# HCU (Ph.D-Physics-2013)

Q1. The eigenvalues of the matrix $A^{55}$ where

$$A=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

are

(a) $1, -1$ (b) $1, 1$ (c) $55, -55$ (d) $0,0$

Q2. There general solution of the second order differential equation

$$4x^2\frac{d^2y}{dx^2}+y=0$$

is

(a) $\sqrt{x}\left(c_1+c_2\log x\right)$ (b) $\sqrt{x}\left(c_1+c_2x\right)$

(c) $\sqrt{x}\left(c_1+c_2e^x\right)$ (d) $c_1\sqrt{x}+c_2\dfrac{1}{\sqrt{x}}$

Q3. If $A=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$, then det $e^A$ is

(a) $e^5-e^2$ (b) $e^4$ (c) $e^5$ (d) $e^6-e$

Q4. The value of exp $\left[i\ (-\hat{n}.\vec{\sigma})\cos\phi\right]$,where $\hat{n}$ is the unit vector $\vec{\sigma}$ is the pauli matrix is

(a) $\sin\phi-i\ \ (\hat{n}.\vec{\sigma})\cos\phi$ (b) $\cos\phi-i\ \ (\hat{n}.\vec{\sigma})\sin\phi$

(c) $\cos\phi+\sin\phi$ (d) $\cos^2\phi-\sin^2\phi$

Q5. If $|\psi_1>$ *and* $|\psi_2>$ are the eigenvectors of $A$ and $B$ corresponding to eigenvalues $\lambda_1$ and $\lambda_2$ respectively then $|\psi_1>-|\psi_2>$ *is*

(a) eigenvector of $A+B$ corresponding to the eigenvalue $\lambda_1+\lambda_2$

(b) eigenvector of $A+B$ corresponding to the eigenvalue $\lambda_1-\lambda_2$

(c) eigenvector of $A+B$ corresponding to the eigenvalue $|\lambda_1-\lambda_2|$

(d) in general not an eigenvector of $A+B$

Q6. If $L_{\pm} = L_x \pm iL_y$ and $R_{\pm} = X \pm iY$, the commutator $\left[L_{\pm,} R_{\mp}\right]$ is

(a) $\pm 2\hbar z$ (b) $\mp 2i\hbar z$ (c) $\pm \hbar L_x$ (d) $\mp i\hbar L_y$

Q7. A spin $-\frac{1}{2}$ particle with magnetic moment $\vec{\mu} = \mu_0 \vec{S}$ and spin $\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$, where $\sigma_i$ are the pauli matrices, is placed in a constant magnetic field $\vec{B}$ pointing along the $x$-axis. The interaction Hamiltonian of the system is

(a) $-\frac{\mu_0 B\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (b) $-\frac{\mu_0 B\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c) $-\frac{\mu_0 B\hbar}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (d) $-\frac{\mu_0 B\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Q8. Which of the following statements is true for the following system (i) spring-mass system, (ii) simple pendulum and (iii) sun-planet system

(a) (i) and (ii) are examples of harmonic oscillators.

(b) all three system are examples of harmonic oscillators.

(c) (i) and (iii) are examples of harmonic oscillators.

(d) only (i) is an example of a harmonic oscillator.

Q9. A vector is transformed from one frame to another by $V' = AV$, where

$$A = \begin{pmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

This transformation represents a rotation of

(a) $30^0$ clockwise about the $z-$axis.

(b) $30^0$ counter-clockwise about the $x-$axis.

(c) $60^0$ counter-clockwise about the $y-$axis.

(d) $60^0$ counter-clockwise about the $x-$axis.

Q10. An observer $(A)$ in the Cartesian frame of reference of sees a particle of mass $m$ moving with a constant velocity. If a second observer $(B)$ is in a frame which rotates with a constant angular velocity with respect to the first observer, which of the following statements is correct?

(a) $B$ will say that force is exerted on the particle.

(b) $B$ will say that no force are exerted on the particle.

(c) $B$ will say that the particle will always experience a positive acceleration.

(d) $B$ will say that the particle will always experience a negative acceleration.

Q11. The optical branch $(q=0)$ of lattice vibrations predicted by analyzing the diatomic linear china of '$+$' and '$-$' ions arises from a coupling of

(a) the elastic vibrations and an elastic field.

(b) motion of ions in-phase to an oscillating electric field.

(c) out-of- phase motion of icon to an oscillating electric field.

