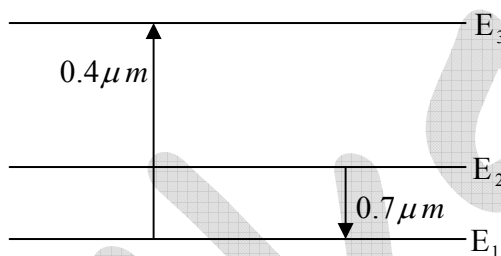


ATOMIC AND MOLECULAR PHYSICSNET/JRF (JUNE-2011)

- Q1. Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with 1.6×10^{19} Chromium ions per cubic centimeter. All the atoms excited by the $0.4 \mu\text{m}$ radiation decay rapidly to level E_2 which has a lifetime $\tau = 3 \text{ ms}$.



A. Assuming that there is no radiation of wavelength $0.7 \mu\text{m}$ present in the pumping cycle and that the pumping rate is R atoms per cm^3 , the population density in the level N_2 builds up as:

(a) $N_2(t) = R\tau(e^{t/\tau} - 1)$

(b) $N_2(t) = R\tau(1 - e^{-t/\tau})$

(c) $N_2(t) = \frac{Rt^2}{\tau}(1 - e^{-t/\tau})$

(d) $N_2(t) = R t$

Ans: (b)

$$N_2(t) = R\tau(1 - e^{-t/\tau})$$

B. The minimum pump power required (per cubic centimeter) to bring the system to transparency, i.e. zero gain, is

(a) 1.52 kW

(b) 2.64 kW

(c) 0.76 kW

(d) 1.32 kW

Ans: (c)

Solution: The Minimum Power required to achieve zero gain is

$$P = \frac{N}{2} \frac{h\nu}{\tau} = \frac{N}{2} \frac{hc}{\lambda\tau} = \frac{1.6 \times 10^{19}}{2} \times \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.7 \times 10^{-6} \times 3 \times 10^{-3}} = 754 \text{ W cm}^{-3}$$

$$P = 0.76 \text{ kW per cubic centimeter}$$

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NET/JRF (DEC-2011)

- Q2. Given that the ground state energy of the hydrogen atom is -13.6 eV , the ground state energy of positronium (which is a bound state of an electron and a positron) is
 (a) $+6.8 \text{ eV}$ (b) -6.8 eV (c) -13.6 eV (d) -27.2 eV

Ans: (b)

The energy expression for Positronium atom is $E_n = -\frac{13.6}{2n^2} (\text{eV})$

For $n = 1, E_1 = \frac{-13.6}{2} (\text{eV}) = -6.8 \text{ eV}, \therefore E_1 = -6.8 \text{ eV}$

- Q3. A laser operating at 500 nm is used to excite a molecule. If the Stokes line is observed at 770 cm^{-1} , the approximate positions of the Stokes and the anti-Stokes lines are
 (a) 481.5 nm and 520 nm (b) 481.5 nm and 500 nm
 (c) 500 nm and 520 nm (d) 500 nm and 600 nm

Ans:

Solution: Given $\lambda_0 = 500 \text{ nm} = 5 \times 10^{-5}, \bar{\nu}_{\text{stoke}} = 770 \text{ cm}^{-1} \therefore \nu_0 = 20,000 \text{ cm}^{-1}$

Raman shift $\Delta\bar{\nu} = \bar{\nu}_0 - \bar{\nu}_{\text{stoke}} = 19230 \text{ cm}^{-1}$

Wave number of anti-stokes line is $\bar{\nu}_{\text{anti-stoke}} = \Delta\bar{\nu} + \bar{\nu}_0 = 39,230 \text{ cm}^{-1}$

In wavelength term $\lambda_{\text{anti-stoke}} = 2.549 \times 10^{-7} = 254.9 \text{ nm}$ and $\lambda_{\text{stoke}} = 12987 \text{ nm}$

- Q4. If the hyperfine interaction in an atom is given by $H = a\vec{S}_e \cdot \vec{S}_p$ where \vec{S}_e and \vec{S}_p denote the electron and proton spins, respectively, the splitting between the 3S_1 and 1S_0 state is
 (a) $a\hbar^2 / \sqrt{2}$ (b) $a\hbar^2$ (c) $a\hbar^2 / 2$ (d) $2a\hbar^2$

Ans: (b)

Solution: Total spin is $\vec{S} = \vec{S}_e + \vec{S}_p \Rightarrow S^2 = S_e^2 + S_p^2 + 2\vec{S}_e \cdot \vec{S}_p \Rightarrow \vec{S}_e \cdot \vec{S}_p = \frac{1}{2}[S^2 - S_e^2 - S_p^2]$

$\Rightarrow H = a\vec{S}_e \cdot \vec{S}_p = \frac{a}{2}[S^2 - S_e^2 - S_p^2]$ where $S_e^2 = S_p^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$

$\Rightarrow H = \frac{a}{2}\left(S^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2\right) = \frac{a}{2}\left[S^2 - \frac{3}{2}\hbar^2\right]$

