**NET December 2015**

## PART 'A'

Q1. In each of the following groups of words is a hidden number, based on which you should arrange them in descending order. Pick the correct answer:

E. Papers I Xeroxed

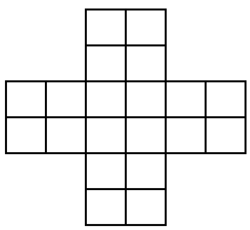
F. Wi-Fi veteran

G. Yourself ourselves

H. Breaks even

(a) H, F, G, H (b) E, G, F, H (c) H, F, G, E (d) H, E, F, G

Q2. The number of squares in the figure is



(a) 30

(b) 29

(c) 25

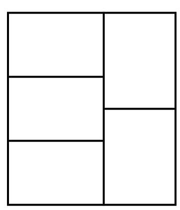
(d) 20

Q3. A shopkeeper purchases a product for Rs. 100 and sells it making a profit of $10\%$. The customer resells it to the same shopkeeper incurring a loss of $10\%$. In these dealings the shopkeeper makes

(a) no profit, no loss (b) Rs. 11

(c) Re. 1 (d) Rs. 20

Q4. Five congruent rectangles are drawn inside a big rectangle of perimeter 165 as shown. What is the perimeter of one of the five rectangles?



(a) 37

(b) 75

(c) 15

(d) 165

**Head office**
fiziks, H.No. 23, G.F, Jia Sarai,
Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498

**Branch office**
Anand Institute of Mathematics,
28-B/6, Jia Sarai, Near IIT
Hauz Khas, New Delhi-16

Website: www.physicsbyfiziks.com
Email: fiziks.physics@gmail.com

Q5. A person walks downhill at $10\,km/h$, uphill at $6\,km/h$ and on the plane at $7.5\,km/h$. If the person takes $3$ hours to go from a place $A$ to another place $B$, and $1$ hour on the way back, the distance between $A$ and $B$ is

(a) $15\,km$

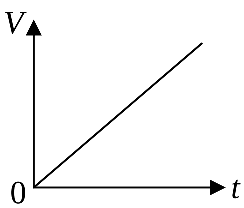
(b) $23.5\,km$

(c) $16\,km$

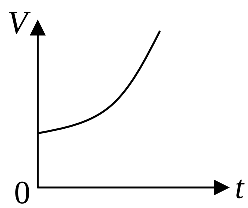
(d) Given data is insufficient to calculate distance.

Q6. A vessel is partially filled with water. More water is added to it at a rate directly proportional to time $\left[\text{i.e.,} \dfrac{dV}{dt} \propto t\right]$. Which of the following graphs depicts correctly the variation of total volume $V$ of water with time $t$?

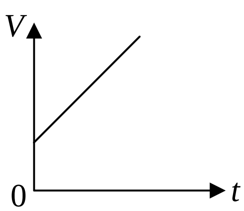
(a)



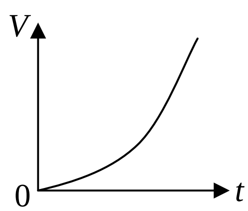

(b)




(c)




(d)




Q7. At one instant, the hour hand and the minute hand of a clock are one over the other in between the markings for $5$ and $6$ on the dial. At this instant, the tip of the minute hand

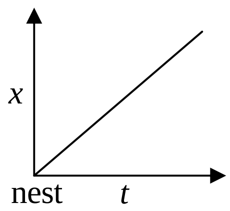
(a) is closer to the marking for $6$

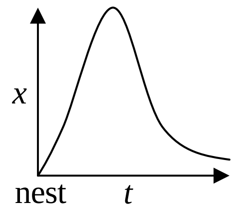
(b) is equidistant from the markings for $5$ and $6$

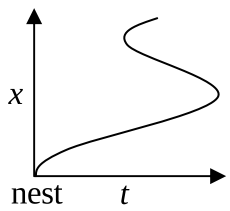
(c) is closer to marking for $5$

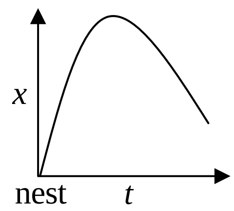
(d) is equidistant from the markings for $11$ and $12$

Q8. A bird leaves its nest and flies away. Its distance $x$ from the nest is plotted as a function of time $t$. Which of the following plots cannot be right?

(a)



(b)



(c)



(d)



Q9. A cubical cardboard box made of $1cm$ thick card board has outer side of $29\,cm$. A tight-fitting cubical box of the same thickness is placed inside it, then another one inside it and so on. How many cubical boxes will be there in the entire set?

(a) 29 (b) 28 (c) 15 (d) 14

Q10. Secondary colours are made by a mixture of three primary colours, Red, Green and Blue, in different proportions; each of the primary colours comes in 8 possible levels. Grey corresponds to equal proportions of Red, Green and Blue. How many shades of grey exist ii this scheme?

