IISc Integrated Ph.D. Entrance Test

Physical Science - 2007

Q1. Consider the function $y(x) = x^4 + ax^3 + bx^2 + c$. This function will have

(a) At least one minimum (b) At least one maximum

(c) Always have three extrema (d) Will never have three extrema

Q2. Consider the series 1 – 1 + ½ – ¼ + 1/3 – 1/9 + ¼ – 1/16....It

(a) Converges conditionally (b) Converges to $\pi^2/6$

(c) Sums to 0 (d) Diverges

Q3. The series $2 + 2ix - (2x^2/2i) - 2ix^3/3! + 2x^4/4! + 2ix^5/5! \ldots$ with $i = \sqrt{-1}$ evalues for $x = \pi/4$ to

(a) $\frac{1+i}{\sqrt{2}}$ (b) $\sqrt{2(1+i)}$ (c) $2i$ (d) $\frac{1-\sqrt{3}}{2}$

Q4. Consider the differential equation given by $y''(t) = 25y(t)$ with the boundary conditions y (0) = 0, y (∞) = 5. The solution to the equation is:

(a) $y(t) = 5e^{5t} + 5e^{-5t}$ (b) $y(t) = 5 + 5e^{5t}$

(c) $y(t) = 5 - 5e^{-5t}$ (d) $y(t) = 5e^{-25t}$

Q5 Your wallet has 10 Rs. 100/- and 16 Rs. 500/- notes. The number of notes you have to take out to ensure atleast two are of the same denomination is :

(a) 26 (b) 3 (c) $^{26}C_{10}$ (d) $^{26}P_{10}$

Q6. The eigenvalues of the matrix $\begin{bmatrix} 0 & 16 \\ 16 & 0 \end{bmatrix}$ are:

(a) 1 and –1 (b) 16 and 16 (c) 16 and –16 (d) 1 and 256

Q7. The $\lim\limits_{x\to 0}\dfrac{1-\cos^n x}{x^2}$ for $n > 1$ is:

(a) 0 (b) n (c) $2n$ (d) $n/2$

Q8. Consider the product $(1 + z)(1 + z^3)(1+z^6)\ldots(1 + z^{3m})$. The derivative of this function at z = 0 is:

(a) 0 (b) 1 (c) m (d) 3m

Q9. Given the equation H' = -aG and G' = bH, G can have oscillatory solutions

(a) $\forall$a, b (b) for no choice of a, b

(c) If a < 0 and b > 0 (d) a < 0 and b < 0

Q10. The equation $x^3 + ax^2 + bx + c$

(a)Will always atleast have a real root (b) Will never have a real root

(c) Will always have atleast two real roots (d) Will always have three real roots

Q11. Given that the co-ordinates of a particle are $y(t) = A\cos(2\omega t)$ and $x(t) = \sin(\omega t)$ the trajectory of the particle is

(a) Circle (b) Ellipse (c) Hyperbola (d) Parabola

Q12. Given that the binding energy per nucleon of an α particle is 7 MeV, and that the proton mass is 938 MeV/ c^2 and the neutron mass is 939 MeV/c^2, the mass of the α particle is

(a) 3726 MeV/c^2 (b) 3748 MeV/c^2 (c) 1870 MeV/c^2 (d) 931 MeV/c^2

Q13. A particle A of mass m_A decays into two particles, one of which is B with mass m_B and the other massless. The magnitude of the momentum of particle B, using relativistic kinetmatics in the rest-frame of the particle A is

(a) $\dfrac{(m_A^2 - m_B^2)c}{m_A}$ (b) $\dfrac{(m_A^2 - m_B^2)c}{2m_A}$ (c) $\dfrac{(m_A^2 - m_B^2)c}{m_B}$ (d) $\dfrac{(m_A^2 - m_B^2)c}{2m_B}$

Q14. Three spin ½ non-interacting electrons are placed in a simple harmonic oscillator potential given by $m\omega^2 x^2 / 2$. The lowest allowed energy of the system is

(a) $\frac{3}{2}\hbar\omega$ (b) 0 (c) $\frac{5}{2}\hbar\omega$ (d) $\frac{7}{2}\hbar\omega$

Q15. Given that the ionization potential of hydrogen is 13.6 eV, that of positronium which is a composite state made of one electron and one position is,