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For ${}^3S_1 : S = 1 \Rightarrow S^2 = S(S+1)\hbar^2 = 2\hbar^2$, For ${}^1S_0 : S = 0 \Rightarrow S^2 = S(S+1)\hbar^2 = 0\hbar^2$

$$\therefore H_1 = \frac{a}{2} \left[2 - \frac{3}{2} \right] \hbar^2 = \frac{a}{4} \hbar^2 \quad \text{for } {}^3S_1$$

$$\text{and } H_2 = \frac{a}{2} \left[0 - \frac{3}{2} \right] \hbar^2 = -\frac{3}{4} a\hbar^2 \quad \text{for } {}^1S_0$$

$$\therefore \text{The splitting between } {}^3S_1 \text{ and } {}^1S_0 \text{ is } \Delta H = H_1 - H_2 = \left(\frac{1}{4} + \frac{3}{4} \right) a\hbar^2 = a\hbar^2$$

Q5. The ratio of intensities of the D_1 and D_2 lines of sodium at high temperature is

- (a) 1:1 (b) 2:3 (c) 1:3 (d) 1:2

Ans: (d)

Solution: The electronic transition for D_2 and D_1 line is

$$D_2 : {}^2P_{3/2} \rightarrow {}^2S_{1/2}, D_1 : {}^2P_{1/2} \rightarrow {}^2S_{1/2} \therefore \frac{I(D_2)}{I(D_1)} = \frac{2J_2 + 1}{2J_1 + 1} = \frac{2 \times \frac{3}{2} + 1}{2 \times \frac{1}{2} + 1} = \frac{4}{2} = 2$$

Q6. An atom of mass M can be excited to a state of mass $(M + \Delta)$ by photon capture. The frequency of a photon which can cause this transition is

- (a) $\frac{\Delta c^2}{2h}$ (b) $\frac{\Delta c^2}{h}$ (c) $\frac{\Delta^2 c^2}{2Mh}$ (d) $\frac{\Delta c^2}{2Mh} (\Delta + 2M)$

Ans: (d)

Solution: The conservation law of energy and Momentum give

$$Mc^2 + h\nu = \left[(M + \Delta)^2 c^4 + p^2 c^2 \right]^{1/2} \text{ and } \frac{h\nu}{c} = p$$

$$M^2 c^4 + h^2 \nu^2 + 2Mc^2 h\nu = M^2 c^4 + \Delta^2 c^4 + 2M\Delta c^4 + h^2 \nu^2 \Rightarrow 2Mc^2 h\nu = \Delta^2 c^4 + 2M\Delta c^4$$

$$\Rightarrow 2Mc^2 h\nu = 2M\Delta c^4 \left(1 + \frac{\Delta}{2M} \right) \Rightarrow \nu = \frac{\Delta c^2}{h} \left(1 + \frac{\Delta}{2M} \right) \Rightarrow \nu = \frac{\Delta c^2}{2Mh} (\Delta + 2M)$$

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NET/JRF (JUNE-2012)

Q7. The first absorption spectrum of $^{12}\text{C}^{16}\text{O}$ is at 3.842 cm^{-1} while that of $^{13}\text{C}^{16}\text{O}$ is at 3.673 cm^{-1} . The ratio of their moments of inertia is

- (a) 1.851 (b) 1.286 (c) 1.046 (d) 1.038

Ans: (c)

Solution: For $^{12}\text{C}^{16}\text{O}$: $2B_1 = 3.842\text{ cm}^{-1} \Rightarrow B_1 = 1.921\text{ cm}^{-1}$

For $^{13}\text{C}^{16}\text{O}$: $2B_2 = 3.673\text{ cm}^{-1} \Rightarrow B_2 = 1.8365\text{ cm}^{-1}$

Where, $B = \frac{h}{8\pi^2 IC} \therefore \frac{I_2}{I_1} = \frac{B_1}{B_2} = \frac{1.921}{1.8365} = 1.046$

Q8. The spin-orbit interaction in an atom is given by $H = a \text{ L.S}$, where L and S denote the orbital and spin angular momenta, respectively, of the electron. The splitting between the levels $^2P_{3/2}$ and $^2P_{1/2}$ is

- (a) $\frac{3}{2}a\hbar^2$ (b) $\frac{1}{2}a\hbar^2$ (c) $3a\hbar^2$ (d) $\frac{5}{2}a\hbar^2$

Ans:

Solution: Given $H = a\vec{L}\cdot\vec{S}$ where $\vec{J} = \vec{L} + \vec{S}$.

$$\therefore J^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S} \Rightarrow \vec{L}\cdot\vec{S} = \frac{1}{2}[J^2 - L^2 - S^2] \therefore H = \frac{a}{2}[J^2 - L^2 - S^2]$$

For $^3P_{3/2}$: $S = 1$ which gives $S^2 = S(S+1)\hbar^2 = 2\hbar^2$

$L = 1$ which gives $L^2 = L(L+1)\hbar^2 = 2\hbar^2$

$$J = \frac{3}{2} \text{ which gives } J^2 = J(J+1)\hbar^2 = \frac{15}{4}\hbar^2 \therefore H_1 = \frac{a}{2}\left[\frac{15}{4} - 2 - 2\right]\hbar^2 = -\frac{a}{8}\hbar^2$$

