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IIT-JAM Physics-2024

Solution-Mathematical Methods

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Section A: Q.1-Q.10 Carry ONE mark each.

Q3. Which of the following matrices is Hermitian as well as unitary?

(A) $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 & 1+i \\ 1-i & 0 \end{pmatrix}$

Ans: (A)

Solution:

(A) $M = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, M^{\dagger} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = M$ [Hermitian]

$MM^{\dagger} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \begin{bmatrix} 0-i^2 & 0 \\ 0 & -i^2+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \Rightarrow M^{\dagger} = M^{-1}$ [Unitary]

(B) $M = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \Rightarrow M^{\dagger} = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = -M$ [Skew-Hermitian]

$MM^{\dagger} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ [Unitary]

(C) $M = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} \Rightarrow M^{\dagger} = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} = M$ [Hermitian]

$MM^{\dagger} = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1-i^2 & -i-i \\ i+i & -i^2+1 \end{bmatrix} = \begin{bmatrix} 2 & -2i \\ 2i & 2 \end{bmatrix} \neq I$

(D) $M = \begin{bmatrix} 0 & 1+i \\ 1-i & 0 \end{bmatrix} \Rightarrow M^{\dagger} = \begin{bmatrix} 0 & 1+i \\ 1-i & 0 \end{bmatrix} = M$ [Hermitian]

$MM^{\dagger} = \begin{bmatrix} 1-i^2 & 0 \\ 0 & 1-i^2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$ [Not Unitary]

Q4. The divergence of a 3-dimensional vector $\frac{\hat{r}}{r^3}$ ($\hat{r}$ is the unit radial vector) is:

(A) $-\frac{1}{r^4}$ (B) Zero

(C) $\frac{1}{r^3}$ (D) $-\frac{3}{r^4}$

Ans: (A)

Solution: $\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^3}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \times \frac{1}{r^3}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{1}{r}\right) = \frac{1}{r^2}\left(-\frac{1}{r^2}\right) \Rightarrow \boxed{\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^3}\right) = -\frac{1}{r^4}}$

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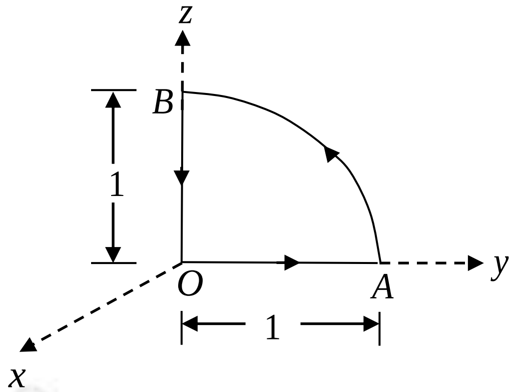
Section A: Q.11-Q.30 Carry TWO marks each.

Q14. The value of the line integral for the vector,

$$\vec{v} = 2\hat{x} + yz^2\hat{y} + \left(3y + z^2\right)\hat{z}$$

along the closed path OABO (as shown in the figure) is: (Path AB is the arc of a circle of unit radius)



(A) $\frac{1}{4}(3\pi - 1)$ (B) $3\pi - \frac{1}{4}$

(C) $\frac{3\pi}{4} - 1$ (D) $3\pi - 1$

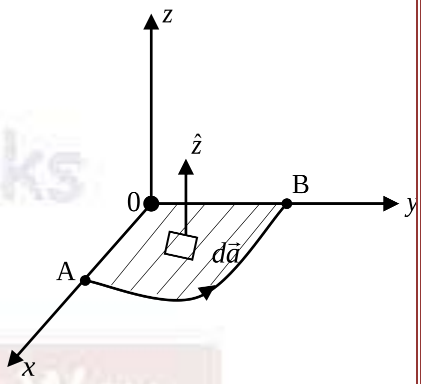
Ans: (A)

Solution:

$$\oint_{\text{line OABO}} \vec{v}\cdot d\vec{l} = \int_s \left(\vec{\nabla}\times\vec{v}\right)\cdot d\vec{a}$$

$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2 & yz^2 & 3y+z^2 \end{vmatrix} = \hat{x}(3-2yz) - \hat{y}(0-0) + \hat{z}(0-0) = (3-2yz)\hat{x}$$

Let us redraw the path in xy-plane, so that we can apply cylindrical coordinate system.



