

ATOMIC AND MOLECULAR PHYSICS SOLUTIONS**GATE-2010**

Q1. To detect trace amounts of gaseous species in a mixture of gases, the preferred probing tool is

- (a) Ionization spectroscopy with X -rays (b) NMR spectroscopy
(c) ESR spectroscopy (d) Laser spectroscopy

Ans: (a)

Q2. A collection of N atoms is exposed to a strong resonant electromagnetic radiation with N_g atoms in the ground state and N_e atoms in the excited state, such that $N_g + N_e = N$. This collection of two-level atoms will have the following population distribution:

- (a) $N_g \ll N_e$ (b) $N_g \gg N_e$
(c) $N_g \approx N_e \approx N/2$ (d) $N_g - N_e \approx N/2$

Ans: (c)

Solution: In two level laser population inversion is possible to achieve at any power level. The maximum possible situation can be $N_g \approx N_e \approx \frac{N}{2}$

Q3. Two states of an atom have definite parities. An electric dipole transition between these states is

- (a) Allowed if both the states have even parity
(b) Allowed if both the states have odd parity
(c) Allowed if the two states have opposite parities
(d) Not allowed unless a static electric field is applied

Ans: (c)

Q4. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the

- (a) Visible range of frequencies (b) Infrared range of frequencies
(c) Ultraviolet range of frequencies (d) Microwave range of frequencies

Ans: (a)

Q5. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotation energy eigenvalues are

- (a) $\frac{\hbar^2}{2I}l(l+1) + \frac{\hbar^2 m_l^2}{2I} \left(\frac{1}{I_z} - \frac{1}{I} \right)$ (b) $\frac{\hbar^2}{2I}l(l+1)$
 (c) $\frac{\hbar^2 m_l^2}{2I} \left(\frac{1}{I_z} - \frac{1}{I} \right)$ (d) $\frac{\hbar^2}{2I}l(l+1) + \frac{\hbar^2 m_l^2}{2} \left(\frac{1}{I_z} + \frac{1}{I} \right)$

Ans: (a)

Solution: CH_3OH is example of symmetric rotator, where $I_x = I_y \neq I_z$, ($I_x + I_y = I$ and $I_z \neq I$)

The classical expression for energy is $E = \frac{1}{2I}(J_x^2 + J_y^2) + \frac{1}{2I_z}J_z^2$.

This can be expressed in term of $J^2 = J_x^2 + J_y^2 + J_z^2$ by adding and subtracting J_z^2

$$E = \frac{1}{2I}J^2 + \left(\frac{1}{2I_z} - \frac{1}{2I} \right) J_z^2.$$

Quantum mechanically $E = \frac{\hbar^2}{2I}J(J+1) + \frac{\hbar^2 m_l^2}{2} \left(\frac{1}{I_z} - \frac{1}{I} \right)$

Q6. Match the typical spectra of stable molecules with the corresponding wave-number range

- | | |
|----------------------------|--------------------------------------|
| 1. Electronic spectra | (i) 10^6 cm^{-1} and above |
| 2. Rotational spectra | (ii) $10^5 - 10^6 \text{ cm}^{-1}$ |
| 3. Molecule dissociation | (iii) $10^8 - 10^2 \text{ cm}^{-1}$ |
| (a) 1 – ii, 2 – i, 3 – iii | (b) 1 – ii, 2 – iii, 3 – i |
| (c) 1 – iii, 2 – ii, 3 – i | (d) 1 – i, 2 – ii, 3 – iii |

Ans: (b)

Q7. Consider the operations $P: \vec{r} \rightarrow -\vec{r}$ (parity) and $T: t \rightarrow -t$ (time reversal). For the electric and magnetic fields \vec{E} and \vec{B} , which of the following set of transformations is correct?

- | | |
|--|---|
| (a) $P: \vec{E} \rightarrow -\vec{E}, \vec{B} \rightarrow \vec{B};$
$T: \vec{E} \rightarrow \vec{E}, \vec{B} \rightarrow -\vec{B}$ | (b) $P: \vec{E} \rightarrow \vec{E}, \vec{B} \rightarrow \vec{B};$
$T: \vec{E} \rightarrow \vec{E}, \vec{B} \rightarrow \vec{B}$ |
| (c) $P: \vec{E} \rightarrow -\vec{E}, \vec{B} \rightarrow \vec{B};$
$T: \vec{E} \rightarrow -\vec{E}, \vec{B} \rightarrow -\vec{B}$ | (d) $P: \vec{E} \rightarrow \vec{E}, \vec{B} \rightarrow -\vec{B};$
$T: \vec{E} \rightarrow -\vec{E}, \vec{B} \rightarrow \vec{B}$ |

Ans: (b)

Common Data Questions 8 and 9:

In the presence of a weak magnetic field, atomic hydrogen undergoes the transition:

$${}^2P_{1/2} \rightarrow {}^2S_{1/2}, \text{ by emission of radiation}$$

Q8. The number of distinct spectral lines that are observed in the resultant Zeeman spectrum is

- (a) 2 (b) 3 (c) 4 (d) 6

Ans: (c)

Solution: ${}^2P_{1/2} \rightarrow {}^2S_{1/2}$ is sodium D1 lines and it has total 4 zeeman components.