(a) 0 eV (b) 6.8 eV (c) 13.6 eV (d) 27.2 eV

Q16. In the hydrogen atom spectrum the ratio of the energy for the transition n = 2 $\rightarrow$ n = 1 to that of n = 3 $\rightarrow$ n = 1 is:

(a) 27/32 (b) 32/27 (c) 27/5 (d) 5/27

Q17. Given that the work-function of a metal is 5 eV and that photons of 9 eV are incident on it the maximum velocity of emitted electrons is approximately

(a) 2.0 m/s (b) 1.2×10^6 m/s (c) 4.0×10^6 m/s (d) 4.0×10^{12} m/s

Q18. You had a sample of 27 g of radioactive material and you found only 1g after 60 minutes. If you had done the measurement 20 minutes earlier, how much material would you have found?

(a) 3g (b) 6g (c) 9 (d) 18 g

Q19. A siren of natural frequency 600Hz is fitted on an ambulance. When it approached a pedestrian, she heard a frequency of 640 Hz. At what speed was the ambulance approaching her? (Note that the speed of sound under those conditions was 360 m/s)

(a) 24.0 km/hr (b) 30.2 km/hr (c) 60.8 km/hr (d) 86.4 km/hr

Q20. A thermally ‘insulated box of volume 2V has a partition which divides it into two chambers of equal volume. Each chamber contains He gas at temperature T and pressure P. On removing the partition, the molecules in the two chambers mix with each other. The change in entropy of the system is

(a) $-\frac{PV}{T}\ln 2$ (b) 0 (c) $\frac{PV}{T}\ln 2$ (d) $\frac{2PV}{T}\ln 2$

Q21. A flask containing 1 mole of oxygen gas is known to contain a mixture of O atoms and O_2 molecules. The specific heat of the gas is found to be $\frac{17}{10}R$, where R is the gas constant. The number n_1 of O atoms and the number n_2 of O_2 molecules are in the ratio $(n_1 : n_2)$

(a) 3:1 (b) 3:2 (c) 4:1 (d) 5:1

Q22. Two bars, both if length l and cross sectional area A, made of different materials with thermal conductivities K_1 and K_2 are joined together to form a composite bar of length $2l$. The free ends of the composite bar are kept at temperatures T_1 and T_2. The temperature at the junction of the two bars is:

(a) $\frac{l^2(T_1+T_2)\sqrt{K_1K_2}}{2A(K_1+K_2)}$ (b) $\frac{T_1T_2(K_1+K_2)}{(T_1+T_2)\sqrt{K_1K_2}}$

(c) $\frac{K_1T_1+K_2T_2}{K_1+K_2}$ (d) $\frac{T_1+T_2}{2}$

Q23. A Carnot engine operates between temperature T_1 and T_2 with $T_1 > T_2$. If W is the work done by the engine in one cycle, then the heat Q_1 absorbed at T_1 and Q_2 rejected at T_2 are

(a) $Q_1 = \frac{T_2W}{T_1+T_2}, Q_2 = \frac{T_1W}{T_1+T_2}$ (b) $Q_1 = \frac{T_1W}{T_1+T_2}, Q_2 = \frac{T_2W}{T_1+T_2}$

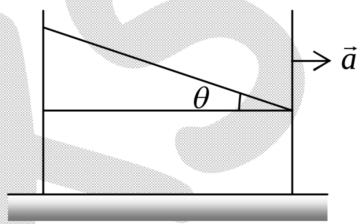
(c) $Q_1 = \frac{T_2W}{T_1-T_2}, Q_2 = \frac{T_1W}{T_1-T_2}$ (d) $Q_1 = \frac{T_1W}{T_1-T_2}, Q_2 = \frac{T_2W}{T_1-T_2}$

Q24. The peaks of emission measured from the different black bodies 1 and 2 are found to be at the frequencies 'v_1' and 'v_2' respectively, with $v_1 = 2v_2$. The temperature of body 1 is known to be T_1 then the temperature T_2 of body 2 is:

(a) $\frac{T_1}{2}$ (b) T_1 (c) $\sqrt{2}T_1$ (d) $2T_1$

Q25. A container carrying water is subjected to an acceleration a to the right as shown in the figure below. The angle θ made by the surface of water with respect to the horizontal is given by