(d) out-of-phase motion of icon to an elastic filed.

Q12. The selection rule that governs the process of optical absorption in an indirect band gap material such as silicon is, (where $E_g$ is the band gap, $K_c$ is the wave vector corresponding to the minimum energy gap; $K_{\text{photon}}$ and $K_{\text{photon}}$ are photon and phonon wave vectors, respectively)

(a) $K_{\text{photon}} = K_c + \ K_{\text{photon}}, \hbar\omega = E_g + \hbar\Omega$

(b) $K_{\text{photon}} = k_c, \hbar\omega = E_g$

(c) $K_{\text{photon}} < K_c, \hbar\omega < E_g$

(d) $K_{\text{photon}} > K_c, \hbar\omega > E_g$

Q13. Consider a closed path of radius $a$ in the $z=0$ plane in a magnetic field $\vec{B} = B_o e^{bt}\hat{a}_z$. The electric field intensity $\overline{E}$ at any point at distance $r$ is

(a) $\overline{E} = -\frac{1}{2} bB_0 e^{bt} r\hat{a}_\phi$

(b) $\overline{E} = bB_0 e^{bt} r\hat{a}_\phi$

(c) $\overline{E} = -bB_0 e^{bt}\pi a^2$

(d) $\overline{E} = -\frac{1}{2} bB_\circ e^{bt}\hat{a}_\phi$

Q14. For an alternating electric field $\vec{E} = \vec{E}_0 \cos \omega t$ within a good conductor of conductivity $\sigma$, with respect to the electric field $\vec{E}$ the phase relationship between the displacement current $\frac{\partial D}{\partial t}$ and the conduction current $J_f$ is described by the statement that

(a) $\frac{\partial D}{\partial t}$ leads by $\frac{\pi}{2}$ and $J_f$ is in phase.
(b) $\frac{\partial D}{\partial t}$ lags by $\frac{\pi}{2}$ and $J_f$ is in phase.
(c) $\frac{\partial D}{\partial t}$ is in phase and $J_f$ leads by $\frac{\pi}{2}$
(d) $\frac{\partial D}{\partial t}$ is in phase and $J_f$ lags by $\frac{\pi}{2}$

Q15. An electromagnetic wave $(\bar{E})$ is incident from a medium of refractive index $n_1$ at the boundary with another medium of refractive index $n_2$. The critical angle $(\theta_{ic})$ and the Brewster angle $(\theta_{iB})$, when $\bar{E}$ is polarized parallel to plane of incidence, are equal when

(a) $n_1 = n_2$
(b) $n_1 >> n_2$
(c) $n_1 < n_2$
(d) $n_1 << n_2$

Q16. The radiation from Sun subtends an angle of 18 mrad at the earth at a wavelength of $0.5\ \mu m$. If it is required to measure the spatial coherence condition for radiation from the Sun, then the slit separation required to observe Young's interference fringes using the Sun as a source is

(a) $14 \mu m$
(b) $7 \mu m$
(c) $28 \mu m$
(d) $1.4 \mu m$

Q17. Consider an isolated system of $N$ identical non-interacting particles each of mass $m$, confined to a volume $V$ with a total energy $E$. If $m$ increases, then the entropy of the system

(a) increases
(b) decreases
(c) remains constant
(d) increases initially and eventually decreases.

Q18. If the temperature and molar volume of one mole of an ideal gas is changed from $T_1V_1$ to $T_{2,}V_2$ the change in entropy, $\Delta S$, is

(a) $\Delta S = Rln\left(\frac{V_2}{V_1}\right)$ (b) $\Delta S = C_v \ln\left(\frac{T_2}{T_1}\right)$

(c) $\Delta S = C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$ (d) $\Delta S = C_v \ln\left(\frac{V_2}{V_1}\right) - R\ln\left(\frac{T_2}{T_1}\right)$

Q19. A paramagnetic system consists of $N$ classical magnetic dipoles. Each dipole carries a magnetic moment $\mu$. If the system is exposed to a uniform magnetic field $H$, at a finite temperature $T$, then the average magnetization $< M >$, of the system is

(a) $< M >= N\mu\left[\coth\left(\frac{\mu H}{k_B T}\right) - \left(\frac{k_B T}{\mu H}\right)\right]$ (b) $< M >= N\mu\left[\tanh\left(\frac{\mu H}{k_B T}\right)\right]$

(c) $< M >= N\mu\left[\tanh\left(\frac{\mu H}{k_B T}\right) - \left(\frac{\mu H}{k_B T}\right)\right]$ (d) $< M >= N\mu\left[\coth\left(\frac{\mu H}{k_B T}\right) - \left(\frac{\mu H}{k_B T}\right)\right]$

Q20. The strangeness quantum number is conserved in

(a) Strong, weak and electromagnetic interaction.

