

## GATE-2013(PH: PHYSICS)

*Duration:* Three Hours

*Maximum Marks:* 100

1. There are a total of 65 questions carrying 100 marks. Questions are of multiple choice type or numerical answer type. A multiple choice type question will have four choices for the answer with only **one** correct choice. For numerical answer type questions, the answer is a number and no choices will be given. **A number as the answer should be entered** using the virtual keyboard on the monitor.
2. Questions Q.1 – Q.25 carry 1 mark each. Questions Q.26 – Q.55 carry 2 marks each. The 2 marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is not attempted, then the answer to the second question in the pair will not be evaluated.
3. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1 mark each, and questions Q.61 – Q.65 carry 2 marks each.
4. Questions not attempted will result in zero mark. Wrong answers for multiple choice type questions will result in **NEGATIVE** marks. For all 1 mark questions,  $\frac{1}{3}$  mark will be deducted for each wrong answer. For all 2 marks questions,  $\frac{2}{3}$  mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question. There is no negative marking for questions of numerical answer type.
5. Calculator is allowed. Charts, graph sheets or tables are **NOT** allowed in the examination hall.
6. Do the rough work in the Scribble Pad provided.

### Head office

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

### Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

**GATE-2013(PH: PHYSICS)**

- Q1.  $f(x)$  is a symmetric periodic function of  $x$  i.e.  $f(x) = f(-x)$ . Then, in general, the Fourier series of the function  $f(x)$  will be of the form
- (a)  $f(x) = \sum_{n=1}^{\infty} (a_n \cos(nkx) + b_n \sin(nkx))$   
 (b)  $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nkx))$   
 (c)  $f(x) = a_0 + \sum_{n=1}^{\infty} (b_n \sin(nkx))$   
 (d)  $f(x) = a_0 + \sum_{n=1}^{\infty} (b_n \sin(nkx))$
- Q2. In the most general case, which one of the following quantities is NOT a second order tensor?
- (a) Stress (b) Strain  
 (c) Moment of inertia (d) Pressure
- Q3. An electron is moving with a velocity of  $0.85c$  in the same direction as that of a moving photon. The relative velocity of the electron with respect to photon is
- (a)  $c$  (b)  $-c$  (c)  $0.15c$  (d)  $-0.15c$
- Q4. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature  $T$  would be ( $k$  is the Boltzmann constant and  $T$  is nonzero)
- (a) zero (b) Infinite (c)  $\frac{3}{2}kT$  (d)  $kT$
- Q5. Across a first order phase transition, the free energy is
- (a) proportional to the temperature  
 (b) a discontinuous function of the temperature  
 (c) a continuous function of the temperature but its first derivative is discontinuous  
 (d) such that the first derivative with respect to temperature is continuous

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
 Near IIT, Hauz Khas, New Delhi-16  
 Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
 28-B/6, Jia Sarai, Near IIT  
 Hauz Khas, New Delhi-16

- Q6. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same
- (a) pressure and temperature (b) volume and temperature  
(c) pressure and volume (d) volume and energy
- Q7. The entropy function of a system is given by  $S(E) = aE(E_0 - E)$  where  $a$  and  $E_0$  are positive constants. The temperature of the system is
- (a) negative for some energies (b) increases monotonically with energy  
(c) decreases monotonically with energy (d) Zero
- Q8. Consider a linear collection of  $N$  independent spin  $\frac{1}{2}$  particles, each at a fixed location. The entropy of this system is ( $k$  is the Boltzmann constant)
- (a) zero (b)  $Nk$  (c)  $\frac{1}{2}Nk$  (d)  $Nk \ln(2)$
- Q9. The decay process  $n \rightarrow p^+ + e^- + \bar{\nu}_e$  violates
- (a) Baryon number (b) lepton number (c) isospin (d) strangeness
- Q10. The isospin ( $I$ ) and baryon number ( $B$ ) of the up quark is
- (a)  $I = 1, B = 1$  (b)  $I = 1, B = 1/3$   
(c)  $I = 1/2, B = 1$  (d)  $I = 1/2, B = 1/3$
- Q11. Consider the scattering of neutrons by protons at very low energy due to a nuclear potential of range  $r_0$ . Given that,  $\cot(kr_0 + \delta) \approx -\frac{\gamma}{k}$  where  $\delta$  is the phase shift,  $k$  the wave number and  $(-\gamma)$  the logarithmic derivative of the deuteron ground state wave function, the phase shift is
- (a)  $\delta \approx -\frac{k}{\gamma} - kr_0$  (b)  $\delta \approx -\frac{\gamma}{k} - kr_0$   
(c)  $\delta \approx \frac{\pi}{2} - kr_0$  (d)  $\delta \approx -\frac{\pi}{2} - kr_0$

