Modern Physics

IIT-JAM 2005

Q1. If M_e, M_p *and* M_H are the rest masses of electron, proton and hydrogen atom in the ground state (with energy $-13.6\ eV$), respectively, which of the following is exactly true? (c is the speed of light in free space)

(a) $M_H = M_p + M_e$

(b) $M_H = M_p + M_e - \frac{13.6\ eV}{c^2}$

(c) $M_H = M_p + M_e + \frac{13.6\ eV}{c^2}$

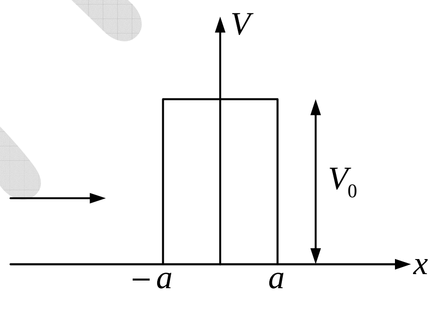
(d) $M_H = M_p + M_e + K$, where $K \neq \pm\frac{13.6\ eV}{c^2}$ or zero

Ans. : (c)

Solution: $B.E. = \left(M_p + M_e - M_H\right)c^2 \Rightarrow M_H = M_p + M_e - \frac{B.E.}{c^2}$ where $B.E. = -13.6eV$.

IIT-JAM 2006

Q2. Electrons of energy E coming from $x = -\infty$ impinge upon a potential barrier of width $2a$ and height V_0 centered at the origin with $V_0 > E$, as shown in the figure below. Let $k = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$. In the region $-a \le x \le a$, the electrons is a linear combination of



(a) e^{kx} and e^{-kx} (b) e^{ikx} and e^{-kx} (c) e^{ikx} and e^{-ikx} (d) e^{ikx} and e^{kx}

Ans. : (a)

Solution: Since, $V_0 > E$ in region $-a \le x \le a$. Thus, Schrodinger equation is given by

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V_o\psi=E\psi \Rightarrow \frac{\partial^2\psi}{\partial x^2}-\frac{2m(V_o-E)}{\hbar^2}\psi=0 \Rightarrow \frac{\partial^2\psi}{\partial x^2}-k^2\psi=0$$

where $k=\frac{\sqrt{2m(V_0-E)}}{\hbar}$.

Thus, the solution of the wave equation is e^{kx} and e^{-kx}, which is exponential in nature.

Q3. The relation between angular frequency ω and wave number k for given type of waves is $\omega^2=\alpha k+\beta k^3$. The wave number k_0 for which the phase velocity equals the group velocity is,

(a) $3\sqrt{\frac{\alpha}{\beta}}$ (b) $\left(\frac{1}{3}\right)\sqrt{\frac{\alpha}{\beta}}$ (c) $\sqrt{\frac{\alpha}{\beta}}$ (d) $\left(\frac{1}{2}\right)\sqrt{\frac{\alpha}{\beta}}$

Ans. : (c)

Solution: Group velocity, $V_g=\frac{d\omega}{dk}$ and phase velocity is $V_p=\frac{\omega}{k}$

$\omega^2 = \alpha k + \beta k^3$ ....(A)

Differentiating both sides we get $2\omega.\frac{d\omega}{dk}=\alpha+3\beta k^2$

Now dividing both sides by k we will get

$$2\frac{\omega}{k}.\frac{d\omega}{dk}=\frac{\alpha}{k}+3\beta k \Rightarrow 2V_p.V_g=\frac{\alpha}{k}+3\beta k$$

For $k=k_0$ and $V_p=V_g$

$$2V_p^2=\frac{\alpha}{k_0}+3\beta k_0 \Rightarrow V_p=\left(\frac{\alpha}{2k_0}+\frac{3\beta k_0}{2}\right)^{1/2}$$

From equation (A) $V_p=\frac{\omega}{k}=\left(\frac{\alpha}{k_0}+\beta k_0\right)^{1/2}$

Thus, $\left(\frac{\alpha}{2k_0}+\frac{3\beta k_0}{2}\right)^{1/2}=\left(\frac{\alpha}{k_0}+\beta k_0\right)^{1/2} \Rightarrow \frac{\alpha}{2k_0}-\frac{\beta k_0}{2}=0 \Rightarrow k_0=\sqrt{\frac{\alpha}{\beta}}$

Q4. A particle of rest mass m_0 is moving uniformly in a straight line with relativistic velocity βc, where c is the velocity of light in vacuum and $0 < \beta < 1$. The phase velocity of the de Broglie wave associated with the particle is,

(a) βc (b) $\frac{c}{\beta}$ (c) c (d) $\frac{c}{\beta^2}$

Ans. : (b)

Solution: $E^2 = p^2c^2 + m_0^2c^4$

$$2E\frac{dE}{dp} = 2pc^2 \Rightarrow E.v_g = pc^2 \Rightarrow \frac{E}{p} = \frac{c^2}{v_g} = v_p \Rightarrow v_p = \frac{c^2}{\beta c} \Rightarrow \frac{c}{\beta}$$

Q5. A neutron of mass, $m_n = 10^{-27}\,kg$ is moving inside a nucleus to be a cubical box of size $10^{-14}\,m$ with impenetrable walls. Take $\hbar \approx 10^{-34}\,Js$ and $1MeV \approx 10^{-13}\,J$. An estimate of the energy in MeV of the neutron is,

(a) $80\,MeV$ (b) $\frac{1}{8}\,MeV$ (c) $8\,MeV$ (d) $\frac{1}{80}\,MeV$

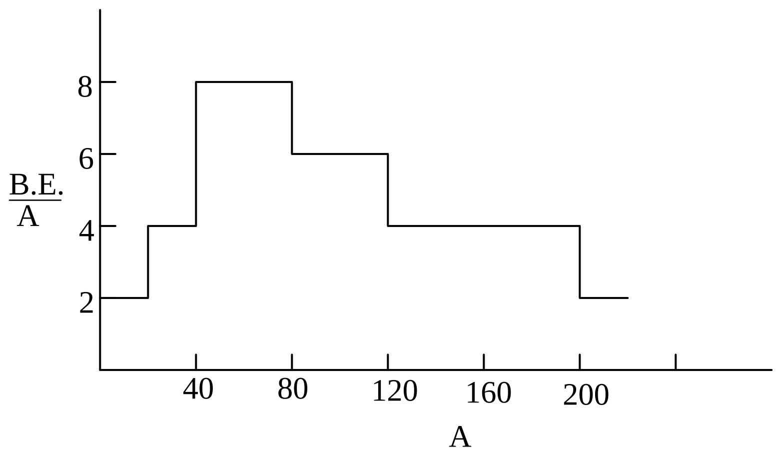
Ans:

Solution: $E = \frac{3\pi^2\hbar^2}{2m_n a^2} = \frac{3\times 10\times\left(10^{-34}\right)^2}{2\times 10^{-27}\times\left(10^{-14}\right)^2} = \frac{3\times 10\times 10^{-68}}{2\times 10^{-27}\times 10^{-28}}$

$$= 15\times 10^{-13}\,J = 15\times 10^{-13}\times 10^{13}\,MeV = 15\,MeV$$

IIT-JAM 2007

Q6. The following histogram represents the binding energy per particle ($B.E./A$) in MeV as a function of the mass number A of a nucleus. A nucleus with mass number $A = 180$ fissions into two nuclei of equal masses. In this process



(a) $180\,MeV$ of energy is released

(b) $180\,MeV$ of energy is absorbed

(c) $360\,MeV$ of energy is released

(d) $360\,MeV$ of energy is absorbed

Ans. : (c)

Solution: $A \rightarrow \frac{A}{2} + \frac{A}{2}$ *or* $180 \rightarrow 90 + 90$

Product B.E $= 90 \times 6 + 90 \times 6 = 1080\ MeV$

B.E. of nucleus $A = 180 \times 4 = 720\ MeV$

Since, B.E of the product nucleus is greater than the nucleus *A*, hence in this process energy is released and that is $= (1080 - 720)MeV = 360\ MeV$.