For $^2P_{3/2}$: $S = \frac{1}{2}$ which gives $S^2 = S(S+1)\hbar^2 = \frac{3}{4}\hbar^2$

$L = 1$ which gives $L^2 = L(L+1)\hbar^2 = 2\hbar^2$

$$J = \frac{1}{2} \text{ which gives } J^2 = J(J+1)\hbar^2 = \frac{3}{4}\hbar^2 \therefore H_2 = \frac{a}{2}\left[\frac{3}{4} - 2 - \frac{3}{4}\right]\hbar^2 = -a\hbar^2$$

$$\therefore \Delta H = H_1 - H_2 = -\frac{a}{8}\hbar^2 + a\hbar^2 = \left(\frac{-1+8}{8}\right)a\hbar^2 \Rightarrow \Delta H = \frac{7}{8}a\hbar^2$$

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- Q9. The spectral line corresponding to an atomic transition from $J = 1$ to $J = 0$ states splits in a magnetic field of 1 kG into three components separated by $1.6 \times 10^{-3} \text{ \AA}$. If the zero field spectral line corresponds to 1849 \AA , what is the g-factor corresponding to the $J = 1$ state?

(You may use $\frac{hc}{\mu_0} \approx 2 \times 10^4 \text{ cm}$)

- (a) 2 (b) 3/2 (c) 1 (d) 1/2

Ans: (c)

Solution: The Zeeman splitting is $\Delta E = (gM_J)(\mu_B B) = g\mu_B B$ for $M_J = +1$

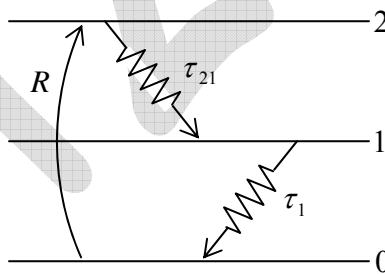
Given, Zeeman splitting separations $\Delta\lambda = \frac{\lambda^2}{c} \Delta\nu = 1.6 \times 10^{-3} \text{ \AA}$

$$\Rightarrow \Delta\nu = \frac{c}{\lambda^2} \Delta\lambda = \frac{3 \times 10^8}{(1849 \times 10^{-10})^2} \times 1.6 \times 10^{-1} \Rightarrow \Delta\nu = 0.1404 \times 10^{10}$$

$$\therefore g = \frac{\Delta E}{\mu_B B} = \frac{6.625 \times 10^{-34} \times 0.1404 \times 10^{10}}{9.27 \times 10^{-24} \times 0.1} = 1.00 \quad \therefore g = 1.0$$

NET/JRF (DEC-2012)

- Q10. Consider the energy level diagram shown below, which corresponds to the molecular nitrogen laser.



If the pump rate R is $10^{20} \text{ atoms cm}^{-3} \text{ s}^{-1}$ and the decay routes are as shown with $\tau_{21} = 20 \text{ ns}$ and $\tau_1 = 1 \mu\text{s}$, the equilibrium populations of states 2 and 1 are, respectively,

- (a) 10^{14} cm^{-3} and $2 \times 10^{12} \text{ cm}^{-3}$ (b) $2 \times 10^{12} \text{ cm}^{-3}$ and 10^{14} cm^{-3} .
(c) $2 \times 10^{12} \text{ cm}^{-3}$ and $2 \times 10^6 \text{ cm}^{-3}$ (d) zero and 10^{20} cm^{-3}

Ans: (b)

Solution: $\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}}$ and $\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1}$.

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Under equilibrium condition $\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$

$$\Rightarrow N_2 = \tau_{21}R = 20 \times 20 \times 10^{-9} = 2 \times 10^{12} \text{ cm}^{-3}$$

$$\Rightarrow N_1 = \frac{\tau_1 N_2}{\tau_{21}} = \frac{10^{-6} \times 2 \times 10^{12} \text{ cm}^{-3}}{20 \times 10^{-9}} = 10^{14} \text{ cm}^{-3}$$

Q11. Consider a hydrogen atom undergoing a $2P \rightarrow 1S$ transition. The lifetime t_{sp} of the $2P$ state for spontaneous emission is 1.6 ns and the energy difference between the levels is 10.2 eV. Assuming that the refractive index of the medium $n_0 = 1$, the ratio of Einstein coefficients for stimulated and spontaneous emission $B_{21}(\omega)/A_{21}(\omega)$ is given by

- (a) $0.683 \times 10^{12} \text{ m}^3 \text{ J}^{-1} \text{ s}^{-1}$ (b) $0.146 \times 10^{-12} \text{ Jsm}^{-3}$
 (c) $6.83 \times 10^{12} \text{ m}^3 \text{ J}^{-1} \text{ s}^{-1}$ (d) $1.463 \times 10^{-12} \text{ Jsm}^{-3}$

Ans: (a)

Solution: $n = 1$, $\Delta E = 10.2 \text{ eV}$ and $\frac{B_{21}}{A_{21}} = \frac{\pi^2 c^3}{\hbar \omega^3 n^3} = \frac{\hbar^2 \pi^2 c^3}{(\Delta E)^3 n_0^3} = 0.67 \times 10^{12}$.