Q9. The spectral line corresponding to the transition

$${}^2P_{1/2} \left(m_j = +\frac{1}{2} \right) \rightarrow {}^2S_{1/2} \left(m_j = -\frac{1}{2} \right)$$

is observed along the direction of the applied magnetic field. The emitted electromagnetic field is

- (a) Circularly polarized (b) Linearly polarized
(c) Unpolarized (d) Not emitted along the magnetic field direction

Ans: (a)

Solution: For ${}^2P_{1/2} \left(m_j = +\frac{1}{2} \right) \rightarrow {}^2S_{1/2} \left(m_j = -\frac{1}{2} \right)$

Here $\Delta m_j = +1$ gives σ^+ component.

In longitudinal observation σ^+ is circularly polarized.

GATE-2011

Q10. The population inversion in a two layer material **CANNOT** be achieved by optical pumping because

- (a) the rate of upward transitions is equal to the rate of downward transitions
(b) the upward transitions are forbidden but downward transitions are allowed
(c) the upward transitions are allowed but downward transitions are forbidden
(d) the spontaneous decay rate of the higher level is very low

Ans: (a)

Q11. A heavy symmetrical top is rotating about its own axis of symmetry (the z -axis). If I_1 , I_2 and I_3 are the principal moments of inertia along x , y and z axes respectively, then

- (a) $I_2 = I_3; I_1 \neq I_2$ (b) $I_1 = I_3; I_1 \neq I_2$ (c) $I_1 = I_2; I_1 \neq I_3$ (d) $I_1 \neq I_2 \neq I_3$

Ans: (c)

Q12. A neutron passing through a detector is detected because of

- (a) the ionization it produces (b) the scintillation light it produces
(c) the electron-hole pairs it produces
(d) the secondary particles produced in a nuclear reaction in the detector medium

Ans: (b)

Q13. An atom with one outer electron having orbital angular momentum l is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is

- (a) $2l + 2$ (b) $2l + 1$ (c) $2l$ (d) $2l - 1$

Ans: (b)

Q14. For a multi-electron atom l , L and S specify the one-electron orbital angular momentum, total orbital angular momentum and total spin angular momentum, respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by l , L and S are

- (a) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta l = 0, \pm 1$ (b) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta l = \pm 1$
(c) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta l = 0, \pm 1$ (d) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta l = \pm 1$

Ans: (b)

Q15. The lifetime of an atomic state is 1 nanosecond. The natural line width of the spectral line in the emission spectrum of this state is of the order of

- (a) 10^{-10} eV (b) 10^{-9} eV (c) 10^{-6} eV (d) 10^{-4} eV

Ans: (c)

Solution: $\Delta E = h\Delta\nu = \frac{h}{\Delta t} = \frac{6.625 \times 10^{-34} \text{ J} \cdot \text{s}}{10^{-9}} = \frac{6.625 \times 10^{-25}}{1.6 \times 10^{-19}} \text{ eV} = 4.14 \times 10^{-6} \text{ eV}$

Q16. The degeneracy of an excited state of nitrogen atom having electronic configuration $1s^2 2s^2 2p^2 3d^1$ is

- (a) 6 (b) 10 (c) 15 (d) 150

Ans: (b)

Solution: Degeneracy = $2(2l + 1)$

Q17. The far infrared rotational absorption spectrum of a diatomic molecule shows equilibrium lines with spacing 20 cm^{-1} . The position of the first Stokes line in the rotational Raman spectrum of this molecule is

- (a) 20 cm^{-1} (b) 40 cm^{-1} (c) 60 cm^{-1} (d) 120 cm^{-1}

Ans: (c)

Solution: Given $2B = 20 \text{ cm}^{-1} \Rightarrow B = 10 \text{ cm}^{-1}$

The position of the first Stokes line in the rotational Raman spectrum = $6B$

$$\therefore \Delta\bar{\nu} = 6B = 6 \times 10 = 60 \text{ cm}^{-1}.$$

GATE-2012

Q18. The ground state of sodium atom (^{11}Na) is a $^2S_{1/2}$ state. The difference in energy levels arising in the presence of a weak external magnetic field B , given in terms of Bohr magneton, μ_B , is

- (a) $\mu_B B$ (b) $2\mu_B B$ (c) $4\mu_B B$ (d) $6\mu_B B$

Ans: (b)

Solution: The energy separation in the Zeeman level is $\Delta E = gM_J(\mu_B B)$

For $^2S_{1/2}$ state; $g = 2$ and $M_J = \pm \frac{1}{2}$. Therefore $\Delta E_1 = +(\mu_B B)$ and $\Delta E_2 = -(\mu_B B)$.

Thus $\Delta E = 2\mu_B B$

Q19. The first Stokes line of a rotational Raman spectrum is observed at 12.96 cm^{-1} .