Q7. The black body spectrum of an object O_1 is such that its radiant intensity (i.e., intensity per unit wavelength interval) is maximum at a wavelength of $200\,nm$. Another object O_2 has the maximum radiant intensity at $600\,nm$. The ratio of power emitted per unit area by O_1 to that of O_2 is

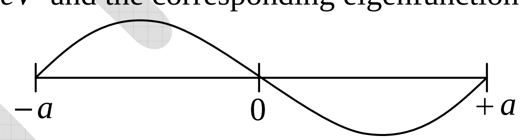
(a) $\frac{1}{81}$ (b) $\frac{1}{9}$ (c) 9 (d) 81

Ans. : (d)

Solution: From Wein's law $\lambda T = k$, where k is a constant. Thus, $\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}$ $\frac{T_1}{T_2} = 3$

Power (P) is proportional to $T^4 \Rightarrow \frac{P_1}{P_2} = \frac{T_1^4}{T_2^4} = 81$

Q8. A particle is confined in a one dimensional box with impenetrable walls at $x = \pm a$. Its energy eigenvalue is $2eV$ and the corresponding eigenfunction is as shown below.



The lowest possible energy of the particle is

(a) $4eV$ (b) $2eV$ (c) $1eV$ (d) $0.5eV$

Ans. : (d)

Solution: The given state is representation of first exited state whose energy is $2eV$.

If E_n is energy of n^{th} state and E_0 is energy of ground state then, $E_n = n^2 E_0$.

So, $E_2 = 4E_0 = 2eV \Rightarrow E_0 = 0.5eV$

IIT-JAM 2008

Q9. A photon of wavelength λ is incident on a free electron at rest and is scattered in the backward direction. The functional shift in its wavelength in terms of the Compton wavelength λc of the electron is,

(a) $\frac{\lambda_C}{2\lambda}$ (b) $\frac{2\lambda_C}{3\lambda}$ (c) $\frac{3\lambda_C}{2\lambda}$ (d) $\frac{2\lambda_C}{\lambda}$

Ans. : (d)

Solution: $\Delta\lambda = \lambda_c(1-\cos\theta)$

When photon scattered in backward direction then $\theta = \pi$. So, $\Delta\lambda = 2\lambda_c$

Functional shift is $\frac{\Delta\lambda}{\lambda} = \frac{2\lambda_C}{\lambda}$

Q10. In an inertial frame S, a stationary rod makes an angle θ with the x-axis. Another inertial frame S' moves with a velocity v with respect to S along the common x-x' axis. As observed from S' the angle made by the rod with the x'- axis is θ'. Which of the following statement is correct?

(a) $\theta' > \theta$

(b) $\theta' > \theta$

(c) $\theta' > \theta$ if v is negative and $\theta' > \theta$ if v is positive

(d) $\theta' > \theta$ if v is negative and $\theta' < \theta$ if v is positive

Ans. : (b)

Solution: $l'_x = l_0 \cos\theta\sqrt{1-\frac{v^2}{c^2}}$, $l'_y = l_0 \sin\theta$

$$\tan\theta' = \frac{l_y}{l_x} = \frac{\tan\theta}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \theta' > \theta$$

Q11. The activity of a radioactive sample is decreased to 75% of the initial value after 30 days. The half-life (in days) of the sample is approximately

[You may use $\ln 3 \approx 1.1$, $\ln 4 \approx 1.4$]

(a) 38 (b) 45 (c) 59 (d) 69

Ans. : (d)

Solution: $\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \frac{1}{30}\ln\left(\frac{R_0}{3/4R_0}\right) = \frac{1}{30}\ln\left(\frac{4}{3}\right) = \frac{1}{30}(1.4-1.1) = \frac{1}{100}$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1/100} = 69.3 \text{ day.}$$

IIT-JAM 2009

Q12. A wave packet in a certain medium is constructed by superposing waves of frequency ω around $\omega_0 = 100$ and the corresponding wave-number k with $k_0 = 10$ as given in the table below,

ω	k
81.00	9.0
90.25	9.5
100.00	10.0
110.25	10.5
121.00	11.0

Find the ratio v_g / v_p of the group velocity v_g and the phase velocity v_p.

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

Ans. : (d)

Solution: For $\omega = \omega_0 = 100$ and $k = k_0 = 10$ the phase velocity is $v_p = \frac{\omega_0}{k_0} = 10$

The group velocity is $v_g = \frac{\Delta\omega}{\Delta k} = \frac{\omega_2 - \omega_1}{k_2 - k_1} = \frac{110.25 - 90.25}{10.5 - 9.5} = 20$

$$\frac{v_g}{v_p} = \frac{20}{10} = 2$$

Q13. Two spherical nuclei have mass numbers 216 and 64 with their radii R_1 and R_2, respectively. The ratio $\frac{R_1}{R_2}$ is

(a) 1.0 (b) 1.5 (c) 2.0 (d) 2.5

Ans. : (d)

Solution: $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{216}{64}\right)^{1/3} = \frac{6}{4} = 1.5$

IIT-JAM 2010

Q14. A particle of mass m is confined in a two-dimensional infinite square well potential of side a. The eigen-energy of the particle in a given state is $E = \frac{25\pi^2\hbar^2}{ma^2}$. The state is

(a) 4 -fold degenerate (b) 3 -fold degenerate

(c) 2 -fold degenerate (d) Non-degenerate

Ans. : (d)

Solution: The eigen-energy of the particle in a given state is given by

$$E = \frac{\pi^2\hbar^2}{2ma^2}(n_x^2 + n_y^2) \text{ where } n_x = 1, 2, 3 \ldots \quad n_y = 1, 2, 3 \ldots$$

$E = \frac{25\pi^2\hbar^2}{ma^2}$ can be obtained by $n_x = 5$ and $n_y = 5$ which is non degenerate .

Q15. For a wave in a medium the angular frequency ω and the wave vector $\vec{k}$ are related by, $\omega^2 = \left(\omega_0^2 + c^2k^2\right)$, where ω_0 and c are constants. The product of group and phase velocities, i.e., $v_g.v_p$ is

(a) $0.25c^2$ (b) $0.4c^2$ (c) $0.5c^2$ (d) c^2

Ans. : (d)

Solution: $\omega^2 = \left(\omega_0^2 + c^2k^2\right)$

$$2\omega\frac{d\omega}{dk} = 2c^2k \Rightarrow \frac{\omega}{k}\cdot\frac{d\omega}{dk} = c^2 \Rightarrow v_p.v_g = c^2$$

Q16. Three identical non-interacting particles, each of spin $\frac{1}{2}$ and mass m, are moving in a one-dimensional infinite potential well given by,

$$V(x)=\begin{cases}0 & \text{for } 0<x<a \\ \infty & \text{for } x\leq 0 \text{ and } x\geq a\end{cases}$$

The energy of the lowest energy state of the system is

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{ma^2}$ (d) $\frac{5\pi^2\hbar^2}{2ma^2}$

Ans. : (c)

Solution: Spin $s=\frac{1}{2}$ means particles are fermions and it will obey Pauli Exclusion Principle. Degeneracy, $g=2s+1\Rightarrow g=2$ means in every state maximum 2 identical particle can be adjusted. If we have three fermions, then in ground state two fermions will be adjusted and one fermion in next higher level will be adjusted. Thus, the energy of the lowest energy state of the system is $2\times\frac{\pi^2\hbar^2}{2ma^2}+\frac{4\pi^2\hbar^2}{2ma^2}=\frac{6\pi^2\hbar^2}{2ma^2}=\frac{3\pi^2\hbar^2}{ma^2}$

IIT-JAM 2011

Q17. The wave function of a quantum mechanical particle is given by

$$\psi(x)=\frac{3}{5}\varphi_1(x)+\frac{4}{5}\varphi_2(x)$$

where $\phi_1(x)$ and $\phi_2(x)$ are eigenfunctions with corresponding energy eigenvalues $-1eV$ and $-2eV$, respectively. The energy of the particle in the state ψ is