(a) $8^3$ (b) 8 (c) $3^8$ (d) $8\times 3$

Q11. The triangle formed by the lines $y=x, y=1-x$ and $x=0$ in a two dimensional plane is ($x$ and $y$ axes have the same scale)

(a) isosceles and right-angled (b) isosceles but not right-angled

(c) right-angled but not isosceles (d) neither isosceles nor right angled

Q12. There are two buckets $A$ and $B$. Initially $A$ has 2 liters of water and $B$ is empty. At every hour 1 liter of water is transferred from $A$ to $B$ followed by returning $\frac{1}{2}$ liter back to $A$ from $B$ half an hour later. The earliest $A$ will get empty is in:

(a) $5h$ (b) $4h$ (c) $3h$ (d) $2h$

Q13. Statement A: The following statement is true

Statement B: The preceding statement is false

Choose the correct inference from the following:

(a) Statements A and B are always true

(b) Statements A and B can be true if there is at least one statement between A and B

(c) Statements A and B can be true if there are at least two statements between A and B

(d) Statements A and B can never be true, independently

Q14. A car is moving at $60\ km/h$. The instantaneous velocity of the upper most points of its wheels is

(a) $60\ km/h$ forward (b) $120\ km/h$ forward

(c) $60\ km/h$ backward (d) $120\ km/h$ backward

Q15. If $D+I+M=1501$

$C+I+V+I+L=157$

$L+I+V+I+D=557$

$C+I+V+I+C=207$

What is $V+I+M=?$

(a) Cannot be found (b) 1009

(c) 1006 (d) 509

Q16. A living cell has a protoplasm which is water based and demarcated by a lipid bilayer membrane. If a cell is pierced up to $\frac{1}{5}th$ of its diameter with a very sharp needle, after taking the needle out

(a) no effect will be observed.

(b) protoplasm will leak out from the hole made by the needle for a few minutes until the cell heals the wound.

(c) protoplasm will keep on leaking out till the cell is dead.

(d) the cell will burst like a balloon.

Q17. Density of a rice grain is 1.5 $g/cc$ and bulk density of rice heap is 0.80 $g/cc$. If a 1 litre container is completely filled with rice, what will be the approximate volume of pore space in the container?

(a) 350 $cc$ (b) 465 $cc$ (c) 550 $cc$ (d) 665 $cc$

Q18. A turtle starts swimming from a point $A$ located on the circumference of a circular pond. After swimming for 4 meters in a straight line it hits point $B$ on the circumference or the pond. From there it changes direction and swims for 3 meters in a straight line and arrives at point $D$ diametrically opposite to point $A$. How far is point $D$ from $A$?

(a) 3 $m$ (b) 4 $m$ (c) 7 $m$ (d) 5 $m$

Q19. Four circles of unit radius each are drawn such that each one touches two others and their centres lie on the vertices of a square. The area of the region enclosed between the circles is

(a) $\pi - 1$ (b) $\pi - 2$ (c) $3 - \pi$ (d) $4 - \pi$

Q20. A film projector and microscope give equal magnification. But a film projector is not used to see living cells because

(a) a living cell cannot be placed in a film projector.

(b) the viewer's eye is close to a microscope whereas it is far away from the projector's screen.

(c) a microscope produces a virtual image whereas a projector produces a real image.

(d) a microscope has greater resolving power than a projector.

## PART 'B'

Q21. In the scattering of some elementary particles, the scattering cross-section $\sigma$ is found to depend on the total energy $E$ and the fundamental constants $h$ (Planck's constant) and $c$ (the speed of light in vacuum). Using dimensional analysis, the dependence of $\sigma$ on these quantities is given by

(a) $\sqrt{\frac{hc}{E}}$ (b) $\frac{hc}{E^{3/2}}$ (c) $\left(\frac{hc}{E}\right)^2$ (d) $\frac{hc}{E}$

Q22. If $y = \dfrac{1}{\tanh(x)}$, then $x$ is

(a) $\ln\left(\dfrac{y+1}{y-1}\right)$

(b) $\ln\left(\dfrac{y-1}{y+1}\right)$

(c) $\ln\sqrt{\dfrac{y-1}{y+1}}$

(d) $\ln\sqrt{\dfrac{y+1}{y-1}}$

Q23. The function $\dfrac{z}{\sin \pi z^2}$ of a complex variable $z$ has

(a) a simple pole at $0$ and poles of order $2$ at $\pm\sqrt{n}$ for $n = 1, 2, 3...$

(b) a simple pole at $0$ and poles of order $2$ at $\pm\sqrt{n}$ and $\pm i\sqrt{n}$ for $n = 1, 2, 3...$

(c) poles of order $2$ at $\pm\sqrt{n}$, $n = 0, 1, 2, 3...$

(d) poles of order $2$ at $\pm n$, $n = 0, 1, 2, 3...$

Q24. The Fourier transform of $f(x)$ is $\tilde{f}(k) = \int_{-\infty}^{+\infty} dx e^{ikx} f(x)$.

If $f(x) = \alpha\delta(x) + \beta\delta'(x) + \gamma\delta''(x)$, where $\delta(x)$ is the Dirac delta-function (and prime denotes derivative), what is $\tilde{f}(k)$?

