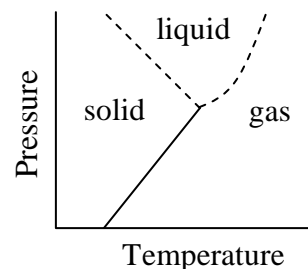


NET-JRF JUNE 2015

PART A

Q1. By reading the accompanying graph, determine the INCORRECT statement out of the following.

- (a) Melting point increases with pressure
 (b) Melting point decreases with pressure
 (c) Boiling point increases with pressure
 (d) Solid, liquid and gas can co-exist at the same pressure and temperature



Q2. A float is drifting in a river 10 m downstream of a boat that can be rowed at a speed of 10 m/minute in still water. If the boat is rowed downstream, the time taken to catch up with the float

- (a) will be 1 minute
 (b) will be more than 1 minute
 (c) will be less than 1 minute
 (d) can be determined only if the speed of the river is known

Q3. Consider a series of letters placed in the following way:

$U_G_C_C_S_I_R$

Each letter moves one step to its right and the extreme right letter takes the first position, completing one operation. After which of the following numbers of operations do the Cs not sit side by side?

- (a) 3
 (b) 10
 (c) 19
 (d) 25

Q4. If you change only one observation from a set of 10 observations, which of the following will definitely change?

- (a) Mean
 (b) Median
 (c) Mode
 (d) Standard deviation

Q5. An inclined plane rests against a horizontal cylinder of radius R . If the plane makes an angle of 30° with the ground, the point of contact of the plane with the cylinder is at a height of

- (a) $1.500 R$
 (b) $1.866 R$
 (c) $1.414 R$
 (d) $1.000 R$

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- Q6. In a fast moving car with open windows, the driver feels a continuous incoming breeze. The pressure inside the car, however, does not keep increasing because,
- (a) air coming in from the front window goes out from the rear
(b) air comes in as well as goes out through every window but the driver only feels the incoming one.
(c) no air actually comes in and the feeling of breeze is an illusion.
(d) Cool air reduces the temperature therefore the pressure does not increase
- Q7. What is the maximum number of parallel, non-overlapping cricket pitches (length 24 m , width 3 m) that can be laid in a field of diameter 140 m , if the boundary is required to be at least 60 m from the centre of any pitch?
- (a) 6 (b) 7 (c) 12 (d) 4
- Q8. The product of the perimeter of a triangle, the radius of its in-circle, and a number gives the area of the triangle. The number is
- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
- Q9. The maximum number of points formed by intersection of all pairs of diagonals of convex octagon is
- (a) 70 (b) 400 (c) 120 (d) 190
- Q10. Each of the following pairs of words hides a number, based on which you can arrange them in ascending order. Pick the correct answer:
- | | |
|----------------|------------------|
| I. Cloth reel | K. Silent wonder |
| J. Good tone | L. Bronze rod |
| (a) L, K, J, I | (b) I, J, K, L |
| (c) K, L, J, I | (d) K, J, I, L |
- Q11. Which of the following values is same as 2^{2^2} ?
- (a) 2^6 (b) 2^8 (c) 2^{16} (d) 2^{222}

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Q12. If

$$\begin{array}{r} 2a \\ \times b2 \\ \hline c6 \\ 84 \\ \hline 8d6 \end{array}$$

Here a, b, c and d are digits. Then $a + b =$

- (a) 4 (b) 9 (c) 11 (d) 16

Q13. A $12\text{ m} \times 4\text{ m}$ rectangular roof is resting on four 4 m tall thin poles. Sunlight falls on the roof at an angle of 45° from the east, creating a shadow on the ground. What will be the area of the shadow?

- (a) 24 m^2 (b) 36 m^2 (c) 48 m^2 (d) 60 m^2




Q14. Find the height of a box of base area $24\text{ cm} \times 48\text{ cm}$, in which the longest stick that can be kept is 56 cm long.




- (a) 8 cm (b) 32 cm (c) 37.5 cm (d) 16 cm



Q15. An infinite row of boxes is arranged. Each box has half the volume of the previous box. If the largest box has volume of 20 cc , what is the total volume of all the boxes?

- (a) Infinite (b) 400 cc (c) 40 cc (d) 80 cc

Q16. Find the missing element based on the given pattern

(1)  (2)  (3) 

(1)  (2)  (3) 

- (a)  (b)  (c)  (d) 

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Q17. A man starts his journey at 0100 Hrs local time to reach another country at 0900 Hrs local time on the same date. He starts a return journey on the same night at 2100 hrs local time to his original place, taking the same time to travel back. If the time zone of his country of visit lags by 10 hours, the duration for which the man was away from his place is

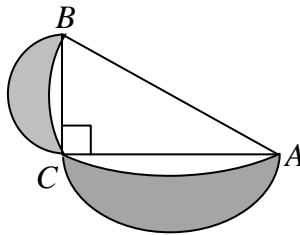
- (a) 48 hours (b) 20 hours (c) 25 hours (d) 36 hours

Q18. Let r be a positive number satisfying $r^{\left(\frac{1}{1234}\right)} + r^{\left(\frac{-1}{1234}\right)} = 2$. Then

$$r^{4321} + r^{-4321} = ?$$

- (a) 2 (b) $2^{\left(\frac{4321}{1234}\right)}$ (c) 2^{3087} (d) 2^{1234}

Q19. ABC is right angled triangle inscribed in a semicircle. Smaller semicircles are drawn on sides BC and AC . If the area of the triangle is a , what is the total area of the shaded lumes?