(a) $\frac{-41}{25}eV$ (b) $\frac{-11}{5}eV$ (c) $\frac{36}{25}eV$ (d) $\frac{-7}{5}eV$

Ans. : (a)

Solution: $\langle E\rangle=\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle}=-1eV\times\frac{9}{25}+-2eV\times\frac{16}{25}=\frac{-41}{25}eV$

Q18. Light described by the equation $E=(90V/m)\left[\sin\left(6.28\times10^{15}s^{-1}\right)t+\sin\left(12.56\times10^{15}s^{-1}\right)t\right]$ is incident on a metal surface. The work function of the metal is $2.0\,eV$. Maximum kinetic energy of the photoelectrons will be,

(a) $2.14\,eV$ (b) $4.28\,eV$ (c) $6.28\,eV$ (d) $12.56\,eV$

Ans. : (c)

Solution: $K_{\max}=\hbar\omega-W$

For given wave maximum kinetic energy is for highest ω so $\omega=12.56\times10^{15}\sec^{-1}$

$$\hbar\omega=\frac{6.6\times10^{-34}\,J\,s\times12.56\times10^{15}s^{-1}}{2\pi}=\frac{82.8\times10^{-19}J}{6.28\times1.6\times10^{-19}}eV=8.24eV$$

$$K_{\max}=\hbar\omega-W\Rightarrow8.24eV-2eV=6.24eV$$

IIT-JAM 2012

Q19. Light takes 4 hours to cover the distance from Sun to Neptune. If you travel in a spaceship at a speed $0.99c$ (where c is the speed of light in vacuum), the time (in minutes) required to cover the same distance measured with a clock on the spaceship will be approximately

(a) 34 (b) 56 (c) 85 (d) 144

Ans. : (a)

Solution: $l=ct_0\sqrt{1-\frac{v^2}{c^2}}=c\times4\times60\times60\sqrt{1-\frac{(0.99c)^2}{c^2}}=c\times4\times60\times60\times.14\ m$

$$t=\frac{c\times4\times60\times60\times.14}{.99c\times60}\min=33.9=34\min$$

Q20. ${}^{60}_{27}Co$ is a radioactive nucleus of half-life $2\ln2\times10^8\,s$. The activity of $10\,g$ of ${}^{60}_{27}Co$ in disintegrations per second is,

(a) $\frac{1}{5}\times10^{10}$ (b) 5×10^{10} (c) $\frac{1}{5}\times10^{14}$ (d) 5×10^{14}

Ans. : (d)

Solution: $R = \lambda N$, where $N = \frac{10}{60} \times \left(6 \times 10^{23}\right) = 10^{23}$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{2\ln 2 \times 10^{8}} = \frac{0.693}{2 \times 2.303 \times 0.3010 \times 10^{8}} = \frac{0.693}{1.386 \times 10^{8}} = \frac{0.693}{1.386 \times 10^{8}} = 5 \times 10^{-9} s^{-1}$$

Thus, $R = 5 \times 10^{-9} \times \frac{10}{60} \times \left(6 \times 10^{23}\right) = 5 \times 10^{14}$.

IIT-JAM 2013

Q21. Electric field component of an electromagnetic radiation varies with time as, $E = a(\cos\omega_0 t + \sin\omega t \cos\omega_0 t)$, where a is a constant and the values of ω and ω_0 are $1 \times 10^{15} s^{-1}$ and $5 \times 10^{15} s^{-1}$ respectively. This radiation falls on a metal of work function $2eV$. The maximum kinetic energy (in eV) of photoelectrons is

(a) 0.64 (b) 1.30 (c) 1.70 (d) 1.95

Ans. : (b)

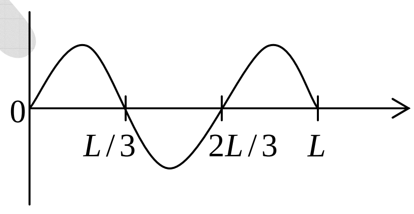
Solution: $K_{\max} = \hbar\omega - W$

For given wave, maximum kinetic energy is for highest $\phi\omega$, so $\omega_0 = 5 \times 10^{15} \sec^{-1}$

$$\hbar\omega_0 = \frac{6.6 \times 10^{-34} J s \times 5 \times 10^{15} s^{-1}}{2\pi} = \frac{33 \times 10^{-19} J}{6.28 \times 1.6 \times 10^{-19}} eV = 3.28$$

$$K_{\max} = \hbar\omega - W = 3.28eV - 2eV = 1.28eV$$

Q22. A free particle of mass m is confined to a region of length L. The de Broglie wave associated with the particle is sinusoidal in nature as given in the figure. The energy of the particle is



Ans. :

Solution: If wavelength of standing wave is λ and length of wall is L then from the figure

$$\frac{3\lambda}{2} = L \Rightarrow \lambda = \frac{2L}{3}.$$

If p is momentum and λ is wavelength, then from de-Broglie hypothesis $p=\frac{h}{\lambda}$, thus $p=\frac{3h}{2L}$.

When particle is confined into a box then total energy is only kinetic energy which is given by $E=\frac{p^2}{2m}$ put the value of $p=\frac{3h}{2L}$ one will get $E=\frac{9h^2}{8mL^2}$.

IIT-JAM 2014

Q23. In a photoelectric effect experiment, ultraviolet light of wavelength 320 nm falls on the photocathode with work function of 2.1 eV. The stopping potential should be close to

(a) 1.8 V (b) 1.6 V (c) 2.2 V (d) 2.4V

Ans. : (a)

Solution: Since, $K.E = eV = \hbar\omega - W \Rightarrow V=\frac{hc}{e\lambda}-\frac{W}{e}$

Now, $\lambda = 320\times10^{-9}m$, $W = 2.1\, eV$

$$\Rightarrow V=\frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times320\times10^{-9}}-2.1$$

$V = 3.867-2.1\approx 3.9-2.1=1.8\,V$.

Thus, option (a) is correct.

Q24. Four particles of mass m each are inside a two dimensional square box of side L. If each state obtained from the solution of the Schrodinger equation is occupied by only one particle, the minimum energy of the system in units of $\frac{h^2}{mL^2}$ is

(a) 2 (b) $\frac{5}{2}$ (c) $\frac{11}{2}$ (d) $\frac{25}{4}$

Ans. : (b)

Solution: For 2 - Dimensional box, possible configurations are $(1,1),(2,1),(2,2)$

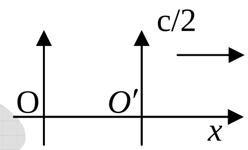
Now, ground state energy $\sum\frac{\pi^2\hbar^2}{2mL^2}\left(n_x^2+n_y^2\right)$; let $E_0=\frac{\pi^2\hbar^2}{2mL^2}$

$$= 2E_0 \times 1 + 2\times 5E_0 + 1\times 8E_0 = 20E_0 = 20\cdot\frac{\pi^2\hbar^2}{2mL^2}$$

$$E = \frac{5}{2}\frac{\hbar^2}{mL^2}$$

Thus, option (b) is correct

Q25. Two frames, O and O', are in relative motion as shown. O' is moving with speed $c/2$, where c is the speed of light. In frame O, two separate events occur at (x_1, t_1) and (x_2, t_2). In frame O', these events occur simultaneously. The value of $(x_2 - x_1)/(t_2 - t_1)$ is