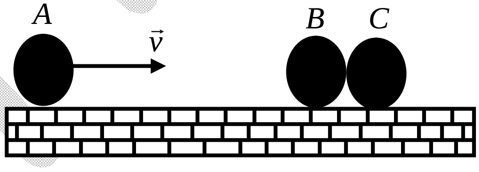
(a) $\cot^{-1}(a/g)$
(b) $\sin^{-1}(a/g)$
(c) $\cos^{-1}(a/g)$
(d) $\tan^{-1}(a/g)$



Q26. The angular momentum of an object points in the direction $\hat{i}+\hat{j}+\hat{k}$. If the object is viewed in a mirror kept in the x-y plane, the angular momentum of the mirror image of the object will point in the direction

(a) $\hat{i}+\hat{j}+\hat{k}$ (b) $-\hat{i}-\hat{j}-\hat{k}$ (c) $-\hat{i}-\hat{j}+\hat{k}$ (d) $\hat{i}+\hat{j}-\hat{k}$

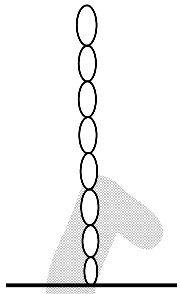
Q27. A ball moving at speed v collide with two balls placed side by side, as shown below



If all the balls have the same mass and all collisions are elastic, then after the collision,

(a) A and B will remain at rest touching each other and C will move with speed v to the right
(b) A will come to rest while B and C will both move at speed $v/2$ to the right
(c) A will rebound with speed $v/3$ and B and C will jointly move at speed $2v/3$
(d) All balls will stick and move at speed $v/3$

Q28. A chain of mass M and length L is hanging vertically over a table with its lowest point touching the surface of the table. It is released and it falls on the table completely inelastically. How much time does it take for the chain to fall completely on the table?



(a) L/g

(b) $2L/g$

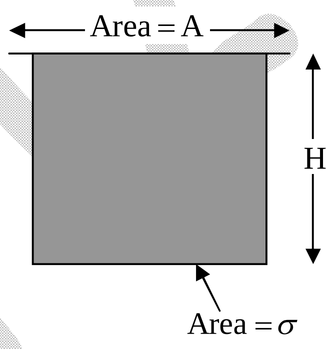
(c) $\sqrt{L/g}$

(d) $\sqrt{2L/g}$

Q29. A planet of mass M_P is in a circular orbit around a star of mass M_S at a radius R. If the star loses a fraction ‘ f ’of its mass in a sudden explosion then what is the minimum fraction of mass that it must lose for the planet to escape to infinity ?

(a) $f = ½$ (b) $f = ¼$ (c) $f = M_p / M_s$ (d) $f = M_p^2 / M_s^2$

Q30. A vessel is filled with water up to a height H as shown in the figure. The cross sectional area of the vessel is A. A tiny hole at the bottom with an area σ slowly drains the vessel.

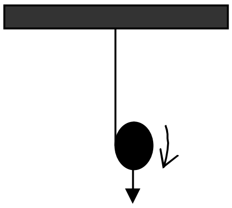


The time taken to completely drain the water is given by

(a) $\frac{A}{\sigma}\sqrt{\frac{8H}{g}}$ (b) $\frac{A}{\sigma}\sqrt{\frac{4H}{g}}$

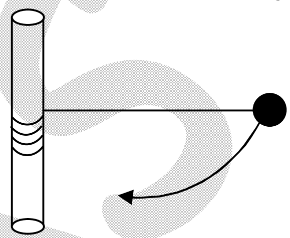
(c) $\frac{A}{\sigma}\sqrt{\frac{2H}{g}}$ (d) ∞

Q31. A cylinder tied to a string of length L (see figure) takes time T to fully unwind. The size of the cylinder is then doubled, thus making its mass 8 times and moment of inertia 32 times their former values. If the length of the string remains uncharged then the time taken to fully unwind the string becomes



(a) 8T (b) 4T (c) $\frac{T}{4}$ (d) T

Q32. A ball tied to a massless string is winding on a cylinder which is fixed firmly to the ground. The angular momentum of the ball



(a) Increases with time

(b) Decreases with time and approaches zero

(c) Remains a constant

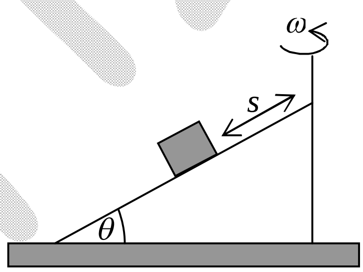
(d) Decreases and then asymptotically approaches a non- zero constant

Q33. A planet is in an elliptical orbit around an star of mass M. If v is the tangential velocity of the planet then at the apogee (farthest distance away form the star) we have

(a) $Mv^2 - 2GM/R = 0$ (b) $Mv^2 - GM/R < 0$

(c) $Mv^2 - GM/R = 0$ (d) $Mv^2 - GM/R > 0$

Q34. An inclined plane is rotating around a pole with angular speed ω as shown in the figure below.



