BHU-2013 (Geo-Physics)

Q.1. Let N coplanar forces each of magnitude F, where each force is making an angle $\frac{2\pi}{N}$ with the preceding one, act from a point. Then the vector sum is

(1) $N\vec{F}$ (2) $\vec{F}$ (3) $\frac{N\vec{F}}{2}$ (4) Zero (vector)

Q.2. A balloon of mass 100 kg is stationary at a height of 100m above the ground. A man of 50 kg is staunding stationary on a rope ladder hanging from the bottom of the balloon. When the man starts climbing up of the ladder, with a speed of 0.3 m/s relative to the rope, the balloon will

(1) remain stationary (2) move downward with a speed of 0.3 m/s

(3) move upwards with a speed of 0.3 m/s (4) move downward with a speed of 0.1 m/s

Q.3. Consider three observers *A*, *B* and *C*, each of whom is at rest in different inertial frames of references. A flash of light is emitted by observer *A*, who observes the light to travel at speed *c*. The frame of the observer *B* is moving away from *A* at a speed of $c/4$ and that of *C* is moving toward *A* with a speed of $c/4$. Then according to Galilean kinematics

(1) *B* will measure the speed of light emitted by *A* as $5c/4$, while *C* will measure it as $3c/4$

(2) *B* will measure the speed of light emitted by *A* $3c/4$, while *C* will measure $5c/4$

(3) Both will measure it as $3c/4$

(4) Both will measure it as *c*

Q.4. If the earth is treated as a sphere of radius *R* and mass M, its angular momentum about the axis of its diurnal motion with period *T* is

(1) $\frac{4\pi MR^2}{5T}$ (2) $\frac{2\pi MR^2}{T}$ (3) $\frac{MR^2T}{2\pi}$ (4) $\frac{\pi MR^3}{T}$

Q.5. Let the displacement of an oscillator be given by the expression

$$y = 4\cos^2\left(\frac{r}{2}\right)\sin(1000t)$$

Then the number of independent simple harmonic motions contained in this expression is

(1) 4 (2) 3 (3) 2 (4) 1

Q.6. The graph of restoring force vs time for a linear harmonic oscillator is

(1) sinusoidal (2) parabolic (3) circular (4) a straight line

Q.7. Conservation of four-vector momentum implies conservation of

(1) momentum only (2) energy only

(3) both momentum and energy (4) mass and energy

Q.8. A pion at rest decays in a time interval of 24 ns. An observer in the laboratory relative to whom the pion is in motion at a speed of $u = 0.8c$ will measure the time interval as

(1) 24 ns (2) 40 ns (3) 19.2 ns (4) 30 ns

Q.9. A constant torque acting on a uniform circular wheel changes its angular momentum from a_0 to $4a_0$ in 4 seconds. The magnitude of this torque is

(1) a_0 (2) $2a_0$ (3) $\frac{3a_0}{4}$ (4) $3a_0$

Q.10. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(1) $t^{1/2}$ (2) $t^{3/4}$ (3) $t^{3/2}$ (4) t^2

Q.11. It is possible that the Newtonian theory of gravitation may need to be modified at short ranges. Suppose that the potential energy between two masses m and m' is given by $V(r) = \frac{Gmm'}{r}\left(1 - ae^{r/\lambda}\right)$. For short distance $r << \lambda$, the force between m and m' is given by

(1) $F = -\frac{Gmm'}{r^2}$ (2) $F = -\frac{Gmm'a}{\lambda r}$

(3) $F = -\frac{Gmm'(1-a)}{r^2}$ (4) $F = -\frac{Gmm'(1+a^2)}{r^2}$

Q.12. A 2 kg body and a 3 kg body are moving along the *x*-axis. At a particular instant the 2 kg body is 2m from the origin and the 3 kg body is 3 m from the origin. The position of the centre of mass from the origin is

(1) 2.0m (2) 2.5m (3) 2.8m (4) 3.0m

Q.13. A particle of mass *m* is moving in a horizontal circle of radius *r* under a centripetal force which is equal to $\frac{-k}{r^2}$, where *k* is a constant. Then the total energy of the particle is

(1) 0 (2) $\frac{-k}{r}$ (3) $\frac{-k}{2r}$ (4) $\frac{k}{2r}$

Q.14. The rotation of a Foucault's pendulum denotes

(1) the spinning motion of earth (2) that each is stationary

(3) the rotational motion of the earth (4) linear motion of the earth

Q.15. The centripetal force required to keep a 4 kg mass moving in a circle of radius 0.8m at a speed of $6m/s$ is

(1) $280\,N$ (2) $180\,N$ (3) $360\,N$ (4) $1000\,N$

Q.16. The co-ordinates of a particle moving in the *xy* plane is given as a function of time by $x = 1 + 2t^2$ and $y = 2t + t^3$, given in metres. What is the magnitude of the velocity of the particle at time $t = 2s$?