(a) $c/4$ (b) $c/2$ (c) $2c$ (d) c

Ans.: (c) $x_2 = \dfrac{x_2' + vt_2'}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$x_1 = \frac{x_1' + vt_1'}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2 = t_2' + \frac{t_2^1 + \frac{Vn_2'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}, t_1 = \frac{t_1' + \frac{vx_1'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_2' = \left(t_2\sqrt{1-\frac{v^2}{c^2}} - t_2'\right)\frac{c^2}{v} \Rightarrow x_2' - x_1' = \left[(t_2 - t_1)\sqrt{1-\frac{v^2}{c^2}}\right]\frac{c^2}{v}$$

$$x_2 - x_1 = \frac{(t_2 - t_1)\sqrt{1-\frac{v^2}{c^2}}\times\frac{c^2}{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$(x_2 - x_y) = (t_2 - t_1)\frac{c^2}{v} \Rightarrow \frac{h_2 - h_1}{t_2 - t_1} = \frac{c^2}{v} = \frac{2c^2}{c} = 2c$$

IIT-JAM 2015

Q26. A particle with energy E is incident on a potential given by

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x \geq 0 \end{cases}.$$

The wave function of the particle for $E < V_0$ in the region $x > 0$ (in terms of positive constants A, B and k) is

(a) $Ae^{kx} + Be^{-kx}$ (b) Ae^{-kx} (c) $Ae^{ikx} + Be^{-ikx}$ (d) Zero

Ans. : (b)

Solution: For $x > 0$; $-\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{d} + V_0\psi_{II} = E\psi_{II}$; $E < V_0$

$$\psi_{II} = Be^{kx} + Ae^{-kx}, \text{ where } k = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

$$\psi_{II} \to 0 \ as \ x \to \infty \Rightarrow A = 0 \Rightarrow \psi_{II} = Ae^{-kx}$$

Q27. A system comprises of three electrons. There are three single particle energy levels accessible to each of these electrons. The number of possible configurations for this system is

(a) 1 (b) 3 (c) 6 (d) 7

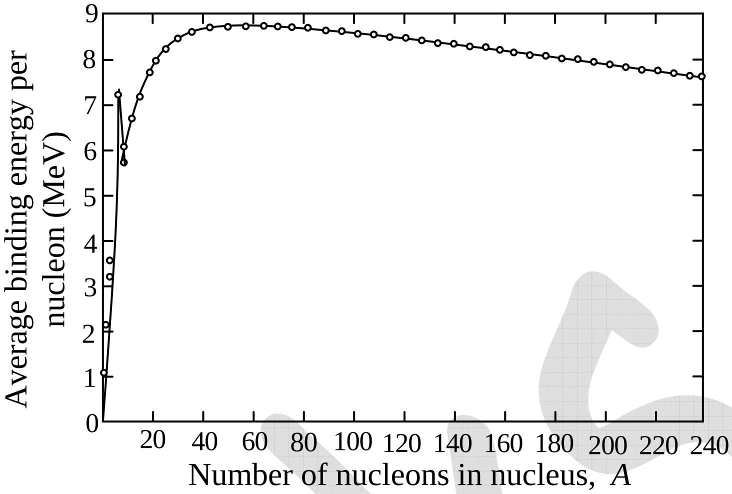
Ans. : (d)

Solution: For electron spin is $\frac{1}{2}$. So in one single state two electrons can be adjusted the number of ways are

	Ground	First	Second
1	2	1	0
2	2	0	1
3	1	2	0
4	1	0	2
5	0	1	2
6	0	2	1
7	1	1	1

So, number of ways are 7.

Q28. The variation of binding energy per nucleon with respect to the mass number of nuclei is shown in the figure.



Consider the following reactions:

(i) ${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 10P + 22n$ (ii) ${}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}He + 6e^{-}$

Which one of the following statements is true for the given decay modes of ${}^{238}_{92}U$?

(a) Both (i) and (ii) are allowed (b) Both (i) and (ii) are forbidden

(c) (i) is forbidden and (ii) is allowed (d) (i) is allowed and (ii) is forbidden

Ans. : (c)

Solution: In reaction (i) all conservation laws are valid. In reaction (ii) charge is not conserved.

Q29. A nucleus has a size of $10^{-15}\,m$. Consider an electron bound within a nucleus. The estimated energy of this electron is of the order of

(a) 1 MeV (b) $10^{2}\,MeV$ (c) $10^{4}\,MeV$ (d) $10^{6}\,MeV$

Ans. : (d)

Solution: $p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{10^{-15}} = 6.6\times10^{-19}\,kgm/\sec$

$$\because E = \frac{p^2}{2m_e} = \frac{44\times10^{-38}}{2\times9.1\times10^{-31}} = 2.4\times10^{-7}\ Joule$$

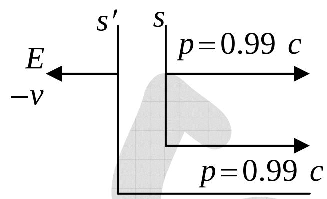
$$\Rightarrow E = \frac{2.4\times10^{-7}}{1.6\times10^{-19}}eV = 1.5\times10^{12}\ eV = 1.5\times10^{6}\ MeV$$

Q30. A proton from outer space is moving towards earth with velocity $0.99c$ as measured in earth's frame. A spaceship, traveling parallel to the proton, measures proton's velocity to be $0.97c$. The approximate velocity of the spaceship in the earth's frame, is

(a) $0.2c$ (b) $0.3c$ (c) $0.4c$ (d) $0.5c$

Ans.: (d)

Solution: Velocity of proton w.r.t. spaceship $= 0.97\ c$



$\because u_x' = 0.99\ c,\ v = -v, u_x = 0.97\ c$

$$\Rightarrow u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} \Rightarrow 0.97c = \frac{0.99c - v}{1 - \frac{0.97v}{c}} \Rightarrow v = 0.5c$$

Q31. A particle is moving in a two dimensional potential well

$$V(x,y) = \begin{cases} 0, & 0 \le x \le L,\ 0 \le y \le 2L \\ \infty, & \text{elsewhere} \end{cases}$$

which of the following statements about the ground state energy E_1 and ground state eigenfunction φ_0 are true?

(a) $E_1 = \frac{\hbar^2 \pi^2}{mL^2}$ (b) $E_1 = \frac{5\hbar^2 \pi^2}{8mL^2}$

(c) $\varphi_0 = \frac{\sqrt{2}}{L} \sin\frac{\pi x}{L} \sin\frac{\pi y}{2L}$ (d) $\varphi_0 = \frac{\sqrt{2}}{L} \cos\frac{\pi x}{L} \cos\frac{\pi y}{2L}$

Ans. : (b) and (c)

Solution: $E_n = \frac{\pi^2 \hbar^2}{2m}\left(\frac{n_x^2}{L^2} + \frac{n_y^2}{4L^2}\right)$

Ground state $n_x = 1,\ n_y = 1 \Rightarrow E_x = \frac{\pi^2\hbar^2}{2m}\left(\frac{1}{L^2} + \frac{1}{4L^2}\right) = \frac{5\pi^2\hbar^2}{8mL^2}$

Wave function $\psi = \sqrt{\frac{2}{L}} \cdot \sqrt{\frac{2}{2L}} \cdot \frac{\sin \pi x}{L} \frac{\sin \pi y}{2L}$

Q32. Muons are elementary particles produced in the upper atmosphere. They have a life time of $2.2\mu s$. Consider muons which are traveling vertically towards the earth's surface at a speed of $0.998c$. For an observer on earth, the height of the atmosphere above the surface of the earth is $10.4\,km$. Which of the following statements are true?

(a) The muons can never reach earth's surface

(b) The apparent thickness of earth's atmosphere in muon's frame of reference is $0.96\,km$

(c) The lifetime of muons in earth's frame of reference is $34.8\mu s$

(d) Muons traveling at a speed greater than $0.998\,c$ reach the earth's surface

Ans.: (c) and (d)

Solution: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow \Delta t = \dfrac{2.2\times10^{-6}}{\sqrt{1-(0.998)^2}} = 34.8\times10^{-6}\sec$

Now distance will be $= \Delta t\times v = 34.8\times10^{-6}\times0.998\times3\times10^8 = 10.4192\,km$

Apparent thickness $\Delta X = \Delta t\times v = 2.2\times10^{-6}\times0.998\times3\times10^8 = 0.658\,km$

Q33. A particle is in a state which is a superposition of the ground state φ_0 and the first excited state φ_1 of a one-dimensional quantum harmonic oscillator. The state is given by $\Phi = \dfrac{1}{\sqrt{5}}\varphi_0 + \dfrac{2}{\sqrt{5}}\varphi_1$. The expectation value of the energy of the particle in this state (in units of $\hbar\omega, \omega$ being the frequency of the oscillator) is...........

