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CSIR NET-JRF Physical Sciences Paper June-2024

Solution-Mathematical Physics

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PART B

Q22. A set of 100 data points yields an average $\bar{x}=9$ and a standard deviation $\sigma_x=4$. The error in the estimated mean is closest to

(1) 3.0 (2) 0.4 (3) 4.0 (4) 0.3

Ans.: (2)

Solution.: $<x>=\bar{x}=9$, $\Delta x=\sigma_x=4$, sample size $n=100$.

The error in the estimated mean $=\dfrac{\sigma_x}{\sqrt{n}}=\dfrac{4}{\sqrt{100}}=0.4$

Q25. An integral is given by $\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dy\exp\left[-\left(x^2+y^2+2axy\right)\right]$, where a is a real parameter. The full range of values of a for which the integral is finite, is

(1) $-\infty<a<\infty$ (2) $-2<a<2$ (3) $-1<a<1$ (4) $-1\le a\le 1$

Ans.: (3)

Q32. Vorticity of vector field $\vec{B}$ is defined as $\vec{V}=\vec{\nabla}\times\vec{B}$. Given $\vec{B}=kxyz\hat{r}$, where k is a constant, which one of the following is correct?

(1) Vorticity is a null vector for all finite x,y,z

(2) Vorticity is parallel to the vector field everywhere

(3) The angle between vorticity and vector field depends on x,y,z

(4) Vorticity is perpendicular to the vector field everywhere

Ans.: (4)

Solution.: $\vec{B}=kxyz\hat{r}=k\left(r\sin\theta\cos\phi\right)\left(r\sin\theta\sin\phi\right)\left(r\cos\theta\right)\hat{r}\Rightarrow\vec{B}=\left(\dfrac{k}{2}r^3\sin^2\theta\cos\theta\sin 2\phi\right)\hat{r}$

Then $\vec{V}=\vec{\nabla}\times\vec{B}=V_\theta\hat{\theta}+V_\phi\hat{\phi}$ and $\vec{V}.\vec{B}=0$ always.

Q39. The matrix A is given by $A=\begin{bmatrix}1 & 2 & -3\\ 0 & 3 & 2\\ 0 & 0 & -2\end{bmatrix}$

The eigenvalues of $3A^3+5A^2-6A+2I$, where I is the identity matrix, are

(1) 4, 9, 27 (2) 1, 9, 44

(3) 1, 110, 8 (4) 4, 110, 10

Ans.: (4)

Solution.: The characteristic equation is given by $\left|A-\lambda I\right|=0\Rightarrow\begin{vmatrix}1-\lambda & 2 & -3\\ 0 & 3-\lambda & 2\\ 0 & 0 & -2-\lambda\end{vmatrix}=0$

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$\Rightarrow (1-\lambda)\left[(3-\lambda)(-2-\lambda)-0\right]=0 \Rightarrow \lambda = 1,\ 3,\ -2$

Eigenvalues of A is $=1,\ 3,\ -2$; Eigenvalues of A^2 is $=1,\ 9,\ 4$

Eigenvalues of A^3 is $=1,\ 27,\ -8$; Eigenvalues of I is $=1,\ 1,1$

For $\lambda=1$, the eigenvalues of $3A^3+5A^2-6A+2I=3+5-6+2=4$.

For $\lambda=3$, the eigenvalues of $3A^3+5A^2-6A+2I=81+45-18+2=110$.

For $\lambda=-2$, the eigenvalues of $3A^3+5A^2-6A+2I=-24+20+12+2=10$.

Q44. Probability density function of a variable x is given by

$P(x)=\frac{1}{2}\left[\delta(x-a)+\delta(x+a)\right]$. The variance of x is

(1) a^2 (2) 0 (3) $2a^2$ (4) $\frac{a^2}{2}$

Ans.: (1)

Solution.:

$$\text{Var}\, x=\sigma_x^2=\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx=\int_{-\infty}^{\infty}x^2\frac{1}{2}\left[\delta(x-a)+\delta(x+a)\right]dx=\frac{1}{2}(a)^2+\frac{1}{2}(-a)^2=a^2$$

PART C

Q48. An integral transform $\tilde{f}(x)$ of a function $f(x)$ can be regarded as a result of applying an operator F to the function such that

$$(Ff)(x)\equiv \tilde{f}(x)=\int_{-\infty}^{\infty}dy\, e^{-ixy}f(y)$$

If I is the identity operator, then the operator F^4 is given by

(1) $(2\pi)^4 I$ (2) $(2\pi)I$ (3) I (4) $(2\pi)^2 I$

Ans.: (4)

Q56. The integral $I=\int_0^1\frac{2x}{1+x^2}dx$ is estimated using Simpson's 1/3rd rule with a grid value of $h=0.5$. The difference $\left(I_{\text{estimated}}-I_{\text{exact}}\right)$ is closest to

(1) 0.007 (2) 0.001 (3) 0.0007 (4) –0.005

Ans.: (1)

Solution: $I_{exact}=\int_0^1\frac{2x}{1+x^2}dx=\left[\ln\left(1+x^2\right)\right]_0^1=\ln 2=0.693$

	$x=$	$y=\dfrac{2x}{1+x^2}$
y_0	0	0
y_1	0.5	$\dfrac{4}{5}$
y_2	1.0	1

$$I=\frac{h}{3}\left[y_0+2\left(y_2+y_4+y_6....\right)+4\left(y_1+y_3+y_5+...\right)+y_n\right]$$

$$I=\frac{h}{3}\left[y_0+4y_1+2y_2\right]=\frac{1/2}{3}\left[0+4\times\frac{4}{5}+1\right]=\frac{1}{6}\left[\frac{16}{5}+1\right]=\frac{21}{30}=0.7$$

So $\left(I_{\text{estimated}}-I_{\text{exact}}\right)=0.7-0.693=0.007$

Q62. The following four matrices form a representation of a group

$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix},\ A=\begin{pmatrix}-1&0\\0&-1\end{pmatrix},\ B=\begin{pmatrix}0&1\\1&0\end{pmatrix},\ C=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$$

Which of the following represents the multiplication table for the same group?

1.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	A	I
C	C	B	I	A

2.

	I	A	B	C
I	I	A	B	C
A	A	B	C	I
B	B	C	I	A
C	C	I	A	B

3.

	I	A	B	C
I	I	A	B	C
A	A	C	I	B
B	B	I	C	A
C	C	B	A	I

4.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	I	A
C	C	B	A	I

Ans.: (4)

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Q68. The general solution for the second order differential equation

$$\frac{d^2y}{dx^2} - y = x\sin x$$

will be

(1) $C_1e^x + C_2e^{-x} - \frac{1}{2}(x\sin x + \cos x)$ (2) $C_1e^x + C_2e^{-x} - \frac{1}{2}(\sin x - x\cos x)$

(3) $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin x - \cos x)$ (4) $C_1e^x + C_2e^{-x} + \frac{1}{2}x(\sin x + \cos x)$

(where C_1 and C_2 are arbitrary constants)

Ans.: (1)

Solution:

Characteristic equation: $\lambda^2 - 1 = 0 \Rightarrow \lambda = +1, -1 \Rightarrow C.F. = C_1e^x + C_2e^{-x}$

$$\Rightarrow P.I. = \frac{1}{D^2-1}x\sin x = \text{Im}\left(\frac{1}{D^2-1}xe^{ix}\right)$$

$$\frac{1}{D^2-1}xe^{ix} = e^{ix}\frac{1}{(D+i)^2-1}x = e^{ix}\frac{1}{D^2-1+2iD-1}x = e^{ix}\frac{1}{D^2+2iD-2}x$$

$$= e^{ix}\frac{1}{-2\left(1-\frac{D^2}{2}-iD\right)}x = -\frac{e^{ix}}{2}\left(1-\frac{D^2}{2}-iD\right)^{-1} = -\frac{e^{ix}}{2}\left(1+\frac{D^2}{2}+iD\right)x$$

$$= -\frac{e^{ix}}{2}(x+0+i) = -\frac{1}{2}(\cos x + i\sin x)(x+i)$$

$$\Rightarrow P.I. = \text{Im}\left(\frac{1}{D^2-1}xe^{ix}\right) = -\frac{1}{2}(x\sin x + \cos x)$$

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Solution-Classical Mechanics

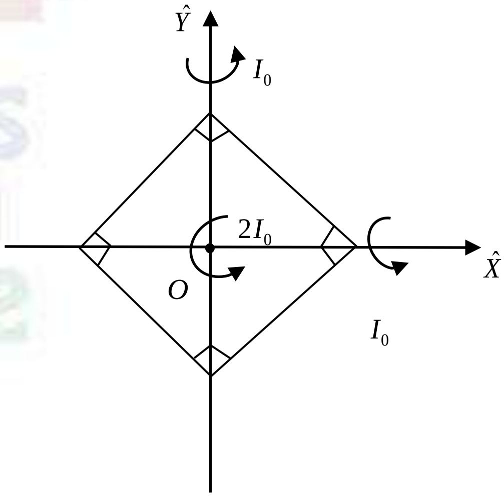
PART B

Q23. A uniform plane square sheet of mass m is centered at the origin of an inertial frame. The sheet is rotating about an axis passing through the origin. At an instant when all its vertices lie on x and y axes, the angular momentum is $\vec{L} = I_0\omega_0\left(2\hat{i}+\hat{j}+2\hat{k}\right)$, where I_0 is the moment of inertia about the x axis. At this instant, the angular velocity of the sheet is

(1) $\left(2\hat{i}+\hat{j}+2\hat{k}\right)\omega_0$ (2) $\left(2\hat{i}+\hat{j}+\hat{k}\right)\omega_0$

(3) $\left(2\hat{i}+\hat{j}\right)\omega_0$ (4) $\left(\hat{i}+\hat{j}\right)\omega_0$

Ans. (2):



Solution: $I_{XX} = I_{YY} = I_0$; $I_{ZZ} = 2I_{XX} = 2I_0$

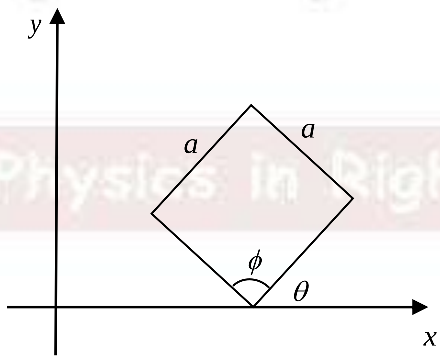
$$\vec{L} = \overset{=}{I}\vec{\omega}$$

$$I_0\omega_0\begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}I_0 & 0 & 0\\0 & I_0 & 0\\0 & 0 & 2I_0\end{pmatrix}\begin{pmatrix}\omega_X\\\omega_Y\\\omega_Z\end{pmatrix}$$

$$\omega_X = 2\omega_0\ ;\ \omega_Y = \omega_0;\ \omega_Z = \omega_0$$

$$\vec{\omega} = \left(2\hat{i}+\hat{j}+\hat{k}\right)\omega_0$$

Q26. A square plate of dimension $a\times a$ makes an angle $\theta = \pi/4$ with the x axis in its rest frame (S) as shown in the figure.



It is moving with a speed $v = \sqrt{\frac{2}{3}}C$ along the x axis with respect to an observer S' (where C is the speed of light in vacuum). The value of the interior angle ϕ indicated in the figure (which is obviously $\pi/2$ in the frame S), as measured in S' is

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\frac{4\pi}{3}$

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Ans.: (1)

Solution:

$$OC = OB = \frac{a}{\sqrt{2}}$$

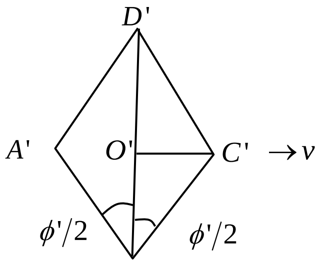
In S'-frame

$$\Rightarrow O'C' = (OC)\sqrt{1-\frac{2}{3}} \Rightarrow O'C' = \frac{a}{\sqrt{6}}$$

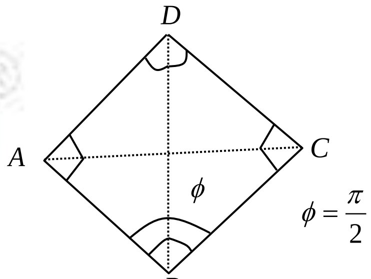
$$\Rightarrow O'B' = OB = \frac{a}{\sqrt{2}}$$

$$\Rightarrow \tan\frac{\phi'}{2} = \frac{O'C'}{O'B'} = \frac{a/\sqrt{6}}{a/\sqrt{2}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\phi'}{2} = \frac{\pi}{6} \Rightarrow \phi' = \frac{\pi}{3}$$



In S'-frame



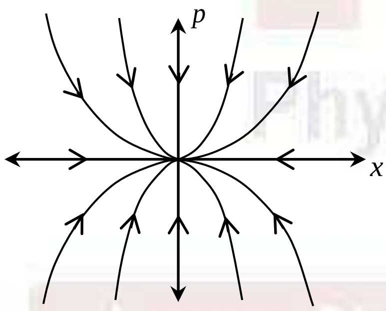
In rest frame

Q28. The evolution of the dynamical variables $x(t)$ and $p(t)$ is given by

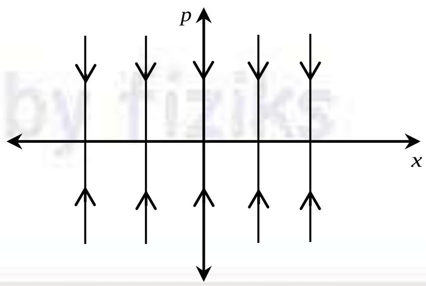
$$\dot{x} = ax, \quad \dot{p} = -p$$

where a is a constant. The trajectory in (x, p) space for $-1 < a < 0$ is best described by

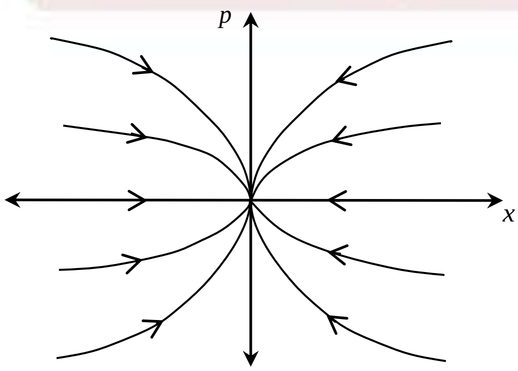
1.