Then $\vec{\nabla}\times\vec{v} = (3-2xy)\hat{z}$, $d\vec{a} = rdrd\phi\hat{z}$, $x = r\cos\phi, y = r\sin\phi$

$$\Rightarrow \left(\vec{\nabla}\times\vec{v}\right)\cdot d\vec{a} = \left(3 - 2r^2\cos\phi\sin\phi\right)rdrd\phi = \left(3 - r^2\sin 2\phi\right)rdrd\phi$$

$$\Rightarrow \int_s \left(\vec{\nabla}\times\vec{v}\right)\cdot d\vec{a} = \int_{r=0}^{1}\int_{\phi=0}^{\pi/2}\left(3 - r^2\sin 2\phi\right)rdrd\phi = 3\times\frac{1}{2}\times\frac{\pi}{2} - \frac{1}{4}\left[-\frac{\cos 2\phi}{2}\right]_0^{\pi/2}$$

$$\Rightarrow \int_s \left(\vec{\nabla}\times\vec{v}\right)\cdot d\vec{a} = \frac{3\pi}{4} - \frac{1}{4}\left(\frac{-\cos\pi + \cos 0}{2}\right) = \frac{3\pi}{4} - \frac{1}{4} = \frac{1}{4}(3\pi - 1)$$

Q15. In the $x-y$ plane, a vector is given by $\vec{F}(x,y) = \frac{-y\hat{x} + x\hat{y}}{x^2 + y^2}$. The magnitude of the flux of $\vec{\nabla}\times\vec{F}$, through a circular loop of radius 2, centered at the origin, is:

(A) π (B) 2π (C) 4π (D) 0

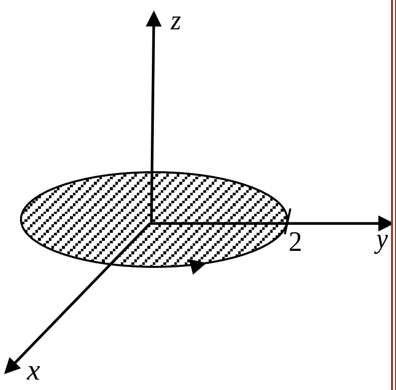
Ans: (B)

Solution:

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0\end{vmatrix}=\hat{x}(0-0)-\hat{y}(0-0)+\hat{z}\left[\frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right)+\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)\right]$$

$$\Rightarrow \vec{\nabla}\times\vec{F}=\hat{z}\left[\frac{(x^2+y^2)\times 1-x\times 2x}{(x^2+y^2)^2}+\frac{(x^2+y^2)\times 1-y\times 2y}{(x^2+y^2)^2}\right]=\hat{z}\left[\frac{-x^2+y^2+x^2-y^2}{(x^2+y^2)^2}\right]=0$$

Magnitude of flux of $\vec{\nabla}\times\vec{F}$, through a circular loop of radius 2 is



$$\int_v(\vec{\nabla}\times\vec{F})\cdot d\vec{a}=\oint_{\text{line}}\vec{F}\cdot d\vec{l}.\quad \because\quad d\vec{l}=dx\hat{x}+dy\hat{y}\Rightarrow\vec{F}\cdot d\vec{l}=-\frac{ydx+xdy}{x^2+y^2}$$

Let $x=r\cos\phi,\ y=r\sin\phi\Rightarrow dx=-r\sin\phi d\phi, dy=r\cos\phi d\phi$

Then $\vec{F}\cdot d\vec{l}=\frac{r\sin\phi(-r\sin\phi d\phi)+r\cos\phi(r\cos\phi d\phi)}{r^2\cos^2\phi+r^2\sin^2\phi}=\frac{r^2d\phi}{r^2}=d\phi$

$$\Rightarrow\oint_{\text{line}}\vec{F}\cdot d\vec{\ell}=\int_0^{2\pi}d\phi=2\pi$$

Q16. The roots of the polynomial, $f(z)=z^4-8z^3+27z^2-38z+26$, are z_1,z_2,z_3 & z_4, where z is a complex variable. Which of the following statements is correct?