Ans. : 1.3

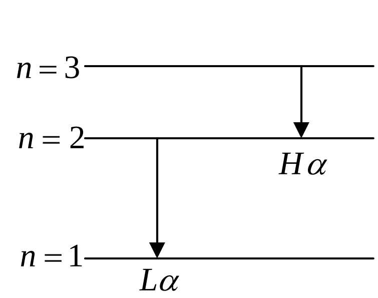
Solution: $\because E_n = \left(n+\dfrac{1}{2}\right)\hbar\omega$ and $P\left(\dfrac{\hbar\omega}{2}\right) = \dfrac{1}{5},\ P\left(\dfrac{3\hbar\omega}{2}\right) = \dfrac{4}{5}$

$$\Rightarrow \langle E\rangle = \frac{\hbar\omega}{2}\times\frac{1}{5} + \frac{3\hbar\omega}{2}\times\frac{4}{5} = \frac{13\hbar\omega}{10} = 1.3\hbar\omega$$

Q34. In the hydrogen atom spectrum. the ratio of the longest wavelength in the Lyman series (final state $n=1$) to that in the Balmer series (final State $n=2$) is…………..

Ans. : 0.185

Solution: According to Bohr Theory $\frac{1}{\lambda_L} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$



The longest wavelength in the Lyman series is

$$\Rightarrow \frac{1}{\lambda_L} = R\left(\frac{1}{1} - \frac{1}{2^2}\right) = R\left(\frac{3}{4}\right) \Rightarrow \lambda_L = \frac{4}{3R}$$

The longest wavelength in the Balmer series is

$$\Rightarrow \frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(\frac{9-4}{36}\right) \Rightarrow \frac{1}{\lambda_B} = R\left(\frac{5}{36}\right) \Rightarrow \lambda_B = \frac{36}{5R}$$

$$\Rightarrow \frac{\lambda_L}{\lambda_B} = \frac{4}{3R} \times \frac{5R}{36} = \frac{5}{27} = 0.185$$

Q35. $X-$rays of wavelength $0.24\,nm$ are Compton scattered and the scattered beam is observed at an angle of 60^o relative to the incident beam. The Compton wavelength of the electron is $0.00243\,nm$. The kinetic energy of scattered elections in eV is..............

Ans. : 25

Solution: $\lambda = 0.24\,nm,\ \lambda_C = 0.00243$ and $\theta = 60^0$

$$\because \lambda' - \lambda = \lambda_C\left(1 - \cos\theta\right) \Rightarrow \lambda' = \lambda + \lambda_C\left(1 - \cos\theta\right)$$

$$\Rightarrow \lambda' = 0.24 + 0.00243\left(1 - \frac{1}{2}\right) = 0.24 + 0.00243 \times \frac{1}{2} = 0.24 + 0.00121 = 0.2412\,nm$$

Kinetic Energy of scattered electron

$$K.E. = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 6.6 \times 10^{-34} \times 3 \times 10^{8}\left(\frac{1}{0.24} - \frac{1}{0.2412}\right) \times \frac{1}{10^{-9}}\ Joules$$

$$\Rightarrow K.E. = \frac{19.8 \times 10^{-26}}{10^{-9}}\left(4.17 - 4.15\right) = \frac{19.8 \times 10^{-26}}{10^{-9}} \times 0.02 = 396 \times 10^{-20}\ \ Joules$$

$$\Rightarrow K.E. = \frac{396 \times 10^{-20}}{1.6 \times 10^{-19}}\ eV = 24.75\ eV$$

IIT-JAM 2016

Q36. Consider a free electron (e) and a photon (ph) both having $10\,eV$ of energy. If λ and P represents wavelength and momentum respectively, then

(mass of electron $=9.1\times10^{-31}\,kg$; speed of light $=3\times10^{8}\,m/s$)

(a) $\lambda_e=\lambda_{ph}$ and $P_e=P_{ph}$ (b) $\lambda_e<\lambda_{ph}$ and $P_e>P_{ph}$

(c) $\lambda_e>\lambda_{ph}$ and $P_e<P_{ph}$ (d) $\lambda_e>\lambda_{ph}$ and $P_e<P_{ph}$

Ans. : (c)

Solution: For photon $p_{ph}=\dfrac{E}{c}$, $\lambda_{ph}=\dfrac{h}{p}=\dfrac{hc}{E}$

For electron $p_e=\dfrac{\sqrt{E^2-m^2c^4}}{c}$, $\lambda_e=\dfrac{h}{p}=\dfrac{hc}{\sqrt{E^2-m^2c^4}}$

Q37. A slit has width 'd' along the x-direction. If a beam of electrons, accelerated in y-direction to a particular velocity by applying a potential difference of $100\pm0.1\,kV$ passes through the slit, then, which of the following statement(s) is (are) correct?

(a) The uncertainty in the position of the electrons in x-direction before passing the slit is zero

(b) The momentum of electrons in x- direction is $\sim\dfrac{\hbar}{d}$ immediately after passing the slit

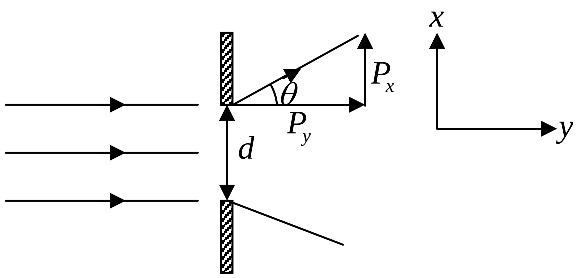
(c) The uncertainty in the position of electrons in y- direction before passing the slit is zero

(d) The presence of the slit does not affect the uncertainty in momentum of electrons in y- direction

Ans. : (b) and (d)

Solution: The electrons beam before slit is collimated in y - direction as shown in figure. Thus, before slit

$P_y=P$ and $P_x=0$



also $\Delta x \to \infty$ as $\Delta P_x = 0$

Thus options (a) and (c) are not correct.

Now, after the slit $\Delta P_x = d$ as a result $\Delta P_x = \frac{\hbar}{\Delta x} = \frac{\hbar}{d}$

i.e., $P_x \cong \frac{\hbar}{d}$

Thus, option (b) is correct.

Whereas presence of slit does not affect the uncertainty in momentum in y - direction.

Thus option (d) is also correct.

Q38. A free particle of energy E collides with a one-dimensional square potential barrier of height V and width W. Which one of the following statement(s) is/are correct?

(a) For $E > V$, the transmission coefficient for the particle across the barrier will always be unity

(b) For $E < V$, the transmission coefficient changes more rapidly with W than with V

(c) For $E < V$, if V is doubled, the transmission coefficient will also be doubled.

(d) Sum of the reflection and the transmission coefficients is always one

Ans. : (b) and (d)

Solution: $R + T = 1$

$$R = \left(\frac{\sqrt{E-V} - \sqrt{E}}{\sqrt{E-V} + \sqrt{E}} \right)^2$$

Q39. A particular radioisotope has a half-life of 5 days. In 15 days the probability of decay in percentage will be…………..

Ans. : 87.5

$$N = N_0 \left(\frac{1}{2} \right)^{t/T_{1/2}} = N_0 \left(\frac{1}{2} \right)^{15/5} = \frac{N_0}{8}$$

In 15 days the probability of decay $= \frac{N_0 - N}{N_0} \times 100 = \frac{7}{8} \times 100 = 87.5\%$

Q40. In photoelectric experiment both sodium (work function $=2.3eV$) and tungsten (work function $=4.5eV$) metals were illuminated by an ultraviolet light of same wavelength. If the stopping potential for tungsten is measured to be $1.8V$, the value of the stopping potential for sodium will be………….. V.