(1) $15.1\,m/s$ (2) $16.1\,m/s$ (3) $8.2\,m/s$ (4) $14.2\,m/s$

Q.17. The moment of inertia of a uniform sphere of radius *R* and mass *M*, with its axis through the center is

(1) MR^2 (2) $\frac{1}{2}MR^2$ (3) $\frac{2}{5}MR^2$ (4) $\frac{3}{5}MR^2$

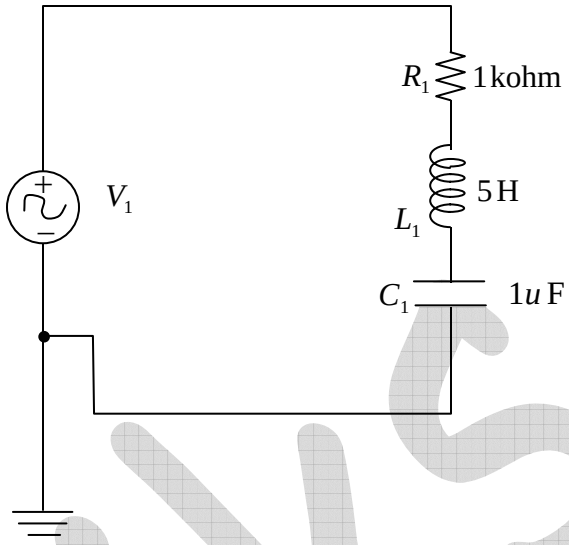
Q.18. Let a force of $4\,N$ causes a displacement of 0.02m. Then the force constant is

(1) $0.08\ N-m$ (2) $200\ N-m$ (3) $200\ N-m^{-1}$ (4) $0.08N-m^{-1}$

Q.19. A simple harmonic oscillator has a period $T = \frac{2\pi}{3}$. If the amplitude of the oscillator's motion is 2 metres, what is its maximum acceleration during its motion?

(1) $6\,m/s^2$ (2) $18\,m/s^2$ (3) $8\,m/s^2$ (4) $20\,m/s^2$

Q.20. What is the quality factor of the given circuit?



(1) 999 (2) 223 (3) 2.2 (4) 0.99

Q.21. A 100 Hz A.C. is flowing in a coil of inductance 7 millihenry. The reluctance of the coil is

(1) 4.44 Ω (2) 4.4 Ω (3) 7 Ω (4) 14 Ω

Q.22. Let a plane electromagnetic wave be associated with a value of electric field $E = 100\ V/m$ and magnetic field $H = 0.25\ A/m$. Then maximum energy flow is

(1) $50\ W/m^2$ (2) $25\ W/m^2$ (3) $12.5\ W/m^2$ (4) $6.25\ W/m^2$

Q.23. The average of the function $f(x) = \sin(x)$ in the interval $(0, \pi)$ is

(1) zero (2) $\frac{2}{\pi}$ (3) $\frac{\pi}{2}$ (4) 2π

Q.24. A plane electromagnetic wave of frequency 25 MHz travels in free space along the *x*-direction. At a particular point in space and time the magnitude of the electric field is $E = 6.3$ V/m. What is the magnitude of the magnetic induction *B* at this point?

(1) 6.3×10^{-10} T (2) 2.1×10^{-8} T (3) 2.1×10^{-10} T (4) 6.3×10^{-8} T

Q.25. A transistor has a current gain factor $\alpha = 0.95$. The transistor is connected with emitter grounded. What is the change in the collector current when the base current is changed by 0.1 mA?

(1) 9.5 mA (2) 19 mA (3) 1.9 mA (4) 0.95 mA

Q.26. An intrinsic Ge crystal has an intrinsic concentration $N_i = 10^{13}/\text{cm}^3$ (at room temperature). When doped with antimony, the hole density N_h is decreased to $10^{11}/\text{cm}^3$ at room temperature. Then the majority carrier density is

(1) $10^{15}/\text{cm}^3$ (2) $10^{2}/\text{cm}^3$ (3) $10^{24}/\text{cm}^3$ (4) $10^{12}/\text{cm}^3$

Q.27. A change of 200 mV in base emitter voltage causes a change of 100μA in the base current. Then the input resistance of the transistor becomes

(1) 20 Ω (2) 200 Ω (3) 1 kΩ (4) 2 kΩ

Q.28. A plane electromagnetic wave $E_x = 100\cos\left(6\times10^8 t + 4x\right)\text{V/m}$ propagates in a medium. Then the dielectric constant of the medium is

(1) 1 (2) 2 (3) 4 (4) 6

Q.29. For a silicon diode the value of the forward bias voltage typically

(1) must be greater than 0.3 V

(2) must be greater than 0.7 V

(3) depends on the width of the depletion region

(4) depends on the concentration of the majority carriers

Q.30. An amplifier that operates in the linear region at all times is

(1) Class A (2) Class AB (3) Class C (4) Class C

Q.31. A band-pass filter has resonant frequency of 950 Hz with the upper cutoff frequency as 3000 Hz. Then the lower cutoff frequency is

(1) 2050 Hz (2) 1500 Hz (3) 300 Hz (4) 100 Hz

Q.32. Reluctance offered by a coil having no resistance in an a.c. circuit is equal to

(1) ωL (2) $\frac{1}{\omega\text{L}}$ (3) $\omega^2\text{L}^2$ (4) $\omega\,\text{LR}$

Q.33. In an $L-C-R$ series resonant circuit the current through the resistance and the current through inductance have a phase difference of value

