IISc Integrated PhD- 2012

PHYSICAL SCIENCE

Q1. Let A and B represent two types of non-interacting spin-1/2 particles. If 3 particles of type A and 4 particles of type B are in a simple harmonic oscillator potential characterized by ω, the ground state energy of the system is

(a) $\frac{9}{2}\hbar\omega$ (b) $\frac{11}{2}\hbar\omega$ (c) $\frac{13}{2}\hbar\omega$ (d) $\frac{15}{2}\hbar\omega$

Q2. The binding energy of a deuteron is 2.2 MeV. In the reaction $d + d \rightarrow \alpha$, 23.6 MeV is released. What is the binding energy per nucleon of an α particle?

(a) 6.2 MeV (b) 6.5 MeV (c) 6.8 MeV (d) 7.0 MeV

Q3. A particle is confined to lie in a one-dimensional potential of width a and situated symmetrically about the origin: $V(x) = 0$ for $-a/2 \le x \le a/2$, and ∞ otherwise. If $\Psi(x)$ denotes the wave function of the particle, which one of the following is true?

(a) $\Psi(-a/2) = \Psi'(a/2) = 0$ (b) $\Psi(-a/2) = \Psi(a/2) = 0$

(c) $\Psi'(-a/2) = \Psi'(a/2) = 0$ (d) $\Psi'(-a/2) = \Psi(a/2) = 0$

Q4. Consider a one-dimensional square well potential of depth V and width a. This potential will

(a) always have at least one bound state

(b) never have a bound state

(c) have a bound state only if $V >> \hbar c/a$

(d) have a bound state only if $V << \hbar c/a$

Q5. A particle ρ of mass 770 MeV $/c^2$ decays into two pions each of mass 140 Mev $/c^2$. The momentum of each pion is

(a) 358.64 MeV $/c$ (b) 179.32 MeV $/c$

(c) 378.58 MeV $/c$ (d) 189.29 MeV $/c$

Q6. A neutron decay in about 9 minutes. The maximum electron energy is

(a) 0.782 eV (b) 0.782 meV (c) 0.782 keV (d) 0.782 MeV

Q7. The ionization potential of positronium, which is the bound state of an electron with a positron, is

(a) 3.4 eV (b) 6.8 eV (c) 13.6 eV (d) 27.2 eV

Q8. In the hydrogen atom spectrum the ratio of the energy for the transition $n = 2 \rightarrow n = 1$ to that of $n = 3 \rightarrow n = 1$ is:

(a) 27/32 (b) 32/27 (c) 27/5 (d) 5/27

Q9. Given that the work-function of metal is 5 eV and that photons of 9 eV are incident on it, the maximum velocity of emitted electrons is approximately

(a) 1.2×10^6 m/s (b) 2.4×10^5 m/s

(c) 4.8×10^6 m/s (d) 6.0×10^4 m/s

Q10. You had a sample of 27 g of a radioactive material and you found only 1 g after 60 minutes. If you had done the measurement 20 minutes earlier, how much material would you have found?

(a) 3 g (b) 6 g (c) 9 g (d) 18 g

Q11. A siren of natural frequency 600 Hz is fitted on an ambulance. When it approached a pedestrian, she hared a frequency of 640 Hz. At what speed was the ambulance approaching her? (Note that speed of sound under those conditions was 360 m/s)

(a) 24.0 km/hr (b) 60.8 km/hr (c) 81.0 km/hr (d) 86.4 km/hr

Q12. Consider a 3 × 3 matrix of the form:

$$\begin{pmatrix} a_1^2 & a_1a_2 & a_1a_3 \\ a_1a_2 & a_2^2 & a_2a_3 \\ a_1a_3 & a_2a_3 & a_3^2 \end{pmatrix}$$

The number of zero eigenvalues for this matrix is

(a) 0 (b) 1 (c) 2 (d) 3

Q13. If $\phi'(x) - m\phi(x) = 0$ where $\phi'(x) = d\phi/dx$, we have for the initial and final points x_{in} and x_{out}, the ratio $\phi(x_{out}) / \phi(x_{in})$ can be expressed as sum of series of the form

(a) $\left(1 + m(x_{in} - x_{out}) + (1/2)m^2(x_{in} - x_{out})^2 + (1/3!)m^3(x_{in} - x_{out})^3 + ...\right)$