- (a) a (b) $\frac{\pi}{a}$ (c) $\frac{a}{\pi}$ (d) $\frac{a}{2\pi}$

Q20. An ant can lift another ant of its size whereas an elephant cannot lift another elephant of its size, because

- (a) ant muscle fibres are stronger than elephant muscle fibres
 (b) ant has proportionately thicker legs than elephant
 (c) strength scales as the square of the size while weight scales as cube of the size
 (d) ants work cooperatively, whereas elephants work as individuals

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PART – ‘B’

Q21. A particle moves in two dimensions on the ellipse $x^2 + 4y^2 = 8$. At a particular instant it is at the point $(x, y) = (2, 1)$ and the x -component of its velocity is 6 (in suitable units).

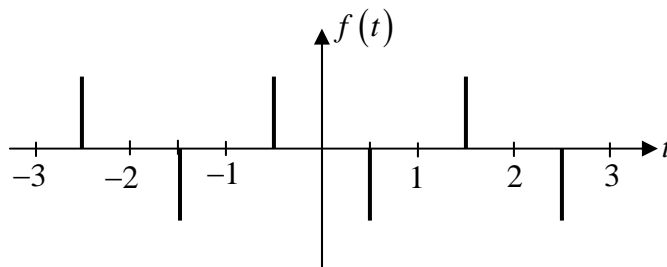
Then the y -component of its velocity is

- (a) -3 (b) -2 (c) 1 (d) 4

Q22. The value of integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$

- (a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\sqrt{2}\pi$ (d) 2π

Q23. Consider the periodic function $f(t)$ with time period T as show in the figure below.



The spikes, located at $t = \frac{1}{2}(2n-1)$, where $n = 0, \pm 1, \pm 2, \dots$, are Dirac-delta function of strength ± 1 . The amplitudes a_n in the Fourier expansion

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{\frac{2\pi i n t}{T}}$$
 are given by

- (a) $(-1)^n$ (b) $\frac{1}{n\pi} \sin \frac{n\pi}{2}$ (c) $i \sin \frac{n\pi}{2}$ (d) $n\pi$

Q24. Consider three inertial frames of reference A, B and C . The frame B moves with a velocity $\frac{c}{2}$ with respect to A , and C moves with a velocity $\frac{c}{10}$ with respect to B in the same direction. The velocity of C as measured in A is

- (a) $\frac{3c}{7}$ (b) $\frac{4c}{7}$ (c) $\frac{c}{7}$ (d) $\frac{\sqrt{3}c}{7}$

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Q25. If the Lagrangian of a dynamical system in two dimensions is $L = \frac{1}{2}m\dot{x}^2 + m\dot{y}$, then its

Hamiltonian is

(a) $H = \frac{1}{m} p_x p_y + \frac{1}{2m} p_y^2$

(b) $H = \frac{1}{m} p_x p_y + \frac{1}{2m} p_x^2$

(c) $H = \frac{1}{m} p_x p_y - \frac{1}{2m} p_y^2$

(d) $H = \frac{1}{m} p_x p_y - \frac{1}{2m} p_x^2$

Q26. Consider the differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$. If $x = 0$ at $t = 0$ and $x = 1$ at $t = 1$,

the value of x at $t = 2$ is

(a) $e^2 + 1$

(b) $e^2 + e$

(c) $e + 2$

(d) $2e$

Q27. A Plan electromagnetic wave is travelling along the positive z -direction. The maximum electric field along the x -direction is 10 V/m . The approximate maximum values of the power per unit area and the magnetic induction B , respectively, are

(a) $3.3 \times 10^{-7} \text{ watts/m}^2$ and 10 tesla

(b) $3.3 \times 10^{-7} \text{ watts/m}^2$ and $3.3 \times 10 \text{ tesla}$

(c) 0.265 watts/m^2 and 10 tesla

(d) 0.265 watts/m^2 and $3.3 \times 10^{-8} \text{ tesla}$

Q28. Which of the following transformations $(V, \vec{A}) \rightarrow (V', \vec{A}')$ of the electrostatic potential

V and the vector potential \vec{A} is a gauge transformation?

(a) $(V' = V + ax, \vec{A}' = \vec{A} + at\hat{k})$

(b) $(V' = V + ax, \vec{A}' = \vec{A} - at\hat{k})$

(c) $(V' = V + ax, \vec{A}' = \vec{A} + at\hat{i})$

(d) $(V' = V + ax, \vec{A}' = \vec{A} - at\hat{i})$

Q29. The ratio of the energy of the first excited state E_1 of a particle in a three-dimensional