(a) $\alpha + i\beta k + i\gamma k^2$

(b) $\alpha + \beta k - \gamma k^2$

(c) $\alpha - i\beta k - \gamma k^2$

(d) $i\alpha + \beta k - i\gamma k^2$

Q25. The solution of the differential equation $\dfrac{dx}{dt} = 2\sqrt{1-x^2}$, with initial condition $x = 0$ at $t = 0$ is

(a) $x = \begin{cases} \sin 2t, & 0 \le t < \dfrac{\pi}{4} \\ \sinh 2t, & t \ge \dfrac{\pi}{4} \end{cases}$

(b) $x = \begin{cases} \sin 2t, & 0 \le t < \dfrac{\pi}{2} \\ 1, & t \ge \dfrac{\pi}{2} \end{cases}$

(c) $x = \begin{cases} \sin 2t, & 0 \le t < \dfrac{\pi}{4} \\ 1, & t \ge \dfrac{\pi}{4} \end{cases}$

(d) $x = 1 - \cos 2t, \quad t \ge 0$

Q26. A particle moves in three dimensional space in a central potential $V(r) = kr^4$ where $k$ is a constant. The angular frequency $\omega$ for a circular orbit depends on it radius $R$ as

(a) $\omega \propto R$ (b) $\omega \propto R^{-1}$ (c) $\omega \propto R^{1/4}$ (d) $\omega \propto R^{-2/3}$

Q27. Two masses $m$ each, are placed at the points $(x, y) = (a, a)$ and $(-a, -a)$ and two masses, $2m$ each, are placed at the points $(a, -a)$ and $(-a, a)$. The principal moments of inertia of the system are

(a) $2m^2, 4ma^2$ (b) $4ma^2, 8ma^2$

(c) $4ma^2, 4ma^2$ (d) $8ma^2, 8ma^2$

Q28. The Lagrangian of a system is given by

$$L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - k\left(\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2\right)$$

where $m$ and $k$ are positive constants. The frequencies of its normal modes are

(a) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{3k}{m}}$ (b) $\sqrt{\frac{k}{2m}}\left(13 \pm \sqrt{73}\right)$

(c) $\sqrt{\frac{5k}{2m}}, \sqrt{\frac{k}{m}}$ (d) $\sqrt{\frac{k}{2m}}, \sqrt{\frac{6k}{m}}$

Q29. Consider a particle of mass $m$ moving with a speed $v$. If $T_R$ denotes the relativistic kinetic energy and $T_N$ its non-relativistic approximation, then the value of $\frac{(T_R - T_N)}{T_R}$ for $v = 0.01\,c$, is

(a) $1.25 \times 10^{-5}$ (b) $5.0 \times 10^{-5}$

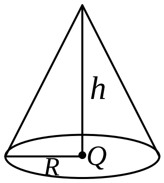
(c) $7.5 \times 10^{-5}$ (d) $1.0 \times 10^{-4}$

Q30. A hollow metallic sphere of radius $a$, which is kept at a potential $V_0$ has a charge $Q$ at its centre. The potential at a point outside the sphere, at a distance $r$ from the centre, is

(a) $V_0$ (b) $\frac{Q}{4\pi \in_0 r} + \frac{V_0 a}{r}$

(c) $\frac{Q}{4\pi \in_0 r} + \frac{V_0 a^2}{r^2}$ (d) $\frac{V_0 a}{r}$

Q31. Consider a charge $Q$ at the origin of $3$- dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height $h$ and a circular base of radius $R$ (as shown in the figure) is




(a) $\frac{Q}{\epsilon_0}$ (b) $\frac{Q}{2\epsilon_0}$ (c) $\frac{hQ}{R\epsilon_0}$ (d) $\frac{QR}{2h\epsilon_0}$

Q32. Given a uniform magnetic field $B = B_0\hat{k}$ (where $B_0$ is a constant), a possible choice for the magnetic vector potential $A$ is

(a) $B_0 y\hat{i}$ (b) $-B_0 y\hat{i}$ (c) $B_0\left(x\hat{j} + y\hat{i}\right)$ (d) $B_0\left(x\hat{i} + y\hat{j}\right)$

Q33. A beam of unpolarized light in a medium with dielectric constant $\epsilon_1$ is reflected from a plane interface formed with another medium of dielectric constant $\epsilon_2 = 3\epsilon_1$. The two media have identical magnetic permeability. If the angle of incidence is $60^0$, then the reflected light

(a) is plane polarized perpendicular to the plane of incidence

(b) is plane polarized parallel to the plane of incidence

(c) is circularly polarized

(d) has the same polarization as the incident light

Q34. A Hermitian operator $\hat{O}$ has two normalized eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues $1$ and $2$, respectively. The two states $|u\rangle = \cos\theta|1\rangle + \sin\theta|2\rangle$ and $|v\rangle = \cos\phi|1\rangle + \sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u|v\rangle = 0$. Which of the following are possible values of $\theta$ and $\phi$?

(a) $\theta = -\frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(c) $\theta = -\frac{\pi}{4}$ and $\phi = \frac{\pi}{4}$ (d) $\theta = \frac{\pi}{3}$ and $\phi = -\frac{\pi}{6}$

Q35. The ground state energy of a particle of mass $m$ in the potential $V(x)=V_0\cosh\left(\frac{x}{L}\right)$, where $L$ and $V_0$ are constants (with $V_0 >> \frac{\hbar^2}{2ml^2}$) is approximately

(a) $V_0+\frac{\hbar}{L}\sqrt{\frac{2V_0}{m}}$ (b) $V_0+\frac{\hbar}{L}\sqrt{\frac{V_0}{m}}$

(c) $V_0+\frac{\hbar}{4L}\sqrt{\frac{V_0}{m}}$ (d) $V_0+\frac{\hbar}{2L}\sqrt{\frac{V_0}{m}}$

