Solid State Physics

JEST-2012

Q1. A beam of X-rays is incident on a BCC crystal. If the difference between the incident and scattered wavevectors is $\vec{K} = n\hat{x} + k\hat{y} + l\hat{z}$ where $\hat{x}, \hat{y}, \hat{z}$ are the unit vectors of the associated cubic lattice, the necessary condition for the scattered beam to give a Laue maximum is

(a) $h+k+l=$ even (b) $h=k=l$

(c) h,k,l are all distinct (d) $h+k+l=$ odd

Ans.: (a)

Solution: In BCC basis $(0, \quad 0, \quad 0), \left(\frac{1}{2}, \quad \frac{1}{2}, \quad \frac{1}{2}\right)$

Crystal structure factor (F) is defined as

$$F = f \cdot S = f\sum_{n=1}^{n_{eff}} e^{2\pi i(hu_n + kV_n + l\omega_n)} = f\left[e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}\right)[h+k+l]}\right] = f\left[1 + e^{\pi i(h+k+l)}\right]$$

$F_{110} = 2f \Rightarrow I = 4f^2$, $F_{111} = 0 \Rightarrow I = 0$, $F_{200} = 2f \Rightarrow I = 4f^2$

Thus, if $h+k+l=$ even, then plane will be present.

If $h+k+l=$ odd, then plane will be absent.

Q2. The second order maximum in the diffraction of X-rays of 0.20 nanometer wavelength from a simple cubic crystal is found to occur at an angle of thirty degrees to the crystal plane. The distance between the lattice planes is

(a) 1 Angstrom (b) 2 Angstrom (c) 4 Angstrom (d) 8 Angstrom

Ans.: (c)

Solution: $2d\sin\theta = n\lambda \Rightarrow 2d\sin\theta = 2\lambda \Rightarrow 2\times d\times\sin 30^\circ = 2\times 0.2\times 10^{-9}m$

$$d = 2\times 0.2\times 10^{-9}m = 0.4\times 10^{-9}m = 4\times 10^{-10}m = 4A^\circ$$

Q3. The Dulong –Petit law fails near room temperature (300 K) for many light elements (such as boron and beryllium) because their Debye temperature is

(a) >> 300 K (b) ~ 300 K (c) << 300 K (d) 0 K

Ans.: (a)

JEST-2013

Q4. A flat surface is covered with non-overlapping disks of same size. What is the largest fraction of the area that can be covered?

(a) $\frac{3}{\pi}$ (b) $\frac{5\pi}{6}$ (c) $\frac{6}{7}$ (d) $\frac{\pi}{2\sqrt{3}}$

Ans.: (d)

Solution: In closed packed hexagonal lattice,

$$n_{eff} = \frac{1}{3}n_C + \frac{1}{2}n_f + 1\times n_i = \frac{1}{3}\times 6 + 1 = 3 \text{ and } a = 2r$$

Now, largest fraction of area i.e., packing fraction $= \dfrac{n_{eff}\times A}{6\times\frac{\sqrt{3}}{4}\times a^2} = \dfrac{3\times\pi r^2}{6\times\frac{\sqrt{3}}{4}\times(2r)^2} = \dfrac{\pi}{2\sqrt{3}}$

Q5. A metal suffers a structural phase transition from face-centered cubic (FCC) to the simple cubic (SC) structure. It is observed that this phase transition does not involve any change of volume. The nearest neighbor distances d_{fcc} and d_{sc} for the FCC and the SC structures respectively are in the ratio $\left(\frac{d_{fcc}}{d_{sc}}\right)$ [Given $2^{\frac{1}{3}} = 1.26$]

(a) 1.029 (b) 1.122 (c) 1.374 (d) 1.130

Ans. : ()

Solution: Nearest neighbour in SC is a and $C.N = 6$

Nearest neighbour in FCC is $\frac{a}{\sqrt{2}}$ and $C.N = 12$

$$\frac{d_{fcc}}{d_{sc}} = \frac{\frac{a}{\sqrt{2}}}{a} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.707$$

JEST-2014

Q6. Circular discs of radius 1 m each are placed on a plane so as to form a closely packed triangular lattice. The number of discs per unit area is approximately equal to

(a) $0.86\,m^{-2}$ (b) $0.43\,m^{-2}$ (c) $0.29\,m^{-2}$ (d) $0.14\,m^{-2}$

Ans.: (c)