A block of mass M is sliding down the plane. At what distance's' along the plane does it fly off the surface

(a) $s = \frac{g}{\omega^2}\cot\theta$ (b) $s = \frac{\omega^2}{g}\cot\theta$ (c) $s = \frac{g}{\omega^2}\text{cosec}\theta$ (d) $s = \frac{g}{\omega^2}\tan\theta$

Q35. Which of the following cannot describe a conservative force field?

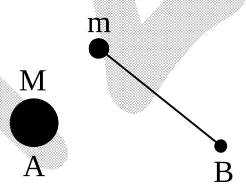
(a) $(x^2 - y^2)\hat{i} - 2xy\hat{j} + 2xy\hat{k}$ (b) $x\hat{i} + y\hat{j} - 2z\hat{k}$

(c) $\sin(x)\hat{i} - \cos(y)\hat{j}$ (d) $x\hat{i} - y\hat{j} + z\hat{k}$

Q36. A bungee jumper tied to a rope with a Young's modulus E = 10^8 Pascal jumps off a bridge. The area of crossection of the rope is 2.4 cm^2 and its unstretched length is 24 m. If the mass of the jumper is 100kg then what is the acceleration of the jumper at the moment when the rope is fully stretched and the jumper has come to a standstill? For this problem assume the rope to be massless and the acceleration due to gravity g is 10 m/s^2.

(a) 0 m/s^2 (b) 10 m/s^2 (c) 70 m/s^2 (d) 100 m/s^2

Q37. An object of mass m, is fixed to a massless rod pivoted at point "B" as show in the figure below. The rod is free to move in any direction without any resistance. A large mass M is fixed to the point A. If the two masses interact with each other via gravity then the general motion of the system will satisfy the condition



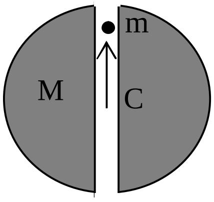
(a) Only the total angular momentum of the system is constant

(b) Only the total energy of the system is constant

(c) Both energy and angular momentum of the system are constant

(d) Neither energy nor the momentum of the system is constant

Q38. A vertical tunnel is dug up through Earth passing through the centre as shown in the figure given. A particle of mass m is thrown inside the tunnel. How should the density vary with radius R inside the Earth if the particle executes perfect simple harmonic motion?



(a) $\rho(R)\alpha 1/R$ (b) $\rho(R)=$ constant

(c) $\rho(R)$ = constant or $\rho(R)\alpha 1/R^2$ (d) $\rho(R)\alpha 1/R^2$

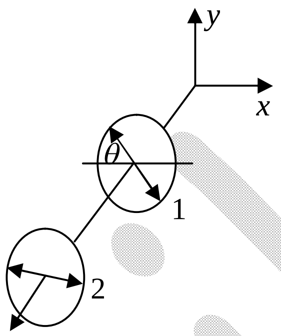
Q39. A telescope of aperture diameter D = 32 mm, and objective focal length f = 24 cm, forms images of two distant stars in the focal plane of the objective lens. If the stars have a minimum resolvable angular separation according to the Rayleigh criterion, then what is the distance, x, between the centers of the image in the focal plane? (Assume light of wavelength 550 nm is used).

(a) 5 μ m (b) 50 μ m (c) 0.5 mm (d) 5 mm

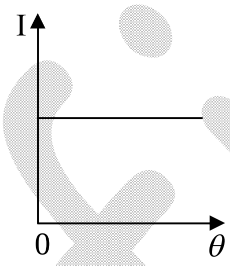
Q40. A mass M is attached to a solid support with an ideal spring of spring constant k. The natural frequency of the system is ω. If the mass now carries a charge $+Q$ and an external electric field E along the axis of the spring is turned on, the frequency for small oscillations

(a) becomes zero (b) increases (c) decreases (d) remains unchanged

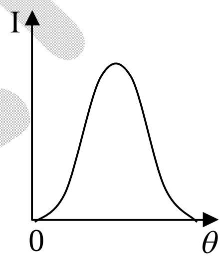
Q41. A plane polarized electromagnetic wave with$_1$polarization along y-axis, is incident on a system of two polarizing sheets as shown below. The polarization axis of polarizer 2 is fixed along the x-axis, while polarizer 1 can be rotated in the x - y plane. If θ is the angle between the polarization axes of polarizers 1 and 2, which of the following figures give the correct description of the intensity at the output of polarizer 2 as a function θ between zero and $\pi / 2$?