Considering the rigid rotor approximation, the rotational constant is given by

- (a) 6.48 cm^{-1} (b) 3.24 cm^{-1} (c) 2.16 cm^{-1} (d) 1.62 cm^{-1}

Ans: (c)

Solution: The first Stokes line of the Rotational Raman spectrum lies at $= 6B$

$$\text{Thus } 6B = 12.96 \text{ cm}^{-1} \Rightarrow B = 2.16 \text{ cm}^{-1}.$$

Q20. Match the typical spectroscopic regions specified in **Group I** with the corresponding type of transitions in **Group II**.

Group I

(P) Infra-red region

(Q) Ultraviolet-visible region

(R) X-ray region

(S) γ -ray region

(a) (P, i); (Q, iii); (R, ii); (S, iv)

(c) (P, iii); (Q, i); (R, iv); (S, ii)

Group II

(i) electronic transitions involving valence electrons

(ii) nuclear transitions

(iii) vibrational transitions of molecules

(iv) transitions involving inner shell electrons

(b) (P, ii); (Q, iv); (R, i); (S, iii)

(d) (P, iv); (Q, i); (R, ii); (S, iii)

Ans: (c)

Q21. The term $\{j_1, j_2\}_J$ arising from $2s^1 3d^1$ electronic in $j-j$ coupling scheme are

(a) $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

(c) $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

(b) $\left\{\frac{1}{2}, \frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2}, \frac{3}{2}\right\}_{2,1}$

(d) $\left\{\frac{3}{2}, \frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2}, \frac{5}{2}\right\}_{3,2}$

Ans: (a)

Q22. The equilibrium vibration frequency for an oscillator is observed at 2990 cm^{-1} . The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96.

Considering the oscillator to be anharmonic, the anharmonicity constant is

(a) 0.005

(b) 0.02

(c) 0.05

(d) 0.1

Ans: (b)

Solution: $\omega_e(1 - 2x_e) = 2990 \text{ cm}^{-1}$ and $\frac{2\omega_e(1 - 3x_e)}{\omega_e(1 - 2x_e)} = 1.96 \Rightarrow \frac{(1 - 3x_e)}{(1 - 2x_e)} = 0.98 \Rightarrow x_e = 0.02$.

GATE-2013

Q23. The number of spectral lines allowed in the spectrum for the $3^2D \rightarrow 3^2P$ transition in sodium is _____.

Ans: 28

Solution: The numbers of Zeeman components for $^2D_{5/2} \rightarrow ^2P_{3/2}$ transition = 12

The numbers of Zeeman components for $^2D_{3/2} \rightarrow ^2P_{3/2}$ transition = 10

The numbers of Zeeman components for $^2D_{3/2} \rightarrow ^2P_{1/2}$ transition = 6

- Q24. In a normal Zeeman Effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition $5^1D_2 \rightarrow 5^1P_1$ of cadmium atoms is to be observed. The spectrometer has a resolution of 0.01 nm. Minimum magnetic field needed to observe this is ($m_e = 9.1 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$, $c = 3 \times 10^8 \text{ m/s}$)
- (a) 0.26T (b) 0.52T (c) 2.6T (d) 5.2T

Ans: (b)

Solution: Separation of Zeeman Components

$$\Delta \nu = \frac{eB}{4\pi m} \Rightarrow \Delta \lambda = \frac{\lambda^2}{c} \Delta \nu = \frac{\lambda^2}{c} \frac{eB}{4\pi m}$$

$$B = \frac{4\pi mc}{e} \frac{\Delta \lambda}{\lambda^2} = \frac{4 \times 3.14 \times 9.1 \times 10^{-31} \times 3 \times 10^8}{1.6 \times 10^{-19}} \times \frac{0.01 \times 10^{-9}}{(643.8 \times 10^{-9})^2} = 0.514T$$

- Q25. The spacing between vibrational energy levels in CO molecule is found to be $8.44 \times 10^{-2} \text{ eV}$. Given that the reduced mass of CO is $1.14 \times 10^{-26} \text{ kg}$, Planck's constant is $6.626 \times 10^{-34} \text{ Js}$ and $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$. The force constant of the bond in CO molecule is
- (a) 1.87 N/m (b) 18.7 N/m (c) 187 N/m (d) 1870 N/m

Ans: (c)

Solution: The energy of the quantum harmonic oscillator is

$$E = h\nu \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2, \dots$$

$$\text{The frequency of oscillation is } \nu = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}}$$

Where k = Spring constant and μ = reduced mass

$$\text{The energy levels are equally spaced with energy separation of } \Delta E = h\nu = \frac{h}{2\pi} \sqrt{\frac{k}{\mu}}$$

$$k = \left(\frac{2\pi}{h} \Delta E \right)^2 \mu = \left(\frac{2 \times 3.14}{6.626 \times 10^{-34}} \times 8.44 \times 10^{-2} \times 1.6 \times 10^{-19} \right)^2 \times 1.14 \times 10^{-26} = 186.7 \text{ N/m}$$

GATE-2014

Q26. The number of normal Zeeman splitting components of $^1P \rightarrow ^1D$ transition is

- (a) 3 (b) 4 (c) 8 (d) 9

Ans: (a)

Solution: This is singlet transition.