Q36. Let $\psi_{nlm}$ denote the eigenstates of a hydrogen atom in the usual notation. The state

$$\frac{1}{5}\left[2\psi_{200}-3\psi_{211}+\sqrt{7}\psi_{210}-\sqrt{5}\psi_{21-1}\right]$$

is an eigenstate of

(a) $L^2$, but not of the Hamiltonian or $L_z$

(b) the Hamiltonian, but not of $L^2$ or $L_z$

(c) the Hamiltonian, $L^2$ and $L_z$

(d) $L^2$ and $L_z$, but not of the Hamiltonian

Q37. The Hamiltonian for a spin-$\frac{1}{2}$ particle at rest is given by $H=E_0\left(\sigma_z+\alpha\sigma_x\right)$, where $\sigma_x$ and $\sigma_z$ are Pauli spin matrices and $E_0$ and $\alpha$ are constants. The eigenvalues of this Hamiltonian are

(a) $\pm E_0\sqrt{1+\alpha^2}$ (b) $\pm E_0\sqrt{1-\alpha^2}$

(c) $E_0$ (doubly degenerate) (d) $E_0\left(1\pm\frac{1}{2}\alpha^2\right)$

Q38. The heat capacity of (the interior of a refrigerator is $4.2\ kJ/K$. The minimum' work that must be done to lower the internal temperature from $18^o\ C$ to $17^o\ C$ when the outside temperature is $27^o\ C$ will be

(a) $2.20\ kJ$ (b) $0.80\ kJ$ (c) $0.30\ kJ$ (d) $0.14\ kJ$

Q39. For a system of independent non interacting one-dimensional oscillators, the value of the free energy per oscillator, in the limit $T \to 0$, is

(a) $\frac{1}{2}\hbar\omega$ (b) $\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) 0

Q40. The partition function of a system of $N$ Ising spins is $Z = \lambda_1^N + \lambda_2^N$ where $\lambda_1$ and $\lambda_2$ are functions of temperature, but are independent of $N$. If $\lambda_1 > \lambda_2$, the free energy per spin in the limit $N \to \infty$ is

(a) $-k_B T \ln\left(\frac{\lambda_1}{\lambda_2}\right)$ (b) $-k_B T \ln \lambda_2$

(c) $-k_B T \ln(\lambda_1 \lambda_2)$ (d) $-k_B T \ln \lambda_1$

Q41. The Hamiltonian of a system of $N$ non interacting spin -$\frac{1}{2}$ particles is $H = -\mu_0 B \sum_i S_i^z$, where $S_i^z = \pm 1$ are components of $i^{th}$ spin along an external magnetic field $B$. At a temperature $T$ such that $e^{\frac{\mu_0 B}{k_B T}} = 2$. the specific heat per particle is

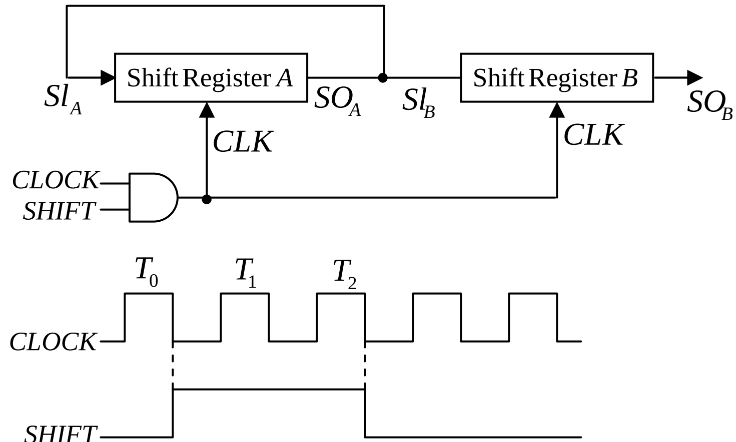
(a) $\frac{16}{25} k_B$ (b) $\frac{8}{25} k_B \ln 2$ (c) $k_B (\ln 2)^2$ (d) $\frac{16}{25} k_B (\ln 2)^2$

Q42. If the reverse bias voltage of a silicon varactor is increased by a factor of $2$, the corresponding transition capacitance

(a) increases by a factor of $\sqrt{2}$ (b) increases by a factor of $2$

(c) decreases h a factor of $\sqrt{2}$ (d) decreases by a factor of $2$

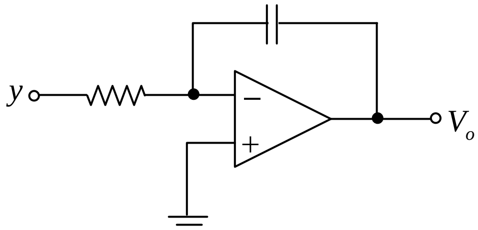
Q43. In the schematic figure given below the initial values of 4 bit shift registers $A$ and $B$ are 1011 and 0010 respectively The values at $SO_A$. and $SO_B$ after the pulse $T_2$ are respectively.

