IISc Integrated PhD- 2011

PHYSICAL SCIENCE

Q1. When a nuclide undergoes β decay, which of these is unchanged?

(a) proton number

(b) neutron number

(c) proton number + neutron number

(d) proton number – neutron number

Q2. Given that the most stable isotope of lead is ^{208}Pb, which of the following is a magic number?

(a) 122 (b) 124 (c) 126 (d) 128

Q3. Given that the binding energy per nucleon of an α-particle is 7 MeV, and that the energy released in the reaction $d + d \rightarrow \alpha$ is 23.6 MeV, total binding energy of a deuteron is

(a) 1.1 MeV (b) 2.2 MeV (c) 3.3 MeV (d) 4.4 MeV

Q4. A pion of mass 140 MeV $/c^2$ decays at rest in the laboratory to a muon of mass 105 MeV/c^2 and a massless neutrino. The momentum of the muon in the laboratory is:

(a) 30.625 MeV/c (b) 15.313 MeV/c

(c) 40.833 MeV/c (d) 20.147 MeV/c

Q5. ^{238}U decays via α-particle emission to

(a) ^{236}U (b) ^{234}U (c) ^{236}Th (d) ^{234}Th

Q6. The following expression

$$\frac{1/1! - 1/3!(\pi/4)^3 + 1/5!(\pi/4)^5 - .....}{1 - 1/2!(\pi/4)^2 + 1/4!(\pi/4)^4 - .....}$$

equals to

(a) 1/2 (b) $1/\sqrt{2}$ (c) $\sqrt{2}$ (d) 1

Q7. Given the three matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which of the following statements is true for all positive integers n and i = 1, 2, 3?

(a) $\sigma_i^n = I$ (b) $\sigma_i^n = \sigma_i$ (c) $\sigma_i^{2n} = I$ (d) $\sigma_i^{2n} = \sigma_i$

Q8. The trace of a 2 × 2 matrix is 1 and its determinant is 1. Which of the following has to be true?

(a) One of the eigenvalues is 0
(b) One of the eigenvalues is 1
(c) Both of the eigenvalues are 1
(d) Neither of the eigenvalues is 1

Q9. Let M by a 3 × 3 Hermitian matrix which satisfies the matrix equation

$$M^2 - 7M + 12I = 0$$

Where I refers to the identity matrix. What is the determinant of the matrix M given that the trace is 10?

(a) 27 (b) 36 (c) 48 (d) 64

Q10. Consider the equation

$$x - \tanh \beta x = 0$$

The change in the number of real solutions to this equation when β is varied from 1/2 to 3/2 is

(a) none
(b) an increase by 2
(c) a decrease by 2
(d) an increase by 3

Q11. A point particle is moving in the (x, y) plane on a trajectory given in polar coordinates by the equation

$$25 + r^2 \cos 2\theta = 0$$

The trajectory of the particle is a

(a) parabola (b) circle (c) ellipse (d) hyperbola

Q12. From the solution of the differential equation

$$\frac{dy}{dx}=\frac{1}{1+x^2},$$

what can you say regarding the following series?

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+.....$$

(a) The series is divergent

(b) The series is absolutely convergent

(c) The series converges to $\frac{\pi}{4}$

(d) The series converges to 0

Q13. Let us suppose there are 2 dice with faces numbered from 1 to 6. One die is such that in any throw the probability of obtaining any number is equal. The other is such that the probability of obtaining 2, 4 or 6 is 1/5 each, while the probability of obtaining odd numbers is equal. In a simultaneous throw of both the dice, what is the probability of obtaining an outcome of 5

(a) $\frac{2}{15}$ (b) $\frac{1}{9}$ (c) $\frac{4}{45}$ (d) $\frac{1}{3}$

Q14. Consider the system of differential equations

$$\frac{dx}{dt}=-y, \qquad \frac{dy}{dt}=x$$

Plotting the various solutions in the x, y-plane one obtains

(a) Hyperbolae (b) Parabolae (c) Circles (d) Straight lines

Q15. The value of the line integral

$$\oint\frac{xdy-ydx}{x^2+y^2}$$

along a circle of radius 3 centered at the origin in the counter clockwise direction is given by