(b)Weak and electromagnetic interaction only.

(c)Strong and weak interactions only.

(d) Strong and electromagnetic interaction only.

Q21. In the $\alpha$ decay process, the speed of the daughter nucleus $v_D$ is related to the speed of the $\alpha$ particle by ($M_\alpha$ and $M_D$ are the corresponding masses)

(a) $v_D = v_\alpha$ (b) $v_D = \left(\frac{M_\alpha}{M_D}\right)v_\alpha$

(c) $v_D = \left(\sqrt{M_a / M_D}\right) v_a$ (d) $v_D = \left(\frac{M_D}{M_\alpha}\right)v_\alpha$

Q22. If the non-inverting terminal of op-amp is held at ground potential, then the inverting terminal to as the virtual ground because

(a) the current drawn by this terminal is very large.

(b) no voltage can be applied at this terminal as it will act a short.

(c) the polarity of the input waveform is inverted

(d) the current drawn by this terminal is very small.

Q23. The number of input combinations and the number of ones in the truth table for the expression $A\bar{B}C + \bar{A}BC + AB\bar{C}$ are respectively,

(a) 1,3 (b) 2,6 (c) 8,3 (d) 8,5

Q24. If an equation of the form $\frac{df}{dx} = f(x)$ has to be solved numerically, the most appropriate method is

(a) Newton-Raphson method (b) Secant method

(c) Euler's method. (d) Method of bisection.

Q25. A voltmeter that can read from 0 to $150\,V$ is guaranteed to have an accuracy of 1% on the full-scale reading. If the voltage measured by this instrument is $45\,V$, then the limiting error in percentage at this value is

(a) 3.3% (b) 1% (c) 0.03% (d) 0.3%

## SECTION - B

Q26. If $A$ is a $3\times3$ matrix with det $[A] = 5$ then det $[2A]$ is

(a) 5 (b) 10 (c) 20 (d) 40

Q27. Which one of the following is not a periodic function of $\theta$

(a) $2\sin\theta + 3\sin\left(\theta + \frac{\pi}{2}\right)$ (b) $2\sin\theta + 3\sin\left(\frac{\pi\theta}{2}\right)$

(c) $\frac{2}{\pi}\sin\theta + 3\sin(\theta)$ (d) $2\sin\theta + \frac{\pi}{2}\sin(3\theta)$

Q28. The value of the contour integral

$$\int_c \frac{d_z}{z^2(z+3)}'$$

Where $c$ denotes the unit circle $|z|=1$ traversed in the counter-clockwise direction is

(a) 0 (b) $2\pi i$ (c) $\frac{2\pi i}{3}$ (d) $\frac{-2\pi i}{9}$

Q29. The residue of $\frac{1}{(z-1)(z^2-5z+4)}$ at $z = 1$ is

(a) zero (b) $\frac{-1}{6}$ (c) $\frac{-1}{9}$ (d) $\frac{-1}{25}$

Q30. The inverse Laplace transform of the function $F(p)=\frac{p}{p^2+\omega^2}$ is

(a) $\cos\omega t$ (b) $\sin\omega t$ (c) $\sinh\omega t$ (d) $\cosh\omega t$

Q31. The vector $\vec{v}=(x+3y)\hat{i}+(y-2z)\hat{j}+(x+az)\hat{k}$ is solenoidal when $a=$

(a) $-1$ (b) $-2$ (c) $-3$ (d) $-4$

Q32. If $u(x,y)=x+\frac{1}{2}(x^2-y^2)$ is the real part of an analytic function $f(z)$ of the complex variable $z=x+iy$, then the imaginary part of $f(z)$ is

(a) $y(1+x)$ (b) $xy$ (c) $y$ (d) $y^2-x^2$

Q33. The value of $\lim_{x\to 1}\frac{x^2-\sqrt{x}}{\sqrt{x}-1}$

(a) 1 (b) 0 (c) 2 (d) 3

Q34. A system of two electrons in a single state is described by an effective Hamiltoian $H=A(s_{1z}+s_{2z})+Bs_1.s_2$ where $s_1$ and $s_2$ are two spins, $s_{1z}$ and $s_{2z}$ are their $z-$components, and $A$ and $B$ are constants. The energy level of this Hamiltonian is