(b) $\left(-m(x_{in} - x_{out}) + (1/3!)m^3(x_{in} - x_{out})^3 - ...\right)$

(c) $\left(1 + (1/2)m^2(x_{in} - x_{out})^2 + ...\right)$

(d) $\left(1 + m(x_{out} - x_{in}) + (1/2)m^2(x_{out} - x_{in})^2 + (1/3!)m^3(x_{out} - x_{in})^3 + ...\right)$

Q14. Given the three matrices

$$T_x = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}; T_y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}; T_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

which of the following statement is true

(a) $T_x^2 + T_y^2 + T_z^2 = 1$ (b) $T_x^2 + T_y^2 + T_z^2 = 21$

(c) $T_x^2 + T_y^2 + T_z^2 = 31$ (d) $T_x^2 + T_y^2 + T_z^2 = 0$

Q15. The series

$$\frac{1 - (\pi/6)^2/2! + (\pi/6)^4/4! - .....}{\pi/6 - (\pi/6)^3/3! + (\pi/6)^5/5! - .....}$$

sums to

(a) $1/\sqrt{3}$ (b) ½ (c) 1 (d) $\sqrt{3}$

Q16. Consider the product $(1 + z)(1 + z^4)(1 + z^8)\ldots.(1 + z^{4n})$. The derivative of the this function at $z = 0$ is

(a) n (b) 1 (c) $4n$ (d) $4n - 1$

Q17. If z is a complex number, then we can express $sin^{-1}z$ as

(a) $-i\ln\left(iz \pm \sqrt{1-z^2}\right)$ (b) $-i\ln\left(z \pm \sqrt{z^2-1}\right)$

(c) $\ln\left(z + \sqrt{z^2+1}\right)$ (d) $\ln\left(z + \sqrt{z^2-1}\right)$

Q18. Consider the function $f(x) = 5x^3 - 6x^2 + 12x + 15$. The graph of this function

(a) Never crosses the x-axis (b) Crosses the x-axis once

(c) Crosses the x-axis-twice (d) Crosses the x-axis thrice

Q19. Which of the following are the solutions of the differential equation $y'' + y = 0$

(a) $y(x) = 4x$ (b) $y(x) = 4.1\, cos(x) + 3.7\, sin(x)$

(c) $y(x) = 3x^2 + 2x - 1$ (d) $y(x) = 4e^x$

Q20. Let us suppose there are 2 dice with faces numbered 1 to 6. One die is such that in any throw the probability of obtaining any number is equal. The other is such that the probability of obtaining 1, 3 or 5 is 1/9 each, while the probability of obtaining even numbers is equal. In a simultaneous throw of both the dice, what is the probability of obtaining an outcome of 5?

(a) 1 / 5 (b) 2 / 15 (c) 1 / 4 (d) 1 / 9

Q21. A massive particle of mass M which is at rest in the laboratory decays into two particles, one of which has a mass $M/4$ while the other is massless. What is the magnitude of the momentum of each of the particles in the laboratory frame?

(a) $3\, Mc / 8$ (b) $15\, Mc / 32$ (c) $Mc / 4$ (d) $Mc / 2$

Q22. An incoherent beam of light strikes a glass plate at normal incidence. It undergoes multiple reflections at the edge before exiting the plate. The intensity reflection coefficient at each interface is R and the glass has negligible absorption. Neglecting interference effects, the ratio of the total reflected intensity to the incident intensity is given by

(a) $\frac{2R}{1+R}$ (b) $R(2 - R)$ (c) R (d) R^2

Q23. A Young's double slit apparatus has a slit separation of 0.5 mm and a screen placed at a distance of 100 cm from the slits. When illuminated by a 632 nm Helium-Neon laser, the position of the first intensity minimum with respect to the correct maximum will be

(a) 63.2 μm (b) 31.6 μm (c) 632 μm (d) 316 μm

Q24. There is a 4×10^6 M_Θ ($M_\Theta = 2 \times 10^{30}$ kg, is the Solar mass) black hole in the center of our Galaxy. A star (with mass much smaller than the black hole) in an elliptical orbit around it is observed to complete a full orbit in 15.78 years. The semi-major axis of the star's orbit is (algebra will be simpler if you recall that the earth takes 1 year to go around the sun and that the distance between earth and the Sun is 1.5×10^{11} m)

(a) 4.9×10^{14} m (b) 2.4×10^{11} m (c) 9.4×10^{11} m (d) 1.5×10^{14} m

Q25. Two plane parallel conducting plates have surface charge density σ each. What is the force per unit area between the two plates?