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) 0

Q.34. In cylindrical co-ordinates the vector magnetic potential is $\vec{A}=50\,r^2\,\vec{a}_x$ Wb/m in a certain region of free space. The value of magnetic induction is

(1) $-100\,\mathrm{r}\,\vec{a}_r$ Wb/m^2 (2) $-100\,\mathrm{r}\,\vec{a}_\phi$ Wb/m^2

(3) $-50\,\mathrm{r}\,\vec{a}_r$ Wb/m^2 (4) $-50\,\mathrm{r}\,\vec{a}_\varpi$ Wb/m^2

Q.35. An 800 MHz traveling plane wave has an average Poynting vector of $8\,\mathrm{mW/m^2}$ and the velocity of the wave is 1×10^8 m/s. Then the wavelength of the plane wave is

(1) 1 m (2) 0.5 m (3) 0.25 m (4) 0.125 m

Q.36. The electric field of a plane EM wave traveling along the z-axis is

$$E_z=\left(E_z x+E_{0y}y\right)\sin\left(\omega t-kz+\phi\right)$$

Then the magnetic field B_z is

(1) $\left(-E_{0x}x+E_{0y}y\right)\cos\left(\omega t-kz+\phi\right)/c$ (2) $\left(E_{0x}x+E_{0y}y\right)\sin\left(\omega t-kz+\phi\right)/c$

(3) $\left(-E_{0y}x+E_{0x}y\right)\sin\left(\omega t-kz+\phi\right)/c$ (4) $\left(E_{0x}x+E_{0y}y\right)\cos\left(\omega t-kz+\phi\right)/c$

Q.37. Which of the following Maxwell equations implies that there is no magnetic monopoles?

(1) $\Delta\cdot E=\rho/\varepsilon_0$ (2) $\Delta\cdot B=0$

(3) $\Delta\times E=-\partial\mathrm{B}/\partial t$ (4) $\Delta\times B=\mu_0 J+\mu_0\varepsilon_0\partial E/\partial t$

Q.38. What is the electric flux density (or electric displacement) through each face of a cube, of side length $2\,\mathrm{metres}_1$ that contains a central point charge of 2 coulombs?

(1) $0.50\,\mathrm{coulomb/m^2}$ (2) $0.33\,\mathrm{coulomb/m^2}$

(3) 0.23 tesla (4) $0.13\,\mathrm{coulomb/m^2}$

Q.39. In an $L-C-R$ circuit with an external forcing signal $f\sin\omega t$, after a sufficiently long time the current will oscillate with a frequency

(1) $\frac{1}{2\pi\sqrt{LC}}$ (2) $\frac{\omega}{2\pi\sqrt{LC}}$ (3) $\frac{\omega}{2\pi}$ (4) $\frac{\omega}{\pi}$

Q.40. If the temperature of a transistor rises by 10 $°C$, which of the following current doubles?

(1) I_C (2) I_B (3) I_{CHO} (4) I_E

Q.41. Which of the following statement is wrong?

(1) Voltmeter should have high resistance

(2) Ammeter should have low resistance

(3) Ammeter should be placed in parallel across the electric circuit

(4) Voltmeter should be placed in parallel across the electric circuit

Q.42. When a current passing through a straight conductor is 1 mA, what is the magnetic induction at a point 20cm from the current carrying conductor?

(1) 1×10^{-7} tesla (2) 2×10^{-7} tesla (3) 1.5×10^{-10} tesla (4) 2×10^{-10} tesla

Q.43. A radially pulsating charged sphere

(1) emits electromagnetic radiation

(2) creates a static magnetic field

(3) can set a nearby charged particle into circular motion

(4) creates a vacuum

Q.44. The avalanche effect is observed in a diode, when

(1) the forward voltage exceeds the break-down voltage

(2) the heavily doped diode is forward biased

(3) the reverse voltage exceeds the break-down voltage

(4) the majority carriers have enough energy compared to valence electrons

Q.45. An electromagnetic wave incident from a rarer to a denser medium undergoes a phase change of

(1) 0 (2) $\frac{\pi}{2}$ (3) π (4) 2π

Q.46. In a Fraunholer diffraction experiment with a slit width of 12×10^{-5} cm and with 6000 Å monochromatic light, the half angular width of the bright central maximum will be found at

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Q.47. The Brewster angle of a material with refractive index 1.732 is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

Q.48. A sound wave travels from air to water. The angle of incidence is α_1 and the angle of refraction is α_2. Assuming Snell's law to be valid

(1) $\alpha_2 < \alpha_1$ (2) $\alpha_2 > \alpha_1$ (3) $\alpha_2 = \alpha_1$ (4) $\alpha_2 = 90°$

Q.49. In a Newton's rings experiment, the diameter of a certain order of the dark ring is measured to be double that of the second dark ring. The order of the dark ring becomes

(1) 2 (2) 6 (3) 4 (4) 8

Q.50. In a Michelson interferometer 200 fringes cross the field of view when the movable mirror is moved through 0.0589 mm. Then the value of wavelength of light used is

(1) 5890 Å (2) 4800 Å (3) 5896 Å (4) 6100 Å

Q.51. Consider the interference between two sources of intensities I and $4I$. What is the intensity at a point where the phase difference is $\frac{\pi}{2}$?