### Head office

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

### Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

- Q12. In the  $\beta$  decay process, the transition  $2^+ \rightarrow 3^+$ , is
- allowed both by Fermi and Gamow-Teller selection rule
  - allowed by Fermi and but not by Gamow-Teller selection rule
  - not allowed by Fermi but allowed by Gamow-Teller selection rule
  - not allowed both by Fermi and Gamow-Teller selection rule
- Q13. At a surface current, which one of the magnetostatic boundary condition is NOT CORRECT?
- Normal component of the magnetic field is continuous.
  - Normal component of the magnetic vector potential is continuous.
  - Tangential component of the magnetic vector potential is continuous.
  - Tangential component of the magnetic vector potential is not continuous.
- Q14. Interference fringes are seen at an observation plane  $z = 0$ , by the superposition of two plane waves  $A_1 \exp[i(\vec{k}_1 \cdot \vec{r} - \omega t)]$  and  $A_2 \exp[i(\vec{k}_2 \cdot \vec{r} - \omega t)]$ , where  $A_1$  and  $A_2$  are real amplitudes. The condition for interference maximum is
- $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = (2m + 1)\pi$
  - $(\vec{k}_1 - \vec{k}_2) \cdot \vec{r} = 2m\pi$
  - $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = (2m + 1)\pi$
  - $(\vec{k}_1 + \vec{k}_2) \cdot \vec{r} = 2m\pi$
- Q15. For a scalar function  $\varphi$  satisfying the Laplace equation,  $\nabla \varphi$  has
- zero curl and non-zero divergence
  - non-zero curl and zero divergence
  - zero curl and zero divergence
  - non-zero curl and non-zero divergence

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

- Q16. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which one of the following statements is correct?
- (a) The reflected light is plane polarized in the plane of incidence and the transmitted light is circularly polarized.
- (b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.
- (c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is elliptically polarized.
- (d) There will be no reflected light and the transmitted light is circularly polarized.
- Q17. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings.
- (a)  $[L^2, L_z] = 0$
- (b)  $[L_x, L_y] = i\hbar L_z$
- (c)  $[L_z, L_+] = \hbar L_+$
- (d)  $[L_z, L_-] = \hbar L_-$
- Q18. The Lagrangian of a system with one degree of freedom  $q$  is given by  $L = \alpha \dot{q}^2 + \beta q^2$ , where  $\alpha$  and  $\beta$  are non-zero constants. If  $p_q$  denotes the canonical momentum conjugate to  $q$  then which one of the following statements is CORRECT?
- (a)  $p_q = 2\beta q$  and it is a conserved quantity.
- (b)  $p_q = 2\beta q$  and it is not a conserved quantity.
- (c)  $p_q = 2\alpha \dot{q}$  and it is a conserved quantity.
- (d)  $p_q = 2\alpha \dot{q}$  and it is not a conserved quantity.
- Q19. What should be the clock frequency of a 6-bit A/D converter so that its maximum converted time is  $32\mu\text{s}$ ?
- (a) 1MHz
- (b) 2MHz
- (c) 0.5MHz
- (d) 4MHz