(a) $\frac{\sigma^2}{2\varepsilon_0}$, attractive (b) $\frac{\sigma^2}{2\varepsilon_0}$, repulsive

(c) $\frac{\sigma^2}{\varepsilon_0}$, repulsive (d) $\frac{2\sigma^2}{2\varepsilon_0}$, repulsive

Q26. A spherically symmetric charge distribution $\rho(r)$ has zero net charge. For an arbitrarily chosen coordinate system, which of the following statements is true?

(a) Only the monopole moment is zero

(b) The monopole and the dipole moments are zero

(c) Only the dipole moment is zero

(d) The monopole and dipole moments are non zero

Q27. Consider the solution to the one dimensional wave equation:

$$\Phi(x,t) = \Phi_0 \exp i(3x + 27t),$$

where x is in meters and t in seconds. The wave velocity is

(a) -9 m/s (b) 9 m/s (c) -1/9 m/s (d) 3 m/s

Q28. A charge is placed at the corner of a cube. The electric flux through a face that does not touch the charge is given by

(a) $\frac{q}{24\varepsilon_0}$ (b) $\frac{q}{18\varepsilon_0}$ (c) $\frac{q}{12\varepsilon_0}$ (d) $\frac{q}{6\varepsilon_0}$

Q29. A horizontally placed hollow tube has a cross-sectional area A at the beginning of the tube that gradually tapers off to $A/2$ at the end. An incompressible, ideal fluid of density ρ enters the tube with a velocity v at the beginning of the tube. What is the difference in pressure at the two ends of the tube?

(a) $2\rho v^2$ (b) ρv^2 (c) $\rho v^2/2$ (d) $3\rho v^2/2$

Q30. A very thick piece of glass with refractive index n has a convex surface with radius of curvature R. For paraxial light rays incident on this surface from vacuum, the glass acts like a lens with focal length f measured from the surface, where f is given by

(a) $\frac{nR}{n-1}$ (b) $\frac{(n-1)R}{n}$ (c) $\frac{(n-1)R}{n+1}$ (d) $\frac{(n+1)R}{n-1}$

Q31. Two metallic sphere of radius 1 cm and 3 cm respectively are separated by a large distance D, (D >> 1 cm). Initially, the smaller sphere carries a charge Q while the large one is unchanged. If a thin metallic wire is connected between the two spheres, the ratio of the charges on the smaller sphere to the large one in equilibrium will be

(a) 1 / 3 (b) 1 / 9 (c) 3 (d) 9

Q32. Which of the following is NOT a physically possible magnetic field configuration (x, y, z are Cartesian coordinates, t is time and γ is a damping rate; sign function is defined as: $sign\,(q) = 1$ for $q > 0$, -1 for $q < 0$, and 0 for $q = 0$):

(a) $\dfrac{B_0 r_0^2\left(x\hat{x}+y\hat{y}+z\hat{z}\right)}{\left(x^2+y^2+z^2\right)^{3/2}}$ (b) $B_0\, sign(z)\hat{x}$

(c) $B_0 e^{-\gamma t}\left[\cos(ky)\hat{x}-\sin(kz)\hat{y}\right]$ (d) $B_0\left[\dfrac{yz}{x^2+y^2}\hat{x}-\dfrac{xz}{x^2+y^2}\hat{y}\right]$

Q33. A sliding metallic cross-bar with length 10 cm and resistance 10 Ω is moving across a perfectly-conducting three-sides metallic frame with magnetic field of 2.0 tesla piercing through the frame (ignore the magnetic field produced by the currents in the circuit). The cross-bar is moved with a velocity of $4te^{-t^2}$ m/s. How much charge has been moved through the wire during the time in which the cross-bar slides from $x = 0$ to $x = 100$ cm?