(1) $2I$ (2) $5I$ (3) $4I$ (4) 0

Q.52. Laser beam is considered to be coherent because it consists of

(1) many wavelength

(2) un-correlated wavelength

(3) co-ordinated waves of exactly the same wavelength

(4) divergent beam

Q.53. A beam of light of wavelength λ having an illumination I falls on a clean surface of sodium. If n photoelectrons with kinetic energy E are emitted, then

(1) $n \propto T^2, E \propto \frac{1}{I}$ (2) $n \propto I, E \propto \frac{1}{\lambda}$

(3) $n \propto I, E \propto \lambda$ (4) $n \propto \lambda, E \propto I$

Q.54. A star emits light with a peak wavelength at 200nm. The surface temperature is

(1) 20490 K (2) 18490 K (3) 16490 K (4) 14490 K

Q.55. The energy of a photon with a wavelength 3cm is

(1) 3.143×10^{-5} (2) 4.143×10^{-5} (3) 2.143×10^{-5} (4) 5.143×10^{-5}

Q.56. An electron accelerated through a potential difference of 10 kV hits a target to produce X-rays. The minimum wavelength of the X-rays produced is

(1) 1.3026×10^{-10} m (2) 1.2306×10^{-10} m

(3) 1.2326×10^{-12} m (4) 1.3326×10^{-12} m

Q.57. The diameters of the 5th and 15th Newton's rings are 0.4 cm and 0.6 cm, respectively. The radius of the plano-convex lens is 100cm and the wavelength of the light used is

(1) 5000 Å (2) 5880 Å (3) 5790 Å (4) 6000 Å

Q.58. The velocity of extraordinary ray in a positive uniaxial crystal is

(1) least in a direction at right angles to optic axis

(2) equal to the velocity of ordinary ray

(3) maximum in a direction at the right angles to optic axis

(4) lesser than the velocity of ordinary ray

Q.59. The areal spread of laser beam of wavelength 8×10^{-7} m and aperture 10^{-3} m traveling 10^6 km is

(1) $4 \times 10^{10}\,\text{m}^2$ (2) $8 \times 10^{8}\,\text{m}^2$ (3) $8 \times 10^{10}\,\text{m}^2$ (4) $4 \times 10^{10}\,\text{m}^2$

Q.60. The readings of a Fahrenheit thermometer and a Centrigrade thermometer will be identical at the value of

(1) 20° (2) – 20° (3) – 40° (4) 40°

Q.61. A turbine in a steam power plant takes steam from a boiler at $450°\,C$ and exhausts it into a condenser at $100°\,C$. The maximum possible efficiency is,

(1) 48% (2) 60% (3) 80% (4) 100%

Q.62. A lump of ice whose mass *m* is 200 gm melts into water. The temperature remains at $0°\,C$ throughout the process. What is the entropy change for the ice? (The latent heat of fusion of ice 300 kJ/kg)

(1) 220 J/K (2) 0.23 J/K (3) 0.023 J/K (4) 22 J/K

Q.63. Isothermal curves correspond to

(1) constant pressure (2) constant volume

(3) constant entropy (4) constant temperature

Q.64. The amount of mechanical work which has to be done to completely melt 1 gm of ice is $(L = 80\ \text{cal/gm})$

(1) 4.2 J (2) 42 J (3) 80 J (4) 336 J

Q.65. Under the steady state, the temperature of a body

(1) increases with time

(2) decreases with time

(3) does not change with time and it has to remain the same at all points of the body

(4) does not change with time, but may be different at different points of the body

Q.66. The temperatures inside and outside of a refrigerator are $273°\,K$ and $303°K$ respectively. Assuming that the refrigerator cycle is reversible, the energy delivered to the surroundings for every joule of work done is

(1) 9.9 J (2) 8.1 J (3) 10.1 J (4) 9.1 J

Q.67. A gas is compressed at a constant pressure of $50\ \text{N/m}^2$ with a change of volume as $6\ \text{m}^3$. Then 100 J energy is added to the gas. What is the change in the internal energy of the gas?

(1) 300 J (2) 200 J (3) 400 J (4) 100 J

Q.68. Two Carnot engines *A* and *B* have their sources at 327°C and 227°C, and sinks at 127°C and 27°C, respectively. Then the ratio of their efficiency is

(1) 1:2 (2) 6:5 (3) 5:6 (4) 2:3

Q.69. Heat sinks are used in power amplifires

(1) to increase output power

(2) to reduce heat losses in the transistor

(3) to increase voltage gain

(4) to increase collector dissipation rating of the transistor

Q.70. The photoelectric threshold for a metal is 3000 Å. The kinetic energy of an electron ejected from it by radiation of wavelength 1200 Å is

(1) 9.93eV (2) 6.2eV (3) 4.5eV (4) 7.5eV

Q.71. The specific heat of a substance is a function of its

(1) mass (2) weight

(3) volume (4) molecular structure

Q.72. Two tuning forks *A* and *B* which are vibrating simultaneously produce 5 beats. Frequency of *B* is 512 Hz. It is seen that if one arm of A is fixed, then the number of beats increases. Frequency of *A* will be

(1) 502 Hz (2) 507 Hz (3) 517 Hz (4) 522 Hz

Q.73. Which of the following set of equations is invariant under Lorentz transformation?