Q12. Consider a He-Ne laser cavity consisting of two mirrors of reflectivities $R_1 = 1$ and $R_2 = 0.98$. The mirrors are separated by a distance $d = 20 \text{ cm}$ and the medium in between has a refractive index $n_0 = 1$ and absorption coefficient $\alpha = 0$. The values of the separation between the modes $\delta\nu$ and the width $\Delta\nu_p$ of each mode of the laser cavity are:

- (a) $\delta\nu = 75 \text{ kHz}, \Delta\nu_p = 24 \text{ kHz}$ (b) $\delta\nu = 100 \text{ kHz}, \Delta\nu_p = 100 \text{ kHz}$
 (c) $\delta\nu = 750 \text{ MHz}, \Delta\nu_p = 2.4 \text{ MHz}$ (d) $\delta\nu = 2.4 \text{ MHz}, \Delta\nu_p = 750 \text{ MHz}$

Ans: (a)

Solution: Mode separation $\delta\nu = \frac{c}{2dn_0} = 750 \text{ MHz}$

where $c = 3 \times 10^8 \text{ m/sec}$, $d = 20 \times 10^{-2} \text{ m}$ and $n_0 = 1$.

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$$\text{Width of each mode } \delta\nu_p = \frac{1}{2\pi t_c} \text{ where } t_c = \frac{2n_0 d}{c \ln\left(\frac{1}{R_1 R_2 e^{-2\alpha d}}\right)}$$

Note: In this question there is no need to calculate $\delta\nu_p$ since in the given options there is only one option with $\delta\nu = 750 \text{ MHz}$ and i.e option (a). You can calculate $\delta\nu_p$ without calculator so use your common sense.

NET/JRF (JUNE-2013)

Q13. A muon (μ^-) from cosmic rays is trapped by a proton to form a hydrogen-like atom. Given that a muon is approximately 200 times heavier than an electron, the longest wavelength of the spectral line (in the analogue of the Lyman series) of such an atom will be

- (a) 5.62 \AA (b) 6.67 \AA (c) 3.75 \AA (d) 13.3 \AA

Ans: (b)

Solution: In case of muonic atom, the reduce mass is $m' = \frac{m_\mu m_p}{m_\mu + m_p} = 180m_e$

$$E'_n = \left(\frac{m'}{m_e}\right) \frac{E_1}{n^2} = 180 \frac{E_1}{n^2} \text{ where, } E_1 = -13.6 \text{ eV}$$

For ground state of muonic atom $n=1$, $E'_1 = 180E_1$

For first excited state of muonic atom $n=2$, $E'_2 = 45E_1$

The longest wavelength of the photon corresponds to the transition between first and ground state of muonic atom.

The energy difference between first excited and ground state is

$$\Delta E = E'_2 - E'_1 = -135E_1 = 1836 \text{ eV} = 2938 \times 10^{-19} \text{ J}$$

In term of wavelength

$$\Delta E = hv = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2938 \times 10^{-19}} = 6.67 \times 10^{-10} \text{ m} = 6.67 \text{ \AA}$$

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Q14. Consider the hydrogen-deuterium molecule HD. If the mean distance between the two atoms is 0.08 nm and the mass of the hydrogen atom is $938 \text{ MeV}/c^2$, then the energy difference ΔE between the two lowest rotational states is approximately

- (a) 10^{-1} eV (b) 10^{-2} eV (c) $2 \times 10^{-2} \text{ eV}$ (d) 10^{-3} eV

Ans: (b)

Solution: Rotational energy expression $E = \frac{\hbar^2}{8\pi^2 I} J(J+1) = BJ(J+1)$

Difference between two lowest energy levels is $\Delta E = 2B$

where $B = \frac{\hbar^2}{8\pi^2 I} = \frac{\hbar^2}{2I}$ and $I = \mu r^2$

Here, $\mu = \frac{M_H M_D}{M_H + M_D} = \frac{M_H \times 2M_H}{M_H + 2M_H} = \frac{2}{3} M_H$, $\mu = \frac{2}{3} \times 938 \text{ MeV}/c^2$

$\hbar = 1.01 \times 10^{-34} \text{ J - sec} = 1.01 \times 10^{-34} \times \frac{1}{1.6} \times 10^{19} \text{ eV - sec} = 6.3 \times 10^{-16} \text{ eV - sec}$

$\therefore \Delta E = \frac{\hbar^2}{I} = \frac{(6.3 \times 10^{-16} \text{ eV - sec})^2}{\frac{2}{3} \times 938 \times 10^6 \text{ eV}/c^2 \times (0.08 \times 10^{-9} \text{ m})^2}$, $\Delta E = 9.2 \times 10^{-3} \text{ eV} \cong 10^{-2} \text{ eV}$

Q15. The electronic energy levels in a hydrogen atom are given by $E_n = -13.6/n^2 \text{ eV}$. If a selective excitation to the $n=100$ level is to be made using a laser, the maximum allowed frequency line-width of the laser is

- (a) 6.5 MHz (b) 6.5 GHz (c) 6.5 Hz (d) 6.5 kHz

Ans: (b)

Solution: $E_n = -13.6/n^2 \Rightarrow \Delta E_n = \frac{2 \times 13.6}{n^3} \text{ eV} \Rightarrow h\Delta\nu = \frac{2 \times 13.6}{n^3} \times 1.6 \times 10^{-19} \Rightarrow \Delta\nu = 6.5 \text{ GHz}$.

