BHU (M.Sc.) 2018 (Solution)

No. of Questions: 120

Time: 2 Hours **Full Marks: 360**

Note: (1) Attempt as many questions as you can. Each question carries **3** marks. **One** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question.

(2) If more than one alternative answers seem to be approximate to the correct answer, choose the closest one.

Q1. In Sommerfeld's model of atom, the orbits characterized by a particular principal quantum number, n and different azimuthal quantum number $n_\theta = 1, 2, 3, ..., n$ have

(a) same energy

(b) energy in increasing order with n_θ

(c) energy in decreasing order with n_θ

(d) no associated energy

Ans. : (a)

Solution: In Sommerfield model of atom both radial and angular quantization has been assumed in order to explain fine structure of atomic line, but energy depends on radial quantization only.

Radial quantization $\rightarrow$ Principle quantum number (Decides energy)

Angular quantization $\rightarrow$ Azimuthal quantum number (Decides orientation only)

Same energy.

Q2. Stern-Gerlach experiment is important because it gives experimental verification of

(a) quantization of energy of atom

(b) orbital motion of electron

(c) electron spin

(d) Sommerfeld model of atom

Ans. : (c)

Solution: Out come of Stern-Gerlach experiment

(i) Quantization of angular momentum (Space quantization)

(ii) Spin of electron

Q3. The ratio of orbital magnetic dipole moment μ_l to the orbital angular momentum L of an electron in an orbit is given by

(a) $\frac{\mu_L}{L}=\frac{\mu_b}{\hbar}$ (b) $\frac{\mu_L}{L}=-\frac{\mu_b}{\hbar}$ (c) $\frac{\mu_L}{L}=-\frac{\mu_b}{2\hbar}$ (d) $\frac{\mu_L}{L}=\frac{\mu_b}{2\hbar}$

Ans. : (b)

Solution: $i=-\frac{e}{2\pi r/v}=-\frac{eV}{2\pi r}\pi r^2$

$$\Rightarrow \mu=-\frac{eVr}{2}=-\frac{emvr}{2m}$$

$$\mu=-\frac{e}{2m}L\Rightarrow \mu=-\frac{e\hbar}{2m}\sqrt{l(l+1)}$$

$$\mu=-\mu_B\sqrt{l(l+1)}$$

$$\mu=-\frac{\mu_B}{\hbar}L\Rightarrow \frac{\mu}{L}=-\frac{\mu_B}{\hbar}$$

Q4. Larmor Frequency is the frequency of precession of

(a) orbital angular momentum L about the external magnetic field, B

(b) spin angular momentum, S about the external magnetic field, B

(c) total angular momentum, J about the external magnetic field, B

(d) orbital angular momentum, L about the total angular momentum, J

Ans. : (c)

Solution: Precession of angular momentum around external field is called Larmor precession and precessional frequency is called Larmor frequency.

Precession of angular momentum L around external magnetic field B (for single electron system)

Precession of J around external magnetic field B (for multi-electron system)

Q5. On application of weak magnetic field the sodium line arising due to the transition ${}^2P_{3/2}\rightarrow {}^2S_{1/2}$ will split ideally into

(a) 2 components (b) 4 components

(c) 6 components (d) 10 components

Ans. : (d)

Q6. The half-width of gain profile of a He-Ne laser is $2\times10^{-3}\ nm$. If the length of the cavity is $30\ cm$, how many longitudinal modes can be excited? The emission wavelength is $6328\ \overset{0}{A}$

(a) 1 (b) 2 (c) 3 (d) 4

Ans. : (c)

Solution: For longitudinal modules

$$n\frac{\lambda}{2}=L\Rightarrow n=\frac{2L}{\lambda}=\frac{2\times0.3}{0.6328\times10^{-6}}=10^{6}\Rightarrow d\lambda=\frac{2L}{n^2}dn$$

$$d\lambda=\frac{2\times0.3}{n^2}\cdot1=\frac{0.6}{10^{12}}=0.6\times10^{-12}$$

$$\text{Number of modes }=\frac{\Delta\lambda}{d\lambda}=\frac{2\times10^{-18}}{0.6\times10^{-12}}=3.33\approx3$$

Q7. At what temperature; pressure remaining unchanged, will the molecular velocity (root mean square velocity) of hydrogen will be double of its value at NTP?

(a) $1092\ ^{o}C$ (b) $819\ ^{o}C$ (c) $1092\ ^{o}F$ (d) $819\ ^{o}K$

Ans. : (b)

Solution: $V_{rms}\propto\sqrt{T}\Rightarrow\frac{2V}{V}=\sqrt{\frac{T_2}{T_1(NTP)}}$

$$\frac{T_2}{T_1}=4\Rightarrow T_2=4\times273=1092^{0}K$$

$$T_2\left(^{0}C\right)=1092-273=819^{0}C$$

Q8. The mean square speed for the following group of particles (N_i represents the number of particles with speed v_i) will be

N_i	v_i (m/sec)
2	1.0
4	2.0
8	3.0

(a) $11.33\,m/\sec$ (b) $16.43\,m^2/\sec^2$

(c) $2.67\,m/\sec$ (d) $3.36\,m/\sec$

Ans. : (b)

Solution: $V_{rms}^2 = \frac{\sum N_i V_i^2}{\sum N_i} = \frac{2\times1^2+4\times2^2+8\times3^2}{2+4+8} = \frac{2+16+72}{14} = \frac{90}{14} = 6.42\,m^2/\sec^2$

Q9. The ratio between most probable speed and root mean square speed of a gas molecule is

(a) $\sqrt{\frac{3}{2}}$ (b) $\sqrt{\frac{3}{8\pi}}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{8}{3\pi}}$

Ans. : (c)

Solution: $V_{imp} = \sqrt{\frac{2kT}{m}} V_{rms} = \sqrt{\frac{3kT}{m}}$

$$\frac{V_{imp}}{V_{rms}} = \sqrt{\frac{2}{3}}$$

Q10. The mean free path of molecules of a gas at pressure p and T temperature is $2\times10^{-5}\,cm$. The mean free path at pressure $\frac{p}{2}$ and temperature $2T$ will be

(a) $10^{-5}\ cm$ (b) $8\times10^{-5}\ cm$ (c) $10^{-5}\ m$ (d) $8\times10^{-5}\ m$

Ans. : (b)

Solution: $\lambda_1 = \frac{k_B T}{\sigma P}$

$$\lambda_2 = \frac{k_B\times2T}{\sigma\times\frac{P}{2}} = \frac{4k_B T}{\sigma P}$$

$$\lambda_2 = 4\lambda_1 = 4\times2\times10^{-5} = 8\times10^{-5}\,cm$$

Q11. For the adiabatic expansion of a blackbody radiation enclosure, which of the following is correct?

(a) $V^{1/3}T =$ constant (b) $V.T =$ constant

(c) $V^{4/3}T =$ constant (d) $\frac{V}{T} =$ constant

where V is the volume and T is the temperature of the enclosure

Ans. : (a)

Solution: For adiabatic expansion

$$PV^{\gamma} = k$$

$\gamma = \frac{4}{3}$ (Assuming photon as monoatomic gas)

$$PV = RT \Rightarrow P = \frac{RT}{V}$$

$$\Rightarrow \frac{RT}{V} V^{\gamma} = k \Rightarrow TV^{\gamma-1} = TV^{1/3} = k'$$

Q12. In throttling process,

(a) the enthalpy remains constant
(b) temperature remains constant
(c) Gibbs' free energy remains constant
(d) entropy remains constant

Ans. : (a)

Solution: When a fluid expands from a region of high pressure to a region of low pressure through a porus plug without exchanging any energy as heat & work with surrounding the fluid is said to have undergone throttling process.

$$\delta\theta = 0 = U_2 - U_1 + p_2V_2 - p_1V_1$$

$$\Rightarrow U_1 + p_1V_1 = U_2 + p_2V_2$$

$$H_1 = H_2$$

i.e. isenthalpic process

i.e. enthalpy remains constant.

Q13. Which one of the following is correct?

(a) $\frac{E_{\lambda}}{T^4} = \text{constant}$
(b) $\frac{E_{\lambda}}{T^5} = \text{constant}$
(c) $\frac{E_{\lambda}}{T^2} = \text{constant}$
(d) $\frac{E_{\lambda}}{T} = \text{constant}$

where E_{λ} is spectral emissive power.