rectangular box of side L, L and $\frac{L}{2}$, is

(a) 3:2

(b) 2:1

(c) 4:1

(d) 4:3

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- Q30. Suppose the yz -plane forms a chargeless boundary between two media of permittivities ϵ_{left} and ϵ_{right} where $\epsilon_{\text{left}} : \epsilon_{\text{right}} = 1:2$. If the uniform electric field on the left is $\vec{E}_{\text{left}} = c(\hat{i} + \hat{j} + \hat{k})$ (where c is a constant), then the electric field on the right \vec{E}_{right} is
- (a) $c(2\hat{i} + \hat{j} + \hat{k})$ (b) $c(\hat{i} + 2\hat{j} + 2\hat{k})$
 (c) $c\left(\frac{1}{2}\hat{i} + \hat{j} + \hat{k}\right)$ (d) $c\left(\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}\right)$
- Q31. If L_i are the components of the angular momentum operator $\sum_{i=1,2,3} \left[[\vec{L}, L_i], L_i \right]$ equals
- (a) \vec{L} (b) $2\vec{L}$ (c) $3\vec{L}$ (d) $-\vec{L}$
- Q32. The Laplace transform of $6t^3 + 3\sin 4t$ is
- (a) $\frac{36}{s^4} + \frac{12}{s^2 + 16}$ (b) $\frac{36}{s^4} + \frac{12}{s^2 - 16}$
 (c) $\frac{18}{s^4} + \frac{12}{s^2 - 16}$ (d) $\frac{36}{s^3} + \frac{12}{s^2 + 16}$
- Q33. A particle of mass m moves in the one dimensional potential $V(x) = \frac{\alpha}{3}x^3 + \frac{\beta}{4}x^4$ where $\alpha, \beta > 0$. One of the equilibrium points is $x=0$. The angular frequency of small oscillations about the other equilibrium point is
- (a) $\frac{2\alpha}{\sqrt{3m\beta}}$ (b) $\frac{\alpha}{\sqrt{m\beta}}$ (c) $\frac{\alpha}{\sqrt{12m\beta}}$ (d) $\frac{\alpha}{\sqrt{24m\beta}}$
- Q34. A particle of unit mass moves in the xy -plane in such a way that $\dot{x}(t) = y(t)$ and $\dot{y}(t) = -x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential
- (a) $\frac{1}{2}(x^2 + y^2)$ (b) $\frac{1}{2}(x^2 - y^2)$
 (c) $x + y$ (d) $x - y$

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- Q35. A proton moves with a speed of 300 m/s in a circular orbit in the xy -plan in a magnetic field 1 tesla along the positive z -direction. When an electric field of 1 V/m is applied along the positive y -direction, the center of the circular orbit
- remains stationary
 - moves at 1 m/s along the negative x -direction
 - moves at 1 m/s along the positive z -direction
 - moves at 1 m/s along the positive x -direction
- Q36. The wavefunction of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p) = \int \psi(x) e^{\frac{-ipx}{\hbar}} dx$ in the momentum representation. If the action of an operator \hat{T} on $\psi(x)$ is given by $\hat{T}\psi(x) = \psi(x+a)$, where a is a constant then $\hat{T}\phi(p)$ is given by
- $-\frac{i}{\hbar} ap\phi(p)$
 - $e^{\frac{-iap}{\hbar}} \phi(p)$
 - $e^{\frac{+iap}{\hbar}} \phi(p)$
 - $\left(1 + \frac{i}{\hbar} ap\right) \phi(p)$
- Q37. A particle moves in one dimension in the potential $V = \frac{1}{2}k(t)x^2$, where $k(t)$ is a time dependent parameter. Then $\frac{d}{dt}\langle V \rangle$, the rate of change of the expectation value $\langle V \rangle$ of the potential energy is
- $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle + \frac{k}{2m} \langle xp + px \rangle$
 - $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle + \frac{1}{2m} \langle p^2 \rangle$
 - $\frac{k}{2m} \langle xp + px \rangle$
 - $\frac{1}{2} \frac{dk}{dt} \langle x^2 \rangle$
- Q38. In a two-state system, the transition rate of a particle from state 1 to state 2 is t_{12} , and the transition rate from state 2 to state 1 is t_{21} . In the steady state, the probability of finding the particle in state 1 is
- $\frac{t_{21}}{t_{12} + t_{21}}$
 - $\frac{t_{12}}{t_{12} + t_{21}}$
 - $\frac{t_{12}t_{21}}{t_{12} + t_{21}}$
 - $\frac{t_{12} - t_{21}}{t_{12} + t_{21}}$

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Q39. A system of N non-interacting classical particles, each of mass m is in a two dimensional harmonic potential of the form $V(r) = \alpha(x^2 + y^2)$ where α is a positive constant. The canonical partition function of the system at temperature T is $\left(\beta = \frac{1}{k_B T}\right)$:

- (a) $\left[\left(\frac{\alpha}{2m}\right)^2 \frac{\pi}{\beta}\right]^N$ (b) $\left(\frac{2m\pi}{\alpha\beta}\right)^{2N}$ (c) $\left(\frac{\alpha\pi}{2m\beta}\right)^N$ (d) $\left(\frac{2m\pi^2}{\alpha\beta^2}\right)^N$

Q40. The concentration of electrons, n and holes p , for an intrinsic semiconductor at a temperature T can be expressed as $n = p = AT^{\frac{3}{2}} \exp\left(-\frac{E_g}{2k_B T}\right)$, where E_g is the band gap and A is a constant. If the mobility of both types of carries is proportional to $T^{\frac{-3}{2}}$, then the log of the conductivity is a linear function of T^{-1} , with slope

- (a) $\frac{E_g}{(2k_B)}$ (b) $\frac{E_g}{k_B}$ (c) $\frac{-E_g}{(2k_B)}$ (d) $\frac{-E_g}{k_B}$

Q41. The viscosity η of a liquid is given by Poiseuille's formula $\eta = \frac{\pi P a^4}{8lV}$. Assume that l and V can be measured very accurately, but the pressure P has an rms error of 1% and the radius a has an independent rms error of 3%. The rms error of the viscosity is closest to