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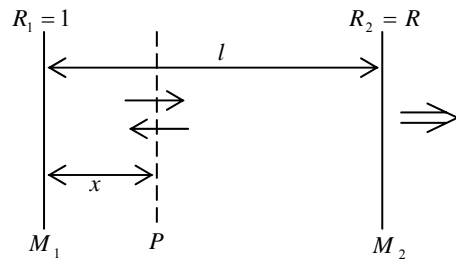
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Q16. Consider the laser resonator cavity shown in the figure.

If I_1 is the intensity of the radiation at mirror M_1 and α is the gain coefficient of the medium between the mirrors, then the energy density of photons in the plane P at a distance x from M_1 is



- (a) $(I_1 / c)e^{-\alpha x}$ (b) $(I_1 / c)e^{\alpha x}$
 (c) $(I_1 / c)(e^{\alpha x} + e^{-\alpha x})$ (d) $(I_1 / c)e^{2\alpha x}$

Ans: (c)

NET/JRF (DEC-2013)

Q17. A perturbation $V_{pert} = aL^2$ is added to the Hydrogen atom potential. The shift in the energy level of the $2P$ state, when the effects of spin are neglected up to second order in a , is

- (a) 0 (b) $2a\hbar^2 + a^2\hbar^4$ (c) $2a\hbar^2$ (d) $a\hbar^2 + \frac{3}{2}a^2\hbar^4$

Ans: (c)

Solution: For $2P$ state, $L=1$

The shift in the energy due to perturbation $V_{pert} = aL^2$ is

$$\Delta E = aL(L+1)\hbar^2 = a1(1+1)\hbar^2 = 2a\hbar^2$$

Q18. A gas laser cavity has been designed to operate at $\lambda = 0.5\mu m$ with a cavity length of $1m$. With this set-up, the frequency is found to be larger than the desired frequency by 100 Hz. The change in the effective length of the cavity required to retune the laser is

- (a) $-0.334 \times 10^{-12} m$ (b) $0.334 \times 10^{-12} m$ (c) $0.167 \times 10^{-12} m$ (d) $-0.167 \times 10^{-12} m$

Ans: (d)

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Q19. The spectroscopic symbol for the ground state of ${}_{13}\text{Al}$ is ${}^2P_{1/2}$. Under the action of a strong magnetic field (when $L-S$ coupling can be neglected) the ground state energy level will split into

- (a) 3 levels (b) 4 levels (c) 5 levels (d) 6 levels

Ans: (d)

Solution: In extremely strong magnetic field coupling between L-S breaks down. J is no longer a valid quantum number.

The ground state energy level will split into $(2S+1)(2L+1) = 6$

NET/JRF (JUNE-2014)

Q20. A spectral line due to a transition from an electronic state p to an s state splits into three Zeeman lines in the presence of a strong magnetic field. At intermediate field strengths the number of spectral lines is

- (a) 10 (b) 3 (c) 6 (d) 9

Ans: (a)

Solution: For p state: $l = 1, s = 1/2 : j = 1/2 \text{ \& } 3/2$.

This gives two spectral terms ${}^2P_{3/2}$ & ${}^2P_{1/2}$

For s state: $l = 0, s = 1/2 : j = 1/2$: This gives spectral terms ${}^2S_{1/2}$

The terms ${}^2P_{3/2}$ and ${}^2S_{1/2}$ corresponding to $J = 3/2$ & $J = 1/2$ will break into $2J+1$ Zeeman levels, which is 4 and 2 respectively.

For ${}^2P_{3/2}$ $M_j = -3/2 \quad -1/2 \quad +1/2 \quad +3/2$

For ${}^2S_{1/2}$ $M_j = -1/2 \quad +1/2$

The selection rule is $\Delta M_j = 0, \pm 1$ ($M_j = 0 \rightarrow M_j = 0$ If $\Delta J = 0$)

$\Delta M_j = 0$ gives π component, $\Delta M_j = \pm 1$ gives σ component

Number of π component = 2, Numbers of σ^+ components = 2

Number of σ^- components = 2

The terms ${}^2P_{1/2}$ and ${}^2S_{1/2}$ corresponding to $J = 1/2$ & $J = 1/2$ will break into $2J+1$ Zeeman levels, which is 2 & 2 respectively.