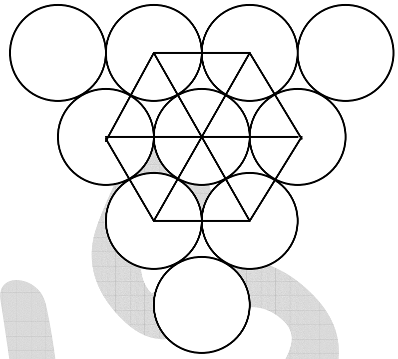
Solution: For closely packed triangular lattice,

$$a = 2r, \quad r = 1 \quad n_{eff} = \frac{1}{6}\times n_C + \frac{1}{2}\times n_f + 1\times n_I$$

$$\Rightarrow n_{eff} = \frac{1}{6}\times 3 + 0\times\frac{1}{2} + 1\times 0 \Rightarrow n_{eff} = 0.5$$

$$\text{Occupancy} = \frac{n_{eff}}{A} \qquad (\because a = 2)$$

$$\Rightarrow \frac{0.5}{\frac{\sqrt{3}}{4}\times 2} = \frac{0.5}{\sqrt{3}} = 0.29\ m^{-2}$$



Closely packed hexagonal

Q7. An ideal gas of non-relativistic fermions in 3-dimensions is at 0K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor

(a) $2^{1/2}$ (b) 1 (c) $2^{1/3}$ (d) $2^{-1/3}$

Ans.: (d)

Solution: $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}}$ at $T = 0\,K$

$$\because n' = 2n \text{ and } m' = 2m \Rightarrow E_F' = \frac{\hbar^2}{4m}\left(3\pi^2 2n\right)^{2/3} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}\times 2^{-1/3}$$

Q8. When two different solids are brought in contact with each other, which one of the following is true?

(a) Their Fermi energies become equal

(b) Their band gaps become equal

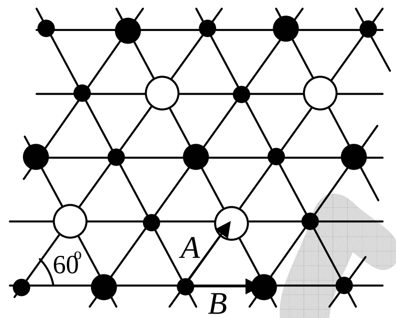
(c) Their chemical potentials become equal

(d) Their work functions become equal

Ans.: (c)

JEST-2015

Q9. What is the area of the irreducible Brillouin zone of the crystal structure as given in the figure?



$|A| = |B| = A$

(a) $\dfrac{2\pi^2}{\sqrt{3}A^2}$

(b) $\dfrac{\sqrt{3}\pi^2}{2A^2}$

(c) $\dfrac{2\pi^2}{A^2}$

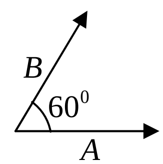
(d) $\dfrac{\pi^2}{\sqrt{3}A^2}$

Ans.: (a)

Solution: Area of the Brillouin zone can be related to the area of normal cell as

$$\text{Area of B.Z.} = \frac{\pi^2}{\text{Area of cell}} = \frac{\pi^2}{\left|\vec{A}\times\vec{B}\right|}$$

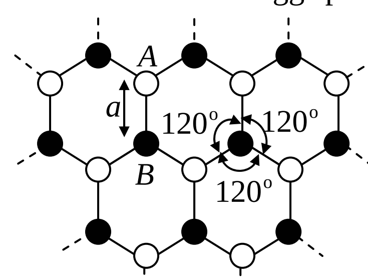
$$\vec{A}\times\vec{B} = |A||B|\sin\theta = A^2\sin\left(60^0\right) = \frac{\sqrt{3}}{2}A^2$$



$|A| = |B| = A$

$$\therefore \text{ Area of Brillouin zone} = \frac{2\pi^2}{\sqrt{3}A^2}$$

Q10. For a 2 - dimensional honeycomb lattice as shown in the figure, the first Bragg spot occurs for the grazing angle θ_1, while sweeping the angle from 0°. The next Bragg spot is obtained at θ_2 given by



(a) $\sin^{-1}\left(3\sin\theta_1\right)$

(b) $\sin^{-1}\left(\dfrac{3}{2}\sin\theta_1\right)$

(c) $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\sin\theta_1\right)$

(d) $\sin^{-1}\left(\sqrt{3}\sin\theta_1\right)$

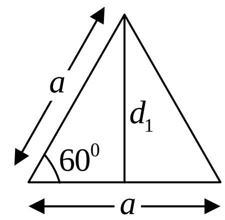
Ans.: (c)

Solution: According to Bragg's law, the condition for first Bragg spot and second spot is

$$2d_1\sin\theta_1 = n\lambda \text{ and } 2d_2\sin\theta_2 = n\lambda$$

$$\therefore 2d_1 \sin\theta_1 = 2d_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left(\frac{d_1}{d_2}\sin\theta_1\right)$$

For 2 - dimensional honeycomb lattice, the lattice constant 'a' and interplanar spacing 'd' is linked as



$$d_1^2 = a^2 - \left(\frac{a}{2}\right)^2 \Rightarrow d_1 = \sqrt{a^2 - \frac{a^2}{4}} = \frac{\sqrt{3}}{2}a \quad \text{and } d_2 = a$$

$$\therefore \theta_2 = \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta_1\right)$$

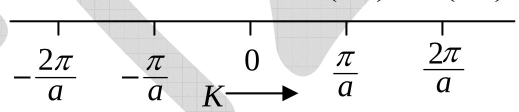
Q11. Given the tight binding dispersion relation $E(k) = E_0 + A\sin^2\left(\frac{ka}{2}\right)$, where E_0 and A are constants and a is the lattice parameter. What is the group velocity of an electron at the second Brillouin zone boundary?