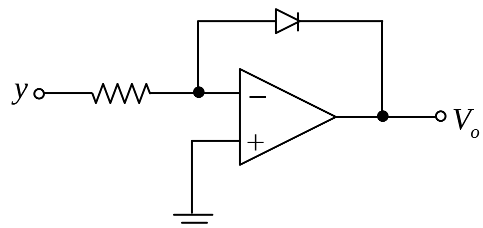


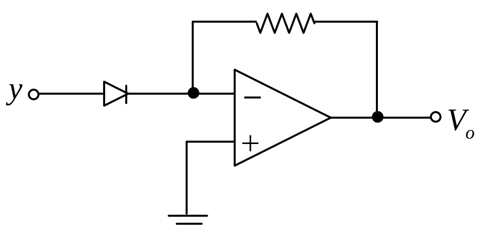

(a) 1110 and 1001 (b) 1101 and 1001

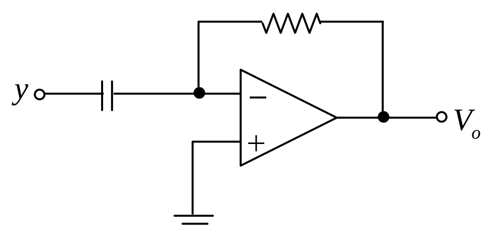
(c) 1101 and 1100 (d) 1110 and 1100

Q44. If the parameters $y$ and $x$ are related by $y = \log(x)$, then the circuit that can be used to produce an output voltage $V_0$ varying linearly with $x$ is

(a)



(b)



(c)



(d)


Q45. Two data sets $A$ and $B$ consist of 60 and 10 readings of a voltage measured using voltmeters of resolution of $1\ mV$ and $0.5\ mV$ respectively. The uncertainty in the mean voltage obtained from the data sets $A$ and $B$ are $U_A$ and $U_B$, respectively. If the uncertainty of the mean of the combined data sets is $U_{AB}$ then which of the following statements is correct?

(a) $U_{AB} < U_A$ and $U_{AB} > U_B$ (b) $U_{AB} < U_A$ and $U_{AB} < U_B$

(c) $U_{AB} > U_A$ and $U_{AB} < U_B$ (d) $U_{AB} > U_A$ and $U_{AB} > U_B$

**PART 'C'**

Q46. The Hermite polynomial $H_n(x)$ satisfies the differential equation

$$\frac{d^2H_n}{dx^2} - 2x\frac{dH_n}{dx} + 2nH_n(x) = 0$$

The corresponding generating function

$G(t,x) = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n$ satisfies the equation

(a) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2t\frac{\partial G}{\partial t} = 0$ (b) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} - 2t^2\frac{\partial G}{\partial t} = 0$

(c) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial G}{\partial t} = 0$ (d) $\frac{\partial^2 G}{\partial x^2} - 2x\frac{\partial G}{\partial x} + 2\frac{\partial^2 G}{\partial x \partial t} = 0$

Q47. A function $f(x)$ satisfies the differential equation

$$\frac{d^2 f}{dx^2} - \omega^2 f = -\delta(x-a)$$

where $\omega$ is positive. The Fourier transform $\tilde{f}(k) = f\int_{-\infty}^{+\infty} dx\, e^{ikx} f(x)$ of $f$, and the solution of the equation are, respectively,

(a) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega}\left(e^{-\omega|x-a|} + e^{\omega|x-a|}\right)$ (b) $\frac{e^{ika}}{k^2+\omega^2}$ and $\frac{1}{2\omega}e^{-\omega|x-a|}$

(c) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2\omega}\left(e^{-i\omega|x-a|} + e^{i\omega|x-a|}\right)$ (d) $\frac{e^{ika}}{k^2-\omega^2}$ and $\frac{1}{2i\omega}\left(e^{-\omega|x-a|} - e^{i\omega|x-a|}\right)$

Q48. For a dynamical system governed by the equation $\frac{dx}{dt} = 2\sqrt{1-x^2}$, with $|x| \le 1$

(a) $x = -1$ and $x = 1$ are both unstable fixed points

(b) $x = -1$ and $x = 1$ are both stable fixed points

(c) $x = -1$ is an unstable fixed point and $x = 1$ is a stable fixed points

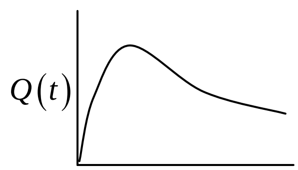
(d) $x = -1$ is a stable fixed point and $x = 1$ is a unstable fixed points

Q49. The value of the integral $\int_0^8 \frac{1}{x^2+5}dx$, valuated using Simpson's $\frac{1}{3}$ rule with $h = 2$ is

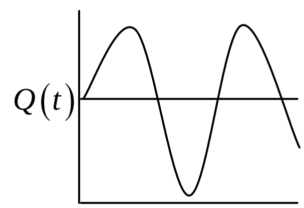
(a) 0.565 (b) 0.620 (c) 0.698 (d) 0.736

Q50. A canonical transformation $(p,q) \to (P,Q)$ is performed on the Hamiltonian $H = \frac{1}{2mp^2} + \frac{1}{2}m\omega^2 q^2$ via the generating function $F = \frac{1}{2}m\omega q^2 \cot Q$. If $Q(0) = 0$, which of the following graphs shows schematically the dependence of $Q(t)$ on $t$?