Ans. : 4

Solution: For tungsten $eV_s = h\nu - W_t \Rightarrow h\nu = eV_s + W_t = 1.8 + 4.5 = 6.3$

For sodium $eV_s = h\nu - W_s = 6.3 - 2.3 = 4eV$

Q41. The de Broglie wavelength of a relativistic electron having $1\,MeV$ of energy is…………..$\times 10^{-12}\ m$. (Take the rest mass energy of the electron to be $0.5\ MeV$. Plank constant $=6.63\times 10^{-34}\ Js$, speed of light $=3\times 10^{8}\ m/s$, Electronic charge $=1.6\times 10^{-19}\ C$)

Ans. : 1.43

Solution: $E^2 = p^2c^2 + \left(m_0c^2\right)^2 \Rightarrow p = \sqrt{\dfrac{E^2 - \left(m_0c^2\right)^2}{c^2}} = \dfrac{\sqrt{1-.25}}{c} = \dfrac{\sqrt{.75}MeV}{c}$

As, $\lambda = \dfrac{h}{p} = \dfrac{6.6\times 10^{-34}\times 3\times 10^{8}}{\sqrt{.75}\times 1.6\times 10^{-13}} = \dfrac{19.8\times 10^{-13}}{1.38} = 14.34\times 10^{-13}m = 1.43\times 10^{-12}m$

Q42. X-ray of 20 keV energy is scattered inelastically from a carbon target. The kinetic energy transferred to the recoiling electron by photons scattered at 90^0 with respect to the incident beam is……………. keV.

(Planck constant = 6.6×10^{-34} Js, Speed of light = $3\times 10^{8}\,m/s$, electron mass = $9.1\times 10^{-31}\,kg$. Electronic charge $=1.6\times 10^{-19}C$)

Ans. : 0.77

Solution: $\because\ \lambda' - \lambda = \dfrac{h}{mc}(1-\cos\theta) \Rightarrow \lambda' - \lambda = \dfrac{h}{mc} \qquad \because \theta = \dfrac{\pi}{2}$

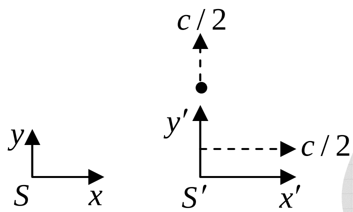
$$\Rightarrow \frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1}{mc^2} \Rightarrow \frac{1}{E'} - \frac{1}{E} = \frac{1}{mc^2} \Rightarrow \frac{1}{E'} = \frac{1}{E} + \frac{1}{mc^2} \Rightarrow \frac{1}{20keV} + \frac{1}{.5MeV}$$

$\Rightarrow E' = 19.23keV$

Recoil velocity of electron $E - E' = 0.77keV$

IIT-JAM 2017

Q43. Consider an inertial frame S' moving at speed $\frac{c}{2}$ away from another inertial frame S along the common x-x' axis, where c is the speed of light. As observed from S', a particle is moving with speed $\frac{c}{2}$ in the y' direction, as shown in the figure. The speed of the particle as seen from S is:



(a) $\frac{c}{\sqrt{2}}$ (b) $\frac{c}{2}$ (c) $\frac{\sqrt{7}c}{4}$ (d) $\frac{\sqrt{3}c}{5}$

Ans. : (c)

Solution: $v = \frac{c}{2}\hat{i} \quad u_x^{'} = 0,\ u_y^{'} = \frac{c}{2},\ u_z^{'} = 0$

$$u_x = \frac{u_x^{'} + v}{1 + \frac{u_x^{'} v}{c^2}} = \frac{c}{2}, \quad u_x = \frac{u_y^{'}\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{u_x^{'} v}{c^2}} = \frac{c}{2}\sqrt{1 - \frac{1}{4}} = \sqrt{3}\frac{c}{4}, \quad u_z = \frac{u_z^{'}\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{u_x^{'} v}{c^2}} = 0$$

$$u = \sqrt{\frac{c^2}{4} + \frac{3c^2}{16}} = \frac{\sqrt{7}c}{4}$$

Q44. Consider Rydberg (hydrogen-like) atoms in a highly excited state with n around 300. The wavelength of radiation coming out of these atoms for transitions to the adjacent states lies in the range:

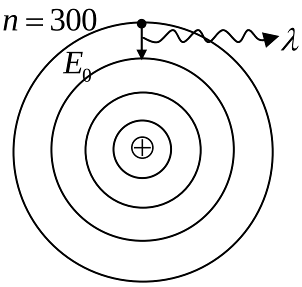
(a) Gamma rays $(\lambda \sim pm)$ (b) $UV\,(\lambda \sim nm)$

(c) Infrared $(\lambda \sim \mu m)$ (d) $RF\,(\lambda \sim m)$

Ans. : (d)

Solution: $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

where $R = 1.097 \times 10^7 m^{-1}$



$n_f = 299$ and $n_i = 300 \Rightarrow \frac{1}{\lambda} = R\left[\frac{n_i^2 - n_f^2}{n_i^2 n_T^2}\right]$

$\Rightarrow \lambda = \frac{1}{R}\left(\frac{n_i^2 n_f^2}{n_i^2 - n_f^2}\right) = \frac{1}{R}\left[\frac{(300)^2 (299)^2}{(300)^2 - (299)^2}\right]$

$= \frac{1}{1.097 \times 10^7}\left[\frac{(300)^2 (299)^2}{599}\right] = \frac{1.34 \times 10^7}{1.097 \times 10^7} = 1.22m \Rightarrow \lambda = 1.22m$

This wavelength corresponds to RF Thus correct option is (d)

Q45. A photon of frequency ν strikes an electron of mass m initially at rest. After scattering at an angle ϕ, the photon loses half of its energy. If the electron recoils at an angle θ, which of the following is (are) true?

(a) $\cos\phi = \left(1 - \frac{mc^2}{h\nu}\right)$

(b) $\sin\theta = \left(1 - \frac{mc^2}{h\nu}\right)$

(c) The ratio of the magnitudes of momenta of the recoiled electron and scattered photon is $\frac{\sin\phi}{\sin\theta}$

(d) Change in photon wavelength is $\frac{h}{mc}(1 - 2\cos\phi)$

Ans. : (a), (c)

Solution: $\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \Rightarrow \frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{mc}(1 - \cos\phi)$

$\Rightarrow \frac{2c}{\nu} - \frac{c}{\nu} = \frac{h}{mc}(1 - \cos\phi) \Rightarrow \frac{c}{\nu} = \frac{h}{mc}(1 - \cos\phi) \Rightarrow \cos\phi = \left(1 - \frac{mc^2}{h\nu}\right)$

From the conservation of momentum in y direction

$\frac{h\nu'}{c}\sin\phi = p\sin\theta \Rightarrow \frac{p}{\frac{h\nu'}{c}} = \frac{\sin\phi}{\sin\theta}$

Q46. For an atomic nucleus with atomic number Z and mass number A, which of the following is (are) correct?

(a) Nuclear matter and nuclear charge are distributed identically in the nuclear volume

(b) Nuclei with $Z > 83$ and $A > 209$ emit α - radiation

(c) The surface contribution to the binding energy is proportional to $A^{2/3}$

(d) β - decay occurs when the proton to neutron ratio is large, but not when it is small

Ans. : (b) and (c)

Solution: From given statement only (b) and (c) are correct.

Q47. Consider a one-dimensional harmonic oscillator of angular frequency ω. If 5 identical particles occupy the energy levels of this oscillator at zero temperature, which of the following statement(s) about their ground state energy E_0 is (are) correct?