(a) 0 (b) $\frac{a}{h}$ (c) $\frac{2a}{h}$ (d) $\frac{a}{2h}$

Ans.: (a)

Solution: Group velocity is defined as, $v_g = \frac{1}{\hbar}\frac{dE}{dk}$

Since $E = E_0 + A\sin^2\left(\frac{ka}{2}\right) \Rightarrow \frac{dE}{dk} = aA\sin\left(\frac{ka}{2}\right)\cos\left(\frac{ka}{2}\right) = \frac{aA}{2}\sin ka$



In one dimension, the Brillouin zone boundary is

The 1st Brillouin zone boundaries lie at $\pm\frac{\pi}{a}$

The 2nd Brillouin zone boundaries lie at $\pm\frac{2\pi}{a}$

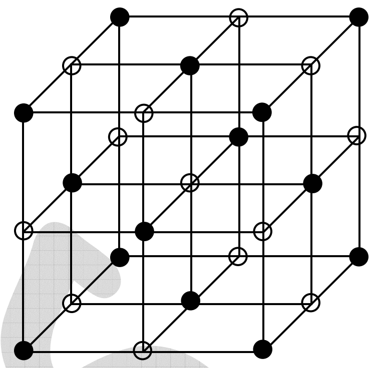
Thus, the group velocity at the second Brillouin zone boundary is

$$v_g\Big|_{\pm\frac{2\pi}{a}} = \frac{aA}{2}\sin\left(\frac{2\pi}{a}\times a\right) = \frac{aA}{2}\sin 2\pi \Rightarrow v_g = 0$$

Q12. The total number of Na^+ and Cl^- ions per unit cell of $NaCl$ is,

(a) 2 (b) 4 (c) 6 (d) 8

Ans.: (d)



Solution: Total number of Na^+ and Cl^- ions per unit (d) is

$$N_{Cl}^- = \frac{1}{8}n_c + \frac{1}{2}n_f, \quad N_{Na}^+ = \frac{1}{4}n_e + 1\times n_i$$

where n_c = number of ions at corner

n_f = number of ions at face

n_e = number of ions at edges

n_i = number of ions inside

$$N = N_{Cl}^- + N_{Na}^+ = \frac{1}{8}\times 8 + \frac{1}{2}\times 6 + \frac{1}{4}\times 12 + 1\times 1 = 1+3+3+1 = 8$$

Q13. For non-interacting Fermions in $d-$dimensions, the density of states $D(E)$ varies as $E^{\left(\frac{d}{2}-1\right)}$. The Fermi energy E_F of an N particle system in $3-$, $2-$ and $1-$dimensions will scale respectively as,

(a) $N^2, N^{2/3}, N$ (b) $N, N^{2/3}, N^2$

(c) $N, N^2, N^{2/3}$ (d) $N^{2/3}, N, N^2$

Ans.: (d)

JEST-2016

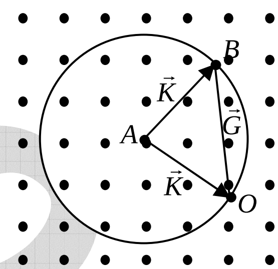
Q14. If $\vec{k}$ is the wavevector of incident light ($\left|\vec{k}\right| = \frac{2\pi}{\lambda}$, λ is the wavelength of light) and $\vec{G}$ is a reciprocal lattice vector, then the Bragg's law can be written as:

(a) $\vec{k} + \vec{G} = 0$ (b) $2\vec{k}.\vec{G} + G^2 = 0$

(c) $2\vec{k}.\vec{G} + k^2 = 0$ (d) $\vec{k}.\vec{G} = 0$

Ans. : (b)

Solution: By means of Eward construction, we can write the Bragg's law in vector form



$$\vec{G} = OB,\ \vec{K}' = AO$$

For diffraction it is necessary that vector $\vec{K}' + \vec{G}$, that is vector AB be equal in magnitude to the vector K or

$$(K + G)^2 = K^2 \Rightarrow 2\vec{K} \cdot \vec{G} + G^2 = 0$$

Q15. The number of different Bravais lattices possible in two dimensions is:

(a) 2 (b) 3 (c) 5 (d) 6

Ans. : (c)

Solution: Five Bravais lattices in $2D$ are:

(i) Square lattice

(ii) Rectangular (P) lattice

(iii) Rectangular (C) lattice

(iv) Hexagonal lattice

(v) Oblique lattice