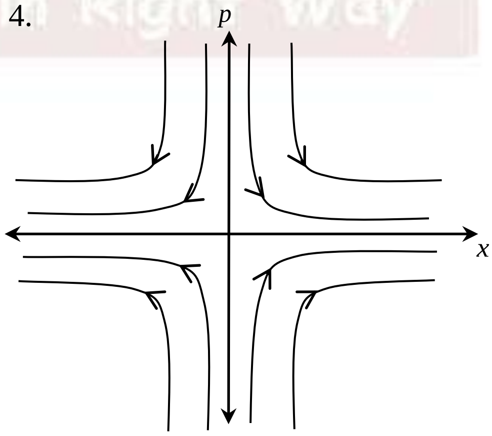
2.



3.



4.



Ans.: (1)

Solution:

$$\dot{x}=\alpha x\Rightarrow\frac{dx}{dt}=\alpha x\,;\;\dot{p}=-p\Rightarrow\frac{dp}{dt}=-p\Rightarrow\frac{dx/dt}{dp/dt}=-\frac{\alpha x}{p}\Rightarrow\int\frac{dx}{x}=-\alpha\int\frac{dp}{p}$$

$$\ln x=-\alpha\ln p+\ln c\Rightarrow\frac{x}{c}=p^{-\alpha}\Rightarrow\boxed{x=cp^{-\alpha}}$$

Here $-1<\alpha<0$

Option (1) is correct.

Q29. A body of mass m is acted upon by a central force $\vec{f}(\vec{r})=-k\vec{r}$, where k is a positive constant. If the magnitude of the angular momentum is l, then the total energy for a circular orbit is

(1) $2\sqrt{\frac{kl^2}{m}}$ (2) $\frac{1}{2}\sqrt{\frac{kl^2}{m}}$ (3) $\frac{3}{2}\sqrt{\frac{kl^2}{m}}$ (4) $\sqrt{\frac{kl^2}{m}}$

Ans.: (4)

Solution.:

$$f(r)=-\frac{\partial V}{\partial r}=-kr$$

Effective Potential $V_{eff}=\dfrac{l^2}{2mr^2}+V(r)$

For circular orbit

$$\left.\frac{\partial V_{eff}}{\partial r}\right|_{r=r_0}=-\frac{l^2}{mr_0^3}+\left.\frac{\partial V}{\partial r}\right|_{r=r_0}=0\Rightarrow\frac{l^2}{mr_0^3}=kr_0\Rightarrow r_0^4=\frac{l^2}{mk}$$

Total energy

$$E=\frac{1}{2}m\dot{r}^2+V_{eff}=0+\frac{l^2}{2mr_0^2}+\frac{1}{2}kr_0^2=\frac{l^2}{2m}\frac{\sqrt{mk}}{l}+\frac{1}{2}k\frac{l}{\sqrt{mk}}=\frac{l\sqrt{k}}{\sqrt{m}}=\sqrt{\frac{kl^2}{m}}$$

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PART C

Q50. A particle of mass m is moving in a potential $V(r) = -\frac{k}{r}$, where k is a positive constant. If $\vec{L}$ and $\vec{p}$ denote the angular momentum and linear momentum respectively, the value of α for which $\vec{A} = \vec{L} \times \vec{p} + \alpha mk\hat{r}$ is a constant of motion, is

(1) –2 (2) –1 (3) 2 (4) 1

Ans.: (4)

Solution: For constant of motion $\frac{d\vec{A}}{dt} = \frac{d}{dt}\left[\vec{L} \times \vec{p} + \alpha mk\hat{r}\right] = 0 \Rightarrow \frac{d\vec{L}}{dt} \times \vec{p} + \vec{L} \times \frac{d\vec{p}}{dt} + \alpha mk\frac{d\hat{r}}{dt} = 0$

$$\Rightarrow \left(mr^2\dot{\theta}\hat{z}\right) \times \left(-\frac{k}{r^2}\hat{r}\right) + \alpha mk\left(\dot{\theta}\hat{\theta}\right) = 0 \qquad \because \frac{d\vec{L}}{dt} = 0 \text{ for central force}$$

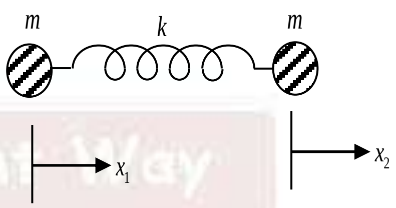
$$\Rightarrow -mk\dot{\theta}\hat{\theta} + \alpha mk\dot{\theta}\hat{\theta} = 0 \Rightarrow mk\dot{\theta}(-1+\alpha)\hat{\theta} = 0 \Rightarrow \alpha = 1$$

Q52. A linear molecule is modelled as two atoms of equal mass m placed at coordinates x_1 and x_2, connected by a spring of spring constant k. The molecule is moving in one dimension under an additional external potential $V(x_1, x_2) = \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right)$. If one frequency of molecular vibration is ω_0, the other frequency is

(1) $\sqrt{\omega_0^2 - \frac{k}{m}}$ (2) $\sqrt{\omega_0^2 + \frac{k}{m}}$ (3) $\sqrt{\omega_0^2 + \frac{2k}{m}}$ (4) $\sqrt{\omega_0^2 - \frac{2k}{m}}$

Ans. (3):

Solution.:



$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 \Rightarrow T = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

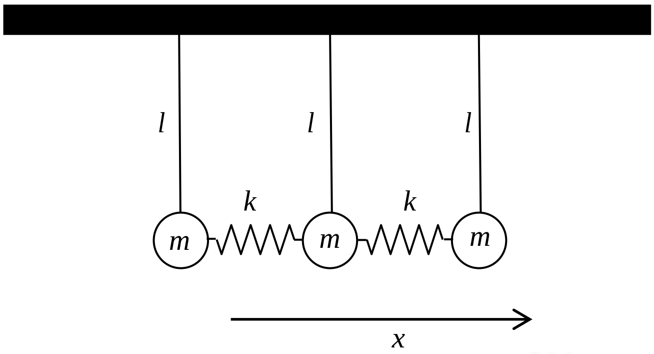
$$V = \frac{1}{2}k\left(x_2 - x_1\right)^2 + \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right)$$

$$\Rightarrow V = \frac{1}{2}k\left(x_1^2 + x_2^2 - 2x_1x_2\right) + \frac{1}{2}m\omega_0^2\left(x_1^2 + x_2^2\right) \Rightarrow V = \begin{pmatrix} k + m\omega_0^2 & -k \\ -k & k + m\omega_0^2 \end{pmatrix}$$

Use Secular equation $\left|V - \omega^2 T\right| = 0 \Rightarrow \begin{vmatrix} k + m\omega_0^2 - \omega^2 m & -k \\ -k & k + m\omega_0^2 - \omega^2 m \end{vmatrix} = 0$

$$\left(k + m\omega_0^2 - \omega^2 m\right)^2 - k^2 = 0 \Rightarrow m\left(\omega_0^2 - \omega^2\right)\left(2k + m\omega_0^2 - \omega^2 m\right) = 0 \Rightarrow \omega_1 = \omega_0 \text{ and } \omega_2 = \sqrt{\frac{2k}{m} + \omega_0^2}$$

Q58. Three identical simple pendula (of mass m and equilibrium string length l) are attached together by springs of spring constant k, as shown in the figure.

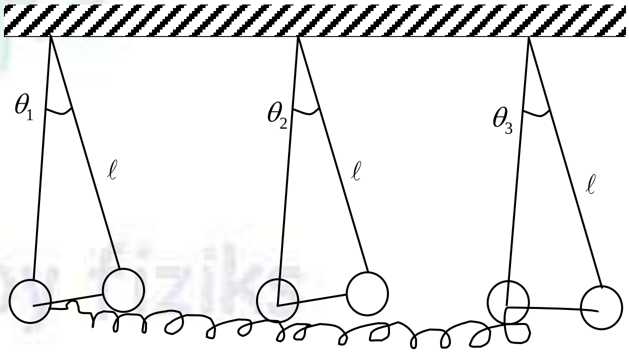


The frequencies of small oscillations are given by $\sqrt{\frac{g}{l}}, \sqrt{\frac{k}{m}+\frac{g}{l}}, \sqrt{\frac{3k}{m}+\frac{g}{l}}$. The normal modes (without normalisation) corresponding to these frequencies respectively are

1. (1,1,1), (1,0,1), (1,–2,1)
2. (1,1,1), (1,0,–1), (1,2,1)
3. (1,1,1), (1,0,–1), (1,–2,1)
4. (1,2,1), (1,0,–1), (1,1,1)

Ans. (3):

Solution.: Kinetic energy



$$T=\frac{1}{2}m\left(\ell^2\dot{\theta}_1^2+\ell^2\dot{\theta}_2^2+\ell^2\dot{\theta}_3^2\right)$$

$$\Rightarrow T=\begin{pmatrix} m\ell^2 & 0 & 0 \\ 0 & m\ell^2 & 0 \\ 0 & 0 & m\ell^2 \end{pmatrix}$$

$$V=\frac{1}{2}k\left(\ell\theta_2-\ell\theta_1\right)^2+\frac{1}{2}k\left(\ell\theta_3-\ell\theta_2\right)^2$$

$$-mg\ell\cos\theta_1-mg\ell\cos\theta_2-mg\ell\cos\theta_3$$

$$\Rightarrow V=\frac{1}{2}k\ell^2\left[\theta_1^2+2\theta_2^2+\theta_3^2-2\theta_1\theta_2-2\theta_2\theta_3\right]-mg\ell\left(1-\frac{\theta_1^2}{2}\right)-mg\ell\left(1-\frac{\theta_2^2}{2}\right)-mg\ell\left(1-\frac{\theta_3^2}{2}\right)$$

$$V=\begin{bmatrix} k\ell^2+mg\ell & -k\ell^2 & 0 \\ -k\ell^2 & 2k\ell^2+mg\ell & -m\ell^2 \\ 0 & -k\ell^2 & k\ell^2+mg\ell \end{bmatrix}$$

$$\left|V-\omega^2T\right|=0;\quad \begin{vmatrix} k\ell^2+mg\ell-m\omega^2\ell^2 & -k\ell^2 & 0 \\ -k\ell^2 & 2k\ell^2+mg\ell-m\ell^2\omega^2 & -k\ell^2 \\ 0 & -k\ell^2 & k\ell^2+mg\ell-m\ell^2\omega^2 \end{vmatrix}=0$$

$$R_1\to R_1+R_2+R_3$$

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$$\begin{vmatrix} mg\ell - m\omega^2\ell^2 & mg\ell - m\ell^2\omega^2 & mg\ell - m\ell^2\omega^2 \\ -k\ell^2 & 2k\ell^2 + mg\ell - m\ell^2\omega^2 & -k\ell^2 \\ 0 & -k\ell^2 & k\ell^2 + mg\ell - m\ell^2\omega^2 \end{vmatrix} = 0$$

$$\left(mg\ell - m\omega^2\ell^2\right)\begin{vmatrix} 1 & 1 & 1 \\ -k\ell^2 & 2k\ell^2 + mg\ell - m\ell^2\omega^2 & -k\ell^2 \\ 0 & -k\ell^2 & k\ell^2 + mg\ell - m\ell^2\omega^2 \end{vmatrix} = 0$$

$$\left(mg\ell - m\omega^2\ell^2\right)\begin{vmatrix} 0 & 1 & 1 \\ 0 & 2k\ell^2 + mg\ell - m\ell^2\omega^2 & -k\ell^2 \\ m\ell^2\omega^2 - k\ell^2 - mg\ell & -k\ell^2 & k\ell^2 + mg\ell - m\ell^2\omega^2 \end{vmatrix} = 0$$

$$\left(mg\ell - m\omega^2\ell^2\right)\left(m\ell^2\omega^2 - k\ell^2 - mg\ell\right)\left(-k\ell^2 - 2k\ell^2 - mg\ell + m\ell^2\omega^2\right) = 0$$

$$\omega_1 = \sqrt{\frac{g}{\ell}},\ \omega_2 = \sqrt{\frac{k}{m} + \frac{g}{\ell}},\ \omega_3 = \sqrt{\frac{3k}{m} + \frac{g}{\ell}}$$

$$\left(V - \omega_1^2 T\right)\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \qquad \left(V - \omega_2^2 T\right)\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\left(V - \omega_3^2 T\right)\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Q67. For a simple harmonic oscillator, the Lagrangian is given by

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

If $H(q,p)$ is the Hamiltonian of the system and $A(p,q) = \frac{1}{\sqrt{2}}(p + iq)$, the Poisson bracket $\{A, H\}$ is

(1) iA (2) A^* (3) $-iA^*$ (4) $-iA$

Ans.: (1)

Solution.: $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$

$$p = \frac{\partial L}{\partial \dot{q}} = \dot{q},\ H = p\dot{q} - L = p\dot{q} - \frac{1}{2}\dot{q}^2 + \frac{1}{2}q^2 = pp - \frac{1}{2}p^2 + \frac{1}{2}q^2 \Rightarrow H = \frac{1}{2}p^2 + \frac{1}{2}q^2$$

$$A = \frac{1}{\sqrt{2}}(p + iq) \Rightarrow [A, H] = \frac{\partial A}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial H}{\partial q} = \frac{i}{\sqrt{2}}p - \frac{1}{\sqrt{2}}q = i\frac{1}{\sqrt{2}}(p + iq) = iA$$

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Solution-Electromagnetic Theory

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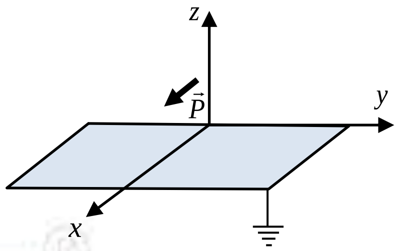
PART B

Q30. A point electric dipole $\vec{P} = p_x \hat{i}$ is placed at a vertical distance d above a grounded infinite conducting xy plane as shown in the figure. At a point $\vec{r}\ (r \gg d, z > 0)$ far away from the dipole, the electrostatic potential $V(r)$ varies approximately as



(1) $\frac{1}{r^2}$ (2) $\frac{1}{r^6}$

(3) $\frac{1}{r^3}$ (4) $\frac{1}{r^4}$

Ans.: (3)

Solution.:

Using image method if we draw image point charges then it's a quadrupole configuration so potential is $\propto \frac{1}{r^3}$.