(a) If the particles are electrons, $E_0 = \frac{13}{2}\hbar\omega$

(b) If the particles are protons, $E_0 = \frac{25}{2}\hbar\omega$

(c) If the particles are spin-less fermions, $E_0 = \frac{25}{2}\hbar\omega$

(d) If the particles are bosons, $E_0 = \frac{5}{2}\hbar\omega$

Ans. : (a), (c) and (d)

Solution: If particles are electrons and protons then ground state energy

$$E_0 = 2\times\frac{\hbar\omega}{2} + 2\times\frac{3\hbar\omega}{2} + 1\times\frac{5\hbar\omega}{2} = \frac{13\hbar\omega}{2}$$

If the particles are spin-less fermions, then energy is

$$E_0 = \frac{\hbar\omega}{2} + \frac{3\hbar\omega}{2} + \frac{5\hbar\omega}{2} + \frac{7\hbar\omega}{2} + \frac{9\hbar\omega}{2} = \frac{25\hbar\omega}{2}$$

If the particles are bosons $E_0 = 5\times\frac{1}{2}\hbar\omega = \frac{5\hbar\omega}{2}$

Q48. A particle of mass m is placed in a three-dimensional cubic box of side a. What is the degeneracy of its energy level with energy $14\left(\frac{\hbar^2\pi^2}{2ma^2}\right)$?

(Express your answer as an integer)

Ans. : 6

Solution: $n_x^2 + n_y^2 + n_z^2 = 14$

$$n_x = 1, n_y = 2, n_z = 3$$
$$n_x = 1, n_y = 3, n_z = 2$$
$$n_x = 2, n_y = 3, n_z = 1$$
$$n_x = 2, n_y = 3, n_z = 1$$
$$n_x = 3, n_y = 1, n_z = 2$$
$$n_x = 3, n_y = 2, n_z = 1$$

So degeneracy is 6

Q49. For a proton to capture an electron to form a neutron and a neutrino (assumed massless), the electron must have some minimum energy. For such an electron the de-Broglie wavelength in pictometers is……………

(Specify your answer to two digits after the decimal point)

Ans. : 1.02

Solution: From conservation of energy

$$E_e = m_e c^2 + K_e = \left(m_n - m_p\right)c^2 = (1.675 - 1.673)\times 10^{-27}\left(3\times 10^8\right)^2 = 1.8\times 10^{-13}\,Joules$$

$$E_e^2 = (pc)^2 + m_e^2c^4 \approx (pc)^2 \quad \Rightarrow p = \frac{E_e}{c} = 0.6\times 10^{-22}\,kg.m/\sec \qquad \left[\because pc >> m_e c^2\right]$$

$$\lambda = \frac{h}{p} = \frac{6.6\times 10^{-34}}{0.6\times 10^{-22}} = 1.1\times 10^{-12}\,m = 1.1\,pm$$

IIT-JAM 2018

Q50. Let T_g and T_e be the kinetic energies of the electron in the ground and the third excited states of a hydrogen atom, respectively. According to the Bohr model, the ratio $\frac{T_g}{T_e}$ is

(a) 3 (b) 4 (c) 9 (d) 16

Ans. : (d)

Solution: From Bohr model the kinetic energy and Total energy $\langle E\rangle$ and kinetic energy $\langle T\rangle$

$\langle T\rangle = -\frac{\langle E\rangle}{2}$ where $E_g = \frac{E_0}{1}$, $E_e = \frac{E_0}{16} \Rightarrow \frac{T_g}{T_e} = \frac{E_g}{F_e} = \frac{16}{1} = 16:1$

Q51. The mean momentum $\vec{p}$ of a nucleon in a nucleus of mass number A and atomic number Z depends on A, Z as

(a) $\vec{p} \propto A^{\frac{1}{3}}$ (b) $\vec{p} \propto Z^{\frac{1}{3}}$ (c) $\vec{p} \propto A^{-\frac{1}{3}}$ (d) $\vec{p} \propto (AZ)^{-\frac{2}{3}}$

Ans. : (c)

Solution: The radius of a nucleus can be combined as $\frac{\lambda}{2\pi}$ (greater than the wavelength of electron)

The moment $p = \frac{h}{\lambda}$

$\lambda - R = R_0 A^{1/3}$ which implies $p \propto \frac{h}{R_0} \cdot A^{-1/3}$.

As, $p \propto A^{-1/3}$

Q52. A particle of mass m is in a one dimensional potential $V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$.

At some instant its wave function is given by $\psi(x) = \frac{1}{\sqrt{3}}\psi_1(x) + i\sqrt{\frac{2}{3}}\psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ are the ground and the first excited states, respectively. Identify the correct statement.

(a) $\langle x\rangle = \frac{L}{2}; \langle E\rangle = \frac{\hbar^2}{2m}\frac{3\pi^2}{L^2}$ (b) $\langle x\rangle = \frac{2L}{3}; \langle E\rangle = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}$

(c) $\langle x\rangle = \frac{L}{2}; \langle E\rangle = \frac{\hbar^2}{2m}\frac{8\pi^2}{L^2}$ (d) $\langle x\rangle = \frac{2L}{3}; \langle E\rangle = \frac{\hbar^2}{2m}\frac{4\pi^2}{3L^2}$

Ans.: (a)

Solution: $\langle E\rangle = \dfrac{\frac{1}{3}\times(E_0)+\frac{2}{3}4E_0}{\frac{1}{3}+\frac{2}{3}} = \dfrac{\frac{9E_0}{3}}{\frac{3}{3}} = 3E_0$ Where, $E_0 = \dfrac{\pi^2\hbar^2}{2mL^2}$

$$\langle E\rangle = \frac{3\cdot\pi^2\hbar^2}{2mL^2} = \frac{3\pi^2\hbar^2}{2mL^2}$$

$$\langle X\rangle = \frac{1}{3}\langle\psi_1|X|\psi_1\rangle + \frac{2}{3}\langle\psi_2|X|\psi_1\rangle + \frac{i}{\sqrt{3}}\sqrt{\frac{2}{3}}\langle\psi_1|X|\psi_2\rangle - \frac{i}{\sqrt{3}}\sqrt{\frac{2}{3}}\langle\psi_2|X|\psi_1\rangle$$

$$\Rightarrow \frac{1}{3}\frac{L}{2} + \frac{2}{3}\frac{L}{2} = \frac{L}{2}$$

Q53. A system of 8 non-interacting electrons is confined by a three dimensional potential $V(r) = \frac{1}{2}m\omega^2 r^2$. The ground state energy of the system in units of $\hbar\omega$ is ______

(Specify your answer as an integer.)

Ans. : 18

Solution: $n=0$ is non degenerate so there will 2 electron in the ground state.

$n=1$ is triple degenerate so there is 6 electron in the first excited state

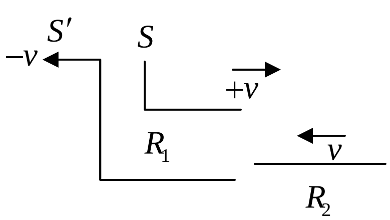
$$E = 2\times\frac{3\hbar\omega}{2} + 6\times\frac{5\hbar\omega}{2} \Rightarrow 3\hbar\omega + 15\hbar\omega = 18\hbar\omega$$

Q54. Rod R_1 has a rest length $1m$ and rod R_2 has a rest length of $2m$. R_1 and R_2 are moving with respect to the laboratory frame with velocities $+v\hat{i}$ and $-v\hat{i}$, respectively. If R_2 has a length of $1m$ in the rest frame of R_1, $\frac{v}{c}$ is given by__________

(Specify your answer upto two digits after the decimal point)

Ans. : 0.48

Solution:



$V=-v\,,u_x'=-v$

$$u_x=\frac{u_x'+V}{1+\frac{u_x'V}{c^2}}=\frac{-2v}{1+\frac{v^2}{c^2}}$$

$$l=l_0\sqrt{1-\frac{u_x^{\;2}}{c^2}}$$

$$1=2\sqrt{1-\frac{u_x^{\;2}}{c^2}}$$

$$\frac{1}{4}=1-\frac{\left(\frac{4v^2}{1+v^2/c^2}\right)}{c^2}\Rightarrow\frac{4v^2/c^2}{1+\frac{v^2}{c^2}}=\frac{3}{4}\Rightarrow 4\left(\frac{v^2}{c^2}\right)=\frac{3}{4}+\frac{3}{4}\left(\frac{v^2}{c^2}\right)$$

$$\frac{13}{4}\left(\frac{v}{c}\right)^2=\frac{3}{4}\Rightarrow\left(\frac{v}{c}\right)^2=\frac{12}{52}\Rightarrow\frac{v}{c}=\sqrt{\frac{12}{52}}\Rightarrow\frac{v}{c}=0.479=0.48\,.$$

Q55. Two events E_1 and E_2 take place in an inertial frame S with respective time space coordinates (in SI units): $E_1\left(t_1=0,\vec{r}_1=0\right)$ and $E_2\left(t_2=0,x_2=10^8,y_z=0,z_2=0\right)$. Another inertial frame S' is moving with respect to S with a velocity $\vec{v}=0.8c\hat{i}$. The time difference $\left(t_2'-t_1'\right)$ as observed in S' is ___________ s .