Ans. : (a)

Solution: $E_{\lambda} = \sigma T^4 \Rightarrow \frac{E_{\lambda}}{T^4} = \sigma$ (constant)

Q14. The numerical value of the slope of an isenthalpic curve at any point on a TP-diagram is called

(a) Joule coefficient
(b) Joule-Kelvin coefficient
(c) Van der Waals' constant
(d) Virial coefficient

Ans. : (b)

Solution: Slope of TP diagram

$$\frac{\partial T}{\partial p}$$

for isenthalpic curve $\left(\frac{\partial T}{\partial p}\right)_H = \mu$ (Joule Kelvin Coefficient)

Q15. Which of the following can be used to produce lowest temperature?

(a) Liquefaction of N_2.

(b) Liquid He

(c) Adiabatic demagnetization of paramagnetic salts

(d) None of these

Ans. : (c)

Solution: In the process of adiabatic demagnetization of a paramagnetic salt. Energy spent in doing magnetic work leads to fall in temperature below 1 K.

So, demagnetization of paramagnetic salt.

Q16. A mass m of water at T_1K is isobarically and adiabatically mixed with an equal mass of water at T_2K the entropy change of the universe is

(a) $2mC_p \ln\frac{(T_1+T_2)/2}{\sqrt{T_1T_2}}$

(b) $2m\ln\frac{(T_1+T_2)/2}{\sqrt{T_1T_2}}$

(c) $2C_p \ln\frac{(T_1+T_2)/2}{\sqrt{T_1T_2}}$

(d) $2mC_p$

Where C_p is specific heat at constant pressure.

Ans. : (a)

Solution: By principle of thermometry

$$mc_p\left(T_1 - T\right) = mc_p\left(T - T_2\right)$$

$\Rightarrow T = \frac{T_1+T_2}{2}$, T is equilibrium temperature

Now ΔS_1 due to cooling from T_1 to T

$$\Delta S_1 = mc_p \int_{T_1}^{T} \frac{dT}{T} = mc_p \ln \frac{T}{T_1}$$

$\Delta S_2 \Rightarrow$ during heating from T_2 to T

$$\Delta S_2 = mc_p \ln \frac{T}{T_2}$$

$$\Delta S = \Delta S_1 + \Delta S_2 = mc_p \ln \frac{T^2}{T_1 T_2} = mc_p \left(\frac{T}{\sqrt{T_1 T_2}}\right)^2$$

$$= 2mc_p \left(\frac{T + T_2/2}{\sqrt{T_1 T_2}}\right)$$

Q17. Thermodynamic equation

$$TdS = C_V dT + \frac{\beta T}{\kappa} dV \text{ is called}$$

(a) 2^{nd} TdS equation (b) 1^{st} TdS equation (c) 3^{rd} TdS equation (d) None of these

Where terms have their usual meanings.

Ans. : (b)

Solution: $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$ (First TdS equation)

Q18. Which of the following is correct?

(a) $C_P = \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_S$

(b) $C_P = T\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_S$

(c) $C_P = \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial P}{\partial T}\right)_S$

(d) $C_P = T\left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial P}{\partial T}\right)_S$

Q19. In one-dimensional elastic collision of two particles, the ratio of velocities of separation and approach is equal to:

(a) coefficient of restitution

(b) negative of coefficient of restitution

(c) zero, if collision is perfectly elastic

(d) infinite

Ans. : (a)

Solution: $e = \frac{v_2 - v_1}{u_2 - u_1}$

Q20. If in an elastic collision, a massive particle collides against a lighter One at rest:

(a) it can never bounce back along its original path

(b) it may bounce back along its original path

(c) the two particles move at right angles to each other after collision

(d) None of the above

Ans. : (a)

Solution: $P_{\text{before collision}} = mu$ but $v = -u$

$$P_{\text{after collision}} = mv \Rightarrow \Delta P = 2mu = mu - (-mu)$$

Momentum not conserved, so it can never bounce back.

Q21. In which of the following conditions, the total linear momentum of the system remains constant?

(a) If the resultant external force acting on the system of particles is zero

(b) If the resultant external force acting on the system of particles is positive

(c) If the resultant external force acting on the system of particles is -ve

(d) None of these

Ans. : (a)

Solution: $F = \dfrac{\partial P}{\partial t}$

Now if $F = 0 \Rightarrow \dfrac{\partial P}{\partial t} = 0 \Rightarrow \Delta P = 0 \Rightarrow P_1 = P_2$

i.e., Total force acting on body should be zero.

Q22. From the nozzle of a rocket $100\,kg$ of gases are exhausted per sec with a velocity of $1000 m/\text{sec}$. What force (thrust) does the gas exert on the rocket?

(a) 100 kg/sec (b) 10^5 Newton (c) 10^3 Newton (d) 100 Newton

Ans. : (b)

Solution: Exhausted force $= u\dfrac{dm}{dt} = 1000 \times 100 = 10^5 N$

Q23. If a particle is at rest relative to an observer at rest at the centre of a rotating frame of reference

(a) centrifugal and Coriolis forces both act (b) only centrifugal force acts

(c) only Coriolis force acts (d) None of these

Ans. : (b)

Solution: $F_{rot} = F + F_{fic}$

$$F_{fic} = -2m\Omega \times V_{rot} - m\Omega \times (\Omega \times r)$$

$V_{rot} = 0$ (given that particle is in rest in rotational form)

$F_{fic} = -m\Omega(\Omega \times r) \rightarrow$ only centrifugal force

Q24. The length of a rod, of length $5m$ in a frame of reference which is moving with $0.6\ c$ velocity in a direction making 30^0 angle with the rod is nearly

(a) $4.3m$ (b) $3.4\ m$ (c) $2.43\ m$ (d) $2.34\ m$

Ans. : (a)

Solution: $v_{along\ rod} = v\cos 30^0 = 0.6 \times \frac{\sqrt{3}}{2}$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}} = 5\sqrt{1-\frac{0.27c^2}{c^2}} = 5\sqrt{0.73} = 4.3m$$

Q25. π^+ meson decays into a μ^+ meson and a neutrino with a mean lifetime of about 2.5×10^{-8} sec in a frame in which it is at rest. If the velocity of the π^+ mesons in the laboratory frame be $0.9c$, then the expected lifetime in this frame is

(a) 5.7×10^{-8} sec (b) $2.5\times10^{-8}\ m/\sec$

(c) 3.1×10^{-8} sec (d) None of these

Ans. : (a)

Solution: $\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.5\times10^{-8}}{\sqrt{1-\frac{0.81c^2}{c^2}}} = \frac{2.5\times10^{8}}{0.43}$

$\approx 5.7\times10^{-8}$ sec

Q26. The speed of an electron having kinetic energy $2\,MeV$ will be

(a) $2.93\times10^{8}\ m/\sec$ (b) 3×10^{8} sec

(c) $10^{8}\ m/\sec$ (d) 1.5×10^{8} sec

Ans. : (a)

Solution: K.E. $= 2MeV$

$$m_0c^2 = 0.511MeV$$

$$KE = (\gamma - 1)m_0c^2$$

$$2 = (\gamma - 1)0.511$$

$$\gamma - 1 \approx 4 \Rightarrow \gamma = 5$$

$$\frac{1}{25} = 1 - \frac{v^2}{c^2}$$

$$v = \sqrt{0.96}\,c = 0.98c \Rightarrow v = 2.93 \times 10^8\ m/\sec$$

Q27. Which of the following relations is correct for modulus of rigidity η bulk modulus K and Poisson's ratio σ?

(a) $\sigma = \frac{K - 2\eta}{6K + 2\eta}$ (b) $\sigma = \frac{3K - 2\eta}{K + 2\eta}$

(c) $\sigma = \frac{3K - 2\eta}{6K + 2\eta}$ (d) $\sigma = \frac{K - 2\eta}{K + 2\eta}$

Ans. : (c)

Solution: $Y = 3K(1 - 2\sigma) = 2\eta(1 + \sigma)$

$$3K - 2\eta = (6K - 2\eta)\sigma$$

$$\sigma = \frac{3K - 2\eta}{6K + 2\eta}$$

Q28. Two wires A and B of the same material and equal length but of radii r and $2r$ are soldered coaxially, The free end of B is twisted by an angle Φ. The ratio of the twist at the junction and angle Φ is

(a) $\frac{16}{1}$ (b) $\frac{17}{16}$ (c) $\frac{16}{17}$ (d) $\frac{1}{16}$

Ans. : (c)

Solution: $Q = \sum \frac{TL}{JG}$

$T \rightarrow$ Shearing stress, $L \rightarrow$ length

$J \rightarrow$ tourque, $\quad G \rightarrow$ Modulus of rigidity

In question G, L and T are same

$$J \propto \frac{1}{r^4}$$

$$\frac{\frac{1}{r^4}}{\frac{1}{r^4}+\frac{1}{(2r)^4}}=\frac{16}{17}$$

Q29. Which of the following is true about liquid flow through a capillary tube?