(A) $\frac{z_1+z_2+z_3+z_4}{z_1z_2z_3z_4}=-\frac{4}{19}$ (B) $\frac{z_1+z_2+z_3+z_4}{z_1z_2z_3z_4}=\frac{4}{13}$

(C) $\frac{z_1z_2z_3z_4}{z_1+z_2+z_3+z_4}=-\frac{26}{27}$ (D) $\frac{z_1z_2z_3z_4}{z_1+z_2+z_3+z_4}=\frac{13}{19}$

Ans: (B)

Solution: $\because$ $f(z)=z^4-8z^3+27z^3-38z+26$. Roots z_1,z_2,z_3 and z_4 are decided by the matrix

$$M=\begin{bmatrix}0&0&0&-26\\1&0&0&+38\\0&1&0&-27\\0&0&1&+8\end{bmatrix}_{4\times4}\quad\Rightarrow \text{Trade M}=8,\ \det\text{M}=26$$

$$\Rightarrow z_1+z_2+z_3+z_4=\sum\lambda_i=\text{Trace}\,M=8\ \text{and}\ z_1\cdot z_2\cdot z_3\cdot z_4=\prod_i\lambda_i=\det M=26$$

$$\Rightarrow\frac{z_1+z_2+z_3+z_4}{z_1\cdot z_2\cdot z_3\cdot z_4}=\frac{8}{26}=\frac{4}{13}$$

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Section B: Q.31 - Q.40 Carry TWO marks arks each.

(No Question)

Section C: Q.41 - Q.50 Carry ONE mark each.

Q42. The co-ordinate system (x, y, z) is transformed to the system (u, v, w), as given by:

$$u = 2x + 3y - z$$

$$v = x - 4y + z$$

$$w = x + y$$

The Jacobian of the above transformation is ____________.

Ans: 4

Solution:

$$J = \frac{\partial(u,v,\omega)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial \omega}{\partial x} & \frac{\partial \omega}{\partial y} & \frac{\partial \omega}{\partial z} \end{vmatrix} = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -4 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 1(3-4) - 1(2+1) = -1 - 3 = -4$$

Jacobian of transformation $|J| = 4$

Q43. Two sides of a triangle OAB are given by:

$$\overrightarrow{OA} = \hat{x} + 2\hat{y} + \hat{z}$$

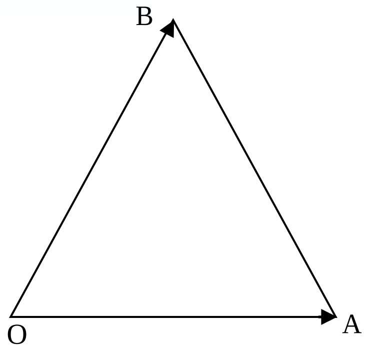
$$\overrightarrow{OB} = 2\hat{x} - \hat{y} + 3\hat{z}$$

The area of the triangle is _________. (Rounded off to one decimal place)

Ans: 4.2 to 4.4

Solution:

Area of triangle $= \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$



$$\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = \hat{x}(6+1) - \hat{y}(3-2) + \hat{z}(-1-4) = 7\hat{x} - \hat{y} - 5\hat{z}$$

$$\Rightarrow \left|\overrightarrow{OA} \times \overrightarrow{OB}\right| = \sqrt{49+1+25} = \sqrt{75} = 8.66$$

Area of triangle $= \frac{1}{2} \times 8.66 = 4.33 \approx 4.3$

Section C: Q.51-Q60 Carry TWO marks each.