Q27. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B . If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are

- (a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$
 (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

Ans: (b)

Solution: $\frac{J_A(J_A+1)}{J_B(J_B+1)} = \frac{I_B}{I_A} = \frac{I_B}{6I_B} \Rightarrow J_A = 6, J_B = 1$

Q28. The value of the magnetic field required to maintain non-relativistic protons of energy $1MeV$ in a circular orbit of radius $100mm$ is _____ Tesla

(Given: $m_p = 1.67 \times 10^{-27} kg$, $e = 1.6 \times 10^{-19} C$)

Ans: 1.44

Solution: $\frac{mv^2}{r} = qvB$, $E = \frac{1}{2}mv^2 \Rightarrow B = \frac{\sqrt{2mE}}{qr} = 1.44$

Q29. Neutrons moving with speed $10^3 m/s$ are used for the determination of crystal structure.

If the Bragg angle for the first order diffraction is 30° the interplanar spacing of the

crystal is _____ \AA . (Given: $m_n = 1.675 \times 10^{-27} kg$ $h = 6.626 \times 10^{-34} J.s$)

Ans: 4

Solution: $2d \sin \theta = \lambda = \frac{h}{mv} \Rightarrow 2d \sin 30^\circ = \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times 10^3} \Rightarrow d = 4 \text{\AA}$

Q30. The emission wavelength for the transition $D_2 \rightarrow F_3$ is 3122\AA . The ratio of population of the final to the initial states at a temperature $5000K$ is
 ($h = 6.626 \times 10^{-34} J.s$, $c = 3 \times 10^8 m/s$, $k_B = 1.380 \times 10^{-23} J/K$)

- (a) 2.03×10^{-5} (b) 4.02×10^{-5} (c) 7.02×10^{-5} (d) 9.83×10^{-5}

Ans: (c)

$$\text{Solution: } \frac{N_F}{N_I} = \frac{(2J_f + 1)}{(2J_i + 1)} e^{-\frac{hc}{\lambda k_B T}} = \frac{5}{7} e^{-9.227641144} = 7.02 \times 10^{-5}$$

GATE-2015

Q31. In a rigid rotator of mass M , if the energy of the first excited state is (1 meV) , then the fourth excited state energy (in meV) is _____.

Ans.: 10

Solution: $\because E \propto J(J+1)$ where $J = 0, 1, 2, 3, \dots$

$$\Rightarrow \frac{E_4}{E_1} = \frac{4(4+1)}{1(1+1)} \Rightarrow E_4 = 10E_1 = 10 \text{ meV}, \text{ where } J = 0, 1, 2, 3, \dots$$

Q32. The binding energy per molecule of NaCl (lattice parameter is 0.563 nm) is 7.956 eV .

The repulsive term of the potential is of the form $\frac{K}{r^9}$, where K is a constant. The value of the Modelung constant is _____ (upto three decimal places)

(Electron charge $e = -1.6 \times 10^{-19} \text{ C}$; $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$)

Ans.: 2.80

Solution: The total energy of one ion due to the presence of all others in NaCl crystal is (considering univalent ions)

$$U(r) = -\frac{Ae^2}{4\pi\epsilon_0 r} + \frac{K}{r^n}, \text{ where } A \text{ is Modelung Constant.}$$

The potential energy will be minimum at the equilibrium spacing r_0 .

$$\text{Thus } \left[\frac{dU}{dr} \right]_{r=r_0} = \left[-\frac{Ae^2}{4\pi\epsilon_0 r_0^2} - \frac{Kn}{r_0^{n+1}} \right] = 0 \Rightarrow K = \frac{Ae^2 r_0^{n-1}}{4\pi\epsilon_0 n}$$

Thus, Binding energy of molecule or lattice energy is

$$U_0 = [U]_{r=r_0} = -\frac{Ae^2}{4\pi\epsilon_0 r_0} + \frac{Ae^2 r_0^{n-1}}{4\pi\epsilon_0 n r_0^n} = -\left[\frac{Ae^2}{4\pi\epsilon_0 r_0} \right] \left[\frac{n-1}{n} \right]$$

Given repulsive term of the potential is $\frac{K}{r^9}$ ($\because n = 9$)

Also binding energy per molecule is $U_0 = 7.95 \text{ eV}$

The Madelung constant is $A = U_0 \times \frac{4\pi\epsilon_0 r_0}{e^2} \times \frac{n}{n-1}$ and the lattice parameter is

$a = 0.563 \text{ nm}$. Thus the interatomic separation is $r_0 = \frac{a}{2} = 0.28 \text{ nm}$.

$$\Rightarrow A = 7.95 \times 1.67 \times 10^{-19} \text{ J} \times \frac{4 \times 3.14 \times (8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}) \times (0.282 \times 10^{-9})}{(1.67 \times 10^{-19} \text{ J})^2} \times \frac{9}{8}$$

$$\Rightarrow A = \frac{7.95 \times 1.67 \times 4 \times 3.14 \times 8.85 \times 0.282 \times 9}{1.67 \times 8} \times 10^{-2} \Rightarrow A = 2.80$$

Q33. Match the phrases in Group I and Group II and identify the correct option.