(a)The velocity of the liquid layer in contact with the capillary tube is least

(b) The velocity of the liquid layer in contact with the capillary tube is maximum

(c) The velocity of the liquid layer at the centre of the capillary tube is minimum

(d) None of these

Ans. : (a)

Solution: Velocity of layer increases as layers moves away from contact surface.

Velocity of liquid layer in contact with capillary is least

Q30. The depletion region is created by

(a) ionization (b) diffusion (c) recombination (d) (a), (b) and (c)

Ans. : (c)

Solution: There is no charge resides in depletion region. It forms due to recombination.

Q31. A silicon diode is in series with a $1k\Omega$ resistor and a $5V$ battery. If the anode is connected to the $+ve$ battery terminal, the cathode voltage with respect to the negative battery terminal is

(a) $0.7\ V$ (b) $0.3\ V$ (c) $5.7V$ (d) $4.3V$

Ans. : (d)

Solution: Silicon diode act like battery of $0.7V$ with opposite polarity in forward bias. So

$V = 5 - 0.7 = 4.3$ volt.

Q32. Where will be the position of the Fermi level of the n-type material when $N_D = N_A$

(a) E_C (b) E_V (c) $\frac{E_C + E_V}{2}$ (d) None of the above

where terms have their usual meanings

Ans. : (c)

Solution: For Dopped semiconductor

$$E_f = \frac{E_C + E_V}{2} + \frac{kT}{2}\ln\left(\frac{N_D}{N_A}\right)$$

$$N_D = N_A \Rightarrow \ln(1) = 0 \Rightarrow E_f = \frac{E_C + E_V}{2}$$

Q33. The mobility of electrons in a material is expressed in unit of

(a) V/s (b) $\left(\frac{m^2}{V\sec}\right)$ (c) m^2/s (d) J/K

Ans. : (b)

Solution: $v_d = \mu E \quad \Rightarrow \mu = \frac{v_d}{E} = \frac{m/\sec}{Volt/m} = \frac{m^2}{V.\sec}$

Q34. A silicon sample is uniformly doped with 10^{16} phosphorous atoms/cm^3 and 2×10^{16} boron atoms/cm^3. If all the dopants are fully ionized, the material is

(a) n-type with carrier concentration of 3×10^{16}/cm^3

(b) p-type with carrier concentration of 10^{16}/cm^3

(c) p-type with carrier concentration of 4×10^{16}/cm^3

(d) intrinsic

Ans. : (b)

Solution: Boron will add a hole on ionization and Phosphorus will add an electron by ionization

Density $= N_h - N_e = 2\times10^{16} - 10^{16} = 10^{16}$ Hole $\Rightarrow$ P type

Q35. The bias condition for a transistor to be used as a linear amplifier is called

(a) forward-reverse (b) forward-forward

(c) reverse-reverse (d) collector bias

Ans. : (a)

Q36. Wien-bridge oscillators are based on

(a) positive feedback (b) negative feedback

(c) the piezoelectric effect (d) high gain

Ans. : (a)

Q37. Which of the following is a universal gate?

(a) OR gate (b)NOR gate (c) AND gate (d) NOT gate

Ans. : (b)

Solution: NAND and NOR gate are universal gate $\Rightarrow$ NOR

Q38. For an ideal dielectric, polarization $\vec{P}$ is given by

(a) $\vec{P} = \varepsilon_0 \vec{E}$ (b) $\vec{P} = (K-1)\varepsilon_0 \vec{E}$

(c) $\vec{P} = (K+1)\varepsilon_0 \vec{E}$ (d) $\vec{P} = \frac{\varepsilon_0}{K-1}\vec{E}$

Ans. : (b)

Solution: $D = \in_0 E + P \Rightarrow P = \in E - \in_0 E = \left(\frac{\in}{\in_0} - 1\right)\in_0 E$

$$P = (k-1)\in_0 E$$

Q39. Clausius-Mossotti relation is represented by the equation

(a) $\alpha = \frac{3\varepsilon_0}{n}\frac{K-1}{K+2}$ (b) $\alpha = \frac{\varepsilon_0}{n}\frac{K-1}{K+2}$

(c) $\alpha = \frac{\varepsilon_0}{3n}\frac{K-1}{K+2}$ (d) $\alpha = \frac{3\varepsilon_0}{n}\frac{K+2}{K-1}$

where symbols have their usual meanings

Ans. : (a)

Solution: $\frac{N\alpha}{3\in_0} = \frac{k-1}{k+2}$

Q40. The dipole moment of water molecule is $6.2\times10^{-30}\,C-m$ at $20\,^0C$. The polarizability α is

(a) $3.17\times10^{-39}\,C-m^2/V$ (b) $3.17\times10^{-37}\,C-m^2/V$

(c) $3.17\times10^{-35}\,C-m^2/V$ (d) $3.17\times10^{-33}\,C-m^2/V$

Ans. : (d)

Q41. The magnetic vectors are related as

(a) $\vec{B} = \mu_0\left(\vec{M} - \vec{H}\right)$ (b) $\vec{B} = \mu_0\left(\vec{M} + \vec{H}\right)$

(c) $\vec{B} = \mu_0^{-1}\left(\vec{M} + \vec{H}\right)$ (d) $\vec{B} = \mu_0^{-1}\left(\vec{M} - \vec{H}\right)$

Ans. : (b)

Solution: $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M} \Rightarrow \vec{B} = \mu_0\left(\vec{M} + \vec{H}\right)$

Q42. For higher values of temperature, the susceptibility of paramagnetic substances is proportional to

(a) T (b) $\frac{1}{T}$ (c) T^2 (d) $\frac{1}{T^2}$

Ans. : (b)

Solution: $\chi = \frac{C}{T-0}$ (Qurie-wises law) $\alpha \frac{1}{T}$

Q43. The loss of energy per hour in the iron core of a transformer, the hysteresis loop of which is equivalent in area to $2500\, ergs/cm^3$, is (given, frequency $= 50 Hz$, density of iron $= 7.5\, g/cm^3$ weight of the iron core $= 10\, kg$)

(a) $5.985 \times 10^2\ J$ (b) $5.985 \times 10^3\ J$

(c) $5.985 \times 10^4\ J$ (d) $5.985 \times 10^5\ J$

Ans. : (c)

Solution: Area of hysteresis loop gives the energy loss per cycle per unit volume.

$$W = 2500 \times 50 \times \frac{10000}{7.5} \times 3600$$

$$= 5.985 \times 10^{11} erg = 5.895 \times 10^4 J$$

Q44. A current i is flowing in a toroidal coil of circular cross-section of radius R with N number of turns distributed uniformly over its circumference. If A is the cross-sectional area of the toroid, its self-inductance will be

(a) $L = \frac{\mu_0 N^2 A}{2\pi R}$ (b) $L = \frac{\mu_0 N^2 A}{\pi R}$

(c) $L = \frac{\mu_0 N^2 A}{4\pi R}$ (d) $L = \frac{\mu_0 N^2 A}{2R}$

Ans. : (a)

Solution: $B = \frac{\mu_0 NI}{2\pi r}$

$$\Phi_B = \int B \cdot ds = \frac{\mu_0 NIA}{2\pi r}$$

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

Q45. Two inductors L_1 and L_2 are connected in series. The total inductance L will be

(a) $L = L_1 + L_2$ (b) $L = L_1 + L_2 + 2M$

(c) $L = L_1 + L_2 + M$ (d) $L = L_1 + L_2 - M$

where M is mutual inductance of two coils.