(a) 2 mC (b) 20 mC (c) 100 mC (d) 200 mC

Q34. The magnetic field of the earth can be approximated as a dipole field. A non-relativistic electron with velocity perpendicular to the magnetic field direction is undergoing circular motion with the Larmor radius of 0.1 cm in the earth's magnetic equatorial plane at a distance of 5 earth radii from the center. An electron with the same kinetic energy and velocity perpendicular to the local magnetic field direction is executing uniform circular motion in the polar direction at the same distance. The electron's Larmor radius is equal to

(a) 0.1 cm (b) 0.05 cm (c) 0.025 cm (d) 0.2 cm

Q35. Which of the following can **NEVER** be a solution of the Maxwell equation for the electric field in free space? (ω and k are constants)

(a) $\vec{E}(x,t)=\hat{y}\left(e^{i(kx-2\omega t)}+e^{i\left(\frac{kx}{\sqrt{2}}-\sqrt{2}\omega t\right)}\right)$ (b) $\vec{E}(x,t)=\hat{y}\left(e^{i(kx-\omega t)}+e^{i(kx+\omega t)}\right)$

(c) $\vec{E}(x,t)=\hat{y}\left(e^{i(kx-\omega t)}+e^{i\left(\frac{kx}{\sqrt{2}}-\sqrt{2}\omega t\right)}\right)$ (d) $\vec{E}(r,t)=\hat{\theta}\frac{e^{i(kr-\omega t)}}{r}$

Q36. A mass m is tethered by a spring of spring constant k to the mid point between two walls separated by a distance d. The relaxed length of the spring is zero and collisions of the mass with the walls are completely elastic. If the mass is released from its equilibrium position with speed v, the time period of its motion is independent of v

(a) for all value of v (b) only for $v<\frac{d}{2}\sqrt{\frac{k}{m}}$

(c) only for $v>\frac{d}{2}\sqrt{\frac{k}{m}}$ (d) only for $\frac{d}{2}\sqrt{\frac{k}{m}}<v<d\sqrt{\frac{k}{m}}$

Q37. A string under tension T can support transverse waves with speed c. A displacement pattern of the form $y(x, t) = A\, cos(2\pi x/\lambda)\, cos(2\pi ct/\lambda)$ as set up in the string.The energy transported in a time interval $\Delta t = \lambda/e$ across $x = 0$ is

(a) 0 (b)TA (c) $T\lambda$ (d) $\frac{TA^2}{2\lambda}$

Q38 A very long rod rotates about a pivot with a constant angular velocity ω. A bead is constrained to slide along the rod without friction. At time $t = 0$, the bead is at rest a distance d away from the pivot. Its distance $r(t)$ from the pivot at time t is

(a) $d\sinh(\omega t)$ (b) $d\sin(\omega t)$ (c) $d\cosh(\omega t)$ (d) $d\cos(\omega t)$

Q39. A damped harmonic oscillator obeys the equation of motion $\ddot{x}+\gamma\dot{x}+\omega_0^2 x=0$ with $\omega_0=2\pi$ rad /s and γ = 5π s^{-1}. The oscillator is started from rest with a displacement of 3 cm at t = 0. How many times is its speed equal to zero between t = 0.25 s and t = 10.25 s?

(a) 0 (b) 2 (c) 10 (d) 20

Q40. A spaceship of mass M enters a stationary dust cloud of mass density ρ with speed $v_0(<<c)$ at time t = 0. If the dust particles are assumed to stick to the face of the spaceship of area A, its speed at a later time t is

(a) $v_0\left(1+\frac{2\rho A v_0 t}{M}\right)^{-1/4}$

(b) $v_0\left(1+\frac{2\rho A v_0 t}{M}\right)^{-1/2}$

(c) $\frac{v_0}{\left(1+\frac{2\rho A v_0 t}{M}\right)}$

(d) $v_0 e^{\frac{-2\rho A v_0 t}{M}}$

Q41. An engine of efficiency η operates between two reservoirs at temperatures T_1 and T_2 with $T_2 > T_1$ doing an amount of work W in one cycle. All of this work is used to drive an ideal Carrot engine in reverse (as a refrigerator) between the two reservoirs. The total heat extracted from hot reservoir in one combined cycle of the two engines is

(a) 0

(b) $W\left(\eta+\frac{T_1}{T_2}-1\right)$

(c) $-W\left(\eta+\frac{T_1}{T_2}-1\right)$

(d) $W\left(\frac{1}{\eta}-\frac{1}{1-\frac{T_1}{T_2}}\right)$

Q42. The specific heat at constant volume of the electrons in a metal $Cv = \alpha T$, where T is the temperature and α is a constant. The specific entropy s is given by