Group I

Group II

(P) Electron spin resonance (ESR)

(i) radio frequency

(Q) Nuclear magnetic resonance (NMR)

(ii) visible range frequency

(R) Transition between vibrational states of a molecule

(iii) microwave frequency

(S) Electronic transition

(iv) far-infrared range

(a) (P-i), (Q-ii), (R-iii), (S-iv)

(b) (P-ii), (Q-i), (R-iv), (S-iii)

(c) (P-iii), (Q-iv), (R-i), (S-ii)

(d) (P-iii), (Q-i), (R-iv), (S-ii)

Ans.: (d)

Solution: (P) Electron spin resonance (ESR) is achieved by Microwave frequency (iii)

(Q): Nuclear magnetic resonance (NMR) is achieved by Radio frequency (i)

(R): Transition between vibrational states of a molecule is achieved by radiation of far infrared range (iv)

(S): Electronic transition is achieved by visible radiation (ii)

Q34. The excitation wavelength of laser in a Raman effect experiment is 546 nm . If the Stokes' line is observed at 552 nm , then the wavenumber of the anti-Stokes' line (in cm^{-1}) is _____

Ans.: 18514

Solution: Raman displacement is

$$\Delta \bar{\nu} = \bar{\nu}_{AS} - \bar{\nu}_0 = \bar{\nu}_0 - \bar{\nu}_S \text{ or } \Delta \bar{\nu} = \frac{1}{\lambda_{AS}} - \frac{1}{\lambda_0} = \frac{1}{\lambda_0} - \frac{1}{\lambda_S}$$

where λ_{AS} , λ_0 , λ_S are wavelength of anti-stoke, exciting & stoke line.

From above relation we can write

$$\frac{1}{\lambda_{AS}} - \frac{1}{\lambda_0} = \frac{1}{\lambda_0} - \frac{1}{\lambda_S} \Rightarrow \frac{1}{\lambda_{AS}} = \frac{2}{\lambda_0} - \frac{1}{\lambda_S} \Rightarrow \frac{1}{\lambda_{AS}} = \frac{2\lambda_S - \lambda_0}{\lambda_0\lambda_S} \Rightarrow \lambda_{AS} = \frac{\lambda_0\lambda_S}{2\lambda_S - \lambda_0}$$

$$\therefore \lambda_{AS} = \frac{(546 \times 10^{-9} m)(552 \times 10^{-9} m)}{(2 \times 552 \times 10^{-9} m - 546 \times 10^{-9} m)} = \frac{546 \times 552}{558} \times 10^{-9} m$$

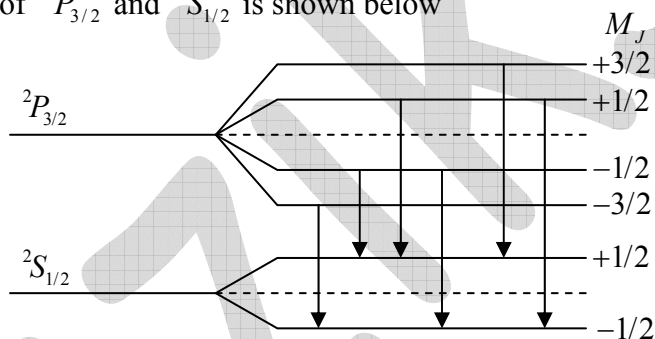
$$\Rightarrow \lambda_{AS} = 540.129 \times 10^{-9} m = 540.129 \times 10^{-7} cm$$

Anti-stoke wavenumber is $\bar{\nu}_{AS} = \frac{1}{\lambda_{AS}} = \frac{1}{540.129 \times 10^{-7} cm} = 18514 cm^{-1}$

Q35. The number of permitted transitions from ${}^2P_{3/2} \rightarrow {}^2S_{1/2}$ in the presence of a weak magnetic field is _____

Ans. : 6

Solution: Zeeman splitting of ${}^2P_{3/2}$ and ${}^2S_{1/2}$ is shown below



The selection rule for Zeeman transitions are

$$\Delta M_J = 0, \pm 1 \quad (0 \rightarrow 0 \text{ if } J=0)$$

There are total six transition in accordance with above selection rules.

GATE-2016

Q36. The molecule ${}^{17}O_2$

- (a) Raman active but not NMR (nuclear magnetic resonance) active.
- (b) Infrared active and Raman active but not NMR active.
- (c) Raman active and NMR active.
- (d) Only NMR active.

Ans.: (c)

Solution: (i) Molecule ${}^{17}O_2$ can not absorb infrared as there is no change in dipole moment during vibration. Thus ${}^{17}O_2$ is infrared inactive.

(ii) Molecule $^{17}\text{O}_2$ shows change in polarizability during rotation. Thus it is Raman active molecule.

(iii) The nucleus of ^{17}O has spin $\frac{5}{2}$, therefore it is NMR active.

Q37. There are four electrons in the $3d$ shell of an isolated atom. The total magnetic moment of the atom in units of Bohr magneton is _____.