(a) $\hbar A+\left(\frac{\hbar^2 B}{4}\right)$ (b) $\frac{\hbar^2 B}{4}$ (c) $-\hbar A+\left(\frac{\hbar^2 B}{4}\right)$ (d) $-\frac{3}{4}\hbar^2 B$

Q35. Let $\hat{x}$ and $\hat{p}$ denote the position and momentum operators obeying the commutation relation $[\hat{x}, \hat{p}] = i\hbar$ and let $|x>$ denote the eigenstate of $\hat{x}$ corresponding to the eigenvalue $x$, then the state

$$|\psi> = e^{\frac{ix_0\hat{p}}{\hbar}}|x>$$

is an eigenstate of

(a) $\hat{x}$ corresponding to the eigenvalue $x + x_0$

(b) $\hat{x}$ corresponding to the eigenvalue $x - x_0$

(c) $\hat{x}$ corresponding to the eigenvalue $xx_0$

(d) $\hat{x}$ corresponding to the eigenvalue $\frac{x}{x_0}$

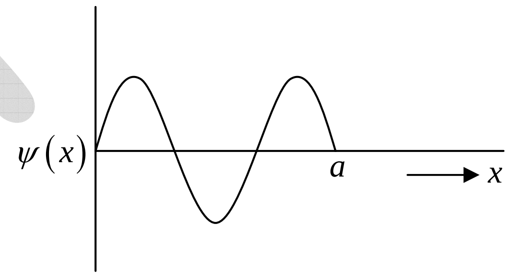
Q36. The energy difference between the states $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ due to the spin-orbit interaction $H_1 \propto \hat{L}.\hat{S}$ is

(a) $\frac{\hbar^2}{2}(2l+1)$ (b) $\hbar^2(2l+1)$ (c) $\hbar^2 l$ (d) $\hbar^2 l.S$

Q37. The eigenfunction $\psi(x)$ of the Schrödinger equation for a particle of mass $m$ in an infinite square well potential

$$V(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

as shown below




corresponds to energy eigenvalue

(a) $\frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{16\pi^2\hbar^2}{2ma^2}$ (c) $\frac{4\pi^2\hbar^2}{2ma^2}$ (d) $\frac{8\pi^2\hbar^2}{2ma^2}$

Q38. The Hamiltonian

$$H = \frac{\vec{p}^2}{2m} + V(r) \qquad \text{where } = \sqrt{x^2 + y^2 + z^2}$$

Commutes

(a) only with $L_z$

(b) with $L_z$ and $L_y$ but not with $L_z$

(c) with $L_z$ but not with $L_x$ and $L_y$

(d) with $L_{x,} L_y$ and $L_z$

Q39. The Lagrangian $L = \frac{m}{2}\left(\omega x + \frac{dx}{dt}\right)^2 + m\omega^2 x \frac{dx}{dt} t$ describes a

(a) particle moving in a harmonic potential.

(b) free particle.

(c) particle moving in a rotation frame.

(d) particle moving in a linear potential.

Q40. For a system described by the Lagrangian

$$L = \frac{\mu d\vec{r}}{2dt} \cdot \frac{d\vec{r}}{dt} + \lambda\left(x^2 + a^2 y^2 + z^2\right), \quad a \in \Re$$

Which one of the following statements is correct

(a) Total energy and total angular moment are conserved.

(b) Total energy and total angular moment about $y$ - axis are conserved.

(c) Total energy and linear moment about $y$ -axis are conserved.

(d) Total energy and angular moment about $x$ -axis and $z$ -axis are conserved.

Q41. The Lagrangian $(L)$ for a system whose Hamiltonian is

$$H = \frac{p^2}{2m} e^{-x/a} + V(x)$$

is

(a) $L = m\left(\frac{dq}{dt}\right)^2 e^{-x/a} + V(x)$

(b) $L = m\left(\frac{dq}{dt}\right)^2 e^{-x/a} - V(x)$

(c) $L = m\left(\frac{dq}{dt}\right)^2 e^{x/a} + V(x)$

(d) $L = m\left(\frac{dq}{dt}\right)^2 e^{-x/a} - V(x)$

Q42. Which of the following statement is the most appropriate for the moment of inertia tensor of a rotating rigid body given in terms of a diagonal matrix.