Ans. : (b)

Q46. A circuit containing resistor R_1, inductor L_1 and capacitor C_1 connected in series gives resonance at the same frequency f as the second similar combination R_2, L_2 and C_2. If the two circuits are connected in series, the whole circuit will resonate at the frequency

(a) $2f$ (b) $\frac{f}{2}$ (c) f (d) $\frac{f}{4}$

Q47. A capacitor of $250\,pF$ is connected in parallel with a coil having inductance of $16\,mH$ and effective resistance $20\ \Omega$. The circuit impedance at resonance is

(a) $3.2\times10^4\ \Omega$ (b) $3.2\times10^3\ \Omega$

(c) $3.2\times10^2\ \Omega$ (d) $3.2\times10^6\ \Omega$

Ans. : (d)

Solution: Impedance $Z_T = \frac{L}{RC} = 3.2\times10^6\ \Omega$

Q48. For dispersive medium, group velocity (v_g) and phase velocity (v_p) are related as

(a) $v_g = v_p + \lambda\frac{dv_p}{d\lambda}$ (b) $v_g = v_p - \lambda\frac{dv_p}{d\lambda}$

(c) $v_g = v_p + \frac{1}{\lambda}\frac{dv_p}{d\lambda}$ (d) $v_g = v_p - \frac{1}{\lambda}\frac{dv_p}{d\lambda}$

Ans. : (b)

Solution: $\omega = kV_p$

$$\frac{d\omega}{dk} = V_p + k\frac{dV_p}{dk} = V_p + k\frac{dV_p}{d\lambda}\frac{d\lambda}{dk}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} \Rightarrow d\lambda = \frac{-2\pi}{k^2}dk \Rightarrow \frac{d\lambda}{dk} = \frac{-2\pi}{k^2}\frac{dw}{dk}$$

$$= V_p + k\left(-\frac{2\pi}{k^2}\right)\frac{dV_p}{d\lambda} \quad = V_p - \lambda\frac{dV_p}{d\lambda}$$

Q49. Photon of energy 1.02 MeV undergoes Compton scattering through 180°. The energy of the scattered photon is

(a) 1.02 MeV (b) 0.204 MeV (c) 0.402 MeV (d) 0.240 MeV

Ans. : (b)

Solution: $E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{hc}{2m_0c^2}$

$$\Delta\lambda = \frac{2h}{m_0c} = \frac{2hc}{m_0c^2}$$

$$\lambda_s = \lambda + \Delta\lambda = \frac{hc}{2m_0c^2} + \frac{2hc}{m_0c^2} = \frac{5hc}{2m_0c^2}$$

$$E_s = \frac{hc}{\lambda_s} = \frac{hc}{5hc/2m_0c^2} = \frac{2m_0c^2}{5} = \frac{1.02}{5} = 0.204\ MeV$$

Q50. In Newton's ring experiment, the diameters of the bright rings are proportional to the

(a) natural number

(b) square root of natural numbers

(c) square root of odd numbers

(d) odd numbers

Ans. : (b)

Solution: $D_n = \sqrt{\frac{4n\lambda R}{\mu}} \Rightarrow D_n \propto \sqrt{n}$

Q51. A thin sheet of a transparent material of refractive index, $\mu = 1.50$ is placed in the path one of the interfering beams in a biprism experiment with a monochromatic source of wavelength,

$\lambda = 5000\,\overset{0}{A}$. The central fringe shifts to a position originally occupied by 10th bright fringe. the thickness of the sheet is

(a) $1\times10^{-5}\ m$ (b) $1.5\times10^{-5}\,m$ (c) $2\times10^{-5}\,m$ (d) $2.5\times10^{-5}\,m$

Ans. : (a)

Solution: $x_n = \dfrac{nD\lambda}{d}$ (without sheet)

$$x_n = \frac{D\{n\lambda + (\mu-1)t\}}{d} \quad \text{(with sheet)}$$

x_{10} (without sheet) $= x_0$ (with sheet)

$$\frac{10D\lambda}{d} = \frac{D(\mu-1)t}{d} \Rightarrow t = \frac{10\lambda}{(\mu-1)} = \frac{10\lambda}{.5} = 20\lambda = 10^{-5}m$$

Q52. Interference pattern is produced by two point sources S_1 and S_2 on a plane perpendicular to the line joining S_1 and S_2. What will be the shape of interference fringes?

(a) straight line (b) circular

(c) Parabolic (d) Hyperbolic

Ans. : (b)

Solution: fringes will be circular

Q53. In order to make a glass plate of refractive index, μ_g non-reflecting over a wide wavelength range around λ, a thin film is deposited on it. The refractive index μ_f and the thickness d of the film should be

(a) $\mu_f = \sqrt{\mu_g \mu_a}, d = \dfrac{3\lambda}{4\mu_f}$ (b) $\mu_f = \sqrt{\mu_g \mu_a}, d = \dfrac{\lambda}{4\mu_f}$

(c) $\mu_f = \sqrt{\mu_g / \mu_a}, d = \dfrac{\lambda}{4\mu_f}$ (d) $\mu_f = \sqrt{\mu_g / \mu_a}, d = \dfrac{3\lambda}{4\mu_f}$

Ans. : (b)

Solution: For non reflecting coating, reflected rays should interfere destructively

$$2\mu_f t\cos\theta = \left(n + \frac{1}{2}\right)\lambda \text{ and, for } n = 0$$

$$\Rightarrow t = \frac{\lambda}{4\mu_f}$$

Total transmission $T = T_{0f} T_{fg}$ should be maximum

$$\frac{dT}{d\lambda} = 0 \Rightarrow \mu_f = \sqrt{\mu_g \mu_a}$$

Q54. When the distance between two mirrors in Michelson interferometer decreased is

(a) the fringe pattern appears to collapse at the centre

(b) the fringe pattern expands

(c) the fringe pattern remains stable

(d) the shape of the fringe changes

Ans. : (a)

Solution: For Michelson interferometer

$$2d = \left(n + \frac{1}{2}\right)\lambda$$

$$d \downarrow \Rightarrow n \downarrow$$

So, fringe pattern will collapse at centre.

Q55. The spread of the central maximum in the Fraunhofer diffraction by a single slit is approximately given by

(a) $\frac{\lambda}{b} \le \theta \le \frac{\lambda}{b}$ (b) $\frac{2\lambda}{b} \le \theta \le \frac{2\lambda}{b}$

(c) $\frac{\lambda}{2b} \le \theta \le \frac{\lambda}{2b}$ (d) $\frac{\lambda}{b} \le \theta \le \frac{\lambda}{2b}$

where θ is diffraction angle, b is width of the slit and λ is the wavelength of the light used.

Ans. : (a)

Solution: In Fraunhofer diffraction by single slit first minima occurs for $b \sin\theta = \lambda$

$$\theta = \sin^{-1}\left(\frac{\lambda}{b}\right)$$

For small angle $\sin\theta \approx \theta \Rightarrow \frac{\lambda}{b} \le \theta \le \frac{\lambda}{b}$

Q56. A $2mW$ laser beams of wavelength $\lambda = 6\times10^{-5}$ cm is focussed on the retina by a human eye lens of focal length $f = 2.5$ cm and pupil diameter $2\,mm$. The intensity on the retina will be of the order of

(a) $10^4 W/m^2$ (b) $10^6 W/m^2$ (c) $10^8 W/m^2$ (d) $10^2 W/m^2$

Ans. : (c)

Solution: For a focused laser pulse

size of focussing spot $= \dfrac{f\lambda}{D}$

$D = 2mm,\ f = 2.5cm,\ \lambda = 6\times10^{-5}\ cm$

$= \dfrac{2.5\times10^{-2}\times6\times10^{-7}}{2\times10^{-3}}$

$2r_0 = 7.5\times10^{-6}\ m$

$$I = \frac{2P}{\pi r_0^2} = \frac{2\times10^{-3}}{\pi\left(7.5\times10^{-6}\right)^2} = 4.5\times10^8\ W/m^2$$

Q57. To increase the resolving power of a grating total number of lines on the grating is increased such that the grating element becomes 2.5λ. How many orders will be seen on the screen?