Q53. In the Taylor expansion of function, $F(x)=e^{x}\sin x$, around $x=0$, the coefficient of x^{5} is ____________________. (Rounded off to three decimal places)

Ans: -0.034 to -0.032

Solution:

$$\because\ F(x)=F(0)+\frac{F'(0)}{1!}x+\frac{F''(0)}{2!}x^{2}+\ldots\frac{F^{v}(0)}{5!}x^{5}+\ldots$$

$$F(x)=e^{x}\sin x;\quad F'(x)=e^{x}\cos x+e^{x}\sin x$$

$$F''(x)=\left(-e^{x}\sin x+e^{x}\cos x\right)+\left(e^{x}\cos x+e^{x}\sin x\right)\Rightarrow F''(x)=2e^{x}\cos x$$

$$\Rightarrow F'''(x)=2\left(-e^{x}\sin x+e^{x}\cos x\right)$$

$$\Rightarrow F^{iv}(x)=-2\left(e^{x}\cos x+e^{x}\sin x\right)+2\left(-e^{x}\sin x+e^{x}\cos x\right)\Rightarrow F^{iv}(x)=-4e^{x}\sin x$$

$$\Rightarrow F^{v}(x)=-4\left(e^{x}\cos x+e^{x}\sin x\right)\Rightarrow F^{v}(0)=-4(1+0)=-4$$

Coefficient of $x^{5}=\dfrac{F^{v}(0)}{5!}=-\dfrac{4}{5\cdot4\cdot3\cdot2\cdot1}=-\dfrac{1}{30}=-0.033$

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Section A: Q.1-Q.10 Carry ONE mark each.

Q10. Which of the following types of motion may be represented by the trajectory,

$$y(x) = ax^2 + bx + c\,?$$

(Here a, b, and c are constants; x, y are the position coordinates)

(A) Projectile motion in a uniform gravitational field

(B) Simple harmonic motion

(C) Uniform circular motion

(D) Motion on an inclined plane in a uniform gravitational field

Ans: (A)

Solution.: $\because y = ax^2 + bx + c$. It is an equation of a parabola. The option (a) is correct because trajectory of projectile motion in a uniform gravitational field is a parabola.

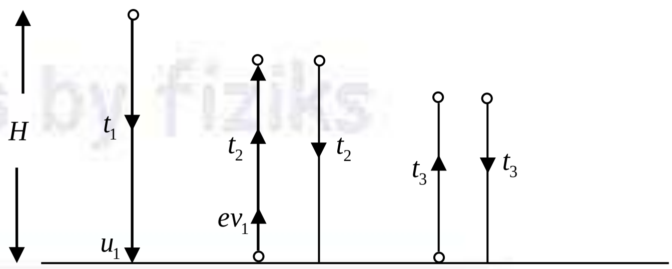
Section A: Q.11-Q.30 Carry TWO marks each.

Q26. A ball is dropped from a height h to the ground. If the coefficient of restitution is e, the time required for the ball to stop bouncing is proportional to:

(A) $\dfrac{2+e}{1-e}$ (B) $\dfrac{1+e}{1-e}$ (C) $\dfrac{1-e}{1+e}$ (D) $\dfrac{2-e}{1+e}$

Ans: (B)

Solution.: $t_1 = \sqrt{\dfrac{2H}{g}}$, $v_1 = \sqrt{2gH}$,



$$0 = ev_1 - gt_2 \Rightarrow t_2 = e\frac{v_1}{g} = e\sqrt{\frac{2H}{g}}$$

$$t_3 = e^2\sqrt{\frac{2H}{g}}$$

$\vdots$

$$T = t_1 + 2t_2 + 2t_3 + 2t_4 + \ldots = \sqrt{\frac{2H}{g}} + 2e\sqrt{\frac{2H}{g}}\left(1 + e + e^2 + \ldots\right)$$

$$\Rightarrow T = \sqrt{\frac{2H}{g}}\left[1 + 2e\frac{1}{1-e}\right] = \sqrt{\frac{2H}{g}}\left(\frac{1+e}{1-e}\right)$$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q35. A particle of mass m, having an energy E and angular momentum L, is in a parabolic trajectory around a planet of mass M. If the distance of the closest approach to the planet is r_m, which of the following statement(s) is(are) true?(G is the Gravitational constant)