(a) The body is rotating rigid about an arbitrary axis which is not a principal axis.

(b) The body is rotating about an arbitrary axis which is a symmetry axis.

(c) The body is an example of a symmetrical top.

(d) The body is an example of a spherical top.

Q43. For a particle of mass $m$ moving under the influence of a central force, $[L_{x,}\ \ [L_{y,}P_z]] + [L_{y,}[P_{z,}L_x]]$ (where $L_x, L_y$ are the components of angular momentum and $P_z$ is the component of the linear momentum of the particle) is equal to

(a) Zero always.

(b) Need not be zero always.

(c) Never zero.

(d) With the given information, no definite statement can be made.

Q44. The collision time and root mean square velocity of an electron temperature are $2.5\times10^{-14}$ second and $5\times10^{5}\ m/\text{sec}$., respectively. The classical value of the mean free path of the electron is

(a) 12.5 $nm$ (b) 5 $nm$ (c) 0.25 $nm$ (d) $2\times10^{19}\,m$

Q45. Given that the structure for a fee lattice is

$$S(hkl) = f\left\{1\pm\exp\left[-i\pi(k+l)\right]+\exp[-i\pi(h+l)]+\exp[-i\pi(h+k)]\right\},$$

where $f$ is the atomic structure factor, which of the following set of Bragg reflections will not occur in the diffraction pattern of such a lattice

(a) $(111), (200), (220),$ (b) $(311), (420), (440)$

(c) $(100), (210), (321)$ (d) $(222), (400), (333)$

Q46. If in two metallic systems the density of states at the Fermi level at a temperature as low as $1\ K$ is in the ratio $1:2$, the electronic to the specific heat at $T=1K$ for the two systems will be in the ratio

(a) $1:4$ (b) $1:8$ (c) $2:1$ (d) $1:2$

Q47. In the Einstein model, the heat capacity of $N$ oscillators in one dimension, all having the same resonance frequency $\omega$, is

(a) $C_v = Nk_B\left(\frac{\hbar\omega}{k_BT}\right)^2 \frac{e^{\hbar\omega/k_BT}}{\left(e^{\hbar\omega/k_BT}-1\right)^2}$

(b) $C_v = Nk_B\left(\frac{\hbar\omega}{k_BT}\right)^2 \frac{e^{-\hbar\omega/k_BT}}{\left(e^{\hbar\omega/k_BT}-1\right)^2}$

(c) $C_v = Nk_B\left(\frac{\hbar\omega}{k_BT}\right) \frac{1}{\left(e^{\hbar\omega/k_BT}-1\right)^2}$

(d) $C_v = Nk_B\left(\frac{\hbar\omega}{k_BT}\right)^{-1} \frac{e^{\hbar\omega/k_BT}}{\left(e^{\hbar\omega/k_BT}-1\right)^2}$

Q48. If the partition function for a classical one-dimensional system is

$$Z = \int\int dpdx \; \exp\left[-E(p,x)/k_BT\right],$$

where $p$ is the momentum, then the average energy of the system is

(a) $\langle E\rangle = k_BT\frac{dZ}{dT}$

(b) $\langle E\rangle = k_BT^2 \overline{\log} Z$

(c) $\langle E\rangle = k_BT^2\frac{d\log Z}{dT}$

(d) $\langle E\rangle = \frac{1}{k_BT^2}.\frac{d\log Z}{dT}$

Q49. The displacement current of a good conductor is given by the ratio of displacement current density $\left(\frac{\partial \bar{D}}{\partial t}\right)$ to conduction current density $\left(\bar{J}_f\right)$. For an alternating electric field $E_0\cos\omega t$ within a good conductor of conductivity $\sigma = 10^7$ mhos/ $m$ and relative permittivity $\varepsilon_r = 1$, the displacement current $\left(\text{in } C/m^2/\text{sec.}\right)$ at optical frequency $(f = 10^{15}Hz)$ is

(a) $5.56\times10^{-3}$ (b) $7.5\times10^{-2}$ (c) $4.8\times10^{-5}$ (d) $6.7\times10^{-4}$

Q50. If a hollow spherical shell carries a charge density $\rho = k/r^2$ in the region $a \le r \le b$ (where $a$ and $b$ are the inner and outer radius of shell, respectively), then the electric field in the region $a < r < b$ is