(1) Maxwell's equation (2) Schrodinger's equation

(3) Newton's equation of motion (4) Laplace equation

Q.74. Which of the following does not support purely wave nature of light?

(1) Interference (2) Diffraction

(3) Polarization (4) Photoelectric effect

Q.75. Light of wavelength 4500 Å is incident on a Na surface for which the threshold wavelength of the photoelectrons is 5420 Å. Then the work function is

(1) 2.76 eV (2) 2.29 eV (3) 1.00 eV (4) 4.76 eV

Q.76. If A and B are non-empty sets with m and n elements respectively, then the number of mapping from A into B is

(1) mn (2) $m+n$ (3) m^n (4) n^m

Q.77. If A has m elements and B has n elements, then the number of elements of $A \times B$ is

(1) m (2) n (3) mn (4) $m+n$

Q.78. A factor of $(x-y)^3+(y-z)^3+(z-x)^3$ is

(1) $x+y+z$ (2) $(x-y)(y-x)(z-x)$

(3) $(x+y)(y+z)(z+x)$ (4) xyz

Q.79. All the complex roots of $z^n=1$ where $n \geq 5$ is a positive integer lie on

(1) a line (2) a circle (3) an ellipse (4) a parabola

Q.80. If a and b are non-zero complex numbers, then $|a+b|=|a|+|b|$ holds if and only if

(1) a and b are real (2) a and b are purely imaginary

(3) a/b is real and positive (4) a/b is real and negative

Q.81. The function $f(x)=\dfrac{x}{1+x}(x>0)$ is

(1) increasing

(2) increasing on $(0, 1)$ and decreasing $(1, \infty)$

(3) decreasing

(4) decreasing on $(0, 1)$ and increasing $(1, \infty)$

Q.82. The function $f(x)=|x|$ $(x-\text{real})$ is

(1) continuous except at $x=0$ (2) continuous at all points

(3) continuous except at $x=\pm 1$ (4) continuous nowhere

Q.83. If f is a real-valued differentiable function of a real variable satisfying $f(x+y)=f(x)f(y)$ the $f'(x)$ is equal to

(1) $f(x)f'(0)$ (2) $f(x)f(0)$ (3) $f(x)$ (4) a constant

Q.84. In the complex plane the equivalence classes under the relation $z \sim \omega \Leftrightarrow \arg z = \arg \omega$ are

(1) circles (2) lines (3) hyperbolas (4) parabolas

Q.85. The range of the strictly increasing function $f:\lfloor a,\ b\rfloor \to R$ is

(1) $\{f(a),\ f(b)\}$ (2) $\{f(b),\ f(a)\}$ (3) $\lfloor f(b),\ f(a)\rfloor$ (4) $\lfloor f(a),\ f(b)\rfloor$

Q.86. If z is a complex number with $\arg\left(\frac{z-1}{z+1}\right)=\frac{z}{2}$, then z lies

(1) one the unit circle (3) inside the unit circle

(3) at all points ± 1 (4) outside the unit circle

Q.87. The number of triangles in the plane whose vertices are order pairs of rational numbers is

(1) 0 (2) 2 (3) 3 (4) 1

Q.88. The value of ${}^{n}C_{k}\ /{}^{n-1}C_{k-1}$ is

(1) nk (2) n/k (3) $n+k$ (4) $n-k$

Q.89. The probability that exactly one of the events A, B to occur is

(1) $p(A)+p(B)$ (2) $p(A)p(B)$

(3) $p(A\cup B)-p(A\cap B)$ (4) $p(A\cup B)+p(A\cap B)$

Q.90. If z is a complex number, then the modulus of e^{iz} is

(1) 1 (2) $e^{\mathrm{Re}\,z}$ (3) e^{-Imz} (4) $e^{|z|}$

Q.91. If A is a bounded subset of the positive real numbers and if $\frac{1}{A}=\left\{\frac{1}{x}/\,x\in A\right\}$ then $\sup\frac{1}{A}$ is equal to

(1) $\frac{1}{\sup A}$ (2) $\sup A$ (3) $\inf A$ (4) $\frac{1}{\inf A}$

Q.92. The number of real solutions of $x^3+3x-4=0$ is

(1) 0 (2) 1 (3) 2 (4) 3

Q.93. The pair of planes represented by $az^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ is perpendicular if

(1) $a-b+c=0$ (2) $a+b-c=0$

(3) $a-b-c=0$ (4) $a+b+c=0$

Q.94. The conjugate of the complex number represented by i^1 (Where $i^2 = -1$) are represented by

(1) i^1 (2) $\pm i$ (3) ± 1 (4) $1+i$ and $1-i$

Q.95. The magnitude of the resultant of two perpendicular forces whose magnitudes are equal and equal to R is

(1) $2R$ (2) $\sqrt{2}R$ (3) $2\sqrt{R}$ (4) $\sqrt{2R}$

Q.96. The product of $n(\geq 4)$ consecutive positive integers divided by $n!$ is

(1) 1
(2) a fraction between 0 and 1
(3) 2
(4) an integer greater than 1

Q.97. The order of the group of permutations of n symbols is

(1) n (2) $(n-1)!$ (3) $n!$ (4) $(n+1)!$

Q.98. The mapping $x \to e^{tz}$ between the group of real numbers under additive and multiplicative group of complex numbers of modulus 1 is