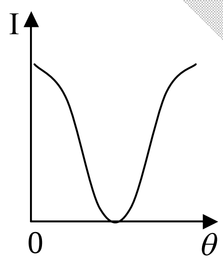
(a)



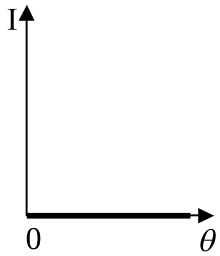
(b)



(c)



(d)

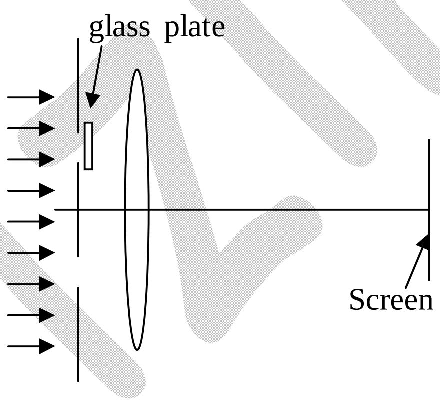


(a) (i) (b) (ii) (c) (iii) (d) (iv)

Q42. A spatially uniform time-dependent magnetic field is changing with time at the constant rate of 1 T/s. A unit positive charge is moved around a circle of radius R = 2m perpendicular to this field. The magnitude of the work done on the charge for one complete revolution is

(a) 0 (b) 2 J (c) 6.28 J (d) 12.56 J

Q43. In a Young's double-slit experiment, the central maximum occurs at zero path difference between interfering beams. A glass plate of thickness 0.005 mm is inserted to cover the upper slit completely (See figure). If the slit separation is 0.1 mm and the diffraction pattern is observed at the focal plane of the converging lens (focal length 200 mm), which of the following statements on the position of the central maximum is true? [Refractive index of glass is 1.5]



(a) The central maximum shifts up by 15 mm.

(b) The position of the central maximum remains unchange

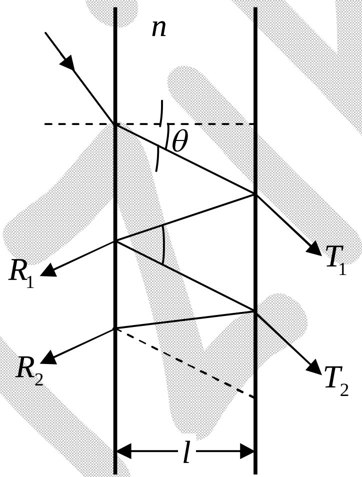
(c) The central maximum shifts down by 15 mm

(d) The central maximum shifts down by 1.5 mm.

Q44. In heat transport, the rate of heat flow dQ/dt through a solid slab is related to the temperature difference ΔT as, $dQ/dt = KA\Delta T/L$. Here K, A and L are the thermal conductivity, cross- sectional area and thickness of the slab, respectively. In an actual experiment, assume that the proportional errors in the measurement of heat flow rate, each length scale and temperature difference are 3%, 1% and 5% respectively. What is the maximum proportional error in the measurement of the thermal conductivity?

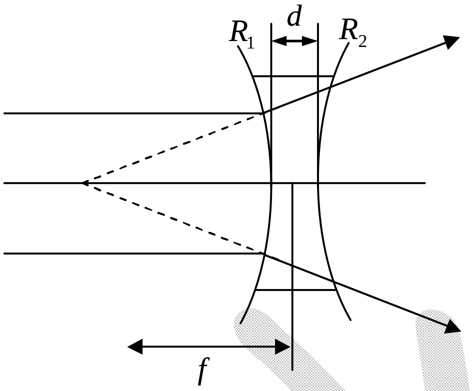
(a) 8% (b) 9% (c) 10% (d) 11%

Q45. In the Fabry-Perot interferometer shown below, n is the refractive index of the glass plate of thickness l and θ is the angle of refraction of the incident light beam of wavelength λ. The path difference between two successive reflections (R_1 and R_2) is given by:



(a) 0 (b) $2nl\cos\theta$ (c) $2nl\sin\theta$ (d) $2nl\cot\theta$

Q46. Assume that a diverging concave lens (refractive index $n > 1$) shown in the figure below has radii of curvature R_1 and R_2 and thickness d. If R_1, R_2 and n are kept constant, how will the focal length f of the lens change with increasing d?