(a) 2% (b) 4% (c) 12% (d) 13%

Q42. A system of N distinguishable particles, each of which can be in one of the two energy levels 0 and ε , has a total energy $n\varepsilon$, where n is an integer. The entropy of the system is proportional to

- (a) $N \ln n$ (b) $n \ln N$ (c) $\ln\left(\frac{N!}{n!}\right)$ (d) $\ln\left(\frac{N!}{n!(N-n)!}\right)$

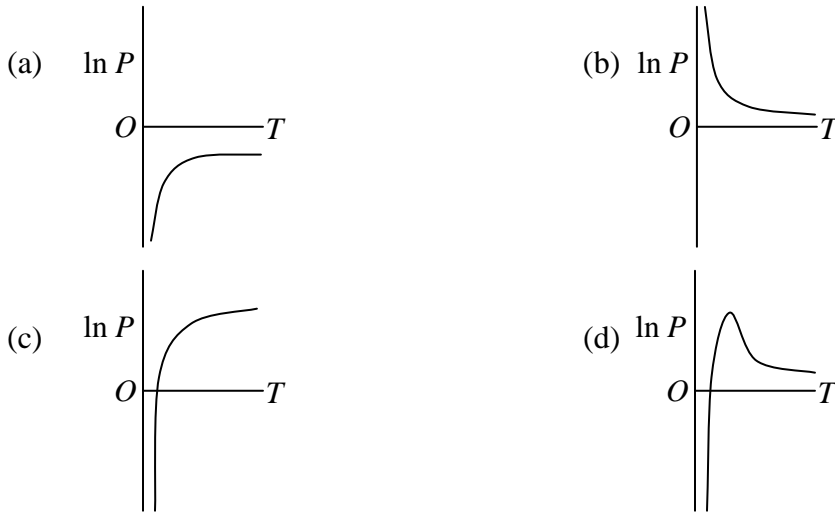
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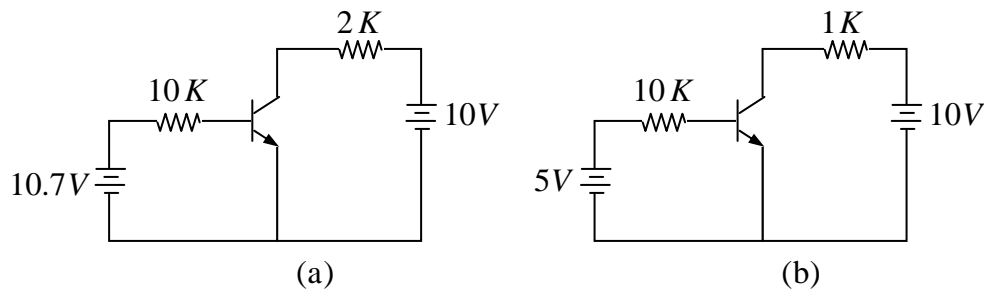
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Q43. The condition for the liquid and vapour phases of a fluid to be in equilibrium is given by the approximate equation $\frac{dP}{dT} \approx \frac{Q_l}{Tv_{vap}}$ (Clausius-Clayperon equation), where v_{vap} is the volume per particle in the vapour phase, and Q_l is the latent heat, which may be taken to be a constant. If the vapour obeys ideal gas law, which of the following plots is correct?



Q44. Consider the circuits shown in figures (a) and (b) below



If the transistors in Figures (a) and (b) have current gain (β_{dc}) of 100 and 10 respectively, then they operate in the

- (a) active region and saturation region respectively
- (b) saturation region and active region respectively
- (c) saturation region in both cases
- (d) active region in both cases

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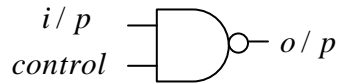
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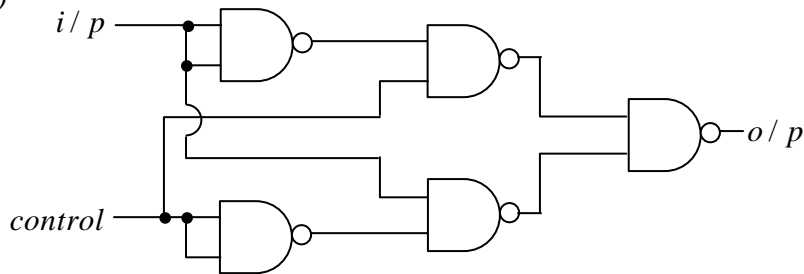
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Q45. Which of the following circuits behaves as a controlled inverter?