(A) $E>0$ (B) $E=0$

(C) $L=\sqrt{2GMm^2r_m}$ (D) $L=\sqrt{2GM^2mr_m}$

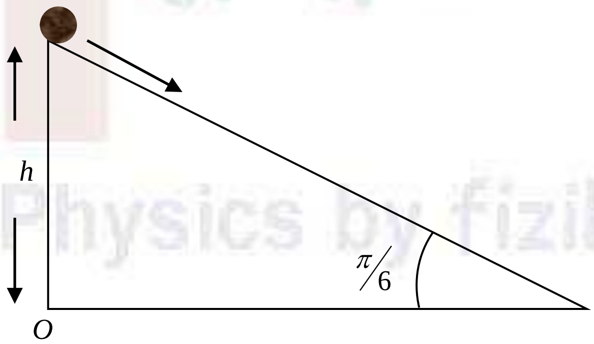
Ans: (B), (C)

Solution.: For parabolic path $E=0$. At distance of closest approach $\dot{r}=0$.

$$E=\frac{1}{2}m\dot{r}^2+\frac{L^2}{2mr^2}-\frac{GMm}{r} \Rightarrow 0=0+\frac{L^2}{2mr_m^2}-\frac{GMm}{r_m} \Rightarrow L=\sqrt{2GMm^2r_m}$$

Section C: Q.41 - Q.50 Carry ONE mark each.

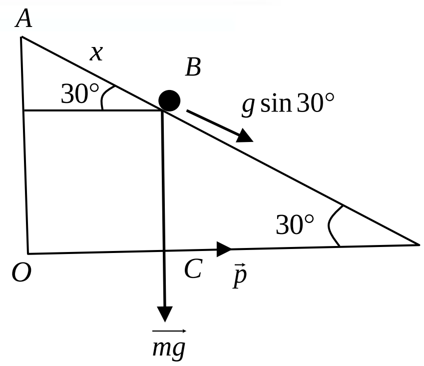
Q44. A particle of mass 1 kg, initially at rest, starts sliding down from the top of a frictionless inclined plane of angle $\pi/6$ (as schematically shown in the figure). The magnitude of the torque on the particle about the point O after a time 2 seconds is ___________N-m. (Rounded off to nearest integer)



$\left(\text{Take } g=10\,\text{m/s}^2\right)$

Ans: 85 to 88

Solution.:



$$x=\frac{1}{2}(g\sin 30^\circ)t^2=\frac{1}{2}\times 10\times\frac{1}{2}\times 2^2=10m$$

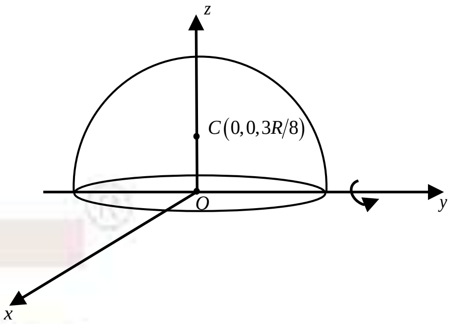
$$\vec{\tau}=\overrightarrow{OC}\times\overrightarrow{mg} \Rightarrow \tau=(x\cos 30^\circ)\times 1\times 10=10\times\frac{\sqrt{3}}{2}\times 10$$

$$\Rightarrow \tau=86.60\,\text{N-m}$$

Q45. The moment of inertia of a solid hemisphere (mass M and radius R) about the axis passing through the hemisphere and parallel to its flat surface is $\frac{2}{5}MR^2$. The distance of the axis from the center of mass of the hemisphere (in units of R) is________. (Rounded off to two decimal places)

Ans: 0.36 to 0.40

Solution:



Coordinate of COM = $(0,0,3R/8)$

$I_x = \frac{2}{5}MR^2$,

$OC = \frac{3R}{8} = 0.375R = 0.38$

Section C: Q.51-Q60 Carry TWO marks each.