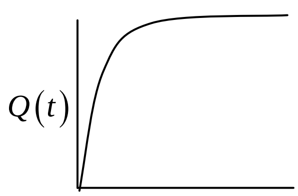
(a)



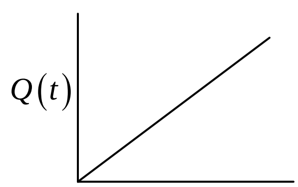

(b)



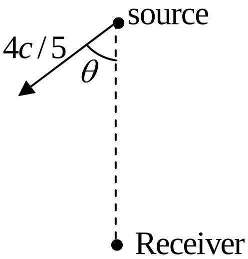

(c)




(d)



Q51. A distant source, emitting radiation of frequency $\omega$ moves with a velocity $\frac{4c}{5}$ in a certain direction with respect to a receiver (as shown in the figure). The upper cut-off frequency of the receiver is $\frac{3\omega}{2}$. Let $\theta$ the angle as shown. For the receiver to detect the radiation, $\theta$ should at least be




(a) $\cos^{-1}\left(\frac{1}{2}\right)$ (b) $\cos^{-1}\left(\frac{3}{4}\right)$ (c) $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (d) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Q52. The Lagrangian of a particle moving in a plane s given in Cartesian coordinates as

$$L = \dot{x}\dot{y} - x^2 - y^2$$

In polar coordinates the expression for the canonical momentum $p_r$ (conjugate to the radial coordinate $r$) is

(a) $\dot{r}\sin\theta + r\dot{\theta}\cos\theta$ (b) $\dot{r}\cos\theta + r\dot{\theta}\sin\theta$

(c) $2\dot{r}\cos\theta - r\dot{\theta}\sin 2\theta$ (d) $\dot{r}\sin 2\theta + r\dot{\theta}\cos 2\theta$

Q53. A small magnetic needle is kept at $(0,0)$ with its moment along the $x$-axis. Another small magnetic needle is at the point $(1,1)$ and is free to rotate in the $xy$- plane. In equilibrium the angle $\theta$ between their magnetic moments is such that

(a) $\tan\theta = \frac{1}{3}$ (b) $\tan\theta = 0$ (c) $\tan\theta = 3$ (d) $\tan\theta = 1$

Q54. A dipole of moment $\vec{p}$, oscillating at frequency $\omega$, radiates spherical waves. The vector potential at large distance is

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r}\vec{p}$$

To order $\left(\frac{1}{r}\right)$ the magnetic field $\vec{B}$ at a point $\vec{r} = r\hat{n}$ is

(a) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{C}(\hat{n}\cdot\vec{p})\hat{n}\frac{e^{ikr}}{r}$ (b) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{C}(\hat{n}\times\vec{p})\frac{e^{ikr}}{r}$

(c) $-\frac{\mu_0}{4\pi}\omega^2 k(\hat{n}\cdot\vec{p})\vec{p}\frac{e^{ikr}}{r}$ (d) $-\frac{\pi_0}{4\pi}\frac{\omega^2}{C}\vec{p}\frac{e^{ikr}}{r}$

Q55. The frequency dependent dielectric constant of a material is given by

$$\varepsilon(\omega) = 1 + \frac{A}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

where $A$ is a positive constant, $\omega_0$ the resonant frequency and $\gamma$ the damping coefficient. For an electromagnetic wave of angular frequency $\omega << \omega_0$ which of the following is true? (Assume that $\frac{\gamma}{\omega_0} << 1$).

(a) There is negligible absorption of the wave

(b) The wave propagation is highly dispersive

(c) There is strong absorption of the electromagnetic wave

(d) The group velocity and the phase velocity will have opposite sign

Q56. A hydrogen atom is subjected to the perturbation

$$V_{pert}(r) = \in \cos\frac{2r}{a_0}$$

where $a_0$ is the Bohr radius. The change in the ground state energy to first order in $\in$

(a) $\frac{\in}{4}$ (b) $\frac{\in}{2}$ (c) $\frac{-\in}{2}$ (d) $\frac{-\in}{4}$

Q57. A positron is suddenly absorbed by the nucleus of a tritium $\left({}_1^3H\right)$ atom to turn the latter into a $He^+$ ion. If the electron in the tritium atom was initially in the ground state, the probability that the resulting $He^+$ ion will be in its ground state is

(a) 1 (b) $\frac{8}{9}$ (c) $\frac{128}{243}$ (d) $\frac{512}{729}$

Q58. The product of the uncertainties $(\Delta L_x)(\Delta L_y)$ for a particle in the state $a|1,1\rangle + b|1,-1\rangle$ where $|l,m\rangle$ denotes an eigenstate of $L^2$ and $L_z$ will be a minimum for

(a) $a = \pm ib$ (b) $a = 0$ and $b = 1$

(c) $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$ (d) $a = \pm b$

Q59. The ground state energy of a particle in potential $V(x)=g|x|$, estimated using the trail wavefunction

$$\psi(x)=\begin{cases}\sqrt{\frac{c}{a^5}}\left(a^2-x^2\right), & x<|a| \\ 0, & x\geq|a|\end{cases}$$

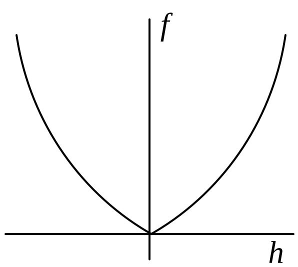
(where $g$ and $c$ are constants) is

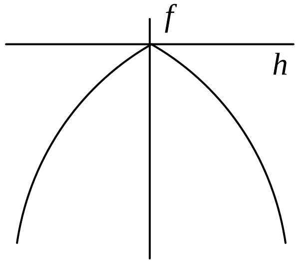
(a) $\frac{15}{16}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (b) $\frac{5}{6}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (c) $\frac{3}{4}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (d) $\frac{7}{8}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$