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For ${}^2P_{1/2}$ $M_j = -1/2, +1/2$, For ${}^2S_{1/2}$ $M_j = -1/2, +1/2$

The selection rule is $\Delta M_j = 0, \pm 1$ ($M_j = 0 \rightarrow M_j = 0$ If $\Delta J = 0$)

$\Delta M_j = 0$ gives π component, $\Delta M_j = \pm 1$ gives σ component

Number of π component = 2, Numbers of σ^+ components = 1

Number of σ^- components = 1

Thus, total number of Zeeman component = 10

Q21. A double slit interference experiment uses a laser emitting light of two adjacent frequencies ν_1 and ν_2 ($\nu_1 < \nu_2$). The minimum path difference between the interfering beams for which the interference pattern disappears is

- (a) $\frac{c}{\nu_2 + \nu_1}$ (b) $\frac{c}{\nu_2 - \nu_1}$ (c) $\frac{c}{2(\nu_2 - \nu_1)}$ (d) $\frac{c}{2(\nu_2 + \nu_1)}$

Ans: (c)

Solution: The condition of maximum intensity for interfering laser beam is: $d \sin \theta = n\lambda$

The condition of dark intensity for interfering laser beam is: $d \sin \theta = \left(n + \frac{1}{2}\right)\lambda$

For interference pattern to vanish the minimum path difference should be $\lambda/2$

The spectral bandwidth of laser is defined as $\Delta\nu = \frac{c\Delta\lambda}{\lambda^2}$ whereas $\frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda}$

For two closely spaced line of wavelength λ_1 and λ_2

$$\frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda} = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} = \frac{1}{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}} = \frac{c}{\frac{c}{\lambda_2} - \frac{c}{\lambda_1}} = \frac{c}{\nu_2 - \nu_1}$$

Since, for interference pattern to vanish for two closely spaced line of wavelength λ_1 and

λ_2 , the minimum path difference should be $= \frac{\lambda}{2} = \frac{c}{2\Delta\nu} = \frac{c}{2(\nu_2 - \nu_1)}$

Q22. How much does the total angular momentum quantum number J change in the transition of $Cr(3d^6)$ atom as it ionize to $Cr^{2+}(3d^4)$?

- (a) Increases by 2 (b) Decreases by 2 (c) Decreases by 4 (d) Does not change

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Ans: (c)

Solution: In $Cr(3d^6)$ state

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$

| | | | | |
|----|---|---|---|---|
| ↑↓ | ↑ | ↑ | ↑ | ↑ |
|----|---|---|---|---|

In this configuration, $S = 4 \times \frac{1}{2} = 2$ and $L = 2$

This is the case of more than half filled subshell, thus state with highest J value will have the lowest energy.

The high J - value is $J = L + S = 4$

Thus, the ground state spectral term for this configuration is 5D_4 .

Now in $Cr^{2+}(3d^4)$

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$

| | | | | |
|---|---|---|---|--|
| ↑ | ↑ | ↑ | ↑ | |
|---|---|---|---|--|

In this configuration, $S = 4 \times \frac{1}{2} = 2$ and $L = 2$

Since this is the case of less than half filled subshell, thus, state with lowest J value will have the lowest energy.

The lowest J - value is $J = L - S = 2 - 2 = 0$.

Thus the ground state spectral term for this configuration is the J - value decreases from $J = 4$ to $J = 0$.

Thus correct answer is option (c).

NET/JRF (DEC-2014)

- Q23. An atomic transition ${}^1P \rightarrow {}^1S$ in a magnetic field 1 Tesla shows Zeeman splitting. Given that the Bohr magneton $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$, and the wavelength corresponding to the transition is 250 nm, the separation in the Zeeman spectral lines is approximately
- (a) 0.01 nm (b) 0.1 nm (c) 1.0 nm (d) 10 nm

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Ans: (a)

Solution: This is the case of Normal Zeeman effect. The Zeeman separation in terms of frequency is

$$\Delta \nu = \frac{\mu_B B}{h} \text{ where } \mu_B \text{ is Bohr magneton}$$

In terms of wavelength it is $\Delta \lambda = \frac{\lambda^2}{c} \Delta \nu = \frac{\lambda^2}{c} \times \frac{\mu_B B}{h}$

$$\Delta \lambda = \frac{(250 \times 10^{-9} \text{ m})^2 \times (9.27 \times 10^{-24} \text{ J/T})(1 \text{ T})}{(3 \times 10^8 \text{ m/s})(6.625 \times 10^{-34} \text{ Js})} \cong 3 \times 10^{-12} \text{ m} \cong 0.003 \text{ nm}$$

None of the answer is matching correctly. But best suitable answer is option (a)

Q24. If the leading anharmonic correction to the energy of n -th vibrational level of a diatomic molecule is $-x_e \left(n + \frac{1}{2} \right)^2 \hbar \omega$ with $x_e = 0.001$, the total number of energy levels possible is approximately

- (a) 500 (b) 1000 (c) 250 (d) 750

Ans: (a)