Q55. A satellite of mass 10kg, in a circular orbit around a planet, is having a speed $v = 200\,\text{m/s}$. The total energy of the satellite is _______kJ. (Rounded off to nearest integer)

Ans: -200

Solution.: $K = \frac{1}{2}mv^2 = \frac{1}{2}\times 10\times(200)^2 = 2\times 10^5 J \;\Rightarrow K = 200\,kJ$

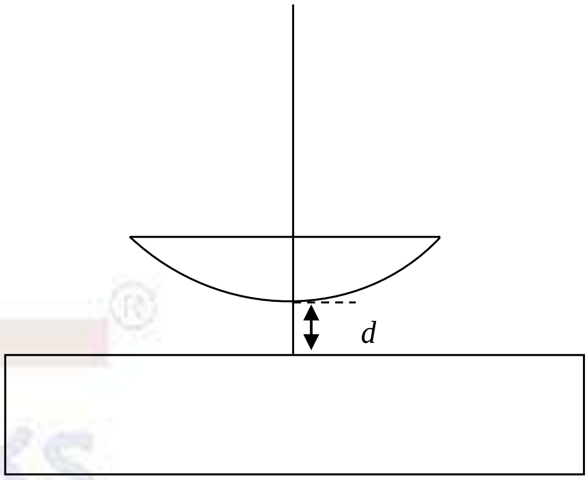
$E = -K = -200\,kJ$

Section A: Q.1-Q.10 Carry ONE mark each. (No Question)

Section A: Q.11-Q.30 Carry TWO marks each.

Q20. In a Newton's rings experiment (using light of free space wavelength 580nm), there is an air gap of height d between the glass plate and a plano-convex lens (see figure). The central fringe is observed to be bright.

The least possible value of d (in nm) is:



(A) 145

(B) 290

(C) 580

(D) 72.5

Ans: (A)

Solution: $d = \frac{\lambda}{4} = \frac{580}{4} = 145\,nm$

Q21. Linearly polarized light (free space wavelength $\lambda_0 = 600$nm) is incident normally on a retarding plate ($n_e - n_0 = 0.05$ at $\lambda_0 = 600$nm). The emergent light is observed to be linearly polarized, irrespective of the angle between the direction of polarization and the optic axis of the plate. The minimum thickness (in μm) of the plate is:

(A) 6 (B) 3 (C) 2 (D) 1

Ans: (A)

Solution: This retarding plate is half wave plate. $(n_e - n_0)t = \frac{\lambda_0}{2} \Rightarrow 0.05t = \frac{600}{2} \times 10^{-9}$

$t = 6 \times 10^{-6} m \Rightarrow t = 6\,\mu m$

Q29. If two traveling waves, given by

$$y_1 = A_0 \sin(kx - \omega t) \text{ and } y_2 = A_0 \sin(\alpha kx - \beta \omega t)$$

are superposed, which of the following statements is correct?

(A) For $\alpha = \beta = 1$, the resultant wave is a standing wave

(B) For $\alpha = \beta = -1$, the resultant wave is a standing wave

(C) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is $\frac{3}{2}\omega$

(D) For $\alpha = \beta = 2$, the carrier frequency of the resultant wave is 3ω

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Ans: (C)

Solution:

For the standing wave, both travelling waves should move in opposite direction, so option (a) and (b) are wrong.

If $\alpha = \beta = 2$, then $y_1 = A_0 \sin(kx - \omega t)$; $y_2 = A_0 \sin(2kx - 2\omega t)$

$$y = y_1 + y_2 = 2A_0 \sin\left(\frac{3}{2}kx - \frac{3}{2}\omega t\right)\cos\left(\frac{k}{2}x - \frac{\omega}{2}t\right) \Rightarrow y = \left[2A_0 \cos\left(\frac{k}{2}x - \frac{\omega}{2}t\right)\right]\sin\left(\frac{3}{2}kx - \frac{3}{2}\omega t\right)$$

The carrier frequency lies between $\omega_1 = \omega$ and $\omega_2 = 2\omega$, so option (c) is correct.