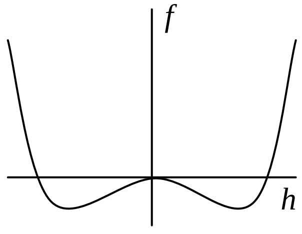
Q60. An ensemble of non-interacting spin -$\frac{1}{2}$ particles is in contact with a heat bath at temperature $T$ and is subjected to an external magnetic field. Each particle can be in one of the two quantum states of energies $\pm\epsilon_0$. If the mean energy per particle is $-\epsilon_0/2$, then the free energy per particle is

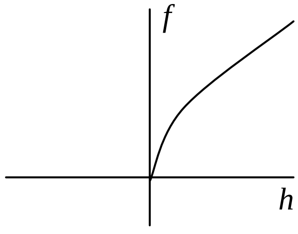
(a) $-2\epsilon_0\frac{\ln\left(4/\sqrt{3}\right)}{\ln 3}$ (b) $-\epsilon_0\ln(3/2)$ (c) $-2\epsilon_0\ln 2$ (d) $-\epsilon_0\frac{\ln 2}{\ln 3}$

Q61. Which of the following graphs shows the qualitative dependence of the free energy $f(h,T)$ of a ferromagnet in an external magnetic field $h$, and at a fixed temperature $T<T_C$, where $T_C$ is the critical temperature?

(a)



(b)


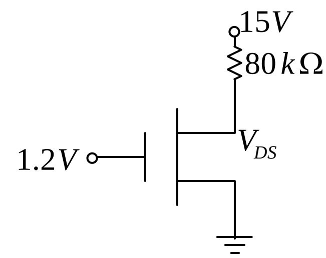

(c)



(d)


Q62. Consider a random walker on a square lattice. At each step the walker moves to a nearest neighbour site with equal probability for each of the four sites. The walker starts at the origin and takes 3 steps. The probability that during this walk no site is visited more than one is

(a) $12/27$ (b) $27/64$ (c) $3/8$ (d) $9/16$

Q63. Consider an $n$- MOSFET with the following parameters: current drive strength $K = 60\mu A/V^2$, breakdown voltage $BV_{DS} = 10V$, ratio of effective gate width to the channel length $\frac{W}{L} = 5$ and threshold voltage $V_{th} = 0.5V$.

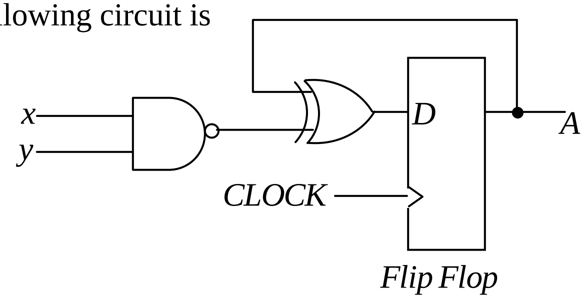


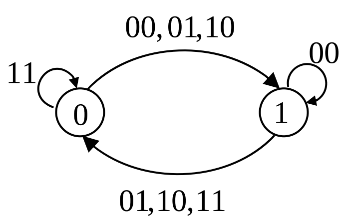

In the circuit given below, this $n$- MOSFET is operating in the

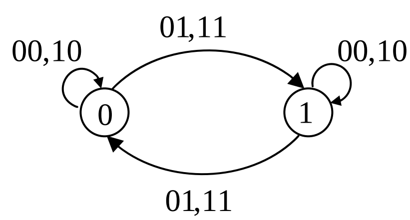
(a) ohmic region (b) cut-off region

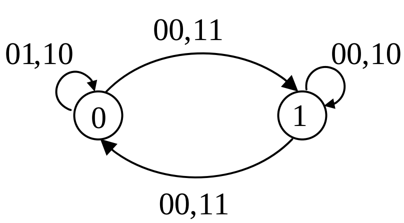
(c) saturation region (d) breakdown

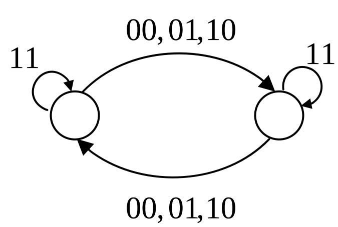
Q64. The state diagram corresponding to the following circuit is



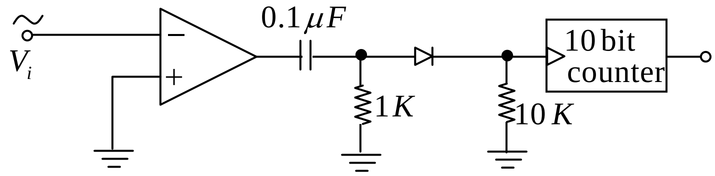

(a)



(b)



(c)



(d)


Q65. A sinusoidal signal of peak to peak amplitude $1V$ and unknown time period is input to the following circuit for 5 seconds duration. If the counter measures a value $(3E8)_H$ in hexadecimal then the time period of the input signal is




(a) $2.5ms$ (b) $4ms$ (c) $10ms$ (d) $5ms$

Q66. The first order diffraction peak of a crystalline solid occurs at a scattering angle of $30^0$ when the diffraction pattern is recorded using an x-ray beam of wavelength $0.15nm$. If the error in measurements of the wavelength and the angle are $0.01nm$ and $1^0$ respectively, then the error in calculating the inter-planar spacing will approximately be

(a) $1.1\times10^{-2}\ nm$ (b) $1.3\times10^{-4}\ nm$ (c) $2.5\times10^{-2}\ nm$ (d) $2.0\times10^{-3}\ nm$

