rightarrow Mc^2 = \frac{3}{10}\frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{3}{10}\frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow Mc^2 = \frac{3}{5}\frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(1-\frac{v^2}{v^2}\right) = \frac{9}{25} \Rightarrow \frac{v^2}{v^2} = \frac{16}{25} \Rightarrow v = 0.8c$$

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Q36. The kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. Its momentum in units of m_0c, where c is the speed of light in vacuum, is _______.

(Give your answer upto two decimal places

Ans. : 1.73

Solution: $\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = 2m_0c^2 \Rightarrow E$

$$E^2 = p^2c^2 + m_0^2c^4 \Rightarrow 4m_0^2c^4 - m_0^2c^4 = p^2c^2 \Rightarrow p = \sqrt{3}m_0c = 1.732m_0c$$

Q37. In an inertial frame of reference S, an observer finds two events occurring at the same time at coordinates $x_1 = 0$ and $x_2 = d$ A different inertial frame S' moves with velocity v with respect to S along the positive x-axis. An observer in S' also notices these two events and finds them to occur at times t_1' and t_2' and at positions x_1' and x_2' respectively. If $\Delta t' = t_2' - t_1', \Delta x' = x_2' - x_1'$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which of the following statements is true?

(a) $\Delta t' = 0, \Delta x' = \gamma d$ (b) $\Delta t' = 0, \Delta x' = \frac{d}{\gamma}$

(c) $\Delta t' = \frac{-\gamma v d}{c^2}, \Delta x' = \gamma d$ (d) $\Delta t' = \frac{-\gamma v d}{c^2}, \Delta x' = \frac{d}{\gamma}$

Ans.: (c)

Solution: $t_2' - t_1' = \left(\frac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) - \left(\frac{t_1 - \frac{vx_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \Rightarrow \Delta t' = \gamma \Delta t - \frac{\gamma v \Delta x}{c^2}$ it is given $\Delta t = 0, \Delta x = d$

$\Rightarrow \Delta t' = -\frac{\gamma v \Delta x}{c^2}$

$x_2' - x_1' = \left(\frac{x_2 - vt_2}{\sqrt{1-\frac{v^2}{c^2}}}\right) - \left(\frac{x_1 - vt_1}{\sqrt{1-\frac{v^2}{c^2}}}\right) \Rightarrow \Delta x' = \gamma\left(\Delta x - v\Delta t\right)$ it is given $\Delta t = 0, \Delta x = d$

$\Rightarrow \Delta x' = \gamma d$

Q38. The Lagrangian of a system is given by

$L = \frac{1}{2} m l^2 \left[\dot{\theta}^2 + \sin^2\theta \dot{\varphi}^2\right] - mgl\cos\theta$, where m, l and g are constants.

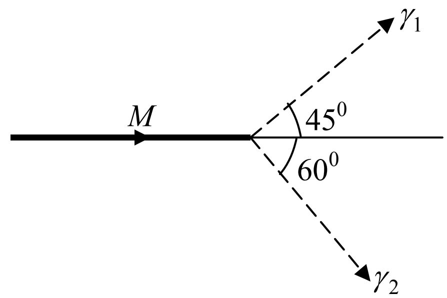
Which of the following is conserved?

(a) $\dot{\varphi}\sin^2\theta$ (b) $\dot{\varphi}\sin\theta$ (c) $\frac{\dot{\varphi}}{\sin\theta}$ (d) $\frac{\dot{\varphi}}{\sin^2\theta}$

Ans.: (a)

Solution: φ is cyclic coordinate so $\frac{\partial L}{\partial \dot{\varphi}} = p_\varphi \Rightarrow ml^2 \sin^2 \dot{\varphi}$ is constant hence m, l and g are constants. Then $\dot{\varphi} \sin^2 \theta$

Q39. A particle of rest mass M is moving along the positive x-direction. It decays into two photons γ_1 and γ_2 as shown in the figure. The energy of γ_1 is $1\ GeV$ and the energy of γ_2 is $0.82\ GeV$. The value of M (in units of $\frac{GeV}{c^2}$) is ________. (Give your answer upto two decimal places)



Ans.: 1.40

Solution: $\sqrt{p^2c^2 + M^2c^4} = E_1 + E_2 = 1.82 GeV$

$$p = \frac{E_1}{c}\cos\theta_1 + \frac{E_2}{c}\cos\theta_2 = \frac{1GeV}{c}\frac{1}{\sqrt{2}} + \frac{.82GeV}{c}\frac{1}{2} = \frac{1.11GeV}{c}$$

$$\Rightarrow p^2c^2 + m^2c^4 = 3.312$$

$$\Rightarrow m^2c^4 = 3.312 - 1.21 = 2.077$$

$$\Rightarrow m = \sqrt{2.076} = 1.40$$