(a) 0 (b) $\frac{3}{2\pi}$ (c) 2π (d) 6π

Q16. Consider two free particles in 3 dimensions of equal mass, the wave functions are given by

$$\Psi_1(\vec{r},t)=\frac{1}{(2\pi)^3}\exp\left(-i\frac{\hbar^2\left|\vec{k}_1\right|^2}{2m}t+i\vec{k}_1\cdot\vec{r}\right),$$

$$\Psi_2(\vec{r},t)=\frac{1}{(2\pi)^3}\exp\left(-i\frac{\hbar^2\left|\vec{k}_2\right|^2}{2m}t+i\vec{k}_2\cdot\vec{r}\right)$$

where m refers to the masses of the particles, and the wave vectors $\vec{k}_1$, $\vec{k}_2$ are given by

$$\vec{k}_1=\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right),\qquad \vec{k}_2=\frac{1}{\sqrt{3}}\left(-\hat{i}+\hat{j}-\hat{k}\right)$$

The direction vector along which the probability density of the wave function $\Psi(\vec{r},t)=\Psi_1(\vec{r},t)-\Psi_2(\vec{r},t)$ vanishes is

(a) $\hat{i}$ (b) $\hat{j}$ (c) $\hat{k}$ (d) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$

Q17. Consider the wave function $\Psi(x,t)$ which satisfies the Schrödinger wave equation in one dimension with the time dependent potential

$$V(x,\ t)=\frac{1}{2}kx^2+E_0\cos\omega t$$

Which of the following is true

(a) $\int_{-\infty}^{\infty}\left|\Psi(x,\ t)\right|^2 dx$ is time independent

(b) $\int_{-\infty}^{\infty}\left|\Psi(x,\ t)\right|^2 dx$ is time dependent

(c) $\left|\Psi(x,\ t)\right|^2 dx$ is time independent

(d) $\int_{-\infty}^{\infty}dx\left(\Psi^*\right)(x,t)\partial_x^2\Psi(x,t)-\partial_x^2\Psi^*(x,t)\Psi(x,t)$ is time dependent

Q18. A hydrogen beam is prepared in the state

$$\Psi(\vec{r},t)=\frac{1}{\sqrt{14}}\exp\left(i\frac{E_1 t}{\hbar}\right)\Psi_1(\vec{r})+\sqrt{\frac{2}{7}}\exp\left(i\frac{E_2 t}{\hbar}\right)\Psi_2(\vec{r})+\frac{3}{\sqrt{14}}\exp\left(i\frac{E_3 t}{\hbar}\right)\Psi_3(\vec{r})$$

where E_1, E_2, E_3 are the energies of the ground state and the first excited state of the hydrogen atom and $\Psi_1(\vec{r}), \Psi_2(\vec{r}), \Psi_3(\vec{r})$ are their normalized wave functions respectively. The beam is incident on a detector which measures their energy. Let E_0 be the ionization energy of the hydrogen atom. The average energy measured by the detector is given by

(a) $-E_0$ (b) $-\frac{3}{14}E_0$ (c) $-\frac{1}{7}E_0$ (d) $-\frac{1}{14}E_0$

Q19. A particle of mass m is confined to move in one dimension and is prepared such that the uncertainty in the measurement of its momentum at time t after it is released from the source is given by $\Delta p=\sqrt{\frac{m\hbar}{2t}}$

where p is the momentum of the particle. An experimentalist has a device which can detect the particle only if the uncertainty in position of the particle Δx is $\Delta x \le \sqrt{\frac{\hbar\tau}{m}}$

where τ is the time scale of the device. What is the maximum time beyond which it is impossible to detect the particle with the experimentalist's device?