(a) First order only

(b) First and second orders only

(c) First, second and third orders only

(d) First, second, third and fourth orders only

Ans. : (b)

Solution: $(a+b)\sin\theta = n\lambda$

$(a+b) \rightarrow$ grating element

$(a+b)\sin\theta = 2.5\lambda\sin\theta = n\lambda$

Now, $\sin\theta|_{\max} = 1 \Rightarrow n = 2.5$

$\Rightarrow$ First and second order only

Q58. The radii of the circles of a zone plate is given by $r_n = 0.1\sqrt{n}\ cm$. The most intense focal point for wavelength $\lambda = 5\times10^{-5}\ cm$ will be at a distance

(a) $50\,cm$ (b) $100\ cm$ (c) $150\ cm$ (d) $200\ cm$

Ans. : (d)

Solution: For zone plates

$$r_n = \sqrt{n\lambda f} = 0.1\sqrt{n}$$

$$r_1 = 0.1,\quad r_2 = 0.1\times\sqrt{2} = 0.141$$

$$r_2^2 - r_1^2 = (2-1)\lambda f$$

$$0.01(2-1) = (2-1)\lambda f$$

$$f = \frac{0.01}{\lambda} = \frac{0.01}{5\times10^{-5}} = \frac{1\times10^{-2}\times10^{5}}{5} = 200\,cm$$

Q59. What is the minimum thickness of the base of a prism that can just resolve the two lines of sodium light centred at $5890\,\mathring{A}$ and $5896\,\mathring{A}$. The given value of refractive index of prism material is 1.6545 at wavelength $6563\,\mathring{A}$ and 1.6635 at wavelength $5270\,\mathring{A}$?

(a) $8\,mm$ (b) $10\,mm$ (c) $12\,mm$ (d)$14\ mm$

Ans. : (d)

Solution: Resolving power $= \dfrac{\lambda}{d\lambda} = b\dfrac{dn}{d\lambda}$

$$\frac{5893}{6} = b\frac{0.0090}{1290} \Rightarrow 1.4\,cm = 14\,mm$$

Q60. An unpolarized light is incident on a glass plate placed in air at polarizing angle. The reflected light is

(a) plane polarized with electric vector perpendicular to the plane of incidence

(b) plane polarized with electric vector parallel to the plane of incidence

(c) partially polarized having more electric field vectors perpendicular to the plane of incidence

(d) partially polarized having more electric field vectors parallel to the plane of incidence

Ans. : (c)

Solution: Partially polarised light with more electric vector perpendicular to plane of incidence.

Q61. A rigid body is constrained to move on plane. Number of degrees of freedom for it will be

(a) 2 (b) 1 (c) 5 (d) 3

Ans. : (c)

Solution: Rotational degree of freedom $=3$

Transitional of $=2$

Total $=5$

Q62. Number of generalized coordinates required. to describe the motion of a solid cylinder rolling without slipping on an inclined plane is

(a) 5 (b) 2 (c) 3 (d) 4

Ans. : (a)

Solution: Degree of freedom $=3$

Number of constants $=1$

Total $=3-1=2$

Q63. The constraints of a rigid body is

(a) conservative and scleronomic (b) conservative and rheonomic

(c) holonomic and rheonomic (d) non-holonomic and scleronomic

Ans. : (c)

Q64. Which one of the following represents the equation of motion for the system described by the Hamiltonian $H(q, p)$?

(a) $\dot{q}=\frac{\partial H}{\partial p}, \dot{p}=\frac{\partial H}{\partial q}$ (b) $-\dot{q}=\frac{\partial H}{\partial p}, \dot{p}=\frac{\partial H}{\partial q}$

(c) $\dot{q}=\frac{\partial H}{\partial p}, \dot{p}=-\frac{\partial H}{\partial q}$ (d) $\dot{q}=\frac{\partial H}{\partial q}, -\dot{p}=\frac{\partial H}{\partial p}$

Ans. : (a)

Q65. A particle of unit mass moves in a potential $V(x)=x^{3}-3x+2$. The angular frequency of small oscillation about the minimum of the potential is

(a) $\sqrt{6}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\frac{1}{\sqrt{3}}$

Ans. : (a)

Solution: $V(x) = x^3 - 3x + 2$

$$\frac{\partial V(x)}{\partial x} = 3x^2 - 3 \Rightarrow x = \pm 1$$

$$\frac{\partial^2 V(x)}{\partial x^2} = 6x \quad \Rightarrow k = \left.\frac{\partial^2 V}{\partial x^2}\right|_{x=1} = 6$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{1}} = \sqrt{6}$$

Q66. A system is described by the Lagrangian $L(r, \theta, \dot{r}, \dot{\theta}) = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{r}$.

Which one of the following is not true?

(a) Total energy of the system is conserved

(b) Angular momentum of the system is conserved

(c) θ is cyclic coordinate

(d) Linear momentum of system is conserved

Ans. : (d)

Solution: Linear momentum of system will not be conserved

Q67. If q_1 and q_2 are generalized coordinates and p_1 and p_2 are corresponding generalized momenta, then the Poisson bracket $\{q_1^2 + q_2^2, 2p_1 + p_2\}$ is

(a) 0 (b) $(q_1 + 2q_2)2p_1$ (c) $3(q_1^2 + q_2^2)$ (d) $2(2q_1 + q_2)$

Ans. : (d)

Solution: $\{f, g\} = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial q}$

$$f = q_1^2 + q_2^2 \quad \frac{\partial f}{\partial q_1} = 2q_1 \quad \frac{\partial f}{\partial q_2} = 2q_2 \quad \frac{\partial f}{\partial p_1} = 0 \quad \frac{\partial f}{\partial p_2} = 0$$

$$g = 2p_1 + p_2 \quad \frac{\partial g}{\partial q_1} = 0 \quad \frac{\partial g}{\partial q_2} = 0 \quad \frac{\partial g}{\partial p_1} = 2 \quad \frac{\partial g}{\partial p_2} = 1$$

$$\{f, g\} = (2q_1 \cdot 2 - 0) + (2q_2 \cdot 1 - 0) = 2(2q_1 + q_2)$$

Q68. Lagrangian for simple harmonic oscillator with frequency ω, mass m in one dimension is given by

(a) $\frac{1}{2}m\left(\dot{x}^2-\omega^2x^2\right)$ (b) $\frac{1}{2}m\left(\dot{x}^2+\omega^2x^2\right)$

(c) $\frac{1}{2}m\left(\ddot{x}+\omega^2x\right)$ (d) $\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2$

Ans. : (a)

Solution: $L=T-V$

$$T=\frac{1}{2}mv^2=\frac{1}{2}m\dot{x}^2,\ V=\frac{1}{2}kx^2=\frac{1}{2}m\omega_0^2x^2,\ L=\frac{1}{2}m\left(\dot{x}^2-\omega_0^2x^2\right)$$

Q69. The probability distribution of variable x in the range $-\infty$ to $+\infty$ is given by $P(x)=10e^{-\left[2x^2-4x-6\right]}$. The maximum probability will correspond to

(a) $x=1$ (b) $x=0$ (c) $x=3$ (d) $x=-1$

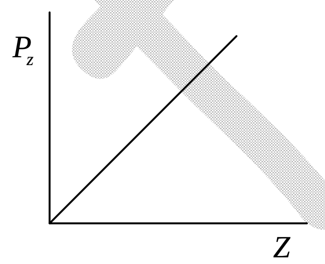
Ans. : (a)

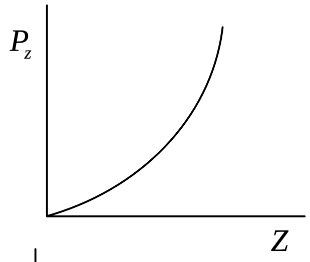
Solution: $P(x)=10\,e^{-\left(2x^2-4x-6\right)}$

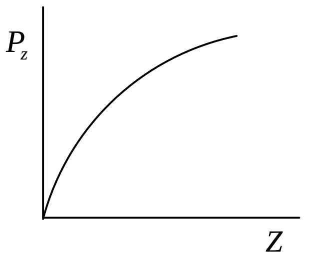
For maxima

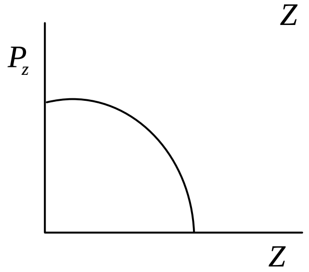
$$\frac{dP}{dx}=0\Rightarrow\frac{10\,e^{-\left(2x^2-4x-6\right)}}{-\left(4x-4\right)}=0\Rightarrow 4x-4=0\Rightarrow x=1$$

Q70. The phase space trajectory of a single particle, falling freely from a height will be

(a)


(b)