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

- Q20. A phosphorous doped silicon semiconductor (doping density:  $10^{17}/\text{cm}^3$ ) is heated from  $100^\circ\text{C}$  to  $200^\circ\text{C}$ . Which one of the following statements is CORRECT?
- (a) Position of Fermi level moves towards conduction band  
 (b) Position of dopant level moves towards conduction band  
 (c) Position of Fermi level moves towards middle of energy gap  
 (d) Position of dopant level moves towards middle of energy gap
- Q21. Considering the BCS theory of superconductors, which one of the following statements is NOT CORRECT?
- ( $h$  is the Plank's constant and  $e$  is the electronic charge)
- (a) Presence of energy gap at temperature below the critical temperature  
 (b) Different critical temperature for isotopes  
 (c) Quantization of magnetic flux in superconduction ring in the unit of  $\left(\frac{h}{e}\right)$   
 (d) Presence of Meissner effect
- Q22. Group I contains elementary excitations in solids. Group II gives the associated field with these exciations. MATCH the excitations with their associated field and select your answer as per codes given below.

**Group I**

- (P) phonon  
 (Q) plasmon  
 (R) polaron  
 (S) polariton

Codes

- (a)  $(P - iv), (Q - iii), (R - i), (S - ii)$   
 (b)  $(P - iv), (Q - iii), (R - ii), (S - i)$   
 (c)  $(P - i), (Q - iii), (R - ii), (S - iv)$   
 (d)  $(P - iii), (Q - iv), (R - ii), (S - i)$

**Group II**

- (i) photon + lattice vibration  
 (ii) electron +elastic deformation  
 (iii) collective electron oscillations  
 (iv) elastic wave

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
 Near IIT, Hauz Khas, New Delhi-16  
 Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
 28-B/6, Jia Sarai, Near IIT  
 Hauz Khas, New Delhi-16

- Q23. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is \_\_\_\_\_.
- Q24. A voltage regulator has ripple rejection of  $-50\text{ dB}$ . If input ripple is  $1\text{ mV}$ , what is the output ripple voltage in  $\mu\text{V}$ ? The answer should be up to two decimal places. \_\_\_\_\_.
- Q25. The number of spectral lines allowed in the spectrum for the  $3^2D \rightarrow 3^2P$  transition in sodium is \_\_\_\_\_.

**Q. 26 to Q. 55 carry two marks each.**

- Q26. Which of the following pairs of the given function  $F(t)$  and its Laplace transform  $f(s)$  is NOT CORRECT?
- (a)  $F(t) = \delta(t), f(s) = 1$  (Singularity at  $+0$ )
- (b)  $F(t) = 1, f(s) = \frac{1}{s}, (s > 0)$
- (c)  $F(t) = \sin kt, f(s) = \frac{s}{s^2 + k^2}, (s > 0)$
- (d)  $F(t) = te^{kt}, f(s) = \frac{1}{(s-k)^2}, (s > k, s > 0)$
- Q27. If  $\vec{A}$  and  $\vec{B}$  are constant vectors, then  $\vec{\nabla}(\vec{A} \cdot (\vec{B} \times \vec{r}))$  is
- (a)  $\vec{A} \cdot \vec{B}$  (b)  $\vec{A} \times \vec{B}$  (c)  $\vec{r}$  (d) zero
- Q28.  $\Gamma\left(n + \frac{1}{2}\right)$  is equal to [Given  $\Gamma(n+1) = n\Gamma(n)$  and  $\Gamma(1/2) = \sqrt{\pi}$ ]
- (a)  $\frac{n!}{2^n} \sqrt{\pi}$  (b)  $\frac{2n!}{n!2^{2n}} \sqrt{\pi}$  (c)  $\frac{2n!}{n!2^{2n}} \sqrt{\pi}$  (d)  $\frac{n!}{2^{2n}} \sqrt{\pi}$