Q30. Suppose that there is a dispersive medium whose refractive index depends on the wavelength as given $n(\lambda) = n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}$. The value of λ at which the group and phase velocities would be the same, is:

(A) $\sqrt{\frac{2b}{a}}$ (B) $\sqrt{\frac{b}{2a}}$ (C) $\sqrt{\frac{3b}{a}}$ (D) $\sqrt{\frac{b}{3a}}$

Ans: (A)

Solution: If $v_p = v_g$ then $\frac{dv_p}{d\lambda} = 0$ i.e. this medium is dispersive.

$$\frac{d}{d\lambda}\left(\frac{c}{n}\right) = -\frac{c}{n^2}\frac{dn}{d\lambda} = 0 \Rightarrow \frac{dn}{d\lambda} = 0 \Rightarrow \frac{d}{d\lambda}\left(n_0 + \frac{a}{\lambda^2} - \frac{b}{\lambda^4}\right) = 0 \Rightarrow -\frac{2a}{\lambda^3} + \frac{4b}{\lambda^5} = 0 \Rightarrow \lambda = \sqrt{\frac{2b}{a}}$$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q33. A spring-mass system (spring constant 80N/m and damping coefficient 40N-s/m), initially at rest, is lying along the y-axis in the horizontal plane. One end of the spring is fixed and the mass (5kg) is attached at its other end. The mass is pulled along the y-axis by 0.5m from its equilibrium position and then released. Choose the correct statement(s). (Ignore mass of the spring)

(A) Motion will be under damped

(B) Trajectory of the mass will be $y(t) = \frac{1}{2}(1+t)e^{-4t}$

(C) Motion will be critically damped

(D) Trajectory of the mass will be $y(t) = \frac{1}{2}(1+4t)e^{-4t}$

Ans: (C), (D)

Solution: $k = 80\,\text{N/m}$, $b = 40\,\text{N-s/m}$, $m = 5kg$, $y_0 = 0.5m$

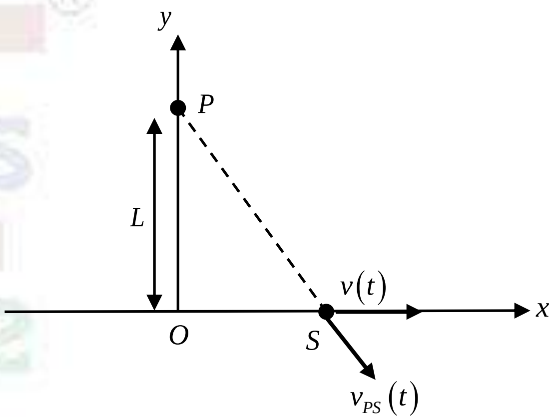
$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{80}{5}} = 4,\ r = \frac{b}{2m} = \frac{40}{2\times 5} = 4$$

This case is critically damped case as $r = \omega_0$.

Trajectory of the mass $y = y_0(1+rt)e^{-rt} \Rightarrow y = 0.5(1+4t)e^{-4t}$

Q39. A whistle S of sound frequency f is oscillating with angular frequency ω along the x-axis. Its instantaneous position and the velocity are given by $x(t) = a\sin(\omega t)$ and $v(t) = v_0\cos(\omega t)$, respectively. An observer P is located on the y-axis at a distance L from the origin (see figure). Let $v_{PS}(t)$ be the component of $v(t)$ along the line joining the source and the observer. Choose the correct option(s):

(Here a and v_0 are constants)



(A) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2\sin^2\omega t + L^2}}\sin(2\omega t)$

(B) The observed frequency will be f when the source is at $x = 0$ and $x = \pm a$

(C) The observed frequency will be f when the source is at position $x = \pm\frac{a}{2}$