(c)


(d)


Ans. : (c)

Solution: $V = \sqrt{2gz}$

$P = m\sqrt{2gz}$

$P \propto \sqrt{z}$

Q71. Number of microstates for a monoatomic ideal gas with N molecules in a volume V and with total energy E is proportional to

(a) E^N (b) $E^{3N/2}$ (c) $E^{N/2}$ (d) E^{3N}

Q72. If Q be the partition function of a system of particles in canonical ensemble, the avenge energy of the system is given by

(a) $\bar{E} = \frac{\partial Q}{\partial \beta}$ (b) $\bar{E} = -\frac{\partial Q}{\partial \beta}$ (c) $\bar{E} = \frac{\partial}{\partial \beta}\ln Q$ (d) $\bar{E} = -\frac{\partial}{\partial \beta}\ln Q$

Ans. : (d)

Q73. Consider a system consisting of two particles each of which can be in any one of three quantum states $0, \varepsilon, 2\varepsilon$. The number of total configurations when the particles are identical bosons

(a) 9 (b) 6 (c) 5 (d) 3

Ans. : (b)

Solution: $W_1 = \frac{(n_i + g_i - 1)!}{n!(g_i - 1)!}, \; n_i = 2, \; g_i = 3$

$$= \frac{(2+3-1)!}{2!2!} = \frac{4\times 3\times 2!}{2!\times 2!} = 6$$

Q74. Consider a gas of photons in a cubical container of edge length L and volume $V = L^3$. The mean pressure in terms of mean energy E is given by

(a) $\frac{E}{V}$ (b) $\frac{2}{3}\frac{E}{V}$ (c) $\frac{1}{3}\frac{E}{V}$ (d) 0

Ans. : (b)

Solution: $PV = nkT$, $P = \frac{nkT}{V} = \frac{2}{3}\cdot\frac{3}{2}\frac{NkT}{V} = \frac{2}{3}\frac{E}{V}$

Q75. The statistical systems in which both energy and number of particles change are best described by

(a) micro-canonical ensemble theory

(b) canonical ensemble theory

(c) grand-canonical ensemble theory

(d) both canonical as well as grand-canonical ensemble theory

Ans. : (c)

Solution: Grand canonical ensemble

Q76. Relative root mean square fluctuation of energy in canonical ensemble theory is

(a) $\propto T^{1/2}$ (b) $\propto T$ (c) $\propto T^2$ (d) $\propto T^{3/2}$

Ans. : (a)

Solution: $\left\langle (\Delta E)^2 \right\rangle = \left\langle E^2 \right\rangle - \left\langle E \right\rangle^2 = kT^2 C_V$

$$\sqrt{\left\langle (\Delta E)^2 \right\rangle} = \sqrt{kT^2 C_V} \qquad \propto T$$

Q77. Given three isobars, namely; ${}^{25}_{11}Na, {}^{25}_{12}Mg$ and ${}^{25}_{13}Al$

(a) ${}^{25}_{11}Na$ is stable and the other two are beta emitters

(b) ${}^{25}_{12}Mg$ is stable and the other two are beta emitters

(c) ${}^{25}_{13}Al$ is stable and the other two are beta emitters

(d) All nuclei are stable

Ans. : (b)

Q78. Radiocarbon dating is done by estimating in the specimen

(a) the ratio of amount of ${}^{14}C$ to ${}^{12}C$ still present

(b) the ratio of amount of ${}^{13}C$ to ${}^{12}C$ still present

(c) the amount of radiocarbon still present

(d) the amount of ${}^{13}C$ still present

Ans. : (b)

Q79. The rate of electron emission from $4\,mg$ of ${}^{210}_{80}Pb$ with half-life 5 days is

(a) 1.84×10^{16} (b) 1.84×10^{13} (c) 9.2×10^{11} (d) 9.2×10^{16}

Ans. : (b)

Decay constant $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5\times 24\times 60\times 60}\sec^{-1}$

Number of atoms in $4\ mg$ is $N = \frac{4\times 10^{-3}\ g}{210}\times 6.023\times 10^{23}$ atoms

Hence, activity $R = \lambda N = 1.84\times 10^{13}\ decay/\sec$

Q80. A proton with $16\,MeV$ energy is bombarded on ${}_{84}^{216}Po$ nucleus. The proton is

(a) scattered
(b) reflected back
(c) captured
(d) transmitted through the nucleus

Ans. : (c)

Q81. The fission rate of ${}^{235}U$ to produce energy of $200\,MW$ is

(a) 6.25×10^{15} fission/sec
(b) 6.25×10^{16} fission/sec
(c) 6.25×10^{18} fission/sec
(d) 3.12×10^{20} fission/sec

Ans. : (c)

Solution: Energy released by fission of single nuclei $= 200\,MeV = 200\times 1.6\times 10^{-19}\ MJ$

$\because\ 3.2\times 10^{-17}\,MJ$ Energy is released by 1 fission/sec

$\therefore\ 1\,MJ$ Energy will be released by $= \frac{1}{3.2\times 10^{-17}}$ fission/sec

$\therefore\ 200\,MW$ Energy will be released by $= \frac{1\times 200}{3.2\times 10^{-17}} = 6.25\times 10^{18}$ fission/sec

Q82. The minimum temperature required to initiate fusion of deuteron and triton is of the order of

(a) $10^{9}\ K$
(b) $10^{6}\ K$
(c) $10^{13}\ K$
(d) $10^{15}\ K$

Ans. : (a)

Solution: According to Lawson criteria

$n\tau T \approx 10^{22}$

$n \approx 10^{19}$

$\tau \approx 10^{-6} \Rightarrow T \approx 10^{9}\,K$

Q83. The average velocity of nucleons inside the nucleus s of the order of

(a) $3\times10^{8}\ m/s$ (b) $6\times10^{7}\ m/s$ (c) $3\times10^{6}\ m/s$ (d) $6\times10^{6}\ m/s$

Ans. : (b)

Solution: $\Delta x\cdot\Delta p_x \approx \hbar$

$$\Delta x \approx 10^{-12}\ cm$$

$$\Delta p_x = m\Delta V \approx \frac{\hbar}{10^{-12}} \approx 5.275\, g\, cm/\sec$$

$$\Delta V \approx 3.17\times10^{7}\ m/\sec$$

Q84. The magnetic dipole and electric quadrupole moment data of deuteron imply that the that the nuclear force is

(a) purely central

(b) central and spin dependent

(c) mixture of central and non-central components

(d) velocity dependent

Ans. : (a)

Solution: $\psi_d = a_1\psi_{l=0} + a_2\psi_{l=1}$

$\Rightarrow$ Mixture of central and non-central forces

Q85. In a crystal, a lattice plane cuts intercepts of $2a, 3b$ and $6c$ along the axes where a, b, c are primitive vectors of the unit cell. The Miller indices of the given plane are

(a) (321) (b) (231) (c) (123) (d) (213)

Ans. : (a)

Solution: $\left(\frac{1}{2},\frac{1}{3},\frac{1}{6}\right)\times 6 \Rightarrow (321)$

Q86. The total number of Bravais lattice are

(a) 7 (b) 14 (c) 21 (d) 26

Ans. : (b)

Q87. Origin of characteristic X -rays is

(a) photoelectric effect (b) inverse photoelectric effect

(c) electronic transitions within atoms (d) Compton effect

Ans. : (c)

Solution: Electronic transition in the inner most shell causes the characteristic X - ray

Q88. The K_α line from Molybdenum has a wavelength of $0.7078\,\overset{0}{A}$. The wavelength of the K_α line of copper (given atomic number of Molybdemum $=42$, atomic number of copper $=29$)

(a) $1.517\,\overset{0}{A}$ (b) $1.157\,\overset{0}{A}$ (c) $1.175\,\overset{0}{A}$ (d) $1.715\,\overset{0}{A}$

Ans. : (a)

Solution: According to Mosselay's law

$$f=a(z-b)^2$$

For k_α lines $b=1$

$$\frac{c}{\lambda}=a(z-1)^2$$

$$\frac{\lambda_{Cu}}{\lambda_{Mo}}=\frac{(z_{Mo}-1)^2}{(z_{Cu}-1)^2}=\left(\frac{41}{28}\right)^2$$

$$\lambda_{Cu}=\left(\frac{41}{28}\right)^2\times 0.7078=1.517\,\overset{0}{\text{A}}$$