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

Q29. The relativistic form of Newton's second law of motion is

$$(a) F = \frac{mc}{\sqrt{c^2 - v^2}} \frac{dv}{dt}$$

$$(b) F = \frac{m\sqrt{c^2 - v^2}}{c} \frac{dv}{dt}$$

$$(c) F = \frac{mc^2}{c^2 - v^2} \frac{dv}{dt}$$

$$(d) F = m \frac{c^2 - v^2}{c^2} \frac{dv}{dt}$$

Q30. Consider a gas of atoms obeying Maxwell-Boltzmann statistics. The average value of  $e^{i\vec{a}\cdot\vec{p}}$  over all the moments  $\vec{p}$  of each of the particles (where  $\vec{a}$  is a constant vector and  $a$  is the magnitude,  $m$  is the mass of each atom,  $T$  is temperature and  $k$  is Boltzmann's constant) is,

(a) one

(b) zero

(c)  $e^{-\frac{1}{2}a^2mkT}$

(d)  $e^{-\frac{3}{2}a^2mkT}$

Q31. The electromagnetic form factor  $F(q^2)$  of a nucleus is given by,

$$F(q^2) = \exp\left[-\frac{q^2}{2Q^2}\right]$$

where  $Q$  is constant. Given that

$$F(q^2) = \frac{4\pi}{q} \int_0^\infty r dr \rho(r) \sin qr$$

$$\int d^3r \rho(r) = 1$$

where  $\rho(r)$  is the charge density, the root mean square radius of the nucleus is given by

(a)  $1/Q$

(b)  $\sqrt{2}/Q$

(c)  $\sqrt{3}Q$

(d)  $\sqrt{6}/Q$

Q32. A uniform circular disk of radius  $R$  and mass  $M$  is rotating with angular speed  $\omega$  about an axis, passing through its center and inclined at an angle 60 degree with respect to its symmetry axis. The magnitude of the angular momentum of the disk is,

(a)  $\frac{\sqrt{3}}{4} \omega MR^2$

(b)  $\frac{\sqrt{3}}{8} \omega MR^2$

(c)  $\frac{\sqrt{7}}{8} \omega MR^2$

(d)  $\frac{\sqrt{7}}{4} \omega MR^2$

## Head office

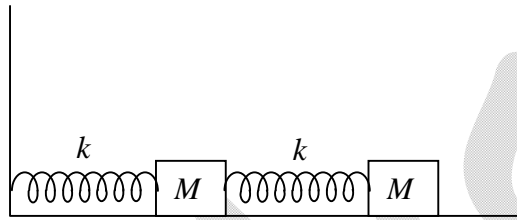
fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

## Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16



- Q33. Consider two small blocks, each of mass  $M$ , attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is  $k$ . The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,



- (a)  $\sqrt{\frac{3+\sqrt{2}}{2}} \sqrt{\frac{k}{M}}$       (b)  $\sqrt{\frac{3+\sqrt{3}}{2}} \sqrt{\frac{k}{M}}$       (c)  $\sqrt{\frac{3+\sqrt{5}}{2}} \sqrt{\frac{k}{M}}$       (d)  $\sqrt{\frac{3+\sqrt{6}}{2}} \sqrt{\frac{k}{M}}$

- Q34. A charge distribution has the charge density given by  $\rho = Q\{\delta(x-x_0) - \delta(x+x_0)\}$ . For this charge distribution the electric field at  $(2x_0, 0, 0)$

- (a)  $\frac{2Q\hat{x}}{9\pi\epsilon_0 x_0^2}$       (b)  $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^3}$       (c)  $\frac{Q\hat{x}}{4\pi\epsilon_0 x_0^2}$       (d)  $\frac{Q\hat{x}}{16\pi\epsilon_0 x_0^2}$

- Q35. A monochromatic plane wave at oblique incidence undergoes reflection at a dielectric interface. If  $\hat{k}_i, \hat{k}_r$  and  $\hat{n}$  are the unit vectors in the directions of incident wave, reflected wave and the normal to the surface respectively, which one of the following expressions is correct?

