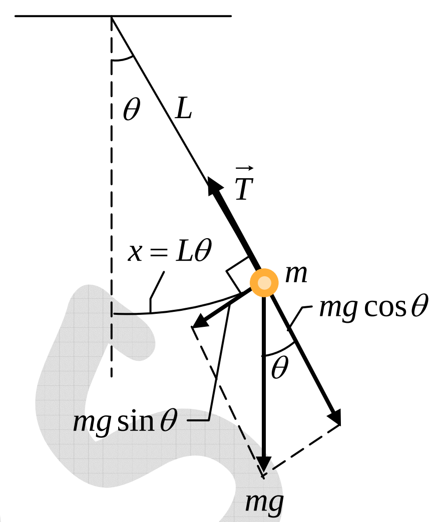
(c) The Simple Pendulum



A simple pendulum is an idealized system containing of a massless inextensible string, fixed rigidly at one end, having a point mass at the other end.

A mass m suspended by a light string of length L from a fixed point. When the mass is displaced slightly from its equilibrium position and released, it oscillates in a vertical plane under gravity. Let θ be the angular displacement of the pendulum. The force acting on m is mg . This force can be resolved into two components, a radial component $mg\cos\theta$ along the string and tangential component $mg\sin\theta$ at right angle to it. The tangential component is the restoring force. Thus, $F = -mg\sin\theta$

Using Newton's law, we get $m\dfrac{d^2x}{dt^2} = -mg\sin\theta$

For small angles of oscillations $\sin\theta = \dfrac{x}{L}$, is governed by $m\dfrac{d^2x}{dt^2} = -mg\dfrac{x}{l}$

The frequency (f) and the period (T) of the pendulum in this case are given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{l}{g}}$$

Therefore, the simple pendulum has the following properties

(i) The period of a pendulum is independent of its own mass and the amplitude of the oscillation.

(ii) The period of the pendulum is depends on its length.

Large Amplitude: When the amplitude of motion of the simple pendulum is large, the approximation $\sin q = q$ does not hold and the pendulum deviates from the simple harmonic behavior. Then the time period

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)\left(1 + \frac{1}{2}\sin^2\frac{\theta}{2} + \frac{1}{2^2}\frac{3^2}{4^2}\sin^4\frac{\theta}{2} + \dots\right)}$$

Thus with increasing amplitude the time period increases.