Q89. The relation of the reciprocal basis vector $\vec{A}$ to the direct basis vector $\vec{a}$ is given by

(a) $\vec{A}.\vec{a}=0$ (b) $\vec{A}.\vec{a}=2\pi$ (c) $\vec{A}.\vec{a}=\pi$ (d) $\vec{A}.\vec{a}=\frac{\pi}{2}$

Ans. : (b)

Solution: $\vec{A}\cdot\vec{a}=2\pi$

Q90. If current carriers are electrons, the Hall coefficient R_H is

(a) $R_H=-\frac{1}{ne}$ (b) $R_H=+\frac{1}{ne}$ (c) $R_H=\frac{n}{e}$ (d) $R_H=ne$

Ans. : (a)

Solution: For electrons $R_H=-\frac{1}{ne}$

Q91. The electron velocity v_F, at the Fermi surface is

(a) $\hbar\left(\frac{3\pi^2 N}{V}\right)^{1/3}$ (b) $\frac{\hbar}{m}\left(\frac{3\pi^2 N}{V}\right)^{1/3}$

(c) $\frac{\hbar}{m}\left(\frac{3\pi N}{V}\right)^{1/3}$ (d) $\frac{\hbar}{m}\left(\frac{\pi^2 N}{V}\right)^{1/3}$

Ans. : (c)

Solution: $E_f = \frac{1}{2}mv_f^2 = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$

$$\Rightarrow mv_f^2 = \frac{\hbar^2}{m^2}\left(\frac{3\pi^2 N}{V}\right)^{2/3} \Rightarrow v_f = \frac{\hbar}{m}\left(\frac{3\pi^2 N}{V}\right)^{1/3}$$

Q92. The Langevin function, $L(\alpha)$ represented by

(a) $L(\alpha) = \cot h\alpha$ (b) $L(\alpha) = \left[\cot h\alpha + \frac{1}{\alpha}\right]$

(c) $L(\alpha) = \left[\cot\ h\alpha - \frac{1}{\alpha}\right]$ (d) $L(\alpha) = (\cot h\alpha - \alpha)$

where the symbols have their usual meaning

Ans. : (c)

Solution: $L(\alpha) = \left[\coth \propto - \frac{1}{\alpha}\right]$

Q93. The curl of the electromagnetic intensity is

(a) conservative (b) rotational (c) divergent (d) static

Ans. : (a)

Q94. The direction of propagation of electromagnetic wave is given by

(a) $\vec{E}.\vec{B}$ (b) $\vec{E}$ (c) $\vec{E}\times\vec{B}$ (d) $\vec{B}$

Ans. : (c)

Solution: $\vec{E}\times\vec{B} = \hat{n}k$

$\hat{n}$ will be in direction of $\vec{E}\times\vec{B}$

Q95. For good conductors the skin depth varies inversely with

(a) ω (b) ω^2 (c) $\sqrt{\omega}$ (d) ω^4

Ans. : (c)

Solution: Skin depth $d=\sqrt{\frac{2}{\mu\omega\sigma}}\Rightarrow d\propto\frac{1}{\sqrt{\omega}}$

Q96. The divergence of the curl of a vector field is

(a) a scalar (b) a vector (c) zero (d) infinity

Ans. : (c)

Solution: $\nabla\cdot\left(\nabla\times\vec{A}\right)=0$ (Vector identity)

Q97. The charge build up in the capacitor is due to which quantity?

(a) Conduction current (b) Displacement current

(c) Convection current (d) Direct current

Ans. : (a)

Solution: Conduction current causes charge piling

Q98. In conductors, which condition will be true?

(a) $\sigma\omega\varepsilon>1$ (b) $\frac{\sigma}{(\omega\varepsilon)}>1$ (c) $\frac{\sigma}{(\omega\varepsilon)}<1$ (d) $\sigma\omega\varepsilon<1$

Ans. : (b)

Solution: $\sigma>>\omega\in\Rightarrow\frac{\sigma}{\omega\in}>>1$

Q99. the relation between the speed of light, permeability and permittivity is

(a) $c=\mu\varepsilon$ (b) $c=\frac{\mu}{\varepsilon}$ (c) $c=\frac{1}{\sqrt{\mu\varepsilon}}$ (d) $c=\frac{1}{\mu\varepsilon}$

Ans. : (c)

Q100. The phenomenon employed in the waveguide operation is

(a) reflection (b) refraction

(c) total internal reflection (d) absorption

Ans. : (c)

Q101. The metric of spherical polar coordinates are

(a) $h_{11}=r,h_{22}=1,h_{33}=r\sin\theta$ (b) $h_{11}=1,h_{22}=r,h_{33}=r\sin\theta$

(c) $h_{11}=r,h_{22}=r\sin\theta,h_{33}=1$ (d) $h_{11}=r^2,h_{22}=r^2\sin^2\theta,h_{33}=r^2\sin^2\theta$

Ans. : (b)

Solution: $h_{11}=1,\ \ h_{12}=r,\ \ h_{13}r\sin\theta$

Q102. Given the transformation $u=x+y, v=x-y$ and $du\ dv=kdx\ dy$, the value of k is

(a) 1 (b) -1 (c) 2 (d) $\frac{1}{2}$

Ans. : (c)

Solution: $u=x+y$, $v=x-y$

$$J=\frac{\partial(u,v)}{\partial(x,y)}=-2$$

$$\iint f(u,v)dudv=\iint f(u(x,y),v(x,y))|J|dxdy$$

$$\Rightarrow dudv=2dxdy\Rightarrow k=2$$

Q103. Given $A=\begin{pmatrix}0&1\\0&0\end{pmatrix}(aI+bA)^n$ is (where I is 2×2 unit vector)

(a) a^nI+b^nA (b) $a^nI+nab^{n-1}A$

(c) $a^nI+nabA$ (d) $a^nI+na^{n-1}bA$

Ans. : (d)

Solution: $A=\begin{pmatrix}0&1\\0&0\end{pmatrix}$

$$(aI+bA)=\begin{pmatrix}a&b\\0&a\end{pmatrix}$$

$$(aI+bA)^2=\begin{pmatrix}a&b\\0&a\end{pmatrix}^2=\begin{pmatrix}a^2&2ab\\0&a^2\end{pmatrix}$$

$$(aI+bA)^3=\begin{pmatrix}a&b\\0&a\end{pmatrix}^3=\begin{pmatrix}a^2&2ab\\0&a^2\end{pmatrix}\begin{pmatrix}a&b\\0&a\end{pmatrix}$$

$$=\begin{pmatrix}a^3&3a^2b\\0&a^3\end{pmatrix}$$

Q104. Eigenvectors of the matrix $\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ are

(a) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(d) $\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$

Ans. : (b)

Solution: $|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda & i \\ -i & \lambda \end{vmatrix} = 0$$

$$\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

Eigen vector corresponding to $+1$

$$\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = +\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} iy \\ -ix \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow y = -ix \Rightarrow \begin{pmatrix} 1 \\ -i \end{pmatrix} = \psi_1$$

Similarly, eigen vector corresponding to -1

$$\begin{pmatrix} 1 \\ i \end{pmatrix} = \psi_2$$

Now from Normalization

$$N^2 \psi_1^* \psi_2 = 1 \Rightarrow N^2\left(1 - i^2\right) = 1$$

$$N^2 \times 2 = 1 \Rightarrow N = \frac{1}{\sqrt{2}}$$

$$\psi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} \text{ and } \psi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

Q105. Given the matrix $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ with one of the eigenvalues equal to -3, the other two eigenvalues are

(a) $0,1$ (b) $0,-1$ (c) $0,2$ (d) $-3,5$

Ans. : (d)

Solution: $\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda = -3, -3, 5$

Q106. In the equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(q^2x^2 - m^2\right)y = 0$

(a) $x = 0$ and $x = \infty$ are ordinary points

(b) $x = 0$ and $x = \infty$ are regular singular points

(c) $x = 0$ is a regular singular point and $x = \infty$ is an irregular singular point

(d) $x = 0$ and $x = \infty$ are irregular singular points

Ans. : (c)

Solution: This is the Bessel's equation

Regular singular point $x = 0$

Irregular singular point $x = \infty$

Q107. One of the solutions of the equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 12y = 0$ is

(a) $H_4(x)$ (b) $P_3(x)$ (c) $L_4(x)$ (d) $J_4(x)$

Ans. : (b)