(a) τ (b) 2τ (c) 4τ (d) $\sqrt{2}\tau$

Q20. A quantum mechanical particle of mass m is confined to move in a circle of radius R. The energy levels are

(a) $E=\frac{\hbar^2}{2mR^2}l(l+1), \quad l\in\{0,1,2,3,.....\}$

(b) $E=\frac{\hbar^2}{2mR^2}l^2, \quad l\in\{....,-3,-2,-1,0,1,2,3,.....\}$

(c) $E=\frac{\hbar^2}{2mR^2}l(l-1), \quad l\in\{0,1,2,3,.....\}$

(d) $E=\frac{\hbar^2 R^2}{2m}l^2, \quad l\in\{....,-3,-2,-1,0,1,2,3,.....\}$

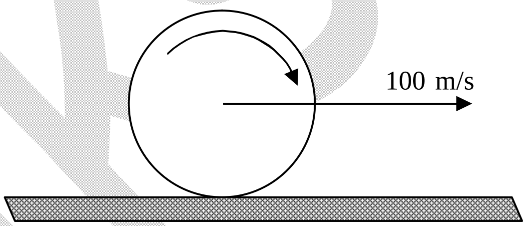
Q21. Consider a one dimensional quantum mechanical oscillator of frequency ω. If the energy levels of this oscillator are populated by 3 electrons, then the lowest energy is

(a) $\frac{3}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{7}{2}\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

Q22. The Planck length is given by

(a) $\sqrt{Gc/h^3}$ (b) $\sqrt{Gh/c^3}$ (c) $\sqrt{hc/G}$ (d) $\sqrt{G/hc}$

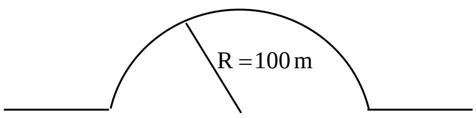
Q23. A wheel of radius R = 1 m is rolling on the ground with slipping. Its angular velocity is 200 rad/s. If it's linear speed is 100 ms^{-1} in the positive x direction then the bottom most part of the wheel is traveling with respect to the ground at



(a) – 300 m/s (b) – 100 m/s

(c) 100 m/s (d) 300 m/s

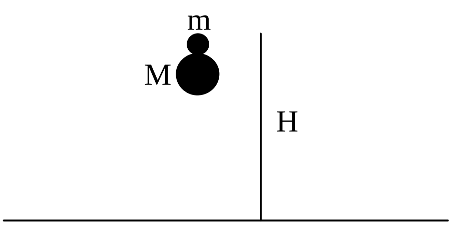
Q24. A man climbs down a hemispherical hill, of radius 100 m from the topmost point. If the coefficient of friction between his shoes and the hill is μ = 0.1 then approximately how much distance does he have to walk before he slips?



(a) 5 m (b) 10 m

(c) 20 m (d) will never slip

Q25. A small ball of mass m is lying on top of a massive ball of mass $M >> m$. They are both released from a given height simultaneously. If the small ball was at height H when it was released then after the balls have collided once with the ground the small ball rises approximately to a height



(a) $3H$ (b) $9H$

(c) H (d) $27H$

Q26. A block attached to a spring moves on a horizontal table with a small coefficient of friction such that it executes damped oscillatory motion. The amplitude as a function of time decreases

(a) Linearly (b) Quadratically

(c) Exponentially (d) Logarithmically

Q27. A pendulum consists of a massive bob suspended from a hook by an elastic band. The spring constant k of the band and the mass m of the bob are such that $k/m = \omega^2 = 10\ \text{rad}^2/\text{s}^2$, and the unstretched length of the band is $L = 1\text{m}$. The pendulum is released from an angle from the vertical of 60 degree. Initially the speed of the bob is zero and the band is in a relaxed (unstretched) state. The total length of the band at the lowest point in the bob's trajectory is

(a) 1.6 m (b) 2.6 m (c) 3.5 m (d) 0.6 m

Q28. Two identical point charges of mass m and charge q are separated by a distance d and are moving at a relative speed u. What is their relative speed when they are at a large distance from each other?