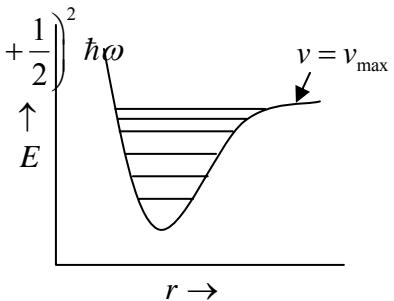
Solution: The energy of anharmonic oscillator is $E_v = \left(v + \frac{1}{2} \right) \hbar \omega - x_e \left(v + \frac{1}{2} \right)^2 \hbar \omega$

where $v = 0, 1, 2, \dots, v_{\max}$ is vibrational quantum number

$$\text{Now, } \left. \frac{dE_v}{dv} \right|_{v=v_{\max}} = 0 \Rightarrow \hbar \omega - 2x_e \left(v_{\max} + \frac{1}{2} \right) \hbar \omega = 0$$

$$\Rightarrow \hbar \omega = 2x_e \left(v_{\max} + \frac{1}{2} \right) \hbar \omega \Rightarrow 1 = 2x_e \left(v_{\max} + \frac{1}{2} \right)$$

$$\Rightarrow v_{\max} = \frac{1}{2x_e} - \frac{1}{2} \cong \frac{1}{2x_e} = \frac{1}{2 \times 0.001} = 500$$



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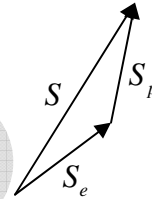
Q25. The effective spin-spin interaction between the electron spin \vec{S}_e and the proton spin \vec{S}_p in the ground state of the Hydrogen atom is given by $H' = a\vec{S}_e \cdot \vec{S}_p$. As a result of this interaction, the energy levels split by an amount

- (a) $\frac{1}{2}a\hbar^2$ (b) $2a\hbar^2$ (c) $a\hbar^2$ (d) $\frac{3}{2}a\hbar^2$

Ans: (c)

Solution: The Hamiltonian is given as $H = a\vec{S}_e \cdot \vec{S}_p$ where S_e and S_p are electron and proton spin.

The coupling between S_e and S_p gives net resultant spin angular momentum



$$\vec{S} = \vec{S}_e + \vec{S}_p$$

$$S^2 = S_e^2 + S_p^2 + 2\vec{S}_e \cdot \vec{S}_p \Rightarrow \vec{S}_e \cdot \vec{S}_p = \frac{1}{2}(S^2 - S_e^2 - S_p^2)$$

$$\therefore H = \frac{a}{2}(S^2 - S_e^2 - S_p^2)$$

where $S^2 = S(S+1)\hbar^2$, $S_e^2 = S_e(S_e+1)\hbar^2$, $S_p^2 = S_p(S_p+1)\hbar^2$

Since $S_e = \frac{1}{2}$ and $S_p = \frac{1}{2} \Rightarrow S = 0, 1$

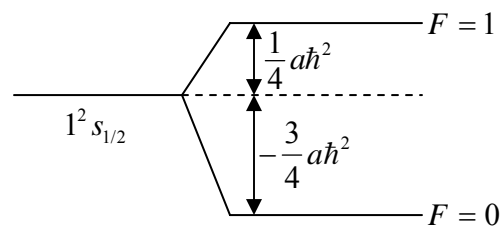
For $S = 0$ (singlet state)

$$H_1 = \frac{a}{2}\left(0 - \frac{3}{4} - \frac{3}{4}\right)\hbar^2 = -\frac{3}{4}a\hbar^2$$

For $S = 1$ (Triplet state)

$$H_2 = \frac{a}{2}\left(2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2\right) = \frac{1}{4}a\hbar^2$$

$$\therefore \Delta H = H_2 - H_1 = \frac{1}{4}a\hbar^2 + \frac{3}{4}a\hbar^2 \Rightarrow \Delta H = a\hbar^2$$



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NET/JRF (JUNE-2015)

Q26. Of the following term symbols of the np^2 atomic configurations, $^1S_0, ^3P_0, ^3P_1, ^3P_2$ and 1D_2 which is the ground state?

- (a) 3P_0 (b) 1S_0 (c) 3P_2 (d) 3P_1

Ans. (a)

Solution: According to Hund's rules

- (i) State with highest multiplicity has lowest energy
 (ii) State with same multiplicity, the state with highest L will have lowest energy
 (iii) State with same multiplicity and L value. The state with lowest J has lowest energy (only if subshell is less than half filled) from the given states $^1S_0, ^3P_0, ^3P_1, ^3P_2, ^1D_2$

The 3P_0 will have the lowest energy

Q27. A diatomic molecule has vibrational states with energies $E_v = \hbar\omega\left(v + \frac{1}{2}\right)$ and rotational states with energies $E_j = Bj(j+1)$, where v and j are non-negative integers. Consider the transitions in which both the initial and final states are restricted to $v \leq 1$ and $j \leq 2$ and subject to the selection rules $\Delta v = \pm 1$ and $\Delta j = \pm 1$. Then the largest allowed energy of transition is