Solution: This is the Legendre equation with $n = 3$

$\Rightarrow P_3(x)$ will be solution

Q108. The Delta function $\delta\left(x^2 - a^2\right)$ is equal to

(a) $\delta(x+a)\delta(x-a)$ (b) $\delta(x+a)+\delta(x-a)$

(c) $\frac{1}{2|a|}\left[\delta(x+a)+\delta(x-a)\right]$ (d) $\delta(x+a)-\delta(x-a)$

Ans. : (c)

Solution: $\delta\left(x^2-a^2\right)=\delta\left\{(x+a)(x-a)\right\}$

$$=\frac{1}{2|a|}\left\{\delta(x+a)+\delta(x-a)\right\}$$

Q109. The Fourier coefficients of the function

$$f(x)=\begin{cases}0 & \text{for } -L\le x\le 0\\ 1 & \text{for } 0\le x\le L\end{cases}$$

expanded in Fourier series are

(a) $a_0=1, a_n=0, b_n=\frac{1}{n\pi}\left[1-(-1)^n\right]$

(b) $a_0=1, a_n=\left[1-(-1)^n\right], b_n=0$

(c) $a_0=1, a_n=0, b_n=0$

(d) $a_0=1, a_n=1, b_n=\frac{1}{n\pi}\left[1-(-1)^n\right]$

Ans. : (a)

Solution: $f(x)=\begin{cases}0 & -L\le x\le 0\\ 1 & 0\le x\le L\end{cases}$

$$a_0=\frac{1}{L}\int_0^{\infty}dx=1$$

$$a_n=\frac{1}{L}\int_0^{\infty}f(x)\sin\frac{n\pi x}{L}dx=0$$

$$b_n=\frac{1}{L}\int_0^{\infty}f(x)\cos\frac{n\pi x}{L}dx=\frac{1}{n\pi}\left[1-(-1)^n\right]$$

Q110. The operator $i\hbar\frac{d}{d\hat{x}}-\hat{x}$ in momentum basis is

(a) $i\hbar\frac{d}{d\hat{p}}-\hat{p}$ (b) $-i\hbar\frac{d}{d\hat{p}}-\hat{p}$ (c) $-i\hbar\frac{d}{d\hat{p}}+\hat{p}$ (d) $i\hbar\frac{d}{d\hat{p}}+\hat{p}$

Q111. If a^+ and a are creation and annihilation operators for SHO, then which of the following is not a Hermitian operator

(a) aa^++a^+a (b) aa^+-a^+a (c) $i\left(a^+-a\right)$ (d) $i\left(a^++a\right)$

Ans. : (a)

Solution: $a^\dagger$ and a are non hermitian

$aa^\dagger = N$ hermitian

$\left[a, a^\dagger\right] = 1 \rightarrow$ unit operator

$aa^\dagger + a^\dagger a \Rightarrow$ non hermitian

Q112. If the expectation value of the momentum operator in the normalized state $\psi(x)$ is $\langle p\rangle$, then expactation value of the momentum operator in the state $\psi_1(x) = e^{\frac{i}{\hbar}p_0 x}\psi(x)$ will be

(a) $\langle p\rangle + p_0$ (b) $\langle p\rangle - p_0$ (c) $\langle p\rangle$ (d) 0

Ans. : (b)

Solution: $\langle\psi|P|\psi\rangle = \langle p\rangle$ *where* $P = i\hbar\dfrac{\partial}{\partial x}$

$|\phi\rangle = e^{\frac{i}{\hbar}p_0 x}|\psi\rangle$

$\langle\psi_1|P|\psi_1\rangle = \langle\psi|p|\psi\rangle - p_0\langle\psi|\psi\rangle = \langle p\rangle - p_0$

Q113. The ground state wave function for a $1-d$ system described by the potential

$$V(x) = 0 \quad \text{for} -\frac{L}{2} \le x \le \frac{L}{2} \text{ is}$$
$$= \infty \qquad \text{elsewhere}$$

(a) $A\cos\dfrac{\pi x}{L}$ (b) $A\sin\dfrac{\pi x}{2L}$ (c) $A\sin\dfrac{\pi x}{L}$ (d) $A\cos\dfrac{\pi x}{2L}$

Ans. : (a)

Solution: Symmetric infinite potential

$A\cos\dfrac{\pi x}{L}$

Q114. A simple harmonic oscillator in one dimension has an eigenfunction (of the Hamiltonian) which vanishes 3 times in the interval $0 < x < \infty$ and is odd under parity. The energy eigenvalue for this state is

(a) $\dfrac{7}{2}\hbar\omega$ (b) $\dfrac{9}{2}\hbar\omega$ (c) $\dfrac{13}{2}\hbar\omega$ (d) $\dfrac{15}{2}\hbar\omega$

Ans. : (b)

Solution: Three cross over after $x=0$ will happen for $n=4$

$$\Rightarrow E_n=\left(4+\frac{1}{2}\right)\hbar\omega=\frac{9}{2}\hbar\omega$$

Q115. The raising and lowering of angular momentum operators are defined as

$L_{\pm}=L_x\pm iL_y$. The commutator $\left|L_-,L_z\right|$ is equal to

(a) $-2\hbar L_-$ (b) $\hbar L_-$ (c) $\hbar L_+$ (d) $-\hbar L_-$

Ans. : (b)

Solution: $\left[L_zL_-\right]=-\hbar L_-$

$$\left[L_-L_z\right]=\hbar L_-$$

Q116. The bound state energy for the state $\psi_{5,4,2}(r,\theta,\phi)$ in a H-atom problem is given by

(a) $-\dfrac{13.6}{5}eV$ (b) $-\dfrac{13.6}{25}eV$ (c) $-13.6\times5eV$ (d) $-13.6\times25eV$

Ans. : (b)

Solution: $\psi_{5,4,2}(r,\theta,\phi)\Rightarrow n=5$

$$E_n=-\frac{13.6}{n^2}=-\frac{13.6}{25}eV$$

Q117. In a H-atom problem if $L_z\psi_{3,2,-2}(r,\theta,\phi)=a\hbar\psi_{3,2,-2}(r,\theta,\phi)$, then a is

(a) 2 (b) -2 (c) $2\sqrt{3}$ (d) $\sqrt{6}$

Ans. : (b)

Solution: $\psi_{3,2,-2}(r,\theta,\phi)\Rightarrow m_l=-2$

$$L_z\left|\psi\right\rangle=m_l\hbar\left|\psi\right\rangle=-2\hbar\Rightarrow-2$$

Q118. ψ_1 and ψ_2 are the wave functions of two orthogonal states of a system belonging to the energy eigenvalues E and $-E$, respectively. In a measurement of energy of another state ψ of the system, the expectation value of energy is found to be $\dfrac{E}{2}$. ψ in terms of ψ_1 and ψ_2 is

(a) $\dfrac{\sqrt{3}}{2}\psi_1+\dfrac{1}{2}\psi_2$ (b) $\dfrac{1}{2}(\psi_1-\psi_2)$ (c) $\dfrac{1}{\sqrt{2}}(\psi_1-\psi_2)$ (d) $\dfrac{3}{4}\psi_1+\dfrac{1}{4}\psi_2$

Ans. : (c)

Solution: $H|\psi_1\rangle = E|\psi_1\rangle, H|\psi_2\rangle = -E|\psi_2\rangle$

$$\langle\psi|H|\psi\rangle = E/2 \Rightarrow \psi = \frac{1}{\sqrt{2}}(\psi_1 - \psi_2)$$

Q119. In any Bohr orbit of hydrogen atom, the ratio of the kinetic energy to the potential energy of the electron is

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{-1}{2}$ (d) -2

Ans. : (c)

Solution: K.E. $= \frac{1}{2}mv^2 = \frac{1}{8\pi\in_0}\frac{e^2}{r}$

P.E. $= -\frac{1}{4\pi\in_0}\frac{e^2}{r}$

$$\frac{K.E.}{P.E.} = -\frac{1}{2}$$

Q120. Considering the nuclear mass finite, the Rydberg constant is maximum for

(a) hydrogen atom (b) deuterium atom

(c) singly ionized helium atom (d) doubly ionized lithium atom

Ans. : (d)

Solution: Rydberg constant depend on the mass of nucleus heaviest nucleus will be doubly ionised lithium.