(Specify your answer in seconds upto two digits after the decimal point)

Ans. : 0.44

Solution: $t_2-t_1=0$ and $x_2-x_1=10^8$

$$t_2'-t_1'=\frac{\left(t_2-t_1\right)}{\sqrt{1-v^2/c_2}}-\left(\frac{x_2-x_1}{\sqrt{1-v^2/c_2}}\right)\frac{v}{c_2}$$

$$=-\frac{\left(x_2-x_1\right)\frac{v}{c^2}}{\sqrt{1-v^2/c^2}}=-\frac{\left(x_{_2}-x_1\right)\frac{v}{c^2}}{\sqrt{1-v^2/c^2}}=-\frac{10^8\times\frac{0.8c}{c^2}}{\sqrt{1-0.64}}=\frac{\cdot 8\times 10^8}{\cdot 6\times 3\times 10^8}=\frac{8}{18}=0.44\sec.$$

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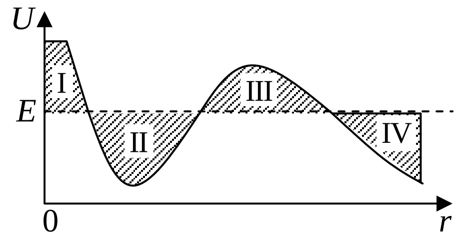
Q56. A classical particle has total energy E. The plot of potential energy (U) as a function of distance (r) from the centre of force located at $r = 0$ is shown in the figure. Which of the regions are forbidden for the particle?



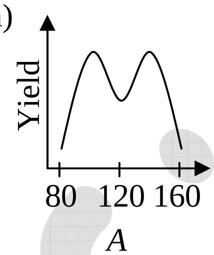
(a) I and II　　(b) II and IV

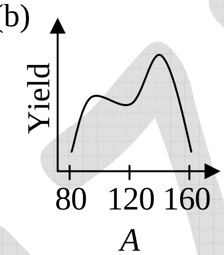
(c) I an IV　　(d) I and III

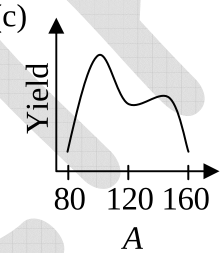
Ans. : (d)

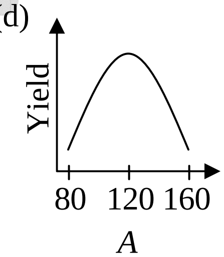
Solution: In the region I and III potential energy is more than total energy .

Q57. In the thermal neutron induced fission of ^{235}U, the distribution of relative number of the observed fission fragments (Yield) versus mass number (A) is given by

(a)


(b)


(c)


(d)


Ans. : (a)

Q58. For a quantum particle confined inside a cubic box of side L, the ground state energy is given by E_0. The energy of the first excited state is

(a) $2E_0$　　(b) $\sqrt{2}E_0$　　(c) $3E_0$　　(d) $6E_0$

Ans. : (d)

Solution: $E_{n_x,n_y,n_z} = \dfrac{\left(n_x^2 + n_y^2 + n_z^2\right)\pi^2\hbar^2}{2ma^2} = \left(n_x^2 + n_y^2 + n_z^2\right)E_0$

$$E_{2,1,1} = E_{1,2,1} = E_{1,1,2} = \frac{(4+1+1)\pi^2\hbar^2}{2ma^2} = 6E_0$$

Q59. A γ -ray photon emitted from a ^{137}Cs source collides with an electron at rest. If the Compton shift of the photon is 3.25×10^{-13} m, then the scattering angle is closets to (Planck's constant $h = 6.626\times10^{-34}$ Js, electron mass $m_\theta = 9.109\times10^{-31}$ kg and velocity of light in free space $c = 3\times10^{8}$ m/s)

(a) 45^{o} (b) 60^{o} (c) 30^{o} (d) 90^{o}

Ans. : (c)

Solution: $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) \Rightarrow \cos\theta = 1 - \dfrac{\Delta\lambda . m_e c}{h}$

$$= 1 - \frac{3.25\times10^{-13}\times9.109\times10^{-31}\times3\times10^{8}}{6.6\times10^{-34}} = 0.866 = \frac{\sqrt{3}}{2}$$

$\theta = 30^{0}$

Q60. The relation between the nuclear radius (R) and the mass number (A), given by $R = 1.2\,A^{1/3}$ fm, implies that

(a) The central density of nuclei is independent of A

(b) The volume energy per nucleon is a constant

(c) The attractive part of the nuclear force has a long range

(d) The nuclear force is charge dependent

Ans. : (a), (b), (d)

Q61. An atomic nucleus X with half-life T_X decays to a nucleus Y, which has half-life T_Y. The condition (s) for secular equilibrium is (are)

(a) $T_X \simeq T_Y$ (b) $T_X < T_Y$ (c) $T_X \ll T_Y$ (d) $T_x \gg T_Y$

Ans. : (d)

Q62. In a typical human body, the amount of radioactive ^{40}K is 3.24×10^{-5} percent of its mass. The activity due to ^{40}K in a human body of mass 70 kg is ______ kBq.

(Round off to 2 decimal places)

(Half-life of $^{40}K = 3.942\times10^{16}$ S, Avogadro's number $N_A = 6.022\times10^{23}$ mol^{-1}

Ans. : 6.0

Solution: $\left|\frac{dN}{dt}\right| = \lambda N$

$$= \frac{0.693}{3.942\times10^{6}(s)} \times \frac{(70\times10^{3})}{40} \times \frac{3.24\times10^{-5}}{100} \times 6.022\times10$$

$= 6.0\times10^{13}$ disintegrations / s

$= 6.0\times10^{13}$ Bq

$= 6.0\times10^{10}$ kBq

Q63. A proton is confined within a nucleus of size 10^{-13} cm. The uncertainty in its velocity is _________ $\times10^{8}$ m/s.

(Round off to 2 decimal places)

(Planck's constant $h = 6.626\times10^{-34}\,J$ and proton mass $m_P = 1.672\times10^{-27}$ kg)

Ans. : 0.31

Solution: $\Delta p\,\Delta x \simeq \frac{h}{4\pi}$

$$\Delta v \simeq \frac{h}{4\pi m\,\Delta x} \simeq \frac{6.6\times10^{-34}}{4\times3.14\times1.672\times10^{-27}\times(10^{-15})} \simeq 0.31\times10^{8}\,m/s$$

Q64. Given the wave function of a particle $\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right) 0 < x < L$ and 0 elsewhere the probability of finding the particle between $x = 0$ and $x = \frac{L}{2}$ is _______.

(Round off to 1 decimal places)

Ans. : 0.5

Solution: $\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{L}x\right) 0 < x < L$, $p\left(0 \le x \le \frac{L}{2}\right) = \int_0^{L/2} |\psi|^2\,dx = \frac{1}{2}$