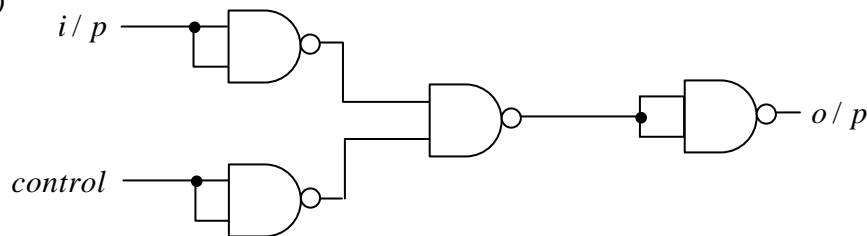
(a)



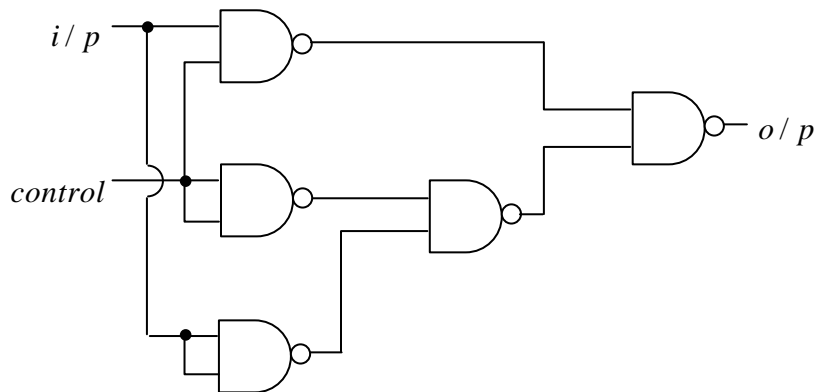
(b)



(c)



(d)



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PART – ‘C’

Q46. The rank-2 tensor $x_i x_j$, where x_i are the Cartesian coordinates of the position vector in three dimensions, has 6 independent elements. Under rotation, these 6 elements decompose into irreducible sets (that is, the elements of each set transform only into linear combinations of elements in that set) containing

- (a) 4 and 2 elements
- (b) 5 and 1 elements
- (c) 3, 2 and 1 elements
- (d) 4, 1 and 1 elements

Q47. Let $f(x, t)$ be a solution of the wave equation $\frac{\partial^2 f}{\partial t^2} = v^2 \frac{\partial^2 f}{\partial x^2}$ in 1-dimension. If at $t = 0$, $f(x, 0) = e^{-x^2}$ and $\frac{\partial f}{\partial t}(x, 0) = 0$ for all x , then $f(x, t)$ for all future times $t > 0$ is described by

- (a) $e^{-(x^2 - v^2 t^2)}$
- (b) $e^{-(x - vt)^2}$
- (c) $\frac{1}{4} e^{-(x - vt)^2} + \frac{3}{4} e^{-(x + vt)^2}$
- (d) $\frac{1}{2} [e^{-(x - vt)^2} + e^{-(x + vt)^2}]$

Q48. The differential cross-section for scattering by a target is given by

$$\frac{d\sigma}{d\Omega}(\theta, \varphi) = a^2 + b^2 \cos^2 \theta$$

If N is the flux of the incoming particles, the number of particles scattered per unit time is

- (a) $\frac{4\pi}{3} N (a^2 + b^2)$
- (b) $4\pi N \left(a^2 + \frac{1}{6} b^2 \right)$
- (c) $4\pi N \left(\frac{1}{2} a^2 + \frac{1}{3} b^2 \right)$
- (d) $4\pi N \left(a^2 + \frac{1}{3} b^2 \right)$

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Q53. Consider the differential equation $\frac{dy}{dx} = x^2 - y$ with the initial condition $y = 2$ at $x = 0$.

Let $y_{(1)}$ and $y_{(\frac{1}{2})}$ be the solutions at $x = 1$ obtained using Euler's forward algorithm with

step size 1 and $\frac{1}{2}$ respectively. The value of $\left(\frac{y_{(1)} - y_{(\frac{1}{2})}}{y_{(\frac{1}{2})}} \right)$ is

- (a) $-\frac{1}{2}$ (b) -1 (c) $\frac{1}{2}$ (d) 1

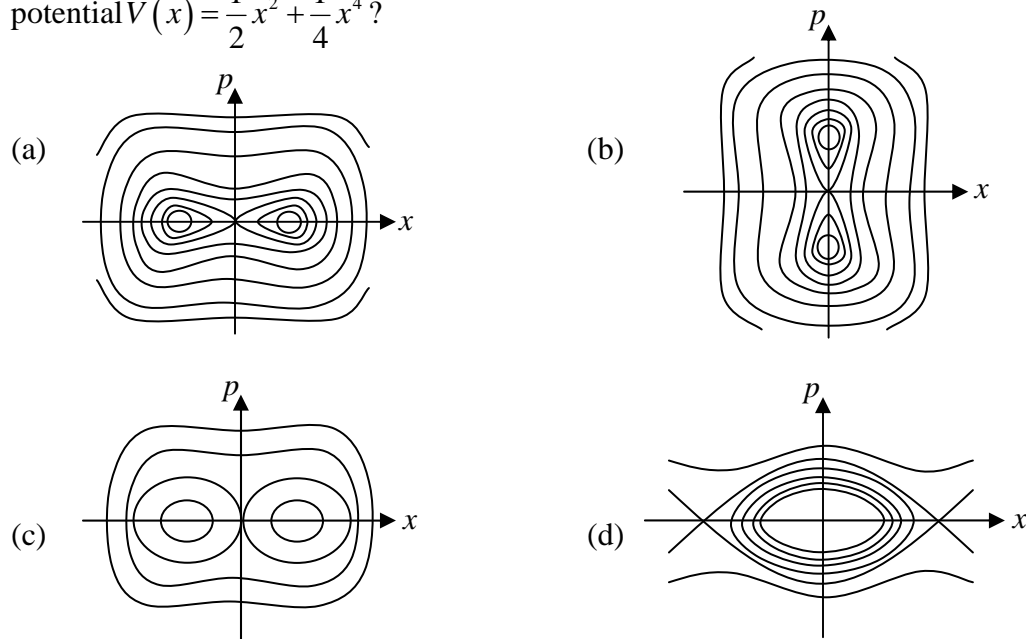
Q54. Let q and p be the canonical coordinate and momentum of a dynamical system. Which of the following transformations is canonical?