- (a)  $(\hat{k}_i - \hat{k}_r) \times \hat{n} \neq 0$       (b)  $(\hat{k}_i - \hat{k}_r) \cdot \hat{n} = 0$   
 (c)  $(\hat{k}_i \times \hat{n}) \cdot \hat{k}_r = 0$       (d)  $(\hat{k}_i \times \hat{n}) \cdot \hat{k}_r \neq 0$

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

- Q36. In a normal Zeeman Effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition  $5^1D_2 \rightarrow 5^1P_1$  of cadmium atoms is to be observed. The spectrometer has a resolution of 0.01 nm. Minimum magnetic field needed to observe this is ( $m_e = 9.1 \times 10^{-31} \text{ kg}$ ,  $e = 1.6 \times 10^{-19} \text{ C}$ ,  $c = 3 \times 10^8 \text{ m/s}$ )
- (a) 0.26T                      (b) 0.52T                      (c) 2.6T                      (d) 5.2T
- Q37. The spacing between vibrational energy levels in CO molecule is found to be  $8.44 \times 10^{-2} \text{ eV}$ . Given that the reduced mass of CO is  $1.14 \times 10^{-26} \text{ kg}$ , Planck's constant is  $6.626 \times 10^{-34} \text{ Js}$  and  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ . The force constant of the bond in CO molecule is
- (a) 1.87 N/m                      (b) 18.7 N/m                      (c) 187 N/m                      (d) 1870 N/m
- Q38. A lattice has the following primitive vector (in Å):  $\vec{a} = 2(\hat{j} + \hat{k})$ ,  $\vec{b} = 2(\hat{k} + \hat{i})$ ,  $\vec{c} = 2(\hat{i} + \hat{j})$ . The reciprocal lattice corresponding to the above lattice is
- (a) BCC lattice with cube edge of  $\left(\frac{\pi}{2}\right) \text{ \AA}^{-1}$   
 (b) BCC lattice with cube edge of  $(2\pi) \text{ \AA}^{-1}$   
 (c) FCC lattice with cube edge of  $\left(\frac{\pi}{2}\right) \text{ \AA}^{-1}$   
 (d) FCC lattice with cube edge of  $(2\pi) \text{ \AA}^{-1}$
- Q39. The total energy of an ionic solid is given by an expression  $E = -\frac{\alpha e^2}{4\pi\epsilon_0 r} + \frac{B}{r^9}$  where  $\alpha$  is Madelung constant,  $r$  is the distance between the nearest neighbours in the crystal and  $B$  is a constant. If  $r_0$  is the equilibrium separation between the nearest neighbours then the value of  $B$  is
- (a)  $\frac{\alpha e^2 r_0^8}{36\pi\epsilon_0}$                       (b)  $\frac{\alpha e^2 r_0^8}{4\pi\epsilon_0}$                       (c)  $\frac{2\alpha e^2 r_0^{10}}{9\pi\epsilon_0}$                       (d)  $\frac{\alpha e^2 r_0^{10}}{36\pi\epsilon_0}$

### Head office

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

### Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

Q40. A proton is confined to a cubic box, whose sides have length  $10^{-12} m$ . What is the minimum kinetic energy of the proton? The mass of proton is  $1.67 \times 10^{-27} kg$  and Planck's constant is  $6.63 \times 10^{-34} Js$ .

- (a)  $1.1 \times 10^{-17} J$       (b)  $3.3 \times 10^{-17} J$       (c)  $9.9 \times 10^{-17} J$       (d)  $6.6 \times 10^{-17} J$

Q41. For the function  $f(z) = \frac{16z}{(z+3)(z-1)^2}$ , the residue at the pole  $z=1$  is (your answer should be an integer) \_\_\_\_\_.