(a) $\sqrt{u^2 - Q^2/\pi\varepsilon_0 md}$ (b) $\sqrt{u^2 + 2Q^2/\pi\varepsilon_0 md}$

(c) $\sqrt{u^2 + Q^2/\pi\varepsilon_0 md}$ (d) $\sqrt{u^2 - 2Q^2/\pi\varepsilon_0 md}$

Q29. A capacitor is made of two conducting spheres each of radius R and covered by a thin insulating sheet. They carry charge Q and $-Q$ respectively and their centers are separated by a distance d. If the capacitance of this capacitor is given by $C = Q/V$ where V is the potential difference between the two spheres then

(a) C is maximum when d is infinity

(b) C is maximum when d approaches $2R$

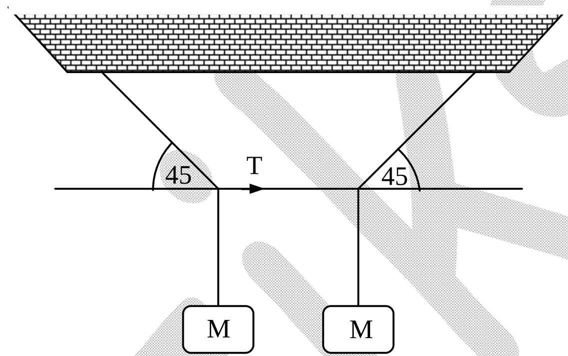
(c) C does not depend on d

(d) C is maximum for $d = 4R$

Q30. A U shaped tube of uniform cross section A contains a liquid of density ρ. The total length of the column is L. If the fluid is displaced then the frequency of oscillation is

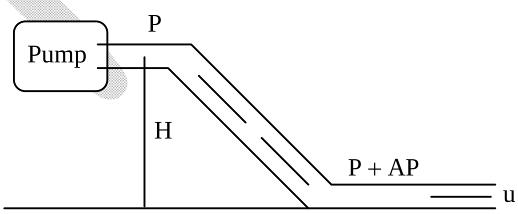
(a) $\frac{1}{2\pi}\sqrt{g/L}$ (b) $\frac{1}{2\pi}\sqrt{gL/A}$ (c) $\frac{1}{2\pi}\sqrt{g\rho A}$ (d) $\frac{1}{2\pi}\sqrt{g/A}$

Q31. If the mechanical system shown below is in static equilibrium then what is the value of tension T?



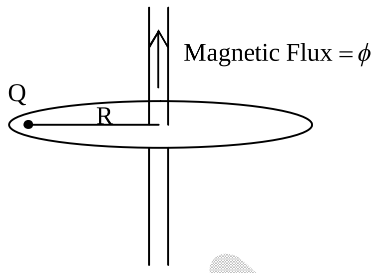
(a) mg (b) $\sqrt{2}mg$ (c) $mg/\sqrt{2}$ (d) $2mg$

Q32. A pump is used to push water of density ρ through a tube of constant crossection A shown below. What is the value of Δp?



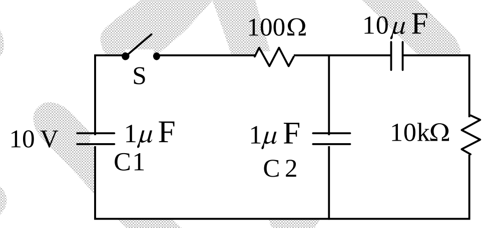
(a) $\rho gH + \rho u^2/2$ (b) 0 (c) ρgH (d) $-\rho gH$

Q33. A freely rotating disc is pierced through by an infinite superconducting solenoid containing a magnetic flux Φ. A charge Q is fixed to the disc at a distance R from the centre of the solenoid. If the solenoid is heated and it loses its magnetic flux in time T then the net angular momentum acquired by the disc is given by

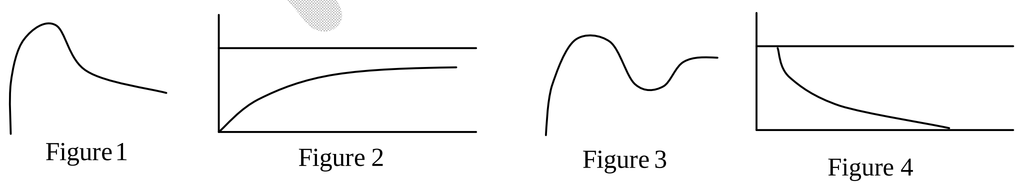


(a) $q\Phi/T$ (b) $q\Phi/2\pi$ (c) $q\Phi$ (d) $q\Phi/2\pi R$

Q34. In the following circuit, capacitor $C1$ is initially charged to 10 Volts while $C2$ and $C3$ are uncharged. At time $t = 0$, the switch S is closed.