Ans.: 0

Solution: The configuration leads to $S = 2$ and $L = 2$

Since it is the case of less than half filled sub shell, thus according to Hund's rules, lower J will be in ground state.

$$\therefore J = L - S = 0 \quad \vec{\mu}_J = -g \left(\frac{e}{2m} \right) \vec{J}$$

$$M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$$

$$3d^4 = \begin{array}{|c|c|c|c|c|} \hline \uparrow & \uparrow & \uparrow & \uparrow & \\ \hline \end{array}$$

Thus, $\vec{\mu} = 0$

Q38. Which of the following transitions is NOT allowed in the case of an atom, according to the electric dipole radiation selection rule?

- (a) $2s - 1s$ (b) $2p - 1s$ (c) $2p - 2s$ (d) $3d - 2p$

Ans.: (a)

Solution: In electron dipole transition, $\Delta l = \pm 1$. Thus in transition $2s \rightarrow 1s$, $\Delta l = 0$. It violate the selection rule and hence not allowed.

Q39. The number of spectroscopic terms resulting from the $L.S$ coupling of a $3p$ electron and a $3d$ electron is _____.

Ans.: 12

Solution: For $3p^1 3d^1$: $s_1 = \frac{1}{2}, s_2 = \frac{1}{2} \therefore S = 0, 1$

$$l_1 = 1, l_2 = 2 \quad \therefore L = 1, 2, 3$$

$$S = 0, L = 1 \Rightarrow J = 1 \quad \therefore \text{Term} = {}^1P_1$$

$$S = 0, L = 2 \Rightarrow J = 2 \quad \therefore \text{Term} = {}^1D_2$$

$$S = 0, L = 3 \Rightarrow J = 3 \quad \therefore \text{Term} = {}^1F_3$$

$$S = 1, L = 1 \Rightarrow J = 0, 1, 2 \quad \therefore \text{Terms} = {}^3P_0, {}^3P_1, {}^3P_2$$

$$S = 1, L = 2 \Rightarrow J = 1, 2, 3 \quad \therefore \text{Terms} = {}^3D_1, {}^3D_2, {}^3D_3$$

$$S = 1, L = 3 \Rightarrow J = 2, 3, 4 \quad \therefore \text{Terms} = {}^3F_2, {}^3F_3, {}^3F_4$$

Thus total number of spectroscopic terms are 12.

GATE-2017

Q40. The wavefunction of which orbital is spherically symmetric:

- (a) p_x (b) p_y (c) s (d) d_{xy}

Ans. : (c)

Solution: For s orbital $l = 0$

Q41. The total energy of an inert-gas crystal is given by $E(R) = \frac{0.5}{R^{12}} - \frac{1}{R^6}$ (in eV), where R is the inter-atomic spacing in Angstroms. The equilibrium separation between the atoms is Angstroms. (up to two decimal places)

Ans. : 1

Solution: Given that $E(R) = \frac{0.5}{R^{12}} - \frac{1}{R^6}$

For equilibrium separation

$$\frac{dE}{dR} = 0 \Rightarrow \frac{dE}{dR} = -\frac{12 \times 0.5}{R^{13}} + \frac{6}{R^7} = 0 \Rightarrow \frac{1}{R^6} \left[\frac{-6}{R^6} + 6 \right] = 0 \Rightarrow R = 1$$

Q42. Which one of the following gases of diatomic molecules is Raman, infrared, and NMR active?

- (a) ${}^1H - {}^1H$ (b) ${}^{12}C - {}^{16}O$ (c) ${}^1H - {}^{35}Cl$ (d) ${}^{16}O - {}^{16}O$

Ans. : (c)

Solution:

(a) ${}^1H - {}^1H$ Infrared inactive

(b) ${}^{12}C - {}^{16}O$ NMR Inactive

(c) ${}^1H - {}^{35}Cl$ Raman, infrared & NMR active

(d) ${}^{16}O - {}^{16}O$ Infrared, Raman inactive

Q43. Using Hund's rule the total angular momentum quantum number J for the electronic ground state of the nitrogen atom is

- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0 (d) 1

Ans. : (b)

Solution: $N : 7 : 1s^2 2s^2 2p^3$

For p^3 : $M_L = \begin{array}{|c|c|c|} \hline -1 & 0 & +1 \\ \hline \uparrow & \uparrow & \uparrow \\ \hline \end{array}$

\therefore spectral term = $^{2s+1}L_J = {}^4S_{3/2}$

Q44. Positronium is an atom made of an electron and a positron. Given the Bohr radius for the ground state of the Hydrogen atom to be 0.53 Angstroms, the Bohr radius for the ground state of positronium is.....Angstroms. (up to two decimal places).