Q35. The electric field of an electromagnetic wave in free space is given by

$$\vec{E} = E_0 \sin(\omega t - k_z z)\,\hat{j}$$

The magnetic field $\vec{B}$ vanishes for $t = \frac{k_z z}{\omega}$. The Poynting vector of the system is

(1) $\frac{k_z}{2\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$ (2) $\frac{4k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$

(3) $\frac{2k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$ (4) $\frac{k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k}$

Ans.: (4)

Solution.: $\because \vec{E} = E_0 \sin(\omega t - k_z z)\,\hat{j} \Rightarrow \vec{B} = -\frac{E_0}{c}\sin(\omega t - k_z z)\,\hat{i}$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right) = \frac{1}{\mu_0}\frac{E_0}{c}\sin(\omega t - k_z z)\,\hat{j} \times \frac{E_0}{c}\sin(\omega t - k_z z)\left(-\hat{i}\right) = \frac{E_0^2}{\mu_0 c}\sin^2(\omega t - k_z z)\hat{k}$$

$$\Rightarrow \vec{S} = \frac{k_z}{\mu_0 \omega} E_0^2 \sin^2(\omega t - k_z z)\hat{k} \qquad \because c = \frac{\omega}{k_z}$$

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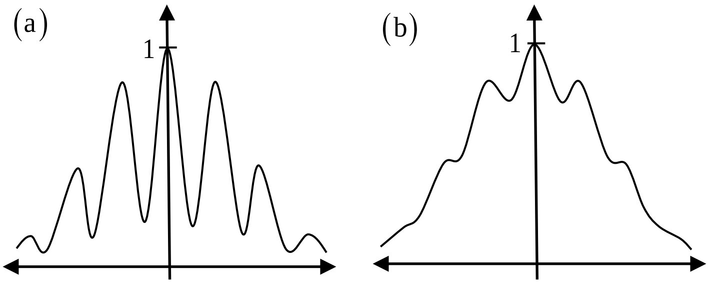
Q42. A finite sized light source is used in a double slit experiment. The observed intensity pattern changes from figure (a) to figure (b), as shown below.



The observed change can occur due to

(1) narrowing of the slits

(2) a reduction in the distance between the slits

(3) a decrease in the coherence length of the light source

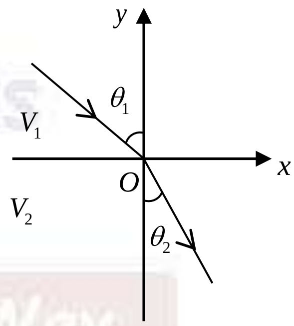
(4) a reduction in the size of the light source

Ans. (3):

Solution:

In figure (b), interference is vanishing. This may occur due to a decrease in the coherence length of the light source.

Q45. The region $y > 0$ has a constant electrostatic potential V_1 and $y < 0$ has a constant electrostatic potential $V_2 \neq V_1$. A charged particle with momentum $\vec{p_1}$ is incident at an angle θ_1 on the interface of the two regions (see figure below). If the particle has momentum $\vec{p_2}$ in the region $y < 0$, then the angle θ_2 is given by



(1) $\cos^{-1}\left(\frac{p_2}{p_1}\cos\theta_1\right)$ (2) $\cos^{-1}\left(\frac{p_1}{p_2}\cos\theta_1\right)$

(3) $\sin^{-1}\left(\frac{p_2}{p_1}\sin\theta_1\right)$ (4) $\sin^{-1}\left(\frac{p_1}{p_2}\sin\theta_1\right)$

Ans.: (4)

Solution.:

As potential is not changing in x-direction; $F_x = 0 \quad \because \vec{F} = -\nabla V$

p_x = conserved so $p_1 \sin\theta_1 = p_2 \sin\theta_2 \Rightarrow \theta_2 = \sin^{-1}\left(\frac{p_1}{p_2}\sin\theta_1\right)$

PART C

Q53. A radio station antenna on the earth's surface radiates $50\,kW$ power isotropically. Assume the electromagnetic waves to be sinusoidal and the ground to be a perfect absorber. Neglecting any transmission loss and effects of earth's curvature, the peak value of the magnetic field (in Tesla) detected at a distance of 100 km is closest to

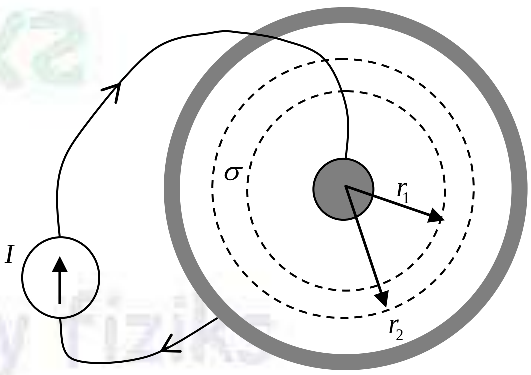
(1) 1.5×10^{-11} (2) 5.5×10^{-11} (3) 8.5×10^{-11} (4) 3.5×10^{-11}

Ans.: (2)

Solution.: $I=\frac{P}{A}=\frac{P}{4\pi r^2}=c\frac{B_0^2}{2\mu_0}\Rightarrow B_0^2=\frac{2\mu_0 P}{c\times4\pi r^2}=\frac{\left(2\times4\pi\times10^{-7}\right)\times50\times10^3}{3\times10^8\times4\pi\times\left(10^5\right)^2}$

$$\Rightarrow B_0^2=\frac{10^{-2}}{3\times10^{18}}=\frac{10^{-20}}{3}\Rightarrow B_0=\frac{10^{-10}}{\sqrt{3}}T=5.8\times10^{-11}T$$

Q60. A two dimensional sheet with a uniform sheet conductivity of σ has a central metallic point contact and a circular metal contact at the boundary as shown in the figure. If a constant current I is injected through the central contact and collected at the boundary, then the voltage difference between two points on the sheet at radius r_1 and r_2 is proportional to



(1) $\frac{I}{\sigma}\left[\tan^{-1}\left(\frac{r_2}{r_1}\right)-\frac{\pi}{4}\right]$ (2) $\frac{I}{\sigma}\left[\ln\left(\frac{r_2}{r_1}\right)\right]$

(3) $\frac{I}{\sigma}\left(\frac{r_2-r_1}{r_2+r_1}\right)$ (4) $\frac{I}{\sigma}\left(\frac{r_2-r_1}{r_2+r_1}\right)^3$

Ans.: (2)

Solution.: Let potential at center with respect to outer layer be V then

$$V=-\int_{r_2}^{r_1}\vec{E}.d\vec{l}=-\int_{r_2}^{r_1}\frac{\lambda}{2\pi\varepsilon_0 r}dr=\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right)=\frac{\varepsilon_0 I}{\sigma l}\frac{1}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right)\propto\frac{I}{\sigma}\ln\left(\frac{r_2}{r_1}\right)$$

Where $I=\oiint\vec{J}.d\vec{a}=\sigma\oiint\vec{E}.d\vec{a}=\sigma\frac{\lambda l}{\varepsilon_0}\Rightarrow\lambda=\frac{\varepsilon_0 I}{\sigma l}$

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Q71. In a non-magnetic material with no free charges and no free currents, the permittivity $\in$ is a function of position. If $\vec{E}$ represents the electric field and $\mu_0, \in_0$ are free space permeability and permittivity respectively, which one of the following expressions is correct?

(1) $\nabla^2\vec{E}-\mu_0\frac{\partial^2\left(\in\vec{E}\right)}{\partial t^2}-\frac{1}{\in_0}\vec{\nabla}\left(\vec{E}\cdot\vec{\nabla}\in\right)=0$ (2) $\nabla^2\vec{E}-\mu_0\frac{\partial^2\left(\in\vec{E}\right)}{\partial t^2}+\frac{1}{\in_0}\vec{\nabla}\left(\vec{E}\cdot\vec{\nabla}\in\right)=0$

(3) $\nabla^2\vec{E}-\mu_0\frac{\partial^2\left(\in\vec{E}\right)}{\partial t^2}+\vec{\nabla}\left(\frac{1}{\in}\vec{E}\cdot\vec{\nabla}\in\right)=0$ (4) $\nabla^2\vec{E}-\mu_0\frac{\partial^2\left(\in\vec{E}\right)}{\partial t^2}-\vec{\nabla}\left(\frac{1}{\in}\vec{E}\cdot\vec{\nabla}\in\right)=0$

Ans.: (3)

Solution.:

There no free charges and no free currents and medium is non-magnetic ($\mu=\mu_0$)

$$\because \vec{\nabla}.\vec{D}=0\Rightarrow\vec{\nabla}.\left(\in\vec{E}\right)=0\Rightarrow\in\vec{\nabla}.\vec{E}+\vec{E}.\vec{\nabla}\in=0\Rightarrow\vec{\nabla}.\vec{E}=\frac{-\vec{E}.\left(\vec{\nabla}\in\right)}{\in}$$

$$\because \vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}\Rightarrow\vec{\nabla}\times\frac{\vec{B}}{\mu_0}=\frac{\partial\left(\in\vec{E}\right)}{\partial t}\Rightarrow\vec{\nabla}\times\vec{B}=\mu_0\frac{\partial\left(\in\vec{E}\right)}{\partial t}$$

$$\because \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\Rightarrow\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)=-\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)\Rightarrow\vec{\nabla}\left(\vec{\nabla}.\vec{E}\right)-\nabla^2\vec{E}=-\mu_0\frac{\partial^2}{\partial t^2}\left(\in\vec{E}\right)$$

$$\Rightarrow\vec{\nabla}\left(\frac{-\vec{E}.\left(\vec{\nabla}\in\right)}{\in}\right)-\nabla^2\vec{E}=-\mu_0\frac{\partial^2}{\partial t^2}\left(\in\vec{E}\right)\Rightarrow\nabla^2\vec{E}-\mu_0\frac{\partial^2}{\partial t^2}\left(\in\vec{E}\right)+\vec{\nabla}\left(\frac{\vec{E}}{\in}.\left(\vec{\nabla}\in\right)\right)=0$$

Q75. A particle of unit mass and unit charge is moving in a magnetic field, which varies as $\vec{B}(\vec{r})=b_0\vec{r}/r^3$ (b_0 is a constant) over a region far away from the origin. If $\vec{L}$ is the instantaneous angular momentum of the particle within that region, then $d\vec{L}/dt$ is

(1) $2b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ (2) $-b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ (3) $b_0\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ (4) 0

Ans.: (3)

Solution.:

Instantaneous angular momentum $\vec{L}=\vec{r}\times\vec{p}=\vec{r}\times m\vec{v}=\vec{r}\times\vec{v}$ $\quad\because m=1$

Force $\vec{F}=m\vec{a}=q\left(\vec{v}\times\vec{B}\right)\Rightarrow\frac{d\vec{v}}{dt}=\vec{v}\times\vec{B}\Rightarrow\frac{d\vec{v}}{dt}=\frac{d\vec{r}}{dt}\times b_0\frac{\vec{r}}{r^3}$ $\quad\because q=1, m=1$

$$\frac{d\vec{L}}{dt}=\vec{r}\times\frac{d\vec{v}}{dt}+\frac{d\vec{r}}{dt}\times\vec{v}=\vec{r}\times\frac{d\vec{v}}{dt}+\vec{v}\times\vec{v}=\vec{r}\times\frac{d\vec{v}}{dt}\qquad\because\vec{v}\times\vec{v}=0$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \vec{r} \times \left[\frac{d\vec{r}}{dt} \times b_0 \frac{\vec{r}}{r^3}\right] = \frac{d\vec{r}}{dt}\left[\vec{r}.b_0 \frac{\vec{r}}{r^3}\right] - b_0 \frac{\vec{r}}{r^3}\left[\vec{r}.\frac{d\vec{r}}{dt}\right] \qquad \because \vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}\left(\vec{A}.\vec{C}\right) - \vec{C}\left(\vec{A}.\vec{B}\right)$$

$$\Rightarrow \frac{d\vec{L}}{dt} = b_0 \frac{d\vec{r}}{dt}\left[\frac{r^2}{r^3}\right] - b_0 \frac{\vec{r}}{r^3}\left[r\frac{dr}{dt}\right] \qquad \because \frac{d}{dt}\left(\vec{r}.\vec{r}\right) = 2r\frac{dr}{dt} = \frac{d\vec{r}}{dt}.\vec{r} + \vec{r}.\frac{d\vec{r}}{dt} \text{ i.e. } \vec{r}.\frac{d\vec{r}}{dt} = r\frac{dr}{dt}$$

$$\Rightarrow \frac{d\vec{L}}{dt} = \frac{b_0}{r}\frac{d\vec{r}}{dt} - b_0 \frac{\vec{r}}{r^2}\frac{dr}{dt} = b_0 \frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

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Solution-Quantum Mechanics

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PART B

Q24. A hydrogen atom is in the state $|\psi\rangle = \sqrt{\frac{8}{21}}|\psi_{200}\rangle + \sqrt{\frac{3}{7}}|\psi_{210}\rangle + \sqrt{\frac{4}{21}}|\psi_{311}\rangle$ where $|\psi_{nlm}\rangle$ are normalised eigenstates. If $\hat{L}^2$ is measured in this state, the probability of obtaining the value $2\hbar^2$ is