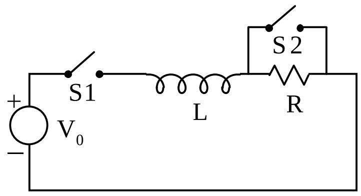


Which of the following waveforms of voltage versus time best represents the voltage across $C2$ for $t > 0$?



(a) Figure 1 (b) Figure 2 (c) Figure 3 (d) Figure 4

Q35. In the following R-L circuit, the switches $S1$ and $S2$ are initially closed at time t = 0.

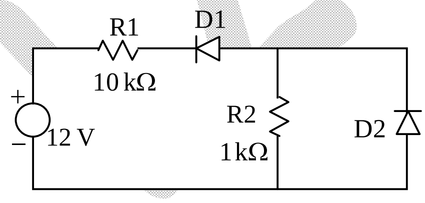


Switch $S2$ is then opened after a time interval T. The peak voltage across the resistor R is given by

(a) $V_0\left(1-\exp\left(-\frac{R}{L}T\right)\right)$ (b) $V_0\exp\left(-\frac{R}{L}T\right)$

(c) $\frac{V_0RT}{L}$ (d) $V_0\left(1-\frac{R}{L}T\right)$

Q36. In the following circuit, D1 and D2 are identical diodes with forward voltage drop of 0.6 Volts and Reverse Breakdown Voltage of 5 Volts.



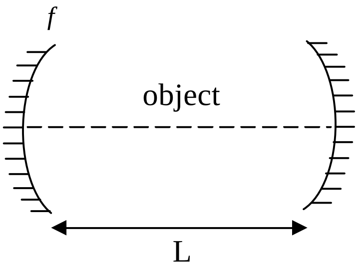
The current through resistor $R2$ is approximately

(a) zero (b) 0.2 mA (c) 0.7 mA (d) 5 mA

Q37. The Boolean algebra expression $A+B+A\bar{B}C+BC$ reduces to

(a) $A+BC$ (b) $A+B$ (c) $AC+\bar{B}C$ (d) $A+C$

Q38. A confocal laser cavity consists of two concave mirrors with equal focal lengths f separated by a distance of L along their axis as shown below:



An object is placed inside the cavity at a distance of L/4 from one of the mirrors. If the final image coincides in position with the object, the focal length f is

(a) $2L$ (b) L (c) $L/2$ (d) $L/4$

Q39. Two beams of monochromatic light with intensities I_1 and I_2 interfere constructively to produce an intensity of 100 mW. If one of the beams is shifted in phase by 60 degrees, the intensity reduces to 84 mW. Then I_1 and I_2 are

(a) 64 mW and 4 mW (b) 36 mW and 16 mW

(c) 10 mW and 9 mW (d) 92 mW and 8 mW

Q40. The refractive index of a medium in which the electric field of an electromagnetic wave is given in MKS units by

$$\vec{E} = E_0 \cos\left(10^7 x + 10^7 y - 10^{15} t\right)\hat{z}$$

is

(a) 1.4 (b) 3.0 (c) 4.2 (d) 6.0

Q41. In a certain process, the temperature (T) and entropy (S) are related by $T \propto S^n$, where n is a number. What is the heat capacity in terms of the entropy for this process?