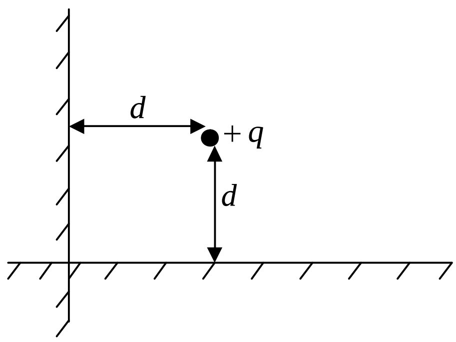
(a) f remains unchanged with increasing d.

(b) f increases with increasing d.

(c) f decreases with increasing d.

(d) f changes non-monotonically with increasing d.

Q47. A particle of charge +q is placed at a distance d form two perpendicular infinite metal plates as shown in the figure below. The magnitude of total force acting on the charge



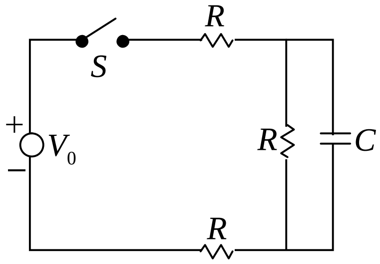
(a) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{2}{d^2}\right)$

(b) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{2d^2}\right)$

(c) $\frac{q^2}{4\pi\varepsilon_0}\left(\frac{2\sqrt{2}-1}{8d^2}\right)$

(d) 0

Q48. An electric circuit is given below:

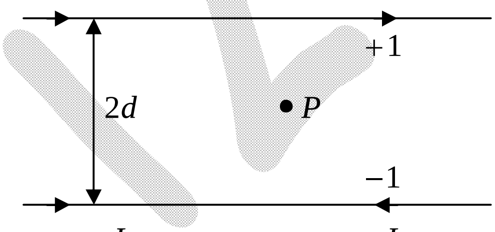


The capacitor is initially uncharged. If the switch S is closed at time t = 0. The voltage across the capacitor as a function of time for t > 0 is given by

(a) $V_0\left[1-\exp\left(-\frac{3t}{RC}\right)\right]$ (b) $\frac{V_0}{3}\left[1-\exp\left(-\frac{3t}{RC}\right)\right]$

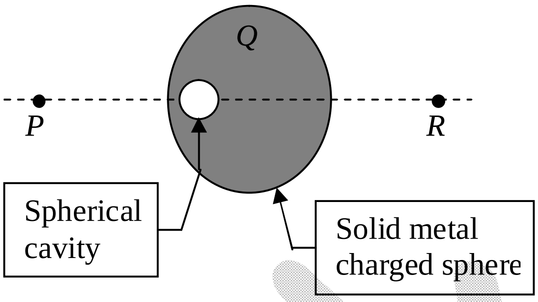
(c) $\frac{2V_0}{3}\left[1-\exp\left(-\frac{3t}{2RC}\right)\right]$ (d) $\frac{V_0}{3}\left[1-\exp\left(-\frac{3t}{RC}\right)\right]$

Q49. Two infinitely long wires carrying equal but opposite current, +I and –I, are separated by a distance 2d as shown below. At the point P equidistant from the wires the strength of the magnetic field B is given by



(a) 0 (b) $\frac{\mu_0 I}{2\pi d}$ (c) $\frac{\mu_0 I}{\pi d}$ (d) $\frac{\mu_0 I}{\pi d^2}$

Q50. A solid metal sphere with a spherical cavity as shown below has a total charge +Q. O is the centre of the sphere, and P and R are two points equidistant form it. If E_P and E_R represent the magnitude of the electric field at P and R respectively, which of the following statements is correct?



(a) $E_P = E_R$
(b) $E_P = 0$ and $E_R = 0$
(c) $E_P > E_R$
(d) $E_P < E_R$