(1) a homomorphism which is not one-to-one
(2) an isomorphism
(3) a homomorphism which is not onto
(4) not a homomorphism

Q.99. The least upper bound of $\left(1+\frac{1}{x}\right)^x \ (x>1)$ is

(1) e (2) $\frac{1}{\mu}$ (3) 2 (4) ∞

Q.100. If $\sin\theta, \cos\theta$ and $\tan\theta$ are in GP, then $\frac{1-\tan^4\theta}{\tan^0\theta}$ is

(1) 0 (2) 1 (3) $\tan\theta$ (4) $\sec^2\theta$

Q.101. A variable chord is drawn through the origin to circle $x^2 + y^2 - 2ax = 0$. The locus of the centre of the circle whose diameter is the variable chord is

(1) $x^2 + y^2 + ax = 0$ (2) $x^2 + y^2 + ay = 0$

(3) $x^2 + y^2 - ax = 0$ (4) $x^2 + y^2 - ay = 0$

Q.102. Defining a simple group as a group having no proper normal subgroups we can say that a homomorphism $f: G \to G'$ where G is a simple group and G' is any group is

(1) trivial (2) injective

(3) surjective (4) either trivial or injective

Q.103. The series $\sum_{n=2}^{\infty} -\frac{1}{n(\log n)^{\mu}}$

(1) converges for $p > 1$ and diverges for $p \le 1$

(2) converges for all p

(3) converges for $p \le 1$ and diverges for $p > 1$

(4) diverges for all p

Q.104. The curve $z = z(t) = e^{i2\pi t}\ (0 \le t \le 2)$ describes

(1) the lower semi-circle excluding ± 1 (2) the unit circle twice

(3) the upper semi-circle including ± 1 (4) the unit circle

Q.105. The eigenvalues of the matrix $\begin{pmatrix} a & a & a \\ a & a & a \\ a & a & a \end{pmatrix} (a - real)$ are

(1) a, a, a (2) $a, 1-a, 2-a$

(3) $0, 0, 3a$ (4) $a, 2a, 3a$

Q.106. The functions whose Laplace transform is $\frac{1}{(n-a)^n} (n = 1, 2, 3, \cdots)$ is

(1) $\frac{e^{nt}}{n!}$ (2) $\frac{t^n e^{at}}{n!}$ (3) $\frac{t^{n-1} e^{at}}{(n-1)!}$ (4) $\frac{t^{n+1} e^{at}}{(n+1)!}$

Q.107. If z is a complex number with $|z| = 1$, then one of the values of arg $\frac{z-1}{z+1}$ is

(1) $\pi/3$ (2) $\pi/6$ (3) $\pi/2$ (4) π

Q.108. The locus of the complex number z such that $|z-1| + |z+1| = 3$ is

(1) a circle (2) a straight hoe (3) an ellipse (4) a hyperbola

Q.109. If A and B are mutually exclusive events, then

(1) $p(A) \le P(B')$ and $p(B) \le P(A')$ (2) $p(A) > P(B')$ and $p(B) < P(A')$

(3) $p(A) < P(B')$ and $p(B) < P(A')$ (4) $p(A) = P(B')$ and $p(B) = P(A')$

Q.110. The function $f(x) = x|x|$ (x -real) is

(1) differentiable everywhere (2) differentiable except at $x = 0$

(3) differentiable except at $x = \pm 1$ (4) nowhere differentiable

Q.111. The function $f(x) = 1$ for $x \ge 1$ and -1 for $x < 1$ is such that

(1) f and $|f|$ are continuous (2) f is continuous but $|f|$ is not

(3) $|f|$ is continuous but f is not (4) neither f nor $|f|$ is continuous

Q.112. The function $f(z) = z^2$ is conformal of both kinds

(1) for all z (2) for all $z \ne 0$ (3) for $|z| < 1$ (4) for $z = \pm 1$

Q.113. The complex number z satisfying $e^z = 0$

(1) is $z = 0$ (2) does not exist (3) is $z = 1$ (4) is $z = i$

Q.114. The shortest distance between the lines $x - 2 = \frac{V - 1}{2} = z + 2$ and $x - 1 = y + 4 = \frac{z - 2}{2}$ is

(1) $\frac{1}{\sqrt{11}}$ (2) $\frac{2}{\sqrt{11}}$ (3) $\frac{3}{\sqrt{11}}$ (4) $\frac{4}{\sqrt{11}}$

Q.115. The equation of the sphere whose center is (1, -2, 2) and radius 4 is

(1) $x^2 + y^2 + z^2 - 2x + 4y - 4z = 7$ (2) $x^2 - y^2 - z^2 - 2x - 4y - 4z = 7$

(3) $x^2 + y^2 + z^2 - 2x + 4y - 4z = -5$ (4) $x^2 + y^2 + z^2 + 2x + 4y + 4z = -5$

Q.116. The directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at the point (1, -2, 1) in the direction of the vector $\vec{i} - 2\vec{j} + 3\vec{k}$ is

(1) $\frac{8}{\sqrt{13}}$ (2) $\frac{4}{\sqrt{3}}$ (3) $-\frac{8}{\sqrt{11}}$ (4) $-\frac{8}{\sqrt{14}}$