Q42. The degenerate eigenvalue of the matrix

$$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$$

is (your answer should be an integer) \_\_\_\_\_

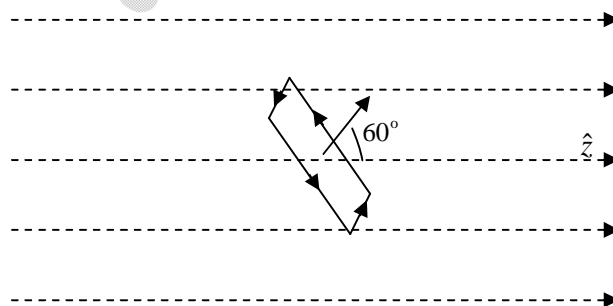
Q43. Consider the decay of a pion into a muon and an anti-neutrino  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$  in the pion rest frame.

$$m_\pi = 139.6 MeV/c^2, \quad m_\mu = 105.7 MeV/c^2, \quad m_\nu \approx 0$$

The energy (in MeV) of the emitted neutrino, to the nearest integer is \_\_\_\_\_

Q44. In a constant magnetic field of 0.6 Tesla along the z direction, find the value of the path integral  $\oint \vec{A} \cdot d\vec{l}$  in the units of  $(Tesla m^2)$  on a square loop of side length  $(1/\sqrt{2})$  meters.

The normal to the loop makes an angle of  $60^\circ$  to the z-axis, as shown in the figure. The answer should be up to two decimal places. \_\_\_\_\_



**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

- Q45. A spin-half particle is in a linear superposition  $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$  of its spin-up and spin-down states. If  $|\uparrow\rangle$  and  $|\downarrow\rangle$  are the eigenstates of  $\sigma_z$  then what is the expectation value up to one decimal place, of the operator  $10\sigma_z + 5\sigma_x$ ? Here, symbols have their usual meanings. \_\_\_\_\_
- Q46. Consider the wave function  $Ae^{ikr}(r_0/r)$ , where  $A$  is the normalization constant. For  $r = 2r_0$ , the magnitude of probability current density up to two decimal places, in units of  $(A^2\hbar k/m)$  is. \_\_\_\_\_
- Q47. An  $n$ -channel junction field effect transistor has  $5\text{ mA}$  source to drain current at shorted gate ( $I_{DSS}$ ) and  $5\text{ V}$  pinch off voltage ( $V_P$ ). Calculate the drain current in  $\text{mA}$  for a gate-source voltage ( $V_{GS}$ ) of  $-2.5\text{ V}$ . The answer should be up to two decimal places.  
\_\_\_\_\_

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

## Common Data Questions

**Common Data for Questions 48 and 49:** There are four energy levels  $E$ ,  $2E$ ,  $3E$  and  $4E$  (where  $E > 0$ ). The canonical partition function of two particles is, if these particles are

Q48. Two identical fermions

- (a)  $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$
- (b)  $e^{-3\beta E} + e^{-4\beta E} + e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$
- (c)  $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$
- (d)  $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Q49. Two distinguishable particles

- (a)  $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$
- (b)  $e^{-3\beta E} + e^{-4\beta E} + e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$
- (c)  $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$
- (d)  $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

**Common Data for Question 50 and 51:** To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

we add a small perturbation given by

$$\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

where  $\varepsilon$  is small quantity.