Ans. : 1.06

Solution: $r_n = a_0 \left(\frac{m_e}{\mu} \right)$

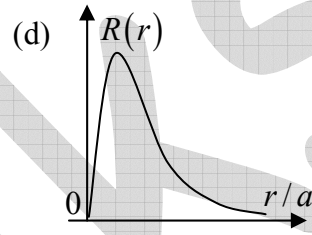
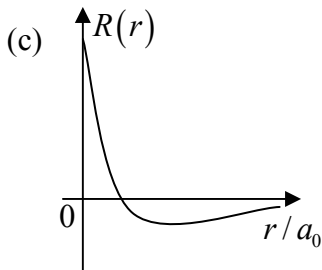
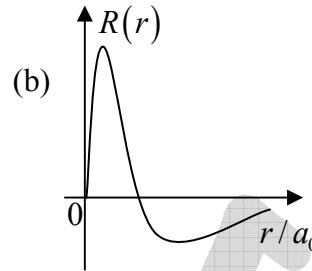
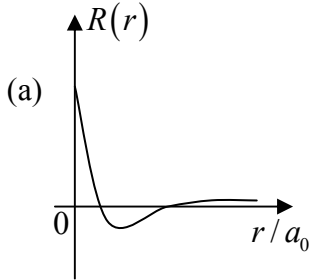
When $\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e^2}{2m_e} = \frac{m_e}{2}$

$\therefore r_n = 2a_0 = 2 \times 0.53 = 1.06 \text{ \AA}$

GATE-2018

Q45. Which one of the following represents the $3p$ radial wave function of hydrogen atom?

(a_0 is the Bohr radius)



Ans. : (b)

Solution: $3p$ radial wave function is $R_{31} \propto r \left(1 - \frac{r}{6a_0}\right) e^{-\frac{r}{3a_0}}$

Q46. Given the following table,

Group I	Group II
P: Stern-Gerlach experiment	1: Wave nature of particles
Q: Zeeman effect	2: Quantization of energy of electrons in the atoms
R: Frank-Hertz experiment	3: Existence of electron spin
S: Davisson-Germer experiment	4: Space quantization of angular momentum

Which one of the following correctly matches the experiments from Group I to their inferences in Group II?

(a) P-2, Q-3, R-4, S-1

(b) P-1, Q-3, R-2, S-4

(c) P-3, Q-4, R-2, S-1

(d) P-2, Q-1, R-4, S-3

Ans. : (c)

Q47. The expression for the second overtone frequency in the vibrational absorption spectra of a diatomic molecule in terms of the harmonic frequency ω_e and anharmonicity constant x_e is

- (a) $2\omega_e(1-x_e)$ (b) $2\omega_e(1-3x_e)$ (c) $3\omega_e(1-2x_e)$ (d) $3\omega_e(1-4x_e)$

Ans. : (d)

Solution: $\varepsilon_v = \omega_e \left(v + \frac{1}{2} \right) - \omega_e x_e \left(v + \frac{1}{2} \right)^2$

Second overtone $v=0 \rightarrow v=3$

$$\therefore \bar{\nu} = \varepsilon_{v=3} - \varepsilon_{v=0} = \frac{7}{2}\omega_e - \omega_e x_e \left(\frac{7}{2} \right)^2 - \frac{\omega_e}{2} + \omega_e x_e \left(\frac{1}{2} \right)^2 = 3\omega_e - 12\omega_e x_e = 3\omega_e(1-4x_e)$$

Q48. Match the physical effects and order of magnitude of their energy scales given below, where $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$ is fine structure constant; m_e and m_p are electron and proton mass, respectively.

Group I	Group II
P: Lamb shift	1: $\sim O(\alpha^2 m_e c^2)$
Q: Fine structure	2: $\sim O(\alpha^4 m_e c^2)$
R: Bohr energy	3: $\sim O(\alpha^4 m_e^2 c^2 / m_p)$
S: Hyperfine structure	4: $\sim O(\alpha^5 m_e c^2)$

(a) P-3, Q-1, R-2, S-4

(b) P-2, Q-3, R-1, S-4

(c) P-4, Q-2, R-1, S-3

(d) P-2, Q-4, R-1, S-3

Ans. : (c)

Solution:- Bohr energy $\Delta E \propto \alpha^2 m_e c^2$

Fine structure $\Delta E \propto \alpha^4 m_e c^2$

Lamb straight $\Delta E \propto \alpha^5 m_e c^2$

Hyperfine structure $\Delta E \propto \frac{\alpha^4 m_e c^2}{m_p}$

Q49. The intrinsic/permanent electric dipole moment in the ground state of hydrogen atom is (a_0 is the Bohr radius)

- (a) $-3ea_0$ (b) zero (c) ea_0 (d) $3ea_0$

Ans. : (b)

Solution: For dipole moment energy is $-eEr \cos \theta$

$$E_1^1 = \langle -eEr \cos \theta \rangle = eE \langle r \rangle \langle \cos \theta \rangle = 0 \quad [\because \langle \cos \theta \rangle = 0]$$

Q50. Which one of the following is an allowed electric dipole transition?