Q.117. The magnitude and the direction of the greatest rate of change of $u = x^2 y z^2$ at (4, 0, 1) are

(1) 4 and x-axis (2) 4 and z-axis

(3) 16 and y-axis (4) 16 and x-axis

Q.118. The vector $x\vec{i} + y\vec{j} + z\vec{k}$ is

(1) irrotational (2) always $\perp$ to its divergence

(3) solenoidal (4) always $\perp$ to its curl

Q.119. Let w be any constant vector and r be the position vector $x\vec{i} + y\vec{j} + z\vec{k}$. If $\mathrm{v} = \mathrm{w} \times \mathrm{r}$, then w is

(1) curl v (2) $(\text{curl v})^2$ (3) $2\,\text{curl v}$ (4) $\frac{1}{2}\text{curl v}$

Q.120. The value of n, if the vector $\mathrm{V} = \left(x\vec{i} + y\vec{j} + z\vec{k}\right)^{n+1}$ is solenoidal is

(1) -1 (2) -3 (3) 1 (4) 3

Q.121. The curve whose torsion and the curvature are both constant is

(1) hyperbola (2) parabola

(3) rectangular hyperbola (4) circular helix

Q.122. The area between the curves $y^2 = 4x$ and $x^2 = 4y$ is

(1) $\frac{16}{3}$ (2) $\frac{8}{3}$ (3) $\frac{4}{3}$ (4) $\frac{2}{3}$

Q.123. Let V be the volume enclosed by the closed surface S, n, the normal to S and r_1 the position vector. Then $\iint_S \mathrm{r}\cdot\mathrm{n}\,dS$ is equal to

(1) V (2) $2V$ (3) $3V$ (4) $4\,V$

Q.124. The torque about the point (1, -2, 2) of the force represented by $\vec{i} + 5\vec{k}$ acting through the point (2, -1, -1) is

(1) $5\vec{i} - 8\vec{j} - \vec{k}$ (2) $5\vec{i} - \vec{j} - 8\vec{k}$ (3) $5\vec{i} + 8\vec{j} + \vec{k}$ (4) $5\vec{i} + \vec{j} + 8\vec{k}$

Q.125. A real function $f : R \to R$ satisfying $|f(x) - f(y)| \le k(x-y)^2$ for all $x, y \in R$, k – a constant must be

(1) identically zero

(2) identically a constant

(3) the identity function i.e., $f(x) = x \forall x \in R$

(4) e^x

Q.126. A particle acted on by the force $2\vec{i} + \vec{j} + 4\vec{k}$ and $\vec{i} + 3\vec{j} - 7\vec{k}$ is displaced from the point (1, 1, 4) to (2, 4, 7). Then the total work done by the forces is

(1) 4 units (2) 6 units (3) 8 units (4) 10 units

Q.127. The solution of the equation $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = x$ is

(1) $\frac{1}{x}(A + B\log x) + \frac{x}{4} + C$ (2) $e^x(A + B\log x) + \frac{x}{4} + C$

(3) $x(A + B\log x) + \frac{x}{4} + C$ (4) $\frac{1}{x}(A + B) + \frac{x}{4} + C$

Q.128. The acceleration of a particle moving along a straight line after t seconds is $12t^2 + 6t + 2\,\text{m/sec}^2$. If the particle starts with the initial velocity of $2\,\text{m/sec}$ and covers a distance 12m in 2 sec, then the distance covered in 3 seconds is

(1) 103 m (2) 123 m (3) 83 m (4) 113 m

Q.129. If $x + y + z = 1$ where x, y, z are all positive real numbers, then

(1) $(1-x)(1-y)(1-z) < 8xyz$ (2) $(1-x)(1-y)(1-z) > 8xyz$

(3) $(1+x)(1-y)(1-z) > 8xyz$ (4) $(1-x)(1+y)(1-z) < 8xyz$

Q.130. The solution of the equation $y = 2x\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^3$ is

(1) $y = cx + c^3$ (2) $y^2 = 2cx + c^3$ (3) $y = cx^2 + c^3$ (4) $y^2 = cx^2 + c^3$

Q.131. The value of $\lim_{n\to\infty} \frac{\sin a}{n}$ is

(1) 1 (2) ∞ (3) 0 (4) 2

Q.132. The series $\frac{1}{3}x+\frac{1}{3}\times\frac{2}{5}x^2+\frac{1}{3}\times\frac{2}{5}\times\frac{3}{7}x^3+\cdots$

(1) converges for all x

(2) converges for $x>2$ and diverges for $x<2$

(3) diverges for all x

(4) converges for $x<2$ and diverges for $x>2$

Q.133. A ball is released from a balloon 20 sec after it starts ascending with a uniform acceleration of 1.96 m/sec^2. If the acceleration due to gravity is 9.8 m/sec^2, then the greatest height above the ground reached by the ball is

(1) 392 m (2) 78.4 m (3) 470.4 m (4) 313.6 m

Q.134. The series $\sum_{n=1}^{\infty}\frac{x^{n\ 1}}{(n-1)!}$

(1) converges absolutely for all x (2) converges absolutely for $x<0$ only

(3) does not converges absolutely (4) converges absolutely for $x>0$ only

Q.135. The solution of the equation $x^2\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$ is

