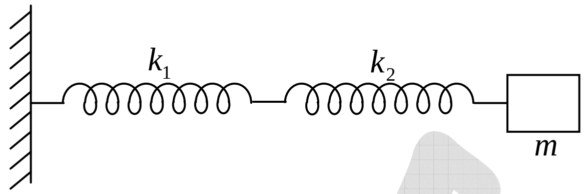
(b) The Basic Mass Loaded Spring System

Series Spring System

Let x be the displacement of m from its equilibrium position at an instant, and x_1, x_2 is extension in the length of springs k_1 and k_2 respectively.



Thus, $x = x_1 + x_2$

The restoring force in k_1 and k_2 will be the same. By Hooke's law we have

$$F = -k_1 x_1 = -k_2 x_2$$

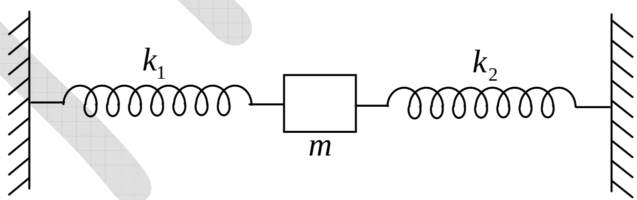
$$x = -\frac{F}{k_1} - \frac{F}{k_2} \Rightarrow F = -\frac{k_1 k_2}{k_1 + k_2} x = -kx$$

Thus the force constant of the system is $k = \frac{k_1 k_2}{k_1 + k_2}$.

Hence, the time period is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1 k_2}}$

Parallel Spring System

Consider the arrangement for the following mass loaded spring having spring constant k_1 and k_2.



During oscillation one spring stretched and other compressed and vice versa. Let x be the displacement of mass m from its equilibrium distance, and this is also the compression and expansion in the springs k_1 and k_2. The restoring force developed in the two springs is $F_1 = -k_1 x$ and $F_2 = -k_2 x$. F_1 and F_2 act on the mass in the same direction

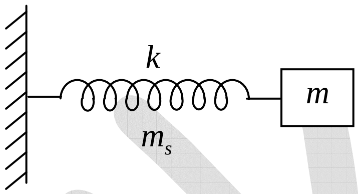
$$F = F_1 + F_2 = -k_1 x - k_2 x = -(k_1 + k_2)x = -kx$$

Thus the force constant of the system is $(k_1 + k_2)$, and the time period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

Finite Mass Spring System

Consider a system of mass m attached with spring of mass $m_s\,(m_s << m)$ and spring constant k.



If $m_s << m$, then the spring will stretch uniformly along its length. Let l be the length of the spring, its mass per unit length will be $\frac{m_s}{l}$. Let us consider an element of length ds at a distance s from the fixed end of the string. The mass of this element is $\left(\frac{m_s}{l}\right)ds$. Let x be the instantaneous displacement of m.

The displacement of an element of spring $= \left(\frac{s}{l}\right)x$

The instantaneous velocity of element $= \left(\frac{s}{l}\right)\frac{dx}{dt}$

The instantaneous kinetic energy of element $= \frac{1}{2}\left(\frac{m_s}{l}ds\right)\left(\frac{sdx}{ldt}\right)^2 = \frac{m_s}{2l^3}\left(\frac{dx}{dt}\right)^2 s^2 ds$

The total kinetic energy of the uniform spring $\frac{m_s}{2l^3}\left(\frac{dx}{dt}\right)^2 \int_0^l s^2 ds = \frac{1}{2}\left(\frac{m_s}{3}\right)\left(\frac{dx}{dt}\right)^2$

The total kinetic energy of the whole system is $K = \frac{1}{2}\left(m + \frac{m_s}{3}\right)\left(\frac{dx}{dt}\right)^2$

This shows the effective mass of the system is $m + \frac{m_s}{3}$.

Hence the time period is $T = 2\pi\sqrt{\dfrac{m + \dfrac{m_s}{3}}{k}}$

Vertical Spring System

Consider l be the length of the spring hanging vertically. When mass m attached to the lower end, it's length extended by y_0. By Hooke's law the restoring force is $F = -ky_0$.

The other force acting on the mass is its weight $+mg$ (downward). Thus the total force on m is $-ky_0 + mg$.

Since mass has no acceleration, the total force on mass m is zero,

$$-ky_0 + mg = 0 \Rightarrow y_0 = mg/k$$

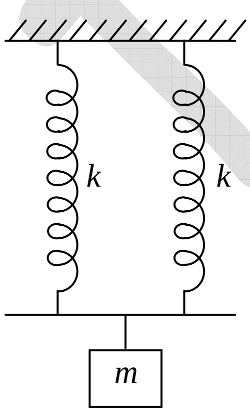
When the body is pulled through a small distance y from the equilibrium position and released, it start oscillating with SHM, since the restoring force is $F = -ky$, imparting an acceleration of $\frac{d^2y}{dt^2}$ is given by $\frac{d^2y}{dt^2} = -\frac{k}{m}y$

Which is a equation of SHM whose time period is $T = 2\pi\sqrt{\frac{m}{k}}$

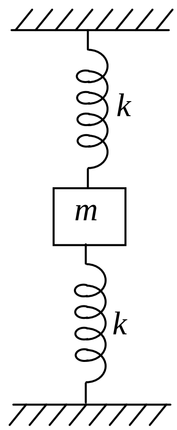
It can also be written as $T = 2\pi\sqrt{\frac{y_0}{g}}$

Example: Find the time period of the following sets of mass loaded springs hanged vertically.

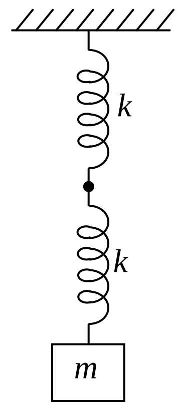
(a)



(b)



(c)



Solution: (a) Since both springs are in parallel combination. The resultant spring constant of the system is $k' = k + k = 2k$. Thus the time period is $T = 2\pi\sqrt{\frac{m}{2k}}$

(b) Since both springs are in series. The resultant spring constant of the system is $k' = \frac{k.k}{(k+k)} = \frac{k}{2}$. Thus the time period is $T = 2\pi\sqrt{\frac{2m}{k}}$

(c) In this case also springs are in series. Thus time period of the system is $T = 2\pi\sqrt{\frac{2m}{k}}$