(a) $\frac{k}{\varepsilon_0}\left(\frac{b-a}{r^2}\right)\hat{r}$ (b) $\frac{k}{\varepsilon_0}\frac{b}{r^2}\hat{r}$ (c) $\frac{k}{\varepsilon_0}\left(\frac{b-a}{r^4}\right)\hat{r}$ (d) $\frac{k}{\varepsilon_0}\left(\frac{r-a}{r^2}\right)\hat{r}$

Q51. The magnetic field corresponding to a vector potential

$$\vec{A} = \frac{1}{2}\vec{F} \times \vec{r} + \frac{10}{r^3}\vec{r}$$

where $\vec{F}$ is a constant vector, is

(a) $\vec{F}$ (b) $-\vec{F}$ (c) $\vec{F} + \frac{30\vec{r}}{r^4}$ (d) $\vec{F} - \frac{30\vec{r}}{r^4}$

Q52. The electric field is an electromagnetic wave is described by the relation

$$E(\vec{r},t) = (\vec{e}_1 E_1 + \vec{e}_2 E_2)\exp i(\vec{k}\cdot\vec{r} - \omega t)$$

where $\vec{e}_1$ and $\vec{e}_2$ are two mutually orthogonal unit vectors perpendicular to $\vec{k}$. If $E_1$ and $E_2$ are any two complex number then the wave is likely to be

(a) Plane polarized (b) Elliptically polarized.

(c) Right circularly polarized. (d) Left circularly polarized.

Q53. If a sphere with a magnetic moment $\mu$ and moment of inertia $I$ about its centre is placed in a uniform magnetic field $B$, then the time period of small oscillations of the dipole is

(a) $I/\mu B$ (b) $2\pi\sqrt{I/\mu B}$ (c) $2\pi\sqrt{\mu/IB}$ (d) $\sqrt{I/\mu B}$

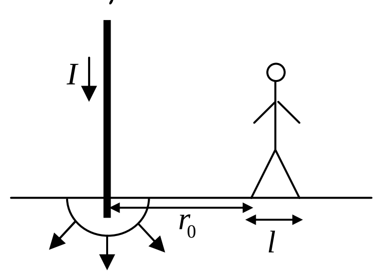
Q54. A hemispherical earth plate with its flat surface parallel of the earth's surface is buried into the earth with its surface (as shown in the figure below). If the current $(I)$ passing through earth plate is $1A$ then the voltage that may be applied to a person at a minimum distance of $r_\circ = 2m$ form the centre of the plate is) given that the length of a step of the person is $l = 0.5m$ and the conductivity of earth is $\gamma = 10^{-2}\,S/m$

(a) 1.59 $V$

(b) 2.7 $V$

(c) 3.18 $V$

(d) 5.4 $V$

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Q55. The heat capacity at constant volume of a canonical ensemble with average energy $<E>, C_v = \left(\partial <E>/\partial T\right)_{N,V}$ is related to the mean-square fluctuation in the energy, $<(E-<E>)^2>$, ,as

(a) $C_v = k_B T <(E-<E>)^2>$ (b) $C_v = <(E-<E>)^2>/k_B TS$

(c) $C_v = <(E-<E>)^2/k_B T^2$ (d) $C_v = 2<(E-<E>)^2>/k_B T$

Q56. The average energy of a quantum mechanical simple harmonic oscillator of frequency $\omega$ at temperature $T$ is

(a) $\frac{1}{2}h\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_BT}+1}$ (b) $\frac{1}{2}h\omega + \frac{\hbar\omega}{e^{-\hbar\omega/k_BT}+1}$

(c) $\frac{1}{2}h\omega - \frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}$ (d) $\frac{1}{2}h\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_BT}-1}$

Q57. Atoms in a moment gas have tow internal energy levels: a good state of degeneracy $g$ and a non-degenerate excited state at an energy $E$ above the ground state. The specific heat at constant volume, $C_v$ for this gas is

(a) $C_{v=} \frac{E^2 e^{E/k_BT}}{k_B T^2\left(1+ge^{E/k_BT}\right)^2}$ (b) $C_{v=} \frac{3}{2}k_B + \frac{gE^2 e^{E/k_BT}}{k_B T^2\left(1+ge^{E/k_BT}\right)^2}$

(c) $C_{v=} \frac{ge^{E/k_BT}}{\left(1+ge^{E/k_BT}\right)^2}$ (d) $C_{v=} \frac{gE^2 e^{E/k_BT}}{k_B T^2\left(1+ge^{E/k_BT}\right)^2}$