1. $Q_1 = \frac{1}{\sqrt{2}}q^2$ and $P_1 = \frac{1}{\sqrt{2}}p^2$ 2. $Q_2 = \frac{1}{\sqrt{2}}(p+q)$ and $P_2 = \frac{1}{\sqrt{2}}(p-q)$

- (a) neither 1 nor 2 (b) both 1 and 2
(c) only 1 (d) only 2

Q55. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional

potential $V(x) = \frac{1}{2}x^2 + \frac{1}{4}x^4$?



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Q56. A rod of length L carries a total charge Q distributed uniformly. If this is observed in a frame moving with a speed v along the rod, the charge per unit length (as measured by the moving observer) is

(a) $\frac{Q}{L} \left(1 - \frac{v^2}{c^2}\right)$

(b) $\frac{Q}{L} \sqrt{1 - \frac{v^2}{c^2}}$

(c) $\frac{Q}{L \sqrt{1 - \frac{v^2}{c^2}}}$

(d) $\frac{Q}{L \left(1 - \frac{v^2}{c^2}\right)}$

Q57. A particle of mass m is in a potential $V = \frac{1}{2} m \omega^2 x^2$, where ω is a constant.

Let $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} + \frac{i\hat{p}}{m\omega} \right)$. In the Heisenberg picture $\frac{d\hat{a}}{dt}$ is given by

(a) $\omega \hat{a}$

(b) $-i\omega \hat{a}$

(c) $\omega \hat{a}^\dagger$

(d) $i\omega \hat{a}^\dagger$

Q58. Two different sets of orthogonal basis vectors

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

are given for a two dimensional real vector space. The matrix representation of a linear operator \hat{A} in these bases are related by a unitary transformation. The unitary matrix may be chosen to be

(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(c) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

(d) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

Q59. Consider three Ising spins at the vertices of a triangle which interact with each other with a ferromagnetic Ising interaction of strength J . The partition function of the system at

temperature T is given by $\left(\beta = \frac{1}{k_B T} \right)$:

(a) $2e^{3\beta J} + 6e^{-\beta J}$

(b) $2e^{-3\beta J} + 6e^{\beta J}$

(c) $2e^{3\beta J} + 6e^{-3\beta J} + 3e^{\beta J} + 3e^{-\beta J}$

(d) $(2 \cosh \beta J)^3$

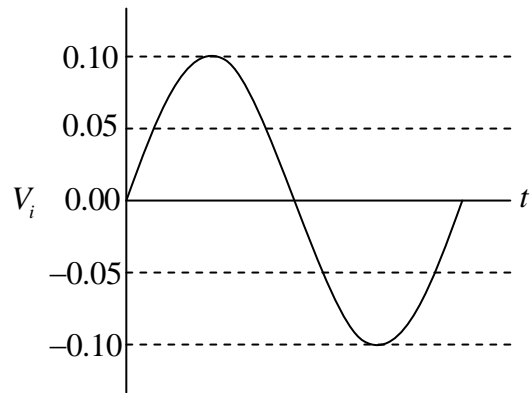
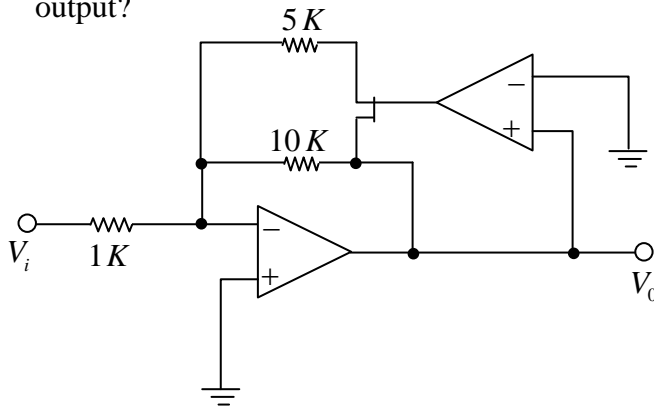
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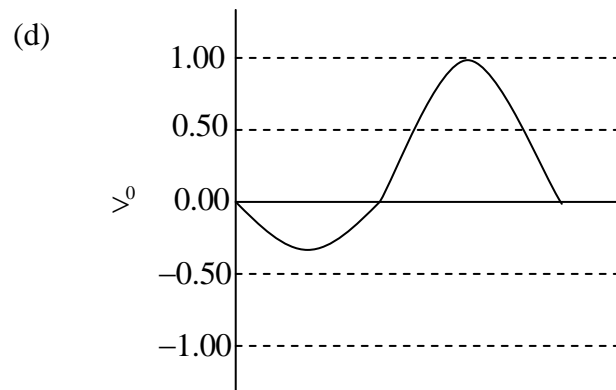
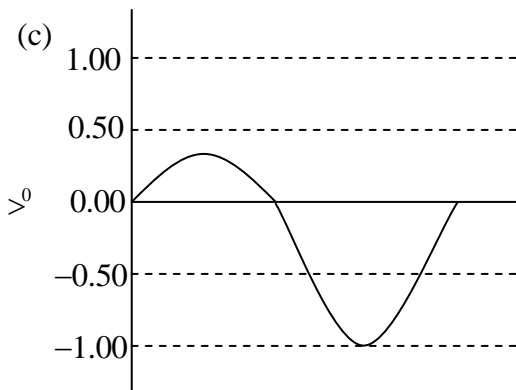
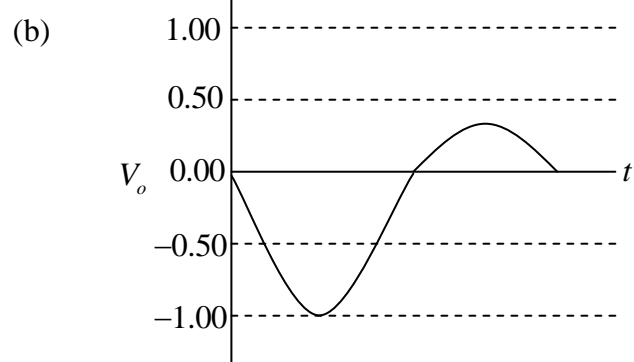
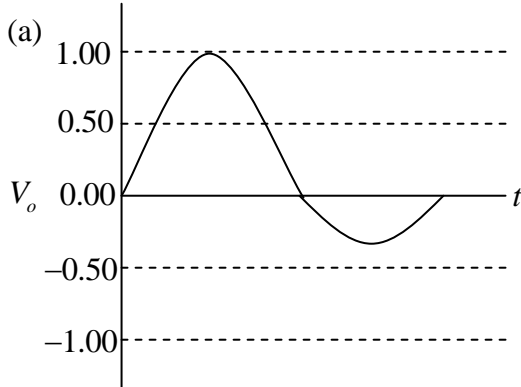
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Q60. For the circuit and the input sinusoidal waveform shown in the correct waveform at the output?