- (a) $^1S_0 \rightarrow ^3S_1$ (b) $^2P_{3/2} \rightarrow ^2D_{5/2}$ (c) $^2D_{5/2} \rightarrow ^2P_{1/2}$ (d) $^3P_0 \rightarrow ^5D_0$

Ans. : (b)

Solution: For electric dipole transition

$$\Delta L = 0, \pm 1 \quad (0 \rightarrow 0), \quad \Delta J = 0, \pm 1, \quad \Delta S = 0$$

Only option (b) satisfies above selection rules

Q51. The term symbol for the electronic ground state of oxygen atom is

- (a) 1S_0 (b) 1D_2 (c) 3P_0 (d) 3P_2

Ans. : (d)

Solution: O: $1s^2, 2s^2, 2p^4$

$$M_L = -1 \quad 0 \quad +1$$

$\uparrow\downarrow$	\uparrow	\uparrow
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Here, $S = 1, L = 2$

According to Hund's rule, for ground state energy

$$J = (L + S) = 2 \quad \therefore {}^{2S+1}L_J = {}^3P_2$$

Q52. 4MeV γ - rays emitted by the de-excitation of ^{19}F are attributed, assuming spherical symmetry, to the transition of protons from $1d_{3/2}$ state to $1d_{5/2}$ state. If the contribution of spin-orbit term to the total energy is written as $C \langle \vec{l} \cdot \vec{s} \rangle$, the magnitude of C is _____

MeV (up to one decimal place).

Ans. : 1.6

Solution: $l = 1, s = \frac{1}{2}, \hat{j}_1 = \frac{3}{2}, \hat{j}_2 = \frac{5}{2}$

$$\vec{j} = (\vec{l} + \vec{s}) \Rightarrow j^2 = l^2 + s^2 + 2\vec{l} \cdot \vec{s} \Rightarrow \vec{l} \cdot \vec{s} = \frac{(j^2 + l^2 - s^2)}{2}$$

$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{[j(j+1) - (l+1) - s(s+1)] \hbar^2}{2}$$

$$\Delta E = \alpha \left[\langle \vec{l} \cdot \vec{s} \rangle_{5/2} - \langle \vec{l} \cdot \vec{s} \rangle_{3/2} \right] = \alpha \left[\frac{5}{2} \cdot \frac{7}{2} - \frac{3}{2} \cdot \frac{5}{2} \right] \frac{\hbar^2}{2} = \alpha \cdot \left(\frac{20}{8} \right) \hbar^2 = \frac{20}{8} \cdot C$$

$$\Delta E = \frac{20}{8} C = 4 \text{ MeV} \Rightarrow C = \frac{32}{20} \text{ MeV}, C = 1.6 \text{ MeV}.$$

Q53. An atom in its singlet state is subjected to a magnetic field. The Zeeman splitting of its 650 nm spectral line is 0.03 nm. The magnitude of the field is _____ Tesla (up to two decimal places).

$$(e = 1.60 \times 10^{-19} \text{ C}, m_e = 9.11 \times 10^{-31} \text{ kg}, c = 3.0 \times 10^8 \text{ ms}^{-1})$$

Ans. : 1.52

Solution: $\Delta \lambda = \frac{\lambda^2}{c} \times \frac{eB}{4\pi m}$

$$\Rightarrow B = \frac{c}{\lambda^2} \cdot \frac{4\pi m}{e} \Delta \lambda = \frac{3 \times 10^8}{(650 \times 10^{-9})^2} \cdot \frac{4\pi \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}} \cdot (0.03 \times 10^{-9}) = 1.52 T$$

GATE-2019

Q54. The spin-orbit interaction term of an electron moving in a central field is written as $f(r) \vec{l} \cdot \vec{s}$, where r is the radial distance of the electron from the origin. If an electron moves inside a uniformly charged sphere, then

- (a) $f(r) = \text{constant}$ (b) $f(r) \propto r^{-1}$ (c) $f(r) \propto r^{-2}$ (d) $f(r) \propto r^{-3}$

Ans. : (a)

Solution: The electric potential of a uniformly charged sphere at $r < R$ is

$$V = \frac{kQ}{2R} \left(3 - \frac{r^2}{R^2} \right)$$

where Q is the electric charge on the sphere of radius R and k is a constant.

The interaction energy is $W = f(r) \vec{l} \cdot \vec{s}$, where for central potential V , $f(r) = \frac{1}{r} \left(\frac{\partial V}{\partial r} \right)$

$$\therefore f(r) = \frac{1}{r} \left[\frac{-kQr}{R^3} \right] = \frac{-kQ}{R^3} = \text{constant. Thus option (a) is correct.}$$

Q55. The ground state electronic configuration of the rare-earth ion (Nd^{3+}) is $[Pd]4f^3 5s^2 5p^6$.

Assuming LS coupling, the Lande g - factor of this ion is $\frac{8}{11}$. The effective magnetic moment in units of Bohr magneton μ_B (rounded off to two decimal places) is

Ans.: 3.62

Solution: For $4f^3$ $M_L = \begin{array}{ccccccc} -3 & -2 & -1 & 0 & +1 & +2 & +3 \\ \uparrow & \uparrow & \uparrow & & & & \end{array}$ $L = 6, S = 3/2, J = 9/2$

$$\therefore \mu = g_J \mu_B \sqrt{J(J+1)} = \frac{8}{11} \times \mu_B \times \sqrt{\frac{9}{2} \left(\frac{9}{2} + 1 \right)}$$

$$= \frac{8}{11} \sqrt{\frac{9}{2} \times \frac{11}{2}} \mu_B = 3.62 \mu_B$$