Q58. A system consisting of particles obeys Boltzmann statistics. If the energy of a particle is $E = (z) = az^2$, where $z$ is the particle moment that can take all values from $-\infty$ to $+\infty$, the average per particle, $\bar{E}$ for such a system is

(a) $\bar{E} = \frac{1}{2}k_BT$ (b) $\bar{E} = k_BT$ (c) $\bar{E} = \frac{3}{2}k_BT$ (d) $\bar{E} = \frac{5}{2}k_BT$

Q59. If $\xi = \langle n_k \rangle$ denotes the average number of Fermions in the quantum state of energy $E_k$ at temperature $T$, chemical potential $\mu$ and $a^2 = \langle n_k^2 \rangle - \langle n_k \rangle^2$, then the relative fluctuations given by $\sigma/\xi$ is

(a) $\sqrt{\frac{1}{\xi}-1}$ (b) $\sqrt{\frac{1}{\xi}+1}$ (c) $\frac{1}{\xi-1}$ (d) $\frac{1}{\sqrt{\xi}}$

Q60. If the $\rho$ and $\omega$ mesons decay as $p \rightarrow 2\pi$ and $\omega \rightarrow 3\pi$ via strong interaction, then the $G$ - parity of $p$ and $\omega$ will be

(a) $+1\ +1$ (b) $+1\ -1$ (c) $-1\ +1$ (d) $-1\ -1$

Q61. The Higgs boson is responsible for providing masses to the elementary particles by coupling to them. The relation that the coupling strengths of Higgs electrons $(g_e)$, Higgs-muons $(g_\mu)$ and Higgs-up quarks $g_u$ satisfy is

(a) $g_e < g_\mu < g_u$ (b) $g_e > g_\mu > g_u$ (c) $g_e = g_\mu < g_u$ (d) $g_e = g_\mu = g_u$

Q62. The ratio of average nuclear radii of $^{8}Be$ and $^{27}Al$ nuclei is

(a) $3:2$ (b) $2:3$ (c) $1:1$ (d) $4:9$

Q63. If the charge distribution of some nucleus has the form

$$p(r) = \begin{cases} p, & r < R \\ 0, & r > R \end{cases}$$

where $R$ is its average nuclear radius, then the form factor, $F(q)$, is (where q is the momentum transfer)

(a) $4\pi\rho_\circ(\sin qR - qR\cos qR)$ (b) $4\pi\rho_\circ(\cos qR - qR\sin qR)$

(c) $\dfrac{4\pi\rho_\circ}{q^2}(\cos qR - qR\sin qR)$ (d) $\dfrac{4\pi\rho_\circ}{q^3}(\sin qR - qR\cos qR)$

Q64. The quadrupole moment of a nucleus measures

(a) The deformation of the charge density distribution.

(b) The number of nucleons.

(c) The spin of nucleons.

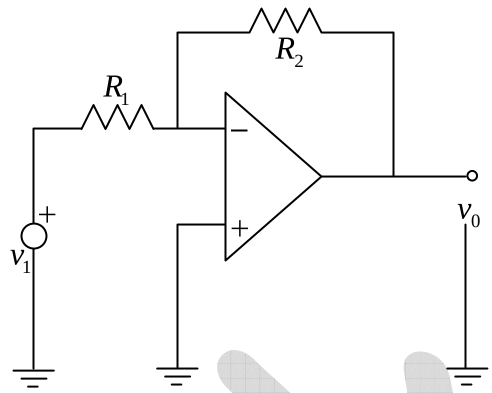
(d) The orbit angular momentum of the nucleus.

Q65. The process $n \rightarrow p + e^-$ is wrong because,

(a) charge is not conserved. (b) parity is not conserved.

(c) angular momentum is not conserved. (d) Baryon number is not conserved.