(The time scales in all the plots are the same.)



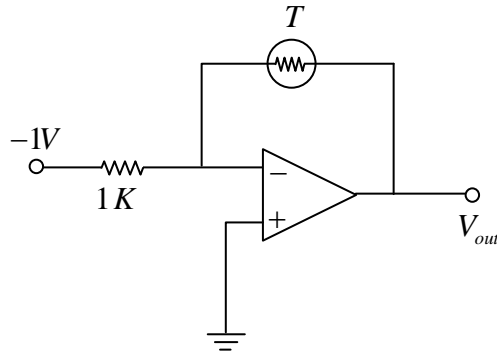
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- Q61. In the circuit given below, the thermistor has a resistance $3\text{ k}\Omega$ at 25°C . Its resistance decreases by $150\ \Omega$ per $^\circ\text{C}$ upon heating. The output voltage of the circuit at 30°C is



- (a) -37.5 V (b) -2.25 V (c) 2.25 V (d) 3.75 V
- Q62. The Dirac Hamiltonian $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$ for a free electron corresponds to the classical relation $E^2 = p^2 c^2 + m^2 c^4$. The classical energy momentum relation of a particle of charge q in a electromagnetic field is

- (a) $c\vec{\alpha}\cdot\vec{p} + \frac{e}{c}\vec{A}\cdot\vec{A} + \beta mc^2 - e\phi$ (b) $c\vec{\alpha}\cdot\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 + e\phi$
- (c) $c\left(\vec{\alpha}\cdot\vec{p} + e\phi + \frac{e}{c}|\vec{A}|\right) + \beta mc^2$ (d) $c\vec{\alpha}\cdot\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 - e\phi$

- Q63. A particle of energy E scatters off a repulsive spherical potential

$$V(r) = \begin{cases} V_0 & \text{for } r < a \\ 0 & \text{for } r \geq a \end{cases}$$

where V_0 and a are positive constants. In the low energy limit, the total scattering cross-

section is $\sigma = 4\pi a^2 \left(\frac{1}{ka} \tanh ka - 1\right)^2$, where $k^2 = \frac{2m}{\hbar^2}(V_0 - E) > 0$. In the limit $V_0 \rightarrow \infty$

the ratio of σ to the classical scattering cross-section off a sphere a radius a is

- (a) 4 (b) 3 (c) 1 (d) $\frac{1}{2}$

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Q64. A large number N of Brownian particles in one dimension start their diffusive motion from the origin at time $t = 0$. The diffusion coefficient is D . The number of particles crossing a point at a distance L from the origin, per unit time, depends on L and time t as

$$(a) \frac{N}{\sqrt{4\pi Dt}} e^{\frac{-L^2}{4Dt}}$$

$$(b) \frac{NL}{\sqrt{4\pi Dt}} e^{\frac{-4Dt}{L^2}}$$

$$(c) \frac{N}{\sqrt{16\pi Dt^3}} e^{\frac{-L^2}{4Dt}}$$

$$(d) Ne^{\frac{-4Dt}{L^2}}$$

Q65. X -ray of wavelength $\lambda = a$ is reflected from the (111) plane of a simple cubic lattice. If the lattice constant is a , the corresponding Bragg angle (in radian) is

$$(a) \frac{\pi}{6}$$

$$(b) \frac{\pi}{4}$$

$$(c) \frac{\pi}{3}$$

$$(d) \frac{\pi}{8}$$

Q66. The critical magnetic fields of a superconductor at temperatures $4K$ and $8K$ are 11 mA/m and 5.5 mA/m respectively. The transition temperature is approximately

$$(a) 8.4 K$$

$$(b) 10.6 K$$

$$(c) 12.9 K$$

$$(d) 15.0 K$$

Q67. Of the following term symbols of the np^2 atomic configurations, 1S_0 , 3P_0 , 3P_1 , 3P_2 and 1D_2 which is the ground state?