(1) $\frac{13}{21}$ (2) $\frac{4}{21}$ (3) $\frac{17}{21}$ (4) $\frac{3}{7}$

Ans.: (1)

Solution.: $|\psi\rangle = \sqrt{\frac{8}{21}}|\psi_{200}\rangle + \sqrt{\frac{3}{7}}|\psi_{210}\rangle + \sqrt{\frac{4}{21}}|\psi_{311}\rangle = C_1|\psi_{200}\rangle + C_2|\psi_{210}\rangle + C_3|\psi_{311}\rangle$

$L^2|\psi_{200}\rangle = \ell(\ell+1)\hbar^2|\psi_{200}\rangle = 0|\psi_{200}\rangle$; $L^2|\psi_{210}\rangle = \ell(\ell+1)\hbar^2|\psi_{210}\rangle = 2\hbar^2|\psi_{210}\rangle$

$L^2|\psi_{311}\rangle = \ell(\ell+1)\hbar^2|\psi_{210}\rangle = 2\hbar^2|\psi_{210}\rangle$

Probability of obtaining $2\hbar^2$ is $P(2\hbar^2) = \frac{|C_2|^2 + |C_3|^2}{|C_1|^2 + |C_2|^2 + |C_3|^2} = \frac{\frac{3}{7} + \frac{4}{21}}{\frac{8}{21} + \frac{3}{7} + \frac{4}{21}} = \frac{13}{21}$

Q31. If A and B are Hermitian operators and C is an anti-Hermitian operator, then

(1) $[[A,B],C]$ is Hermitian and $[[A,C],B]$ is anti-Hermitian

(2) $[[A,B],C]$ and $[[A,C],B]$ are both anti-Hermitian

(3) $[[A,B],C]$ and $[[A,C],B]$ are both Hermitian

(4) $[[A,B],C]$ is anti-Hermitian and $[[A,C],B]$ is Hermitian

Ans.: (2)

Solution.: Given, $A^\dagger = A, B^\dagger = B$ and $C^\dagger = -C$

Now $[A,B]^\dagger = (AB - BA)^\dagger = B^\dagger A^\dagger - A^\dagger B^\dagger = BA - AB = -(AB - BA) = -[A,B]$

and $[A,C]^\dagger = [AC - CA]^\dagger = C^\dagger A^\dagger - A^\dagger C^\dagger = -CA + AC = AC - CA = [A,C]$

Thus, $[(A,B),C]^\dagger = ([A,B]C - C[A,B])^\dagger = C^\dagger [A,B]^\dagger - [A,B]^\dagger C^\dagger$

$[(A,B),C]^\dagger = (-C)(-[A,B]) - (-[A,B](-C)) = C[A,B] - [A,B]C = -[[A,B],C]$

Therefore, $[[A,B],C]$ is anti-Hermitian

and $[[A,C],B]^\dagger = ([A,C]B - B(A,C))^\dagger = B^\dagger [A,C]^\dagger - [A,C]^\dagger B^\dagger$

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$$\Rightarrow\left[\left[A,C\right],B\right]^{\dagger}=B\left[A,C\right]-\left[A,C\right]B=-\left(\left[A,C\right]B-B\left[A,C\right]\right)=-\left[\left[A,C\right],B\right]^{\dagger}$$

Therefore, $\left[\left[A,C\right],B\right]$ is anti-Hermitian. Thus, correct option is (2).

Q33. Two non-interacting classical particles having masses m_1 and m_2 are moving in a one-dimensional box of length L. For total energy not exceeding a given value E, the phase space "volume" is given by

(1) $\pi L^2E\left(\frac{m_1m_2}{m_1+m_2}\right)$ (2) $\pi L^2E\sqrt{m_1m_2}$ (3) $2\pi L^2E\left(\frac{m_1m_2}{m_1+m_2}\right)$ (4) $2\pi L^2E\sqrt{m_1m_2}$

Ans.: (4)

Q34. The Hamiltonian for a one-dimensional simple harmonic oscillator is given by $H=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2$. The harmonic oscillator is in the state $|\psi\rangle=\frac{1}{\sqrt{1+\lambda^2}}\left(|1\rangle+\lambda e^{i\theta}|2\rangle\right)$, where $|1\rangle$ and $|2\rangle$ are the normalized first and second excited states of the oscillator and λ,θ are positive real constants. If the expectation value $\langle\psi|x|\psi\rangle=\beta\sqrt{\frac{\hbar}{m\omega}}$, the value of β is

(1) $\frac{1}{\sqrt{2}\left(1+\lambda^2\right)}$ (2) $\frac{\sqrt{2}\lambda\cos\theta}{1+\lambda^2}$ (3) $\frac{2\lambda\cos\theta}{1+\lambda^2}$ (4) $\frac{\lambda^2\cos\theta}{1+\lambda^2}$

Ans.: (3)

Solution.: $\hat{x}=\sqrt{\frac{\hbar}{2m\omega}}\left(a+a^{\dagger}\right)$, $a|n\rangle=\sqrt{n}|n-1\rangle$ and $a^{\dagger}|n\rangle=\sqrt{n+1}|n+1\rangle$

Given, $|\psi\rangle=\frac{1}{\sqrt{1+\lambda^2}}\left(|1\rangle+\lambda e^{i\theta}|2\rangle\right)$ and $\langle\psi|=\frac{1}{\sqrt{1+\lambda^2}}\left(\langle1|+\lambda e^{-i\theta}\langle2|\right)$

$$\hat{x}|\psi\rangle=\sqrt{\frac{\hbar}{2m\omega}}\frac{1}{\sqrt{1+\lambda^2}}\left(a|1\rangle+a^{\dagger}|1\rangle+\lambda e^{i\theta}a|2\rangle+\lambda e^{i\theta}a^{\dagger}|2\rangle\right)$$

$$=\sqrt{\frac{\hbar}{2m\omega}}\frac{1}{\sqrt{1+\lambda^2}}\left(|0\rangle+\sqrt{2}|2\rangle+\lambda e^{i\theta}\sqrt{2}|1\rangle+\lambda e^{i\theta}\sqrt{3}|3\rangle\right)$$

Now $\langle\psi|\hat{x}|\psi\rangle=\sqrt{\frac{\hbar}{2m\omega}}\frac{1}{1+\lambda^2}\left(\sqrt{2}\lambda e^{-i\theta}+\sqrt{2}\lambda e^{i\theta}\right)=\sqrt{\frac{\hbar}{m\omega}}\cdot\frac{2\lambda}{1+\lambda^2}(\cos\theta)$

$=\frac{2\lambda\cos\theta}{1+\lambda^2}\cdot\sqrt{\frac{\hbar}{m\omega}}=\beta\sqrt{\frac{\hbar}{m\omega}}$. Thus, $\beta=\frac{2\lambda\cos\theta}{1+\lambda^2}$, the correct option is (3)

Q37. If $\vec{L}$ is the orbital angular momentum operator and $\vec{\sigma}$ are the Pauli matrices, which of the following operators commutes with $\vec{\sigma}\cdot\vec{L}$?

(1) $\vec{L}-\frac{\hbar}{2}\vec{\sigma}$ (2) $\vec{L}+\frac{\hbar}{2}\vec{\sigma}$ (3) $\vec{L}+\hbar\vec{\sigma}$ (4) $\vec{L}-\hbar\vec{\sigma}$

Ans.: (2)

Solution.: Since $\vec{S}=\frac{\hbar}{2}\sigma$, therefore $\vec{\sigma}\cdot\vec{L}=\frac{2}{\hbar}\vec{S}\cdot\vec{L}=\frac{2}{\hbar}\vec{L}\cdot\vec{S}$

We also know $\left[\vec{L}\cdot\vec{S},L_i\right]=i\hbar\left(L_jS_k-L_kS_j\right)$ and $\left[\vec{L}\cdot\vec{S},S_i\right]=i\hbar\left(L_kS_j-L_jS_k\right)$

Now option (1) $\left[\vec{\sigma}\cdot\vec{L},\vec{L}-\frac{\hbar}{2}\vec{\sigma}\right]=\left[\frac{2}{\hbar}\vec{L}\cdot\vec{S},\vec{L}-\vec{S}\right]=\frac{2}{\hbar}\left[\vec{L}\cdot\vec{S},\vec{L}-\vec{S}\right]=\frac{2}{\hbar}\left[\vec{L}\cdot\vec{S},\vec{L}\right]-\frac{2}{\hbar}\left[\vec{L}\cdot\vec{S},\vec{S}\right]$

where, $\left[\vec{L}\cdot\vec{S},\vec{L}\right]=\left[\vec{L}\cdot\vec{S},L_x\hat{i}+L_y\hat{j}+L_z\hat{k}\right]=\left[\vec{L}\cdot\vec{S},L_x\right]\hat{i}+\left[\vec{L}\cdot\vec{S},L_y\right]\hat{j}+\left[\vec{L}\cdot\vec{S},L_z\right]\hat{k}$

$$=i\hbar\left(L_yS_z-L_zS_y\right)\hat{i}+i\hbar\left(L_zS_x-L_xS_z\right)\hat{j}+i\hbar\left(L_xS_y-L_yS_x\right)\hat{k}$$

also $\left[\vec{S}\cdot\vec{L},\vec{S}\right]=\left[\vec{S}\cdot\vec{L},S_x\hat{i}+S_y\hat{j}+S_z\hat{k}\right]=\left[\vec{S}\cdot\vec{L},s_x\right]\hat{i}+\left[s\cdot L,s_y\right]\hat{j}+\left[s\cdot L,s_z\right]\hat{k}$

$$=i\hbar\left(L_zS_y-L_yS_z\right)\hat{i}+i\hbar\left(L_xS_z-L_zS_x\right)\hat{j}+i\hbar\left(L_yS_x-L_xS_y\right)\hat{k}$$

$\left[\vec{\sigma}\cdot\vec{L},\vec{L}-\frac{\hbar}{2}\vec{\sigma}\right]=4i\left\{\left(L_ys_z-L_zs_y\right)\hat{i}+\left(L_zs_x-L_xs_z\right)\hat{j}+\left(L_xs_y-L_ys_x\right)\hat{k}\right\}$. It is non-commuting

Now option (2); $\left[\vec{\sigma}\cdot\vec{L},\vec{L}+\frac{\hbar}{2}\vec{\sigma}\right]=\frac{2}{\hbar}\left\{\left[\vec{L}\cdot\vec{S},L\right]+\left[\vec{L}\cdot\vec{S},\vec{S}\right]\right\}=0$.

It is commuting. Thus option (2) is correct.

Q41. A quantum mechanical system is in the angular momentum state $\left|l=4,l_z=4\right\rangle$. The uncertainty in L_x is

(1) $\hbar\sqrt{2}$ (2) $2\hbar$ (3) 0 (4) $\hbar$

Ans.: (1)

Solution.: The uncertainty in L_x is $\Delta L_x=\sqrt{\left\langle L_x^2\right\rangle-\left\langle L_x\right\rangle^2}$

Since $\left\langle l,l_z\left|L_x\right|l,l_z\right\rangle=0$

and $\left\langle l,l_z\left|L_x^2\right|l,l_z\right\rangle=\frac{\hbar^2}{2}\left(l\left(l+1\right)-l_z^2\right)=\frac{\hbar^2}{2}\left(4\left(4+1\right)-4^2\right)=\frac{\hbar^2}{2}\left(20-10\right)=2\hbar^2$

$\therefore\ \Delta L_x=\sqrt{2}\hbar$

Thus, correct option is (1).

Q43. Quantum particles of unit mass, in a potential

$$V(x)=\begin{cases}\frac{1}{2}\omega^2x^2 & x>0\\ \infty & x\le 0\end{cases}$$

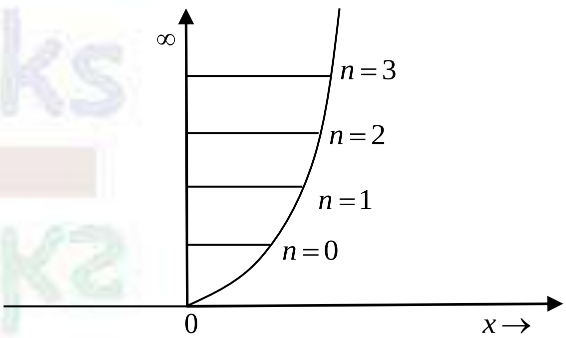
are in equilibrium at a temperature T. Let n_2 and n_3 denote the numbers of the particles in the second and third excited states respectively. The ratio n_2/n_3 is given by

(1) $\exp\left(\frac{2\hbar\omega}{k_BT}\right)$ (2) $\exp\left(\frac{\hbar\omega}{k_BT}\right)$ (3) $\exp\left(\frac{3\hbar\omega}{k_BT}\right)$ (4) $\exp\left(\frac{4\hbar\omega}{k_BT}\right)$

Ans.: (1)

Solution.:

For half harmonic oscillator



$E_n=\left(n+\frac{1}{2}\right)\hbar\omega,\ n=1,3,5,7,...$

or $E_n=\left(2n+\frac{3}{2}\right)\hbar\omega$, $n=0,1,2,3,....$

Now $\frac{n_2}{n_3}=e^{\Delta E/kT}=e^{(E_3-E_2)/k_BT}$ where

$E_3-E_2=\left(\frac{15}{2}-\frac{11}{2}\right)\hbar\omega=2\hbar\omega$

$\therefore\ \frac{n_2}{n_3}=\exp\left(\frac{2\hbar\omega}{k_BT}\right)$

Thus correct option is (1)

PART C

Q55. Five classical spins are placed at the vertices of a regular pentagon. The Hamiltonian of the system is $H = J\sum S_i S_j$, where $J > 0$, $S_i = \pm 1$ and the sum is over all possible nearest neighbour pairs. The degeneracy of the ground state is

(1) 8 (2) 5 (3) 4 (4) 10

Ans.: (4)

Q63. The Hamiltonian of a particle of mass m is given by $H = \dfrac{p^2}{2m} + V(x)$, with

$$V(x) = \begin{cases} -\alpha x & \text{for } x \le 0 \\ \beta x & \text{for } x > 0 \end{cases}$$

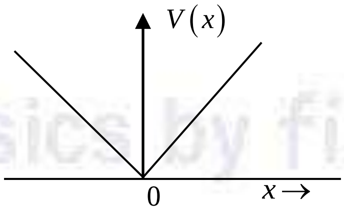
where α, β are positive constants. The n^{th} energy eigenvalue E_n obtained using WKB approximation is $E_n^{3/2} = \dfrac{3}{2}\left(\dfrac{\hbar^2}{2m}\right)^{1/2} \pi\left(n - \dfrac{1}{2}\right) f(\alpha,\beta) \quad (n = 1, 2, \ldots)$

The function $f(\alpha,\beta)$ is

(1) $\sqrt{\dfrac{\alpha^2\beta^2}{2(\alpha^2+\beta^2)}}$ (2) $\dfrac{\alpha\beta}{\alpha+\beta}$ (3) $\dfrac{\alpha+\beta}{4}$ (4) $\dfrac{1}{2}\sqrt{\dfrac{\alpha^2+\beta^2}{2}}$

Ans. (2):

Solution.: $V(x) = \begin{cases} -\alpha x & \text{for } x \le 0 \\ \beta x & \text{for } x > 0 \end{cases}$



$V(x)$ is an even potential.