- (a) $\hbar\omega - 3B$ (b) $\hbar\omega - B$ (c) $\hbar\omega + 4B$ (d) $2\hbar\omega + B$

Ans. (c)

Solution: $E = \hbar\omega\left(v + \frac{1}{2}\right) + BJ(J+1)$

For vibrational transition with $\Delta v = \pm 1$ and rotational transition with $\Delta J = \pm 1$ we get

$$\Delta E = E_{\text{initial}} - E_{\text{final}} = \hbar\omega + 2B(J+1)$$

where J is lowest quantum number

according to question in rotational states two transition is possible one for $J = 0 \rightarrow J = 1$

and second for $J = 1 \rightarrow J = 2$ but second transition will have photon of higher energy

\therefore For $J = 1 \rightarrow J = 2$

$$\Delta E = \hbar\omega + 2B(1+1) = \hbar\omega + 4B$$

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NET/JRF (DEC-2015)

Q28. The LS configurations of the ground state of ^{12}Mg , ^{13}Al , ^{17}Cl and ^{18}Ar are, respectively,

- (a) 3S_1 , $^2P_{1/2}$, $^2P_{1/2}$ and 1S_0 (b) 3S_1 , $^2P_{3/2}$, $^2P_{3/2}$ and 3S_1
 (c) 1S_0 , $^2P_{1/2}$, $^2P_{3/2}$ and 1S_0 (d) 1S_0 , $^2P_{3/2}$, $^2P_{1/2}$ and 3S_1

Ans.: (c)

Solution: $^{12}\text{Mg} : 1s^2 2s^2 2p^6 2s^2$

Ground state: 1s_0

$^{13}\text{Al} : 1s^2 2s^2 2p^6 3s^2 3p^1$

the terms are $2p_{1/2}$ and $2p_{3/2}$. Since its less than half filled. Thus

Ground state: $2p_{1/2}$

$^{17}\text{Cl} : 1s^2 2s^2 2p^6 3s^2 3p^5$

The terms are $2p_{1/2}$ and $2p_{3/2}$. Since it shell is more than half filled.

Thus Ground state: $2p_{3/2}$

$^{18}\text{Ar} : 1s^2 2s^2 2p^6 3s^2 3p^6$

Ground state: 1s_0

Q29. For a two level system, the population of atoms in the upper and lower levels are 3×10^{18} and 0.7×10^{18} , respectively. If the coefficient of stimulated emission is $3.0 \times 10^5 \text{ m}^3 / \text{W-s}^3$ and the energy density is $9.0 \text{ J} / \text{m}^3\text{-Hz}$, the rate of stimulated emission will be

- (a) $6.3 \times 10^{16} \text{ s}^{-1}$ (b) $4.1 \times 10^{16} \text{ s}^{-1}$ (c) $2.7 \times 10^{16} \text{ s}^{-1}$ (d) $1.8 \times 10^{16} \text{ s}^{-1}$

Ans.: **None of the answer is matching.**

Solution: The rate of stimulated emission is

$$\frac{dN_2}{dt} = N_2 B_{21} u(w)$$

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$$\text{where } N_2 = 3 \times 10^{18}, B_{21} = 3 \times 10^5 \frac{m^3}{W - s^3} = 3 \times 10^5 \frac{m^3}{J - s^2}$$

$$\text{and } u(w) = 9.0 \text{ J/n}^3 - \text{Hz} = 9.0 \frac{J - s}{m^3}$$

$$\frac{dN_2}{dt} = 3 \times 10^{18} \times 3 \times 10^5 \frac{m^3}{J - s^2} \times 9 \frac{J - s}{m^3} = 8.1 \times 10^{24} s^{-1}$$

Q30. The first ionization potential of K is 4.34 eV , the electron affinity of Cl is 3.82 eV and the equilibrium separation of KCl is 0.3 nm . The required to dissociate a KCl molecule into a K and a Cl atom is

- (a) 8.62 eV (b) 8.16 eV (c) 4.28 eV (d) 4.14 eV

Ans.: (c)

Solution: Energy required to dissociate kcl is ($kcl \rightarrow k^+ + cl^-$)

$$V = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_{12}} = \left(9 \times 10^9 \frac{Nm^2}{c^2} \right) \frac{(1.6 \times 10^{-19} c)}{0.3 \times 10^{-9} m} = 7.7 \times 10^{-19} \text{ J} = 4.79 \text{ eV}$$

The band dissociation energy is the energy required to dissociate a molecule into its component atom $kcl \rightarrow k + cl$

To find the energy required to dissociate kcl into k and cl , we must add an electron to the k^+ ion, which releases the atomic potassium ionization energy. Remove one electron from cl^- ion which requires the atomic chlorine electron affinity energy

Given ionization energy of $k = E_{ie} = 4.34 \text{ eV}$

Electron affinity of $cl = E_{ai} = 3.82 \text{ eV}$

Thus the energy gained in the transformation from ion to atom is

$$= E_{ie} - E_{ai} = 4.34 - 3.82 = 0.52 \text{ eV}$$

Thus dissociation energy = $4.79 - 0.52 = 4.27 \text{ eV}$

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