Q66. In the circuit given below, if the open loop gain is $2\times10^5$, then percentage error in magnitude of the closed loop gain relative to the ideal value, when $R_2/R_1=100$, is




(a) $-0.05\%$ (b) $-0.5\%$ (c) $+0.05\%$ (d) $0\%$

Q67. The minimum number of gates required to implement the Boolean expression $XY+X(X+Z)+Y(X+1)$ after simplification is

(a) 1 (b) 2 (c) 4 (d) 5

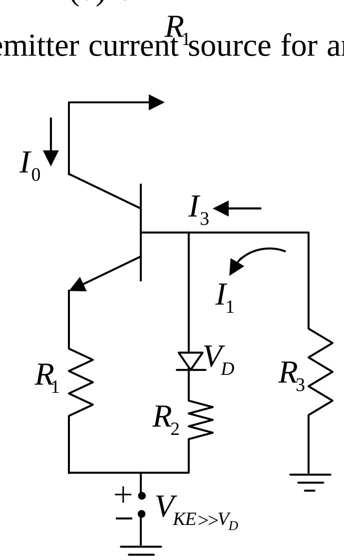
Q68. If the transistor in the circuit given below acts as a constant emitter current source for an amplifier stage, then the current $I_o$, across the resistor $R_1$ is

(a) $\dfrac{R_2V_{EE}}{R_!(R_2+R_3)}$

(b) $\dfrac{V_{EE}}{R_1}$

(c) $\dfrac{R_2V_D}{R_1(R_2+R_3)}$

(d) $\dfrac{R_2V_{BE}}{R_1(R_2+R_3)}$




Q69. If the diode current of a $p-n$ junction diode has doubled then the diode voltage, $V_D$ has to be increased by (assume threshold voltage $V_T=25mV$ and the diode is operating in the region where $V_D/nV_T \gg 1$ where $n$ is the ideality factor=1

(a) $25.0mV$ (b) $34.7mV$ (c) $50mV$ (d) $1725.3mV$

Q70. Consider a laser with 20 gigawatts power and a beam diameter of 2 *mm*. The peak values of the electric field $E_0$ and magnetic field $B_0$ are:

(a) $E_0 = 2.2\times10^9 V/m$ and $B_0 = 7.3$ Tesla

(b) $E_0 = 1.2\times10^7 V/m$ and $B_0 = 1.6$ Tesla

(c) $E_0 = 3.6\times10^5 V/m$ and $B_0 = 0.7$ Tesla

(d) $E_0 = 7.4\times10^8 V/m$ and $B_0 = 3.7$ Tesla

Q71. Consider a sandwhich of three polarizers wherein the first and third polarizers are crossed and the middle polarizer makes an angle $\theta$ with the axis of the first polarizer. If circularly polarized light of intensity $I_0$ is incident on this polarizer sandwich, the output intensity is

(a) $I_\circ \cos^{2\theta}$ (b) $I_\circ \cos^{2\theta}\sin^2\theta$ (c) $\frac{1}{2}I_\circ \cos^{2\theta}\sin^2\theta$ (d) $2I_\circ \cos^{2\theta}\sin^2\theta$

Q72. The lowest kinetic energy required for the neutrons to excite molecular rotation in a molecule of gaseous oxygen is (bond length of oxygen molecules is $1.2\overset{0}{\mathrm{A}}$

(a) $3.6\times10^{-4}eV$ (b) $3.6\times10^{-2}eV$ (c) $3.6\times10^{-6}eV$ (d) $7.2\times10^{-4}eV$

Q73. Consider the spectral line corresponding to a $j=1\to j=0$ transition which is split into three component spectral lines separated by $0.0016\overset{0}{\mathrm{A}}$ in a magnetic field of 1000 gauss. If the zero-field spectral line is at $1849\overset{0}{\mathrm{A}}$, the

(a) 1 (b) 2 (c) 0.5 (d) 1.5

Q74. The following 10 readings were obtained in an experiment to measure resistance in $\Omega$. 100.1, 100.2 100.3, 100.1, 100.4, 100.2, 100.3 100.4,100.1 *and* 100.1 and 100.1. The probable error in measurement is

(a) $0.4\Omega$ (b) $0.2\Omega$ (c) $0.8\Omega$ (d) $0.08\Omega$

Q75. Three methods are available to numerically solve a polynomial $f(x)=0$, with convergence relations $(1)\left|\varepsilon_{n+1}\right| = M_1\varepsilon^2{}_n$, $(2)\left|\varepsilon_{n+1}\right| = M_2\varepsilon_n$ and $(3)\left|\in_{n+1}\right| = M \in_n^{\alpha}$ where $\alpha = 1.618$.

The order in which they converge, from slowest to fastest, would be

(a) methods $1, 2$ and $3$ (b) methods $2, 3$ and $1$

(c) methods $1, 3$ and $2$ (d) methods $3, 2$ and $1$