### Head office

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

### Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16

- Q50. The ground state eigenvector of the unperturbed Hamiltonian is
- (a)  $(1/\sqrt{2}, 1/\sqrt{2}, 0)$  (b)  $(1/\sqrt{2}, -1/\sqrt{2}, 0)$   
 (c)  $(0, 0, 1)$  (d)  $(1, 0, 0)$
- Q51. A pair of eigenvalues of the perturbed Hamiltonian, using first order perturbation theory, is
- (a)  $3 + 2\varepsilon, 7 + 2\varepsilon$  (b)  $3 + 2\varepsilon, +2 + \varepsilon$   
 (c)  $3, 7 + 2\varepsilon$  (d)  $3, 2 + 2\varepsilon$

### Linked Answer Questions

**Statement for linked Answer Questions 52 and 53:** In the Schmidt model of nuclear magnetic moments, we have,

$$\vec{\mu} = \frac{e\hbar}{2Mc} (g_l \vec{L} + g_s \vec{S})$$

where the symbols have their usual meaning

- Q52. For the case  $J = l + 1/2$ , where  $J$  is the total angular momentum, the expectation value of  $\vec{S} \cdot \vec{J}$  in the nuclear ground state is equal to,
- (a)  $(J - 1)/2$  (b)  $(J + 1)/2$  (c)  $J/2$  (d)  $-J/2$
- Q53. For the  $O^{17}$  nucleus ( $A = 17, Z = 8$ ), the effective magnetic moment is given by,

$$\vec{\mu}_{eff} = \frac{e\hbar}{2Mc} g \vec{J},$$

Where  $g$  is equal to, ( $g_s = 5.59$  for proton and  $-3.83$  for neutron)

- (a) 1.12 (b) -0.77 (c) -1.28 (d) 1.28

### Head office

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

### Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16



Q58. Which of the following options is the closest in meaning to the word given below:

Primeval

(a) Modern

(b) Historic

(c) Primitive

(d) Antique

Q59. Friendship, no matter how \_\_\_\_\_ it is, has its limitations.

(a) cordial

(b) intimate

(c) secret

(d) pleasant

Q60. Select the pair that best expresses a relationship similar to that expressed in the pair:

**Medical: Health**

(a) Science: Experiment

(b) wealth: Peace

(c) Education: Knowledge

(d) Money: Happiness

**Q. 61 to Q. 65 carry two marks each.**

Q61.  $X$  and  $Y$  are two positive real numbers such that  $2X + Y \leq 6$  and  $X + 2Y \leq 8$ . for which of the following values of  $(X, Y)$  the function  $f(X, Y) = 3X + 6Y$  will give maximum value?

(a)  $(4/3, 10/3)$

(b)  $(8/3, 20/3)$

(c)  $(8/3, 10/3)$

(d)  $(4/3, 20/3)$

Q62. If  $|4X - 7| = 5$  then the value of  $2|X| - |-X|$  is

(a)  $2, 1/3$

(b)  $1/2, 3$

(c)  $3/2, 9$

(d)  $2/3, 9$

**Head office**

fiziks, H.No. 23, G.F, Jia Sarai,

Near IIT, Hauz Khas, New Delhi-16

Phone: 011-26865455/+91-9871145498

**Branch office**

Anand Institute of Mathematics,

28-B/6, Jia Sarai, Near IIT

Hauz Khas, New Delhi-16



Q63. Following table provides figures (in rupees) on annual expenditure of a firm for two years-2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

- (a) Raw material and Salary & wages
- (b) Salary & wages and Advertising
- (c) Power & fuel and Advertising
- (d) Raw material and Research & Development

Q64. A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

- (a) 5
- (b) 10
- (c) 15
- (d) 30

Q65. Abhishek is elder to Savar.  
Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

- (a) Abhishek is elder to Anshul
- (b) Anshul is elder to Abhishek
- (c) Abhishek and Anshul are of the same age
- (d) No conclusion follows

#### Head office

fiziks, H.No. 23, G.F, Jia Sarai,  
Near IIT, Hauz Khas, New Delhi-16  
Phone: 011-26865455/+91-9871145498

#### Branch office

Anand Institute of Mathematics,  
28-B/6, Jia Sarai, Near IIT  
Hauz Khas, New Delhi-16