(1) $y=c'+c\int e^{\frac{1}{x}}dx$ (2) $y=c'+c\int e^{\frac{-1}{x}}dx$

(3) $y=c'+c\int e^{x}dx$ (4) $y=c'+c\int e^{-x}dx$

Q.136. The solution of the equation $xyp^2+\left(3x^2-2y^2\right)p-6xy=0$ where $p=\frac{dy}{dx}$ are

(1) $y=cx$ and $y=3x^2+c$ (2) $y^2=cx$ and $y=3x^2+c$

(3) $y=cx^2$ and $y=-3x^2+c$ (4) $y=cx^2$ and $y=3x^2+c$

Q.137. The vector equation of the line $6x-2=3y+1=2z-2$ is

(1) $\vec{r}=\vec{i}-\vec{j}+3\vec{k}+\lambda\left(\vec{i}+2\vec{j}+3\vec{k}\right)$ (2) $\vec{r}=\vec{i}+2\vec{j}+3\vec{k}+\frac{\lambda}{3}\left(\vec{i}-\vec{j}+3\vec{k}\right)$

(3) $\vec{r}=\frac{1}{3}\vec{i}-\frac{1}{3}\vec{j}+\vec{k}+\lambda\left(\vec{i}+2\vec{j}+3\vec{k}\right)$ (4) $\vec{r}=2\vec{i}+\vec{j}-2\vec{k}+\lambda\left(6\vec{i}+3\vec{j}+2\vec{k}\right)$

Q.138. The equation of a sphere which passes through (1, 0, 0), (0, 1, 0) and (0, 0, 1) and whose centre lies on the curve $4xy=1$ is

(1) $x^2+y^2+z^2-x-y-z=0$ (2) $x^2+y^2+z^2+x-y-z=0$

(3) $x^2+y^2+z^2+x+y-z=0$ (4) $x^2+y^2+z^2+x-y+z=0$

Q.139. Let $f, g: R \to R$ be functions. If $g\,o\,f$ is bijective, then

(1) both f and g are bijective (2) f is bijective but not g

(3) g is bijective but not f (4) f is injective and g is surjective

Q.140. The angle between the lines whose direction cosines are proportional to $1, 2, 3$ and $-2, 2, 5$ is

(1) $\cos^{-1}\frac{7}{\sqrt{462}}$ (2) $\cos^{-1}\frac{17}{\sqrt{462}}$

(3) $\cos^{-1}\frac{17}{\sqrt{461}}$ (4) $\cos^{-1}\frac{7}{\sqrt{461}}$

Q.141. The co-ordinates of the point equidistant from the points $(a, 0, 0), (0, b, 0), (0, 0, c)$ and $(0, 0, 0)$ are

(1) (a, b, c) (2) $(a, 0, c)$

(3) $(a/2, b/2, c/2)$ (4) $(a/2, b, c/2)$

Q.142. If $\frac{1+4p}{4}, \frac{1-p}{3}, \frac{1-2p}{3}$ are probabilities of three mutually exclusive events, then the value of p is

(1) 1/3 (2) ¼ (3) 1/5 (4) 1/2

Q.143. A bag contains 12 pairs of socks. 4 socks are picked up at random. The probability that there is at least one pair is

(1) 41/161 (2) 40/160 (3) 8/25 (4) 20/161

Q.144. If $f(x)=\int_0^x \frac{1-\sqrt{t}}{1+\sqrt{i}}dt\,(0\le x\le 1)$, then $f(1)$ is

(1) 1 (2) $1/\sqrt{2}$ (3) $\sqrt{2}$ (4) 0

Q.145. The value of $\int \frac{\sin x-\cos x}{\sqrt{\sin 2x-1}}dx$ is

(1) $-\log\left[(\sin x+\cos x)+\sqrt{\sin 2x-1}\right]+c$ (2) $-\log\left[(\sin x+\cos x)+\sqrt{\sin 2x}\right]+c$

(3) $-\log\left[(\sin x-\cos x)+\sqrt{\sin 2x-1}\right]+c$ (4) $-\log\left[(\sin x-\cos x)+\sqrt{\sin 2x}\right]+c$

Q.146. If $\Delta=\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$ then $\sqrt{\frac{\Delta}{4}}$ is

(1) $(a+b)abc$ (2) $(b+c)abc$ (3) $(c+a)abc$ (4) abc

Q.147. In a group G if $a^2*b^2=(a*b)^2$ for all $a, b\in G$, then which of the following is true?

(1) G is Abelian (2) G is cyclic

(3) G is infinite (4) Every element of G is its own inverse

Q.148. $\int \frac{\log x+\log 2x+\cdots\log nx}{x}dx$ is

(1) a polynomial in x (2) a polynomial in $\log x$

(3) a constant (4) a polynomial in x and $\log x$

Q.149. If $f(x)$ is a differentiable function such that $\lim_{x\to\infty} f'(x)=0$, then $\lim_{x\to\infty}(g)$ where $g(x)=f(x+1)-f(x)$ is

(1) 0 (2) 1 (3) ∞ (4) Need not exist

Q.150. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value at least one is

(1) 2/3 (2) 4/5 (3) 7/8 (4) 15/16