(a) k_B (b) $\sqrt{k_B \alpha T}$ (c) αT (d) $\frac{\alpha^2 T^2}{k_B}$

Q43. N_1 moles of an ideal monoatomic gas are mixed with N_2 moles of an ideal diatomic gas. If the mixture is allowed to expand adiabatically, the relation between its pressure and volume is $PV^\gamma =$ constant, where γ is equal to

(a) $5/3$ (b) $\frac{\frac{5}{3}N_1 + \frac{7}{5}N_2}{N_1 + N_2}$

(c) $\frac{5N_1 + 7N_2}{3N_1 + 5N_2}$ (d) $\frac{N_1 + N_2}{\frac{3}{5}N_1 + \frac{5}{7}N_2}$

Q44. Two identical containers A and B are connected by a small tube with a valve that allows gas flow when the pressure difference across it is greater than 0.6 atm. Initially, A contains an ideal gas at pressure 0.5 atm and B a vacuum with the entire system at 27° C. The system is now heated to a temperature of 207° C. What is the final pressure in B?

(a) 1 atm (b) 0.7 atm (c) 0.4 atm (d) 0.1 atm

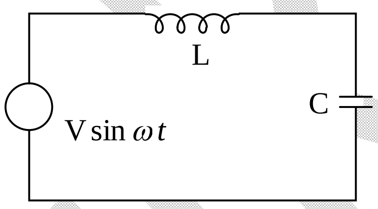
Q45. Assume that a planet and the star it orbits can both be thought of as perfect black bodies. The surface temperature of the star is T_s and that of the planet is T_p. Suppose there is another star-planet system identical in all respects but for the two temperature. If the surface temperature of the other star is $T_s/2$, the surface temperature of its planet is

(a) T_p (b) $\frac{3T_p}{2}$ (c) $\frac{T_p}{2}$ (d) $\frac{T_p}{16}$

Q46. A certain gas has equation of state $p = \alpha N^2 T / V^2$, where p is the pressure, N, the number of moles, V, the volume and T, the temperature, α is a constant. One mole of the gas undergoes expansion from volume V to $2V$ at constant temperature T. Given that $\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial p}{\partial T}\right)_{V,N}$, the change in energy in the isothermal expansion is

(a) 0 (b) $\frac{\alpha T}{V}$ (c) $\frac{-\alpha T}{V}$ (d) $\frac{\alpha T}{V}\ln 2$

Q47. An inductor L and capacitor C are connected across a signal source of frequency $f = 110$ KHz to form a low-pass filter as shown in the figure.

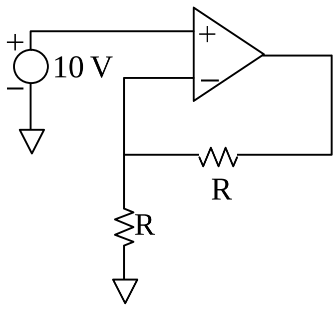


The output is measured across the capacitor. Which combination of L and C gives the smallest output?

(a) $L = 1\mu$H and $C = 0.2\ \mu$F (b) $L = 1\mu$H and $C = 2\ \mu$F

(c) $L = 10\mu$H and $C = 2\ \mu$F (d) $L = 10\mu$H and $C = 20\ \mu$F

Q48. An ideal op-amp powered by +15 V and – 15V dual supplies is connected with equal resistors R as shown in the figure.



The non-inverting input is connected to a DC voltage of +10V. The voltage at the inverting input will be

(a) +5 V (b) +7.5V (c) +10V (d) +20V

Q49. If A, B and C are three Boolean variables, the function $A\overline{B}+C$ can be realized using which of the following sets of four standard logic gates?

(a) two NAND gates and two OR gates

(b) two NOR gates and two AND gates

(c) two NAND gates and two NOR gates

(d) two AND gates and two OR gates

Q50. A classical electron gas with electron charge e, number density n, electric permittivity ε_0 and temperature T has a Debye screening length L_D. Using dimensional analysis, this length (in SI units) is proportional to

(a) $\sqrt{\frac{\varepsilon_0 k_B T}{e^2 n}}$ (b) $\frac{\varepsilon_0 k_B T}{e^2 n}$ (c) $\frac{e^2 n}{\varepsilon_0 k_B T}$ (d) $\sqrt{\frac{\varepsilon_0 k_B T}{en}}$