$$(a) ^3P_0$$

$$(b) ^1S_0$$

$$(c) ^3P_2$$

$$(d) ^3P_1$$

Q68. The reaction $^2_1D + ^2_1D \rightarrow ^4_2He + \pi^0$ cannot proceed via strong interactions because it violates the conservation of

(a) angular momentum

(b) electric charge

(c) baryon number

(d) isospin

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Q69. Let us approximate the nuclear potential in the shell model by a three dimensional isotropic harmonic oscillator. Since the lowest two energy levels have angular momenta $l=0$ and $l=1$ respectively, which of the following two nuclei have magic numbers of protons and neutrons?

- (a) ${}^4_2\text{He}$ and ${}^{16}_8\text{O}$ (b) ${}^2_1\text{D}$ and ${}^8_4\text{Be}$
 (c) ${}^4_2\text{He}$ and ${}^8_4\text{Be}$ (d) ${}^4_2\text{He}$ and ${}^{12}_6\text{C}$

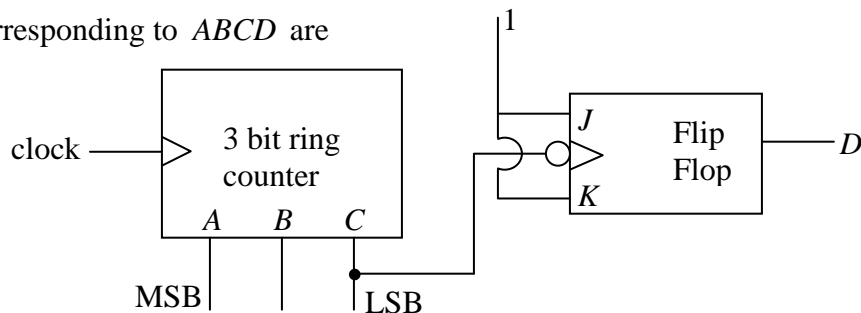
Q70. An ideal Bose gas in d -dimensions obeys the dispersion relation $\varepsilon(\vec{k}) = Ak^s$, where A and s are constants. For Bose-Einstein condensation to occur, the occupancy of excited states

$$N_e = c \int_0^\infty \frac{e^{-\beta\varepsilon}}{e^{\beta(\varepsilon-\mu)} - 1} d\varepsilon$$

where c is a constant, should remain finite even for $\mu = 0$. This can happen if

- (a) $\frac{d}{s} < \frac{1}{4}$ (b) $\frac{1}{4} < \frac{d}{s} < \frac{1}{2}$
 (c) $\frac{d}{s} > 1$ (d) $\frac{1}{2} < \frac{d}{s} < 1$

Q71. For the logic circuit given below, the decimal count sequence and the modulus of the circuit corresponding to $ABCD$ are



- (a) $8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 9 \rightarrow 5 \pmod{6}$
 (b) $8 \rightarrow 4 \rightarrow 2 \rightarrow 9 \rightarrow 5 \rightarrow 3 \pmod{6}$
 (c) $2 \rightarrow 5 \rightarrow 9 \rightarrow 1 \rightarrow 3 \pmod{5}$
 (d) $8 \rightarrow 5 \rightarrow 1 \rightarrow 3 \rightarrow 7 \pmod{5}$

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Q72. The low-energy electronic excitations in a two-dimensional sheet of grapheme is given by $E(\vec{k}) = \hbar v k$, where v is the velocity of the excitations. The density of states is proportional to

- (a) E (b) $E^{\frac{3}{2}}$ (c) $E^{\frac{1}{2}}$ (d) E^2

Q73. The charm quark assigned a charm quantum number $C = 1$. How should the Gellmann-Nishijima formula for electric charge be modified for four flavors of quarks?

- (a) $I_3 + \frac{1}{2}(B - S - C)$ (b) $I_3 + \frac{1}{2}(B - S + C)$
 (c) $I_3 + \frac{1}{2}(B + S - C)$ (d) $I_3 + \frac{1}{2}(B + S + C)$

Q74. A diatomic molecule has vibrational states with energies $E_v = \hbar\omega\left(v + \frac{1}{2}\right)$ and rotational states with energies $E_j = B j(j+1)$, where v and j are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \leq 1$ and $j \leq 2$ and subject to the selection rules $\Delta v = \pm 1$ and $\Delta j = \pm 1$. Then the largest allowed energy of transition is

- (a) $\hbar\omega - 3B$ (b) $\hbar\omega - B$
 (c) $\hbar\omega + 4B$ (d) $2\hbar\omega + B$

Q75. A $He-Ne$ laser operates by using two energy levels of Ne separated by 2.26 eV . Under steady state conditions of optical pumping, the equivalent temperature of the system at which the ratio of the number of atoms in the upper state to that in the lower state will be $\frac{1}{20}$, is approximately (the Boltzmann constant $k_B = 8.6 \times 10^{-5} \text{ eV} / K$)

- (a) 10^{10} K (b) 10^8 K (c) 10^6 K (d) 10^4 K

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