(c) Rayleigh and Jeans Theory of Black Body Radiation (Classical Theory)

Classical theory of Black Body Radiation is explained by different people but popularly explained by first Rayleigh and Jeans theory then Wien's distribution law.

Rayleigh and Jeans Theory of Black Body Radiation

Consider a cavity with metallic walls heated uniformly up to temperature T. The walls emits electromagnetic radiation in the thermal range of frequencies.

Basic assumption and methodology

(1) Classical electromagnetic theory is used to show that radiation inside the cavity must exist in the form of standing waves with nodes at the metallic surfaces.

(2) By using geometrical argument, a count is made of the number of such standing waves $dN(\nu)$ in the frequency interval ν to $\nu + d\nu$

According to Rayleigh Jeans radiation waves in black body can be compared to the standing waves in cubical cavity. Let the equation of standing wave is

$y = A\sin(\vec{k}.\vec{r})\cos\omega t = A\sin(k_x x + k_y y + k_z z)\cos\omega t$ where $\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$ is wave vector.

For standing wave at the wall their must be only nodes so $k_x = \frac{n_x\pi}{a}, k_y = \frac{n_y\pi}{a}, k_z = \frac{n_z\pi}{a}$

Where $n_x = 1, 2, 3....,$ $n_y = 1, 2, 3....,$ and $n_z = 1, 2, 3....,$

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \frac{n_x^2 + n_y^2 + n_z^2}{\pi^2 a^2} \Rightarrow n_x^2 + n_y^2 + n_z^2 = \frac{k^2 a^2}{\pi^2}$$

where $k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c}$

$n_x^2 + n_y^2 + n_z^2 = \frac{4a^2\nu^2}{c^2}$ which represents the equation of sphere of radius $\frac{2a\nu}{c}$ with positive values of n_x, n_y and n_z lies in the one 8^{th} part of this sphere, which is shown in figure 1.

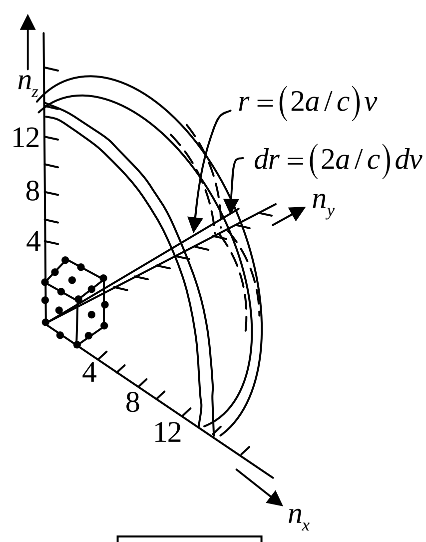

Figure 1

We know that one set of n_x, n_y and n_z represent one mode of vibration.

This implies that the total number of mode N in frequency range 0 to ν

$$N(\nu)=\frac{1}{8}\times\frac{4\pi}{3}\left(\frac{2a\nu}{c}\right)^3=\frac{4\pi a^3\nu^3}{3c^3}$$

$$dN\,d\nu=\frac{4\pi a^3\nu^2}{c^3}d\nu \text{ put } a^3=V \;\Rightarrow dN\,d\nu=\frac{4\pi V\nu^2}{c^3}d\nu$$

We also account that two polarization state of vibration for electromagnetic wave per mode of vibrations. So $dN\,d\nu=2\times\frac{4\pi Va^3\nu^2}{c^3}d\nu\Rightarrow dNd\nu=\frac{8\pi Va^3\nu^2}{c^3}d\nu$

In term of wavelength we can calculate number of modes in range λ to $\lambda+d\lambda$ one will get

$$N d\lambda=\frac{8\pi V}{\lambda^4}d\lambda$$

(3) From using kinetic theory of gases each standing wave (Equivalent harmonic oscillator) at equilibrium absolute temperature T contribute energy kT.

So the energy per unit Volume in the frequency interval ν to $\nu+d\nu$ of the black body spectrum of cavity at absolute temperature T is just product of average energy per standing wave times the number of standing waves in the frequency interval, is given by

$\rho(\nu)d\nu=\frac{8\pi\nu^2}{c^3}kT$ which is popularly known as Rayleigh-Jeans formula for black body radiation.

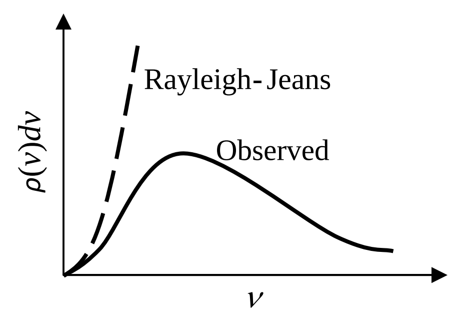

Figure 2: Energy density versus frequency according to Rayleigh-Jeans

The equation does accurately describe the low frequency (high wavelength) spectrum of thermal emission from objects, but it fails to accurately fit the experimental data for high frequency (short wavelengths) emission.