For even potential of type $V(x) = \alpha |x|^p$, the energy is $E_n = \alpha\left[\left(n - \dfrac{1}{2}\right)\sqrt{\dfrac{\pi\hbar^2}{2m\alpha}}\dfrac{\Gamma\left(\frac{1}{p}+\frac{3}{2}\right)}{\Gamma\left(\frac{1}{p}+1\right)}\right]^{\frac{2p}{p+2}}$

For $V(x) = -\alpha x \quad (x \le 0)$; $E_n = \alpha\left[\left(n - \dfrac{1}{2}\right)\sqrt{\dfrac{\pi\hbar^2}{2m\alpha}} \cdot \dfrac{\Gamma\left(\frac{5}{2}\right)}{\Gamma(2)}\right]^{\frac{2}{3}}$ where $\Gamma\left(\dfrac{5}{2}\right) = \dfrac{3}{4}\sqrt{\pi}$ and $\sqrt{2} = 1$

$$\Rightarrow E_n^{3/2} = \alpha^{3/2}\left[\left(n - \frac{1}{2}\right)\sqrt{\frac{\pi\hbar^2}{2m\alpha}} \cdot \frac{3}{4}\sqrt{\pi}\right] = \alpha\left[\pi\left(n - \frac{1}{2}\right)\sqrt{\frac{\hbar^2}{2m}} \cdot \frac{3}{4}\right] \Rightarrow \frac{E_n^{3/2}}{\alpha} = \frac{3}{4}\left(\frac{\hbar^2}{2m}\right)^{1/2}\pi\left(n - \frac{1}{2}\right) \quad \ldots(1)$$

Now, For $V(x)=\beta x \quad (x>0)$; $\frac{E_n^{3/2}}{\beta}=\frac{3}{4}\left(\frac{\hbar^2}{2m}\right)^{1/2}\pi\left(n-\frac{1}{2}\right)$ ...(2)

Add (1) and (2), we get $E_n^{3/2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2}\pi\left(n-\frac{1}{2}\right)\Rightarrow E_n^{3/2}=\frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2}\pi\left(n-\frac{1}{2}\right)\cdot\frac{\alpha\beta}{\alpha+\beta}$

$\Rightarrow E_n^{3/2}=\frac{3}{2}\left(\frac{\hbar^2}{2m}\right)^{1/2}\pi\left(n-\frac{1}{2}\right)f(\alpha,\beta)$. Thus, $f(\alpha,\beta)=\frac{\alpha\beta}{\alpha+\beta}$, the correct option is (2)

Q66. A particle of energy E is scattered off a one-dimensional potential $\lambda\delta(x)$, where λ is a real positive constant, with a transmission amplitude t_+. In a different experiment, the same particle is scattered off another one-dimensional potential $-\lambda\delta(x)$, with a transmission amplitude t_-. In the limit $E\to 0$, the phase difference between t_+ and t_- is

(1) $\pi/2$ (2) π (3) 0 (4) $3\pi/2$

Ans.: (2)

Solution.: The Transmission amplitudes t_+ for $V(x)=\lambda\delta(x)$ is $t_+=\frac{1}{1+\frac{i\lambda}{2k}}$ where $E=\frac{\hbar^2k^2}{2m}$

The transmission amplitude t_- for $V(x)=-\lambda\delta(x)$ is $t_-=\frac{1}{1-i\frac{\lambda}{2k}}$

The phases of a complex number $z=a+ib$ is $\phi=\tan^{-1}\left(\frac{b}{a}\right)$

$\therefore$ For t_+, $\phi_+=-\tan^{-1}\left(\frac{\lambda}{2k}\right)$; For t_-, $\phi_-=\tan^{-1}\left(\frac{\lambda}{2k}\right)$

Phase difference is $\Delta\phi=\phi_+-\phi_-=-2\tan^{-1}\left(\frac{\lambda}{2k}\right)$

In the limit, $E\to 0, k\to 0$; $\Delta\phi=-2\tan^{-1}(\infty)=-2\times\frac{\pi}{2}=-\pi$

Thus correct option is (2).

Q70. Using a normalized trial wavefunction $\psi(x)=\sqrt{\alpha}e^{-\alpha|x|}$ (α is a positive real constant) for a particle of mass m in the potential $V(x)=-\lambda\delta(x),(\lambda>0)$, the estimated ground state energy is

(1) $-\frac{m\lambda^2}{\hbar^2}$ (2) $\frac{m\lambda^2}{\hbar^2}$ (3) $\frac{m\lambda^2}{2\hbar^2}$ (4) $-\frac{m\lambda^2}{2\hbar^2}$

Ans.: (4)

Q70. Ans. (3)

Solution: Given $\psi(x)=\sqrt{\alpha}e^{-\alpha|x|}$ and $V(x)=-\lambda\delta(x)$

The Schrodinger equation is $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=E\psi \Rightarrow \frac{\partial^2\psi}{\partial x^2}+\frac{2mE}{\hbar^2}\psi=\frac{2mV}{\hbar^2}\psi$

Now $\int_{-\epsilon}^{+\epsilon}\frac{\partial^2\psi}{\partial x^2}dx+\frac{2mE}{\hbar^2}\int_{-\epsilon}^{+\epsilon}\psi(x)dx=\frac{-2m\lambda}{\hbar^2}\int_{-\epsilon}^{+\epsilon}\delta(x)\psi(x)dx$

For $\epsilon\to 0$, $\int_{-\epsilon}^{+\epsilon}\psi(x)dx\to 0$; $\therefore\ \left.\frac{d\psi}{dx}\right|_{x>0}-\left.\frac{d\psi}{dx}\right|_{x<0}=\frac{-2m\lambda}{\hbar^2}\psi(0)$

$$\therefore\ -\alpha\sqrt{\alpha}e^{-\alpha x}\Big|_{x=0}-\alpha\sqrt{\alpha}e^{\alpha x}\Big|_{x=0}=\frac{-2m\lambda}{\hbar^2}\psi(0)\Rightarrow -\sqrt{\alpha}\alpha-\alpha\sqrt{\alpha}=-\frac{2m\lambda}{\hbar^2}\cdot\sqrt{\alpha}$$

$$\Rightarrow \alpha=\frac{m\lambda}{\hbar^2}\Rightarrow \alpha^2=\frac{m^2\lambda^2}{\hbar^4}\qquad \left(\because \alpha=k\Rightarrow \alpha^2=k^2=\frac{2mE}{\hbar^2}\right)$$

$$\Rightarrow \frac{2mE}{\hbar^2}=\frac{m^2\lambda^2}{\hbar^4}\ \Rightarrow E=\frac{m\lambda^2}{2\hbar^2}$$

Thus correct option is (3).

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Solution-Thermodynamics and Statistical Mechanics

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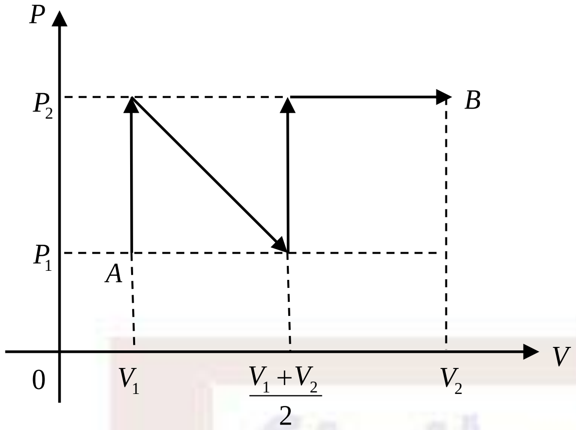
PART B

Q27. The following $P-V$ diagram shows a process, where an ideal gas is taken quasi-statically from A to B along the path as shown in the figure.



The work done W in this process is

(1) $\frac{1}{4}(V_2-V_1)(3P_2+P_1)$ (2) $\frac{1}{4}(V_2-V_1)(3P_2-P_1)$

(3) $\frac{1}{2}(V_2-V_1)(P_1+P_2)$ (4) $\frac{1}{2}(V_2+V_1)(P_2-P_1)$

Ans.: (1)

Solution.: The work done W in this process is equal to area under the curve.

$$W=\frac{1}{2}(P_2-P_1)\left(\frac{V_1+V_2}{2}-V_1\right)+(P_2-P_1)\left(V_2-\frac{V_1+V_2}{2}\right)+P_1(V_2-V_1)$$

$$\Rightarrow W=\frac{1}{2}(P_2-P_1)\left(\frac{V_2-V_1}{2}\right)+(P_2-P_1)\left(\frac{V_2-V_1}{2}\right)+P_1(V_2-V_1)$$

$$\Rightarrow W=\left(\frac{V_2-V_1}{2}\right)\left[\frac{1}{2}(P_2-P_1)+(P_2-P_1)+2P_1\right]=\left(\frac{V_2-V_1}{2}\right)\left[\frac{3}{2}(P_2-P_1)++2P_1\right]$$

$$\Rightarrow W=\frac{1}{4}(V_2-V_1)(3P_2+P_1)$$

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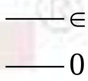
Q38. A single particle can exist in two states with energies 0 and E respectively. At high temperatures $(k_BT \gg E)$ the specific heat of the system (C_V) will be approximately

(1) proportional to $\frac{1}{T}$ (2) proportional to $\frac{1}{T^2}$

(3) proportional to $e^{\frac{E}{k_BT}}$ (4) constant

Ans.: (2)

Solution.: Partition function $Z_n = \Sigma g_i e^{-\beta \in_i n_i}$



$Z_1 = \left(1+e^{-\beta\in}\right)$ [For single particle]

Mean energy $\langle E \rangle = -\frac{\partial}{\partial\beta}\ln Z_1 = \left[\frac{\in e^{-\beta\in}}{1+e^{-\beta\in}}\right]$

Specific heat $C_V = \frac{\partial\bar{E}}{\partial T} = \frac{\partial}{\partial T}\left\{\frac{\in e^{-\in/k_BT}}{1+e^{-\in/k_BT}}\right\} = \frac{\in^2}{k_BT^2}\left\{\frac{\left(1+e^{-\in/k_BT}\right)\left(e^{-\in/k_BT}\right) - e^{-\in/k_BT}e^{-\in/k_BT}}{\left(1+e^{-\in/k_BT}\right)^2}\right\}$

$\Rightarrow C_V = \frac{\in^2}{k_BT^2}\left\{\frac{1+e^{-\in/k_BT} - e^{-\in/k_BT}}{\left(1+e^{-\in/k_BT}\right)^2}\right\}e^{-\in/k_BT} = \frac{\in^2}{k_BT^2}\left\{\frac{1-\in/k_BT+\ldots}{\left(1+1-\in/k_BT+\ldots\right)^2}\right\}$

But $k_BT \gg \in$ then $\in/k_BT \ll 1$; $C_V = \frac{\in^2}{4k_BT^2} \Rightarrow C_V \propto \left(\frac{1}{T^2}\right)$

PART C

Q51. A random walker takes a step of unit length towards right or left at any discrete time step. Starting from $x=0$ at time $t=0$, it goes right to reach $x=1$ at $t=1$. Hereafter if it repeats the direction taken in the previous step with probability p, the probability that it is again at $x=1$ at $t=3$ is

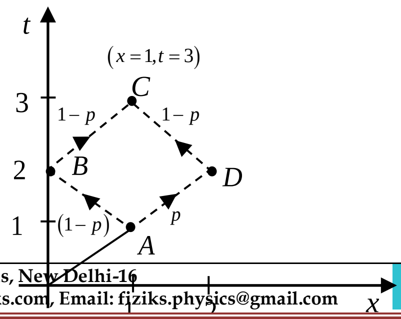
(1) $1-p$ (2) $(1-p)^2$ (3) $2p(1-p)$ (4) $4p^2(1-p)$

Ans.: (1)

Solution.:

Probability $(=x=1, t=3)$

$= P(ABC)$ or $P(ADC)$



$$=(1-p)(1-p)+p(1-p)$$

$$=1-p-\cancel{p}+\cancel{p^2}+\cancel{p}-\cancel{p^2}=(1-p)$$

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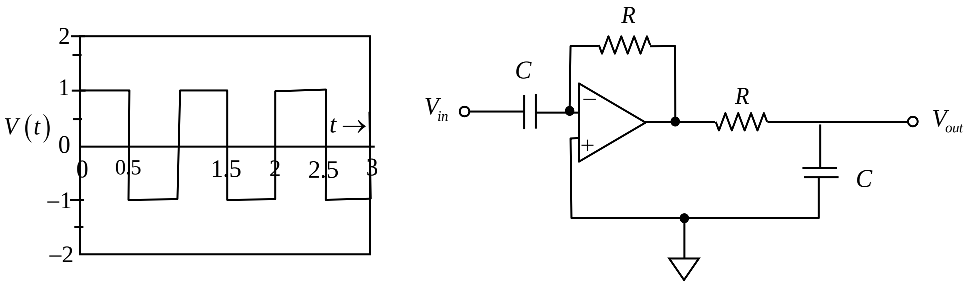
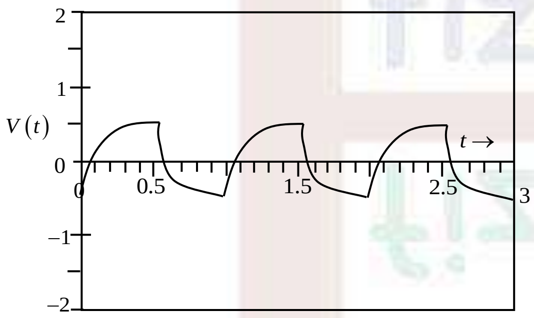
PART B

Q21. A train of square wave pulses is given to the input of an ideal opamp circuit shown below.

