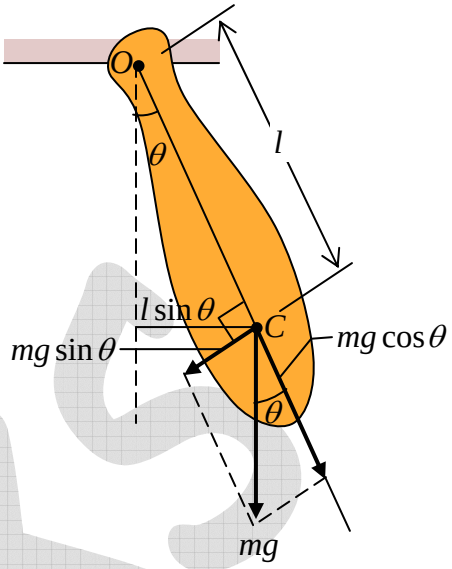
(d) Compound Pendulum

A compound pendulum is a rigid body, of any shape, capable of oscillating about a horizontal axis passing through it.



Let m be the mass of the body and l be the distance of C from O. When body displaced by an angle θ with the vertical, at this instant the **moment** of its weight mg about the axis of oscillation through O is $mg\left(l\sin\theta\right)$. This is the restoring torque which brings the body to its equilibrium position.

Thus, $\tau = -mgl\sin\theta$

For small angle, we have; $\tau = -mgl\theta$

If I is the moment of inertia of the pendulum about the axis of suspension O, the torque is also equal to

$$I\frac{d^2\theta}{dt^2} = -mgl\theta \quad \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\theta$$

Thus the time period is $T = 2\pi\sqrt{\dfrac{I}{mgl}}$

It is sometime more useful to write the time period in term of radius of gyration of pendulum. If we write the moment of inertia about a parallel axis through C, then

$$I = mk^2 + ml^2 = m\left(k^2 + l^2\right)$$

Thus, the time period is

$$T = 2\pi\sqrt{\frac{\left(\dfrac{k^2}{l} + l\right)}{g}} = 2\pi\sqrt{\frac{L}{g}}$$

Here L is called the "length of the equivalent simple pendulum."