Q67. The dispersion relation of electrons in a 3-dimensional lattice in the tight binding approximation is given by,

$$\varepsilon_k = \alpha\cos k_x a + \beta\cos k_y a + \gamma\cos k_z a$$

where $a$ is the lattice constant and $\alpha,\beta,\gamma$ are constants with dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $\left(\frac{\pi}{a},\frac{\pi}{a},\frac{\pi}{a}\right)$ is

(a) $\frac{\hbar^2}{a^2}\begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$ (b) $\frac{\hbar^2}{a^2}\begin{pmatrix} -\frac{1}{\alpha} & 0 & 0 \\ 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$

(c) $\frac{\hbar^2}{a^2}\begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & \frac{1}{\gamma} \end{pmatrix}$ (d) $\frac{\hbar^2}{a^2}\begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\beta} & 0 \\ 0 & 0 & -\frac{1}{\gamma} \end{pmatrix}$

Q68. A thin metal film of dimension $2\,mm \times 2\,mm$ contains $4\times10^{12}$ electrons. The magnitude of the Fermi wavevector of the system, in the free electron approximation, is

(a) $2\sqrt{\pi}\times10^{7}\,cm^{-1}$ (b) $\sqrt{2\pi}\times10^{7}\,cm^{-1}$ (c) $\sqrt{\pi}\times10^{7}\,cm^{-1}$ (d) $2\pi\times10^{7}\,cm^{-1}$

Q69. For an electron moving through a one-dimensional periodic lattice of periodicity $a$, which of the following corresponds to an energy eigenfunction consistent with Bloch's theorem?

(a) $\psi(x)=A\exp\left(i\left[\frac{\pi x}{a}+\cos\left(\frac{\pi x}{2a}\right)\right]\right)$ (b) $\psi(x)=A\exp\left(i\left[\frac{\pi x}{a}+\cos\left(\frac{2\pi x}{a}\right)\right]\right)$

(c) $\psi(x)=A\exp\left(i\left[\frac{2\pi x}{a}+i\cosh\left(\frac{2\pi x}{a}\right)\right]\right)$ (d) $\psi(x)=A\exp\left(i\left[\frac{\pi x}{a}+i\left|\frac{\pi x}{2a}\right|\right]\right)$

Q70. The $LS$ configurations of the ground state of $^{12}Mg$, $^{13}Al$, $^{17}Cl$ and $^{18}Ar$ are, respectively,

(a) $^{3}S_{1}$, $^{2}P_{1/2}$, $^{2}P_{1/2}$ and $^{1}S_{0}$ (b) $^{3}S_{1}$, $^{2}P_{3/2}$, $^{2}P_{3/2}$ and $^{3}S_{1}$

(c) $^{1}S_{0}$, $^{2}P_{1/2}$, $^{2}P_{3/2}$ and $^{1}S_{0}$ (d) $^{1}S_{0}$, $^{2}P_{3/2}$, $^{2}P_{1/2}$ and $^{3}S_{1}$

Q71. For a two level system, the population of atoms in the upper and lower levels are $3\times10^{18}$ and $0.7\times10^{18}$ , respectively. If the coefficient of stimulated emission is $3.0\times10^{5}\,m^{3}/W\text{-}s^{3}$ and the energy density is $9.0\,J/m^{3}\text{-}Hz$, the rate of stimulated emission will be

(a) $6.3\times10^{16}\,s^{-1}$ (b) $4.1\times10^{16}\,s^{-1}$ (c) $2.7\times10^{16}\,s^{-1}$ (d) $1.8\times10^{16}\,s^{-1}$

Q72. The first ionization potential of $K$ is $4.34\,eV$ , the electron affinity of $CI$ is $3.82\,eV$ and the equilibrium separation of $KCI$ is $0.3\,nm$. The required to dissociate
a $KCI$ molecule into a $K$ and a $CI$ atom is

(a) $8.62\,eV$ (b) $8.16\,eV$ (c) $4.28\,eV$ (d) $4.14\,eV$

Q73. Consider the following processes involving free particles

(i) $\bar{n} \to \bar{p} + e^{+} + \bar{\nu}_e$ (ii) $\bar{p} + n \to \pi^{-}$

(iii) $p + n \to \pi^{+} + \pi^{0} + \pi^{0}$ (iv) $p + \bar{\nu}_e \to n + e^{+}$

Which of the following statements is true?

(a) Process (i) obeys all conservation laws

(b) Process (ii) conserves baryon number, but violates energy-momentum conservation

(c) process (iii) is not allowed by strong interaction but is allowed by weak interactions

(d) Process (iv) conserves baryon number, but violates lepton number conservation

Q74. The electric quadrupole moment of an odd proton nucleus is $\frac{(2j-1)}{2(j+1)}\langle r^2 \rangle$, where $j$ is the total angular momentum. Given that $R_0 = 1.2\ fm$, what is the value in barn, of the quadrupole moment of the $^{27}Al$ nucleus in the shell model?

(a) 0.043 (b) 0.023 (c) 0.915 (d) 0

Q75. Of the nuclei of mass number $A = 125$, the binding energy calculated from the liquid drop model (given that the coefficients for the Coulomb and the asymmetry energy are $a_c = 0.7\ MeV$ and $a_{sym} = 22.5\ MeV$ respectively) is a maximum for

(a) $^{125}_{54}Xe$ (b) $^{124}_{53}I$ (c) $^{125}_{52}Te$ (d) $^{125}_{51}Sb$