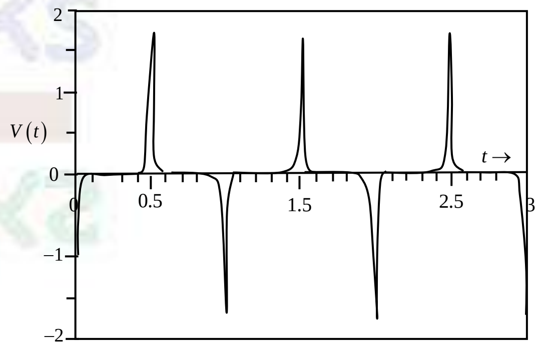


Given that the period of the input pulses T □ RC and the opamp does not get into saturation, which of the following best represents the output waveform?

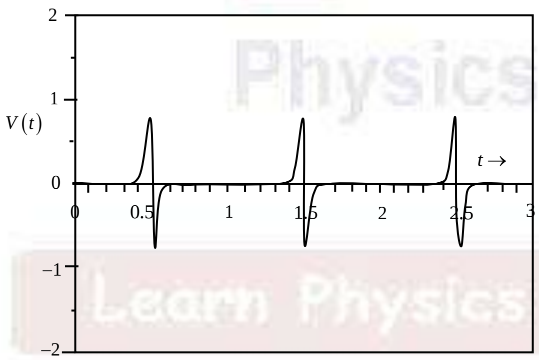
1.



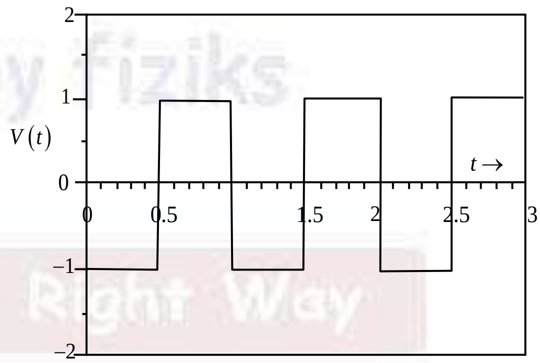
2.



3.



4.



Ans.: (4)

Solution.:

It's a combination of active differentiator and passive integrator, so output remains same.

Q36. A battery with an open circuit voltage of 10 V is connected to a load resistor of $485\,\Omega$ and the voltage measured across the battery terminals using an ideal voltmeter is 9.7 V. The internal resistance of the battery is closest to

(1) 30Ω (2) 15Ω (3) 20Ω (4) 40Ω

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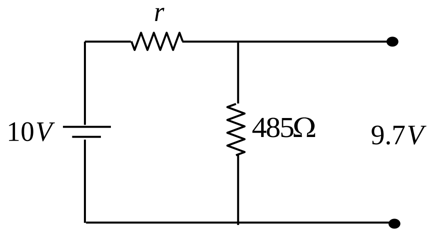
Ans.: (2)

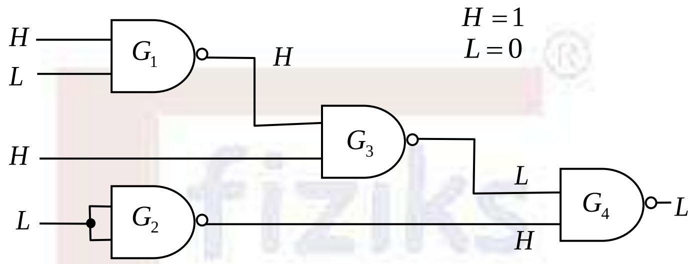
Solution.:



$$\frac{485}{485+r}\times 10V = 9.7V \Rightarrow 4850 = 9.7r + 4704.5$$

$$\Rightarrow 9.7r = 145.5 \Rightarrow r = 15\Omega$$

Q40. The logic levels H and L at different locations in a digital circuit are found to be as shown in the figure.



Based on these observations, which of the logic gates is not behaving as an ideal NAND gate?

(1) G_2 (2) G_3 (3) G_4 (4) G_1

Ans.: (3)

Solution.: G_1 : output $= \overline{H.L} = H$; G_2 : output $= \overline{L.L} = H$

G_3 : output $= \overline{H.H} = L$; G_4 : output $= \overline{H.L} = H$ (Not correct)

PART C

Q54. An astronomer observes 500 objects and classifies them as either of type A or type B. She finds 148 objects to be of type B. Assuming a binomial distribution, the best estimate of the fraction of type A objects and its associated standard deviation respectively are

(1) 0.704, 0.002 (2) 0.70, 0.02

(3) 0.704, 0.031 (4) 0.72, 0.03

Ans.: (2)

Solution.:

Type-A + Type-B $= 500 = n$, Type-B $= 148$

fraction of type A objects $= \frac{500-148}{500} = \frac{352}{500} = 0.704$.

$P_A = 0.704,\ P_B = 1-0.704 = 0.296$; standard deviation $\sigma = \sqrt{\frac{P_A P_B}{n}} = 0.0204$.

Q73. A piezoresistive pressure sensor utilizes change in electrical resistance (ΔR) with change in pressure (ΔP) as $\Delta R = -R_0 \log_{10}\left(\frac{\Delta P}{P_0}\right)$, where $R_0 = 500\Omega$ and $P_0 = 1000\,\text{mbar}$. A current of $2\mu A$ is passed through the sensor and the resultant voltage drop is measured using an analog-to-digital (ADC) converter having a range 0 to 1V. If a pressure change of 1 mbar is to be measured, amongst the given options, the minimum number of bits needed for the ADC is

(1) 12 (2) 14 (3) 8 (4) 10

Ans.: (4)

Solution.:

Here $R_0 = 500\Omega$, $P_0 = 1000\,\text{mbar}$, $\Delta P = 1\text{mbar}$ and $I = 2\times10^{-6} A$

So $\Delta R = -R_0 \log_{10}\left(\frac{\Delta P}{P_0}\right) = -500\log_{10}\left(\frac{1}{1000}\right) = 1500\Omega$

The resultant voltage drops $\Delta V = I\times\Delta R = 2\times10^{-6}\times1500 = 3\times10^{-3}V = 3mV$

For ADC, resolution should be greater than or equal to $3mV$.

Since $R \ge \frac{V_{full\,scale}}{2^n - 1} \Rightarrow 3mV \ge \frac{1V}{2^n-1} \Rightarrow 3\times10^{-3} \ge \frac{1}{2^n-1} \Rightarrow (2^n-1) \ge \frac{1}{3\times10^{-3}}$

$\Rightarrow (2^n-1) \ge 333.33 \Rightarrow 2^n \ge 334.33 \Rightarrow n\ln 2 \ge \ln(334.33) \Rightarrow n \ge \frac{\ln(334.33)}{\ln 2}$

$\Rightarrow n \ge \frac{\ln(334.33)}{\ln 2} \Rightarrow n \ge \frac{5.81}{0.693} \Rightarrow n \ge 8.386$

So possible values of $n = 9, 10, 11, 12...$

Thus, amongst the given options, the minimum number of bits needed for the ADC is 10.

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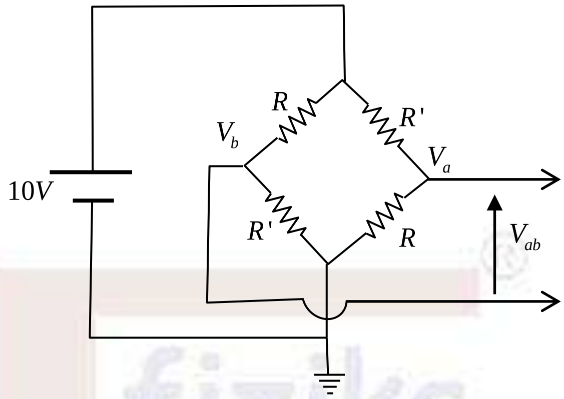
Q74. In the circuit shown in the figure, the resistances R and R' change due to strain. While R increases, R' decreases by the same amount ΔR due to the applied strain. The unstrained values of R and R' are 100Ω each. If same strain is applied to all the resistors, and the output voltage (V_{ab}) changes to 0.3 V, then ΔR is closest to



(1) 3Ω (2) 1.5Ω (3) 4.5Ω (4) 6Ω

Ans.: (1)

Solution.: $$V_a = \frac{R+\Delta R}{(R+\Delta R)+(R'-\Delta R)} \times 10V = \frac{100+\Delta R}{200} \times 10 \Rightarrow V_a = \frac{100+\Delta R}{20}$$

$$V_b = \frac{R'-\Delta R}{(R+\Delta R)+(R'-\Delta R)} \times 10V = \frac{100-\Delta R}{200} \times 10 \Rightarrow V_b = \frac{100-\Delta R}{20}$$

$$\because V_{ab} = V_a - V_b = 0.3V \Rightarrow \frac{(100+\Delta R)-(100-\Delta R)}{20} = 0.3V \Rightarrow 2\Delta R = 6\Omega \Rightarrow \Delta R = 3\Omega$$

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PART C

Q46. The Debye temperature of a two-dimensional insulator is 150 K. The ratio of the heat required to raise its temperature from 1 K to 2 K and from 2 K to 3 K is

(1) 7:19 (2) 3:13 (3) 1:1 (4) 3:5

Ans.: (1)

Solution.: In two dimensions $C_V = BT^2$

The heat required to raise the temperature from T_1 to T_2 is

$$Q = \int_{T_1}^{T_2} C_V dT = B\int_{T_1}^{T_2} T^2 dT = B\left[\frac{T^3}{3}\right]_{T_1}^{T_2}$$

From $T_1 = 1K$ to $T_2 = 2K$: $Q_1 = \frac{B}{3}\left[T^3\right]_1^2 = \frac{B}{3}\left[2^3 - 1\right] = \frac{B}{3}[8-1] = \frac{7B}{3}$

From $T_1 = 2K$ to $T_2 = 3K$: $Q_2 = \frac{B}{3}\left[T^3\right]_2^3 = \frac{B}{3}\left[3^3 - 2^3\right] = \frac{B}{3}[27-8] = \frac{19B}{3}$

$\therefore \frac{Q_1}{Q_2} = \frac{7B/3}{19B/3} = \frac{7}{19}$. Thus $Q_1 : Q_2 = 7:19$. The correct option is (1).

Q61. Consider a body-centered tetragonal lattice with lattice constants $a = b = a_0$ and $c = \frac{a_0}{2}$.

The number of nearest neighbours, the nearest neighbour distance, the number of next nearest neighbours and the next nearest neighbour distance, respectively, are

(1) $6, \frac{1}{2}a_0, 8\frac{\sqrt{3}}{2}a_0$ (2) $8, \frac{\sqrt{3}}{2}a_0, 6, a_0$

(3) $2, \frac{1}{2}a_0, 8, \frac{3}{4}a_0$ (4) $8, a_0, 6, \frac{4}{3}a_0$

Ans.: (3)

Solution:

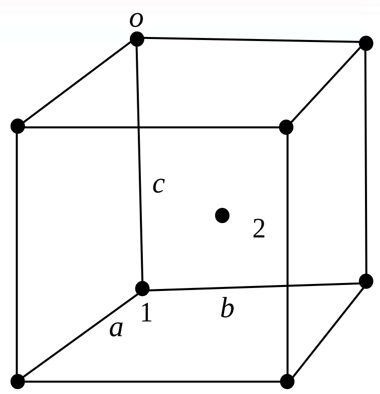
$a = b = a_0$ and $c = \frac{a_0}{2}$

Number of first nearest neighbors = 2

distance of first nearest neighbor $= c = \frac{a_0}{2}$

Number of second nearest neighbors = 8

distances of second nearest neighbor $= \frac{3}{4}a_0$



Thus, correct option is (3).

Q65. The band dispersion of electrons in a two dimensional square lattice (lattice constant a) is given by, $E(k_x, k_y) = -2(t_x \cos k_x a + t_y \cos k_y a)$

where $t_x, t_y > 0$. The effective mass tensor $m^* = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix}$ of electrons at $\vec{k} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right)$ is

1. $\begin{pmatrix} 0 & \frac{\hbar^2}{2a^2\sqrt{t_x t_y}} \\ \frac{\hbar^2}{2a^2\sqrt{t_x t_y}} & 0 \end{pmatrix}$

2. $\begin{pmatrix} \frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & \frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$

3. $\begin{pmatrix} -\frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & -\frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$

4. $\begin{pmatrix} 0 & -\frac{\hbar^2}{2a^2(t_x + t_y)} \\ -\frac{\hbar^2}{2a^2(t_x + t_y)} & 0 \end{pmatrix}$

Ans.: (3)

Solution.: Given $E = -2(t_x \cos k_x a + t_y \cos k_y a)$

$$\frac{\partial E}{\partial k_x} = 2a(t_x \sin k_x a), \frac{\partial^2 E}{\partial k_x^2} = 2a^2 t_x \cos k_x a \text{ and } \frac{\partial^2 E}{\partial k_y^2} = 2a^2 t_y \cos k_y a$$

$\frac{\partial E}{\partial k_x \partial k_y} = 0$ and $\frac{\partial E}{\partial k_y \partial k_x} = 0$. The effective mass tensor $m^* = \begin{pmatrix} m_{xx} & m_{xy} \\ m_{yx} & m_{yy} \end{pmatrix}$

where $m_{xx} = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k_x^2}} = \frac{\hbar^2}{2a^2 t_x \cos k_x a} = -\frac{\hbar^2}{2a^2 t_x} \left(\text{at } k_x = \frac{\pi}{a}\right)$

$$m_{yy} = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k_y^2}} = \frac{\hbar^2}{2a^2 t_y \cos k_y a} = -\frac{\hbar^2}{2a^2 t_y} \left(\text{at } k_y = \frac{\pi}{a}\right)$$

$$m_{xy} = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k_x \partial k_y}} = 0 \text{ and } m_{yx} = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k_y \partial k_x}} = 0$$

$\therefore\ m^* = \begin{pmatrix} -\frac{\hbar^2}{2a^2 t_x} & 0 \\ 0 & -\frac{\hbar^2}{2a^2 t_y} \end{pmatrix}$. Thus, correct option is (3).