(a) $\frac{S^2}{k_B n^2}$ (b) $\frac{S}{n}$ (c) $nk_B e^{S/k_B}$ (d) $n\sqrt{Sk_B}$

Q42. The probability of a system to be in a state with energy E at temperature T is proportional to e^{-E/k_BT}. If the system in question is a one dimensional quantum harmonic oscillator of frequency ω, the average energy at temperature T is

(a) $\frac{\hbar\omega}{2}$

(b) $\hbar\omega\left(e^{-\hbar\omega/k_BT}+\frac{1}{2}\right)$

(c) $\hbar\omega\left(\frac{1}{e^{-\hbar\omega/k_BT}-1}+\frac{1}{2}\right)$

(d) $\hbar\omega\left(\frac{1}{e^{-\hbar\omega/k_BT}+1}+\frac{1}{2}\right)$

Q43. Two solid blocks, one at temperature T_1 and the other T_2 ($T_1 > T_2$), with the same temperature independent heat capacity C are put in contact with each other. The change in entropy of the universe after they have equilibrated is

(a) $C\ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$

(b) $C\ln\left(\frac{T_1}{T_2}\right)$

(c) $C\ln\left(\frac{T_1-T_2}{T_1+T_2}\right)$

(d) C

Q44. A thin spherical rubber balloon is blown up to a radius R by filling it with air, which can be assumed to be an ideal gas of average molecular mass m. If the temperature of the balloon is T, the pressure outside is P and the surface tension of rubber is γ, the density of air inside the balloon is

(a) $\frac{mP}{k_BT}$

(b) $\frac{mP}{k_BT}\left(P-\frac{4\gamma}{R}\right)$

(c) $\frac{mP}{k_BT}\left(P+\frac{4\gamma}{R}\right)$

(d) $\frac{mP}{k_BT}\sqrt{\frac{4P\gamma}{R}}$

Q45. N molecules of an ideal monoatomic gas and diatomic gas are kept in identical containers at the same pressure. If the temperatures of the monoatomic gas and diatomic gas are T_M and T_D respectively and their entropies are S_M and S_D, in general

(a) $T_M = T_D$ and $S_M = S_D$

(b) $T_M = T_D$ and $S_M \neq S_D$

(c) $T_M \neq T_D$ and $S_M = S_D$

(d) $T_M \neq T_D$ and $S_M \neq S_D$

Q46. A shooter fires a bullet with velocity u in the $\hat{x}$ direction at a target. The target is moving with velocity v in the $\hat{x}$ direction relative to the shooter and is at a distance L from him at the instant the bullet is fired. If $\gamma = 1\sqrt{1-v^2/c^2}$, how long from the bullet is fired will it take to hit the target in the target's frame of reference?

(a) L/c (b) $\gamma Lc/uv$

(c) $\gamma L\left(1-uv/c^2\right)/(u-v)$ (d) $L\left(1+uv/c^2\right)/[\gamma(u-v)]$

Q47. A source of light S and detector D are approaching an observer O from opposite directions with speed v related to O. If S is emitting light of frequency v in its rest frame in the direction of D, the frequency observed by D is

(a) v (b) $v\sqrt{\dfrac{1-v/c}{1+v/c}}$ (c) $v\sqrt{\dfrac{1+v/c}{1-v/c}}$ (d) $v\left(\dfrac{1+v/c}{1-v/c}\right)$

Q48. A beam of relativistic particles of mass m0 and kinetic energy K is normally incident upon a perfectly absorbing surface. If the particle flux (number of particles per unit area pre unit time) is J, the pressure on the surface is

(a) $\dfrac{JK}{c}$ (b) $\dfrac{J\sqrt{K\left(K+m_0c^2\right)}}{c}$

(c) $\dfrac{J\left(K+m_0c^2\right)}{c}$ (d) $\dfrac{J\sqrt{K\left(K+2m_0c^2\right)}}{c}$

Q49. A spherical black body has the following relations between its energy E, temperature T and radius R; T $\propto$ 1/E and R $\propto$ E. If the initial energy of the black body is E_0 the time t after which it evaporates due to black body radiation (as given by Stefan's law) are related as

(a) t $\propto$ E_0 (b) $t \propto 1/E_0^2$ (c) $t \propto E_0^3$ (d) $t \propto 1/E_0^{3/2}$

Q50. Water rises to a certain height in a thin vertical capillary of radius r. Assuming that the contact angle is 0° throughout and the density and surface tension of water are ρ and γ respectively, the heat released in the process is

(a) 0 (b) γr^2 (c) $4\pi r^3\sqrt{\rho g \gamma}$ (d) $\dfrac{2\pi\gamma^2}{\rho g}$