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Solution-Nuclear and Particle Physics

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PART C

Q47. In a scattering experiment, a beam of e^- with an energy of 420 MeV scatters off an atomic nucleus. If this first minimum of the differential cross section is observed at a scattering angle of 45°, the radius of the nucleus (in fermi) is closest to

(1) 0.4 (2) 8.0 (3) 2.5 (4) 0.8

Ans. (3):

Solution: $E = 420\,MeV$, $E_0 = m_0c^2 = 0.51\,MeV$

$E > E_0$ Ultra-relativistic $E \approx pc$

For scattering

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{hc}{E} = \frac{1240\times10^{-9}\,(\text{eV-m})}{E(eV)} \Rightarrow \lambda = \frac{1240\times10^{-9}}{420\times10^{6}} \approx 3\times10^{-15}m = 3\,fm$$

For first order minima

$$d\sin\theta = n\lambda \Rightarrow d\sin45° = 1\times3\,fm \Rightarrow d = 3\sqrt{2}\,fm$$

$$R = \frac{d}{2} = \frac{3}{2}\sqrt{2} = 1.5\times1.414\,fm \Rightarrow R = 2.5\,fm$$

Q57. π^- has spin 0 and negative intrinsic parity. In a reaction a deuteron in its ground state ($J = 1$, parity +1) captures a π^- in p-wave to produce a pair of neutrons (intrinsic parity is +1). The neutrons will be produced in a state with

(1) $l = 1, S = 0$ (2) $l = 0, S = 1$ (3) $l = 1, S = 1$ (4) $l = 0, S = 0$

Ans. (4):

Solution: $d + \pi^- \to n + n$

(i) $\vec{J}_d = \vec{1}$, $\vec{J}_\pi = \vec{\ell}_\pi + \vec{S}_\pi = \vec{1} + \vec{0} = \vec{1}$ $\because$ π^- is in p-wave

$$\vec{J}_n = \vec{J}_d + \vec{J}_\pi = \vec{1} + \vec{1}$$

$J_n = 0, 1, 2$

Neutrons are produced in a state

(ii) $s_{n_1} = \frac{1}{2}$, $s_{n_2} = \frac{1}{2} \to S_n = 0, 1$

$S_n = 1$ is not possible as neutron pair is in a state. They will have opposite spins. So, $S_n = 0$.

Apply conservation of parity principle

$$\pi_d\pi_\pi(-1)^{\ell_\pi} = \pi_n\pi_n(-1)^{L_n} \Rightarrow (+)(-)(-1)^1 = (+)(+)(-1)^{L_n} \Rightarrow (-1)^{L_n} = +1 \Rightarrow L_n = 0, 2, 4$$

So, possible state is $L = 0, S = 0$

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Q69. The Δ^{++} can be produced by colliding a pion beam onto a H_2 target, in a reaction $\pi^+ + p \to \Delta^{++} \to \pi^+ + p$. In the rest frame of Δ^{++}, the energy and momentum of the pion in the final state (in MeV) are closest to

(assume $c=1$, and $m_\pi \approx 140\,\text{MeV}$, $m_p \approx 1\,\text{GeV}$, $m_{\Delta^{++}} \approx 1.2\,\text{GeV}$)

(1) 210, 156 (2) 230, 182 (3) 175, 105 (4) 190, 130

Ans. (4):

Solution:

$\Delta^{++} \to \pi^+ + p$

In the rest frame of Δ^{++}: $P_\Delta = 0$ and $|\vec{P}_\pi| = |\vec{P}_p| = P$; $E_\Delta = E_\pi + E_p$,

$c=1, E_\Delta = m_\Delta c^2 = m_\Delta$

$$\Rightarrow m_\Delta = \sqrt{P^2+m_\pi^2} + \sqrt{P^2+m_p^2} \Rightarrow \left[m_\Delta - \sqrt{P^2+m_p^2}\right]^2 = P^2 + m_\pi^2$$

$$m_\Delta^2 - 2m_\Delta\sqrt{P^2+m_p^2} + \cancel{P^2} + m_p^2 = \cancel{P^2} + m_\pi^2 \Rightarrow P^2 + m_p^2 = \left[\frac{m_\Delta^2 + m_p^2 - m_\pi^2}{2m_\Delta}\right]^2$$

$$\Rightarrow P^2 = \left[\frac{(1200)^2 + (1000)^2 - (140)^2}{2(1200)}\right]^2 - (1000)^2 \Rightarrow P = 130.66\,MeV$$

$$E_\pi = \sqrt{P^2+m_\pi^2} \Rightarrow E_\pi = \sqrt{(130.66)^2 + (140)^2} = 191.5\,MeV$$

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Solution-General Aptitude

PART A

Q1. Canals A and B join to form canal C, all having semi-circular cross-sections of radii which are in the ratio 3:4:5, respectively. Assume smooth merger of A and B, and ignore the possibility of flooding. If the speed s of water is the same and uniform in both A and B then the speed of water flowing in C is

(1) s (2) $7s/5$ (3) $2s$ (4) $5s/7$

Ans. (1):

Solution.: The flow rate of water in a canal depends on its cross-sectional area and the velocity of the water. The relationship can be written as: $Q = A \cdot s$

where Q is the flow rate, A is the cross-sectional area, s is the speed of water.

For canals A, B and C, we use the fact that the cross-sectional area A of a semi-circular canal is: $A = \frac{1}{2}\pi r^2$

where r is the radius of the semi-circular cross-section.

The radii of the canals are given in the ratio 3:4:5, so let the radii be $3k, 4k$ and $5k$ for canals A, B and C, respectively.

Step 1: Flow rates in A and B

The flow rates in A and B are: $Q_A = A_A \cdot s = \frac{1}{2}\pi(3k)^2 \cdot s = \frac{9}{2}\pi k^2 \cdot s$

$$Q_B = A_B \cdot s = \frac{1}{2}\pi(4k)^2 \cdot s = 8\pi k^2 \cdot s$$

The total flow rate into canal C is the sum of the flow rates from A and B:

$$Q_C = Q_A + Q_B = \frac{9}{2}\pi k^2 \cdot s + 8\pi k^2 \cdot s \Rightarrow Q_C = \left(\frac{9}{2} + 8\right)\pi k^2 \cdot s = \frac{25}{2}\pi k^2 \cdot s$$

Step 2: Speed of water in canal C

The flow rate in canal C is also given by: $Q_C = A_C \cdot s_C$

where A_C is the cross-sectional area of canal C and s_C is the speed of water in canal C. The area of canal C is: $A_C = \frac{1}{2}\pi(5k)^2 = \frac{25}{2}\pi k^2$

Substitute Q_C and A_C into the equation: $\frac{25}{2}\pi k^2 \cdot s = \frac{25}{2}\pi k^2 \cdot s_C$

Simplify to solve for s_C: $s_C = s$

Thus the speed of water in canal C is the same as the speed in canals A and B.

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Q2. A patient requires administration of 500 ml of an intravenous fluid in 1 hour. What is the approximate drip rate (number of drops per minute) at which the fluid should be administered, if the volume of a drop is 0.05 ml?

(1) 76 (2) 152 (3) 167 (4) 332

Ans.: (3)

Solution.: To calculate the drip rate (number of drops per minute), we need to determine the total number of drops required and divide it by the time in minutes.

Step 1: Calculate the total number of drops

The total volume to be administered is $500\,\text{ml}$. Since each drop has a volume of $0.05\,\text{ml}$, the total number of drops is:

$$\text{Total drops} = \frac{\text{Total volume}}{\text{Volume per drop}} = \frac{500}{0.05} = 10{,}000\,\text{drops}$$

Step 2: Determine the time in minutes

The fluid needs to be administered in 1 hour, which is: 1 hour = 60 minutes.

Step 3: Calculate the drip rate

The drip rate (number of drops per minute) is given by:

$$\text{Drip rate} = \frac{\text{Total drops}}{\text{Time in minutes}} = \frac{10{,}000}{60} \approx 166.67 \text{ drops per minute.}$$

Thus, the drip rate is approximately: 167 drops per minute.

Q3. A large number of birds, half of which belong to specie A and the other half of specie B, rest on a tree where they are distributed randomly across the branches. In a random sample of 5 birds from the tree, what is the probability that at least one is from specie A?

(1) 0.03125 (2) 0.15625 (3) 0.84375 (4) 0.96875

Ans.: (4)

Solution.:

Step 1: Probability of selecting no birds of species A

The probability of selecting a bird of species B in a single trial is $\frac{1}{2}$. If all 5 birds in the sample are from species B, the probability is: $P(\text{all 5 are species B}) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Step 2: Probability of selecting at least one bird of species A

The complement of "all 5 are species B" is "at least one is species A". Therefore:

$$P(\text{at least one species } A) = 1 - P(\text{all 5 are species } B)$$

Substitute $P(\text{all 5 are species } B)=\frac{1}{32}$: $P(\text{at least one species } A)=1-\frac{1}{32}=\frac{31}{32}$

Thus, the probability that at least one bird in the sample is from species A is: $\boxed{\frac{31}{32}}$ or approximately 0.96875 (96.875%).

Q4. How many three-digit numbers exist whose first and last digits add up to 9?

(1) 90 (2) 81 (3) 80 (4) 72

Ans.: (1)

Solution.: A three-digit number can be written as: $ABC-100A+10B+C$

where:

- A is the first digit (hundreds of places, $A\in\{1,2,...,9\}$)
- B is the middle digit (tens place, $B\in\{0,1,...,9\}$)
- C is the last digit (units places, $C\in\{0,1,...,9\}$)

Step 2: Condition for the sum of first and last digits

The condition is: $A+C=9$

Step 3: Find valid pairs of A and C

Since A is a digit from 1 to 9, and C is a digit from 0 to 9, the valid pairs are:

$(A,C)=(1,8),(2,7),(3,6),(4,5),(5,4),(6,3),(7,2),(8,1),(9,0)$

This gives 9 valid pairs of (A,C).

Step 4: Consider the middle digit B

The middle digit B can be any digit from 0 to 9, regardless of A and C. Thus, there are 10 possible choices for B for each pair (A,C).

Step 5: Total number of three-digit numbers

For each of 9 valid pairs (A,C), there are 10 possible values of B. Therefore, the total number of three-digit numbers is: $9\times10=90$

Q5. On a one-way road, broken lines consisting of 2.5 m length segments separated by 2.5 m gaps are painted along the length of the road to demarcate 3 lanes, and continuous lines are painted along both the borders. What is the total length of the painted lines (in m) over a 250 m stretch of the road?

(1) 500 (2) 625 (3) 750 (4) 1000

Ans.: (3)

Solution.:

Step 1: Broken lines for lane demarcation

• The broken lines alternate between $2.5m$ painted segments and $2.5m$ gaps, meaning the cycle length is $2.5+2.5=5m$.

• In a $250m$ stretch of road, the number of full cycles of broken lines is:

$$\text{Number of cycles} = \frac{250}{5} = 50$$

• Each cycle includes $2.5m$ of painted line. Therefore, the total length of the painted segments in one broken line is: $50\times 2.5=125m$

• Since there are 2 broken lines to demarcate 3 lanes, the total length of painted broken lines is: $125\times 2=250m$

Step 2: Continuous lines along the borders

• The road has 2 continuous lines, one along each border.

• Each continuous line spans the entire $250m$ stretch.

• Therefore, the total length of the continuous lines is: $250\times 2=500m$

Step 3: Total length of painted lines

The total length of all painted lines is the sum of the lengths of the broken lines and the continuous lines: Total length $=250+500=750m$

Q6. A record player stylus moves along a spiral groove cut on an annular portion of a disc with inner radius 4 cm and outer radius 10 cm. If the record turns 100 times when playing, the stylus travels approximately

(1) 2.2 m (2) 4.4 m (3) 22 m (4) 44 m

Ans.: (4)

Solution.: To calculate approximate distance traveled by the stylus, let us consider the geometry of the spiral groove.

Step 1: Understand the setup

• The groove is cut on an annular portion of the disc with:

Inner radius $r_{inner}=4cm$; Outer radius $r_{outer}=10cm$

• The record turns $N=100$ times while the stylus moves from the inner radius to the outer radius.

• The groove forms a spiral and the stylus moves outward uniformly.

Step 2: Total radial distance traveled

The radial distance traveled by the stylus (from inner to outer radius) is:

$$\Delta r = r_{outer} - r_{inner} = 10 - 4 = 6\,cm$$

The stylus completes $N = 100$ revolutions, so the radial distance traveled per revolution is:

$$\Delta r_{\text{per revolution}} = \frac{\Delta r}{N} = \frac{6}{100} = 0.06\,cm \text{ per revolution.}$$

Step 3: Approximate length of the groove

The length of the spiral groove can be approximated by considering the average radius of the annular region. $r_{avg} = \frac{r_{inner} + r_{outer}}{2} = \frac{4+10}{2} = 7\,cm$

The approximate circumference for each revolution is:

Circumference $= 2\pi r_{avg} = 2\pi \times 7 = 14\pi\,cm$

The total length of the groove over $N = 100$ revolution is:

Total length $\approx N \times \text{Circumference} = 100 \times 14\pi = 1400\pi\,cm$

Step 4: Convert to meters

Using $\pi \approx 3.14$, the total length in centimeters is:

$1400\pi \approx 1400 \times 3.14 = 4396\,cm$; Total length $\approx 43.96\,m$

The stylus travels approximately: $\boxed{44\,m}$

Q7. In how many distinct ways can 128 identical marbles be arranged in a complete rectangular grid (disregarding the orientation of the grid)?

(1) 7 (2) 6 (3) 5 (4) 4

Ans.: (4)

Solution.: Step 1: Prime factorization of 128: The prime factorization of 128 is: $128 = 2^7$

Step 2: Total number of factors

The total number of factors of 128 is given by adding 1 to each exponent in the prime factorization and multiplying: Number of factors $= (7+1) = 8$

The factors of 128 are: 1, 2, 4, 8, 16, 32, 64, 128

Step 3: Pairing factors to form rectangles

The factor pairs are: $(1,128),(2,64),(4,32),(8,16)$

Since we disregard orientation, only pairs with $r \le c$ are distinct.

Step 4: Count distinct pairs: The distinct pairs are: $(1,128),(2,64),(4,32),(8,16)$

This gives 4 distinct ways to arrange the marbles.

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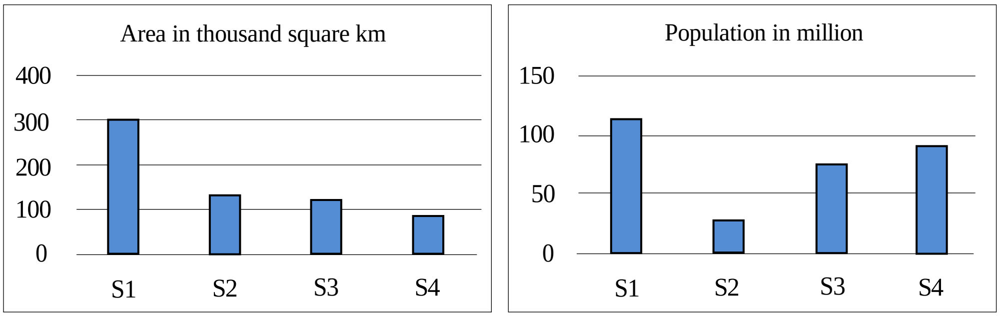
Q8. Areas and populations of four states S1, S2, S3 and S4 are shown.



Their arrangement **in decreasing order** of population density would be

(1) S4, S3, S1, S2 (2) S1, S2, S3, S4

(3) S4, S1, S3, S2 (4) S2, S1, S3, S4

Ans.: (1)

Q9. The population of a town is increasing at a uniform rate. If its population was 90,000 and 96000 in 2022 and 2023 respectively, what would be its population in 2024?

(1) 102,000 (2) 102,400 (3) 102,720 (4) 102,960

Ans.: (2)

Solution.: The population of the town is increasing at a uniform rate. The increase in population per year can be calculated as:

Increase per year = Population in 2023 – Population in 2022

Increase per year $= 96,000 - 90,000 = 6,000$

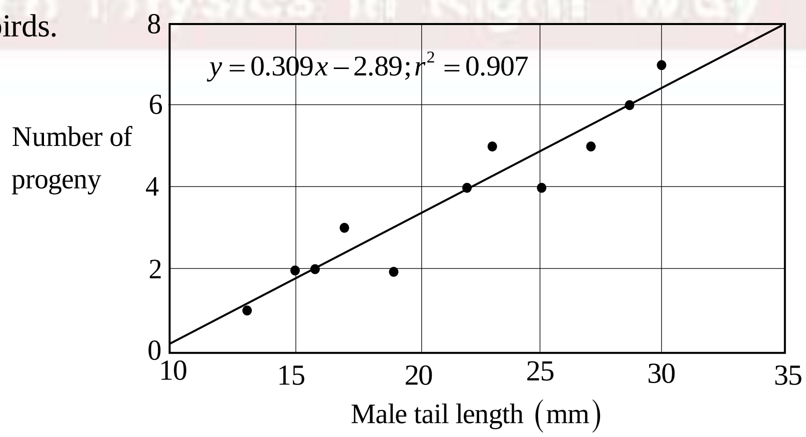
The population in 2024 can be found by adding another year's increase to the population in 2023:

Population in 2024 = Population in 2023 + Increase per year

Population in 2024 $= 96,000 + 6,000 = 102,000$

The population in 2024 is expected to be: $\boxed{102,000}$

Q10. The graph shows observations and a regression line of the number of progeny on the tail length of male birds.



Which of the following can be inferred from the graph?

(1) Producing less progeny decreases the tail length of the males.

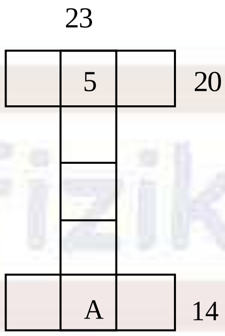
(2) Males cannot have a tail length lesser than 10 mm.

(3) Males with longer tails tend to father more progeny.

(4) For a male with a 25 mm tail, the expected number of progeny is 4.

Ans.: (3)

Q11. The squares in the following grid are filled with numbers 1 to 9, without repetition, such that the numbers in the squares forming the top and bottom rows add to 20 and 14 respectively and those forming the column to 23. What is the value of A?



(1) 4 (2) 6 (3) 7 (4) 8

Ans.: (3)

Q12. Among A, B, C, D, E and F, D is taller than B but shorter than F. E is taller than B, but shorter than C. B is not the shortest of all. Then A is

(1) the shortest of all (2) the tallest of all

(3) taller than E, but shorter than C (4) taller than C, but shorter than F

Ans.: (1)

Solution.:

Step 1: Analyze the given statements

1. D is taller than B but shorter than F : $B < D < F$

2. E is taller than B but shorter than C : $B < E < C$

3. B is not the shortest of all: This implies there is someone shorter than B.

Step 2: Arrange heights relative to each other

From the information, we know: $B < D < F$ and $B < E < C$

- F is taller than D and C is taller than E, so F and C are among the tallest.
- B is not the shortest, so A must be shorter than B.

Step 3: Determine $A's$ position

Since A is shorter than B, A must be the shortest of all.

Q13. An egg tray has 30 cavities to hold eggs in 5 rows and 6 columns. Each cavity is surrounded by 4 raised corners shared by adjacent cavities. How many raised corners does the egg tray have?

(1) 30 (2) 35 (3) 36 (4) 42

Ans.: (4)

Solution.: Understanding the Problem

- The egg tray has 5 rows and 6 columns, resulting in $5\times6=30$ cavities.
- Each cavity is surrounded by 4 raised corners.
- Corners are shared between adjacent cavities, so we must count only the unique corners.

Analyzing the Raised Corners

- Each row of the tray has **6 cavities**, which means it has **7 vertical lines of corners** (one at each end and one between each column).
- Similarly, each column of the tray has 5 cavities, which means it has 6 horizontal lines of corners (one at each end and one between each row).

Hence, the total number of raised corners is equivalent to the number of intersections of these vertical and horizontal lines.

Total Number of Corners

The grid formed by the vertical and horizontal lines has: $7\times6=42$ raised corners.

Thus egg tray has 42 raised corners.

Q14. Among 1000 squirrel babies, 200 have three stripes on their back, 500 have two stripes on their back and the rest have four stripes on their back. While 90% of the three-striped babies survive to adulthood, only 80% of the two-striped and 70% of the four-striped babies survive to adulthood. The fraction of four-striped squirrels among the adults is nearest to

(1) 0.21 (2) 0.3 (3) 0.266 (4) 0.288

Ans.: (3)

Solution.:

Step 1: Survival to adulthood for each group

1. Three-striped squirrels:

Total = 200, Survival rate = 90%, Survivors $=200\times0.9=180$

2. Two-striped squirrels:

Total = 500, Survival rate = 80%, Survivors = $500\times0.8=400$

3. Four-striped squirrels:

$\text{Total}=1000-200-500=300$, Survival rate = 70%, Survivors $=300\times0.7=210$

Step 2: Total number of surviving adults

Total adults $=180+400+210=790$

Step 3: Fraction of four-striped squirrels among adults

Fraction of four-striped squirrels = $\frac{\text{Four-striped survivors}}{\text{Total adults}}=\frac{210}{790}\approx 0.266$

Q15. A referendum on a proposal involved 7000 participants. Among the participants 3600 were women and the rest were men. 2900 participants, of whom 1300 were women, voted against while 3000 participants voted in favour. 400 women abstained. The ratio of the number of men that did not vote to the total number of participants is

(1) 11:70 (2) 17:35 (3) 1:10 (4) 8:70

Ans.: (3)

Solution.: Step 1: Total number of men

Total men = Total participants – Total women = 7000 –3600 = 3400

Step 2: Number of men who voted

- Total participants who voted = 2900 (against) + 3000 (in favor) = 5900
- Women who voted = 3600 – 400 = 3200
- Men who voted = Total participants who voted – Women who voted = 5900 – 3200 =2700

Step 3: Number of men who did not vote

Men who did not vote =Total men – Men who voted = 3400 – 2700= 700

Step 4: Ratio of men who did not vote to total participants

$$\text{Ratio}=\frac{\text{Men who did not vote}}{\text{Total participants}}=\frac{700}{7000}=1:10$$

Q16. A rectangular tray of 30 cm $\times$60 cm size is used for baking circular biscuits. The diameter of each biscuit is 3 cm before banking, which increases by 10% on baking. What is the maximum number of biscuits that can be baked in the tray such that the base of each biscuit is in contact with the tray?

(1) 171 (2) 162 (3) 180 (4) 200

Ans.: (2)

Solution.:

Step 1: Final diameter of the biscuits

The diameter of each biscuit increases by 10% during baking:

Final diameter $=3cm+0.1\times 3cm=3.3cm$

The radius of each biscuit becomes: $\text{Radius}=\frac{3.3}{2}cm=1.65cm$

Step 2: Arrangement of biscuits

To maximize the number of biscuits, we will arrange them in a rectangular grid pattern:

• Each biscuit occupies a square of side $3.3cm$ (equal to its diameter) in the tray.

• The number of biscuits along the length and width of the tray can be calculated as:

Along the length $(60cm)$: Biscuits along length $=\left\lfloor\frac{60}{3.3}\right\rfloor=\lfloor 18.18\rfloor=18$

Along the width $(30cm)$: Biscuits along width $=\left\lfloor\frac{30}{3.3}\right\rfloor=\lfloor 9.09\rfloor=9$

Step 3: Total number of biscuits

The total number of biscuits that can fit is: Total biscuits $=18\times 9=162$

Thus, maximum number of biscuits that can be baked in the tray is 162.

Q17. If 32XY6 is divisible by 9, X and Y being even decimal digits, then X =

(1) 2 (2) 4 (3) 6 (4) 8

Ans.: (4)

Solution.:

$32XY6=3+2+x+y+6=11+x+y=11+8+8=27$

Q18. In a class of 70 students, 20% of girls have spectacles and 40% of boys have spectacles. If the total number of students having spectacle is 23, the number of boys in the class is

(1) 45 (2) 14 (3) 18 (4) 25

Ans.: (1)

Solution.:

Step 1: Spectacle counts for boys and girls

• 40% of boys have spectacles: Boys with spectacles $=0.4B$

• 20% of girls have spectacles: Girls with spectacles $=0.2(70-B)$

Step 2: Total students with spectacles

The total number of students with spectacles is given as 23: $0.4B+0.2(70-B)=23$

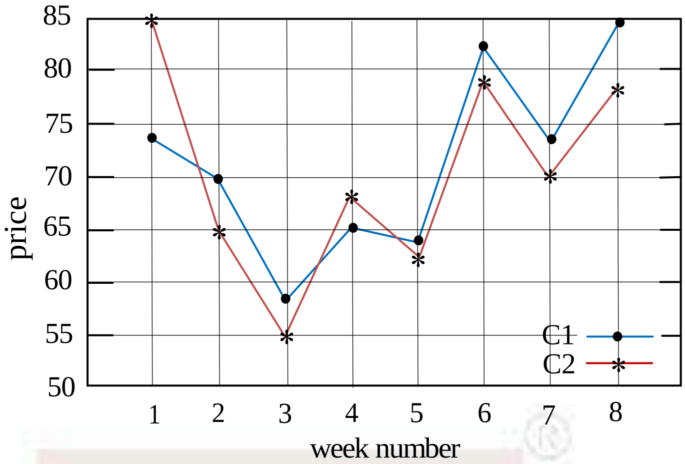
Step 3: Solve for B

Expand and simplify the equation: $0.4B+0.2\times 70-0.2B=23\Rightarrow 0.4B-0.2B+14=23$,

$\Rightarrow 0.2B=23-14\Rightarrow 0.2B=9\Rightarrow B=\frac{9}{0.2}=45$

Thus number of boys in the class is 45.

Q19. The two graphs show the change in price of two commodities C1 and C2 over 8 weeks.



Which of the statements is correct?

1. C1 has higher fluctuation than C2
2. Average price of C1 is lower than that of C2
3. The largest change in a week is shown by C2
4. C1 shows a tendency of reduction

Ans.: (3)

Q20. Suppose that the increase in a population can be modelled as $\left(\frac{dN}{dt}\right) = rN\frac{(K-N)}{K}$

where N is the size of the population, K is the carrying capacity, r is the per capita growth rate and t is time. Which of the following statements is correct?

(1) When $N \approx 0$, the change in population N is nearly exponential.

(2) When $N = K$, the population goes extinct as dN/dt goes to zero.

(3) When $N \approx 0$, the population growth dN/dt is maximum.

(4) When $N \approx K/4$, the population growth dN/dt is maximum.

Ans.: (1)

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