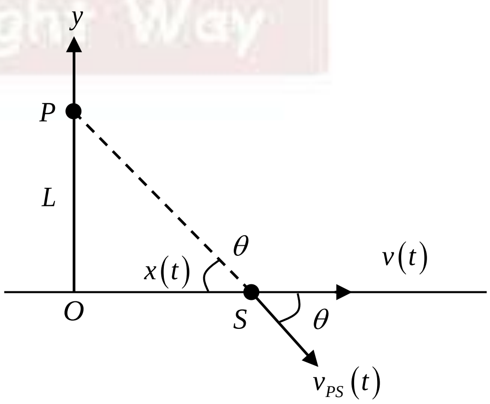
(D) $v_{PS}(t) = \frac{1}{2}\frac{av_0}{\sqrt{a^2 + L^2}}\sin(2\omega t)$

Ans: (A), (B)

Solution:

(a) $v_{PS}(t) = v(t)\cos\theta = v_0\cos\omega t\frac{x(t)}{\sqrt{L^2 + x^2(t)}}$

$$\Rightarrow v_{PS}(t) = \frac{1}{2}\frac{v_0 a}{\sqrt{L^2 + a^2\sin^2\omega t}}\sin 2\omega t$$



(b) (i) At point O, velocity $v(t)$ is perpendicular to OP, so

$f_{\text{app}} = f_{\text{actual}} = f$

(ii) At $x = \pm a$, velocity is zero, so there will not be Doppler's effect at these positions $f_{\text{app}} = f_{\text{actual}} = f$

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Section C: Q.41 - Q.50 Carry ONE mark each.

(No Question)

Section C: Q.51-Q60 Carry TWO marks each.

Q56. When a system of multiple long narrow slits (width $2\mu m$ and period $4\mu m$) is illuminated with a laser of wavelength 600nm. There are 40 minima between the two consecutive principal maxima observed in its diffraction pattern. Then maximum resolving power of the system is _______.

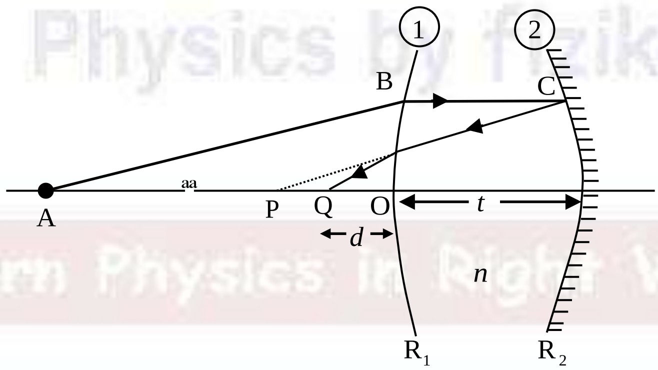
Ans: 246

Solution: $n_{\max} = \frac{e+d}{\lambda} = \frac{4\times10^{-6}}{600\times10^{-9}} = 6.66 \Rightarrow n_{\max} = 6$

$N - 1 = 40 \Rightarrow N = 41$

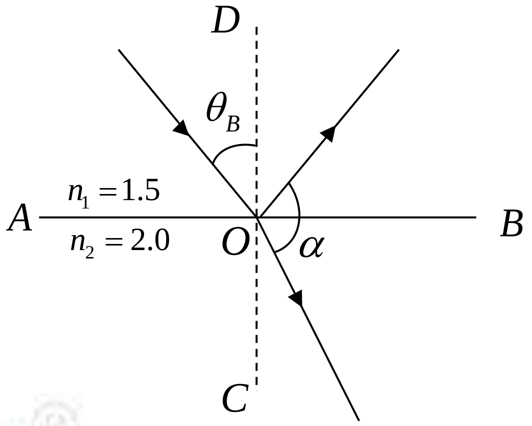
$RP_{\max} = n_{\max} N \Rightarrow RP_{\max} = 41\times6 = 246$

Q57. Consider a thick biconvex lens (thickness $t = 4$cm and refractive index $n = 1.5$) whose magnitudes of the radii of curvature R_1 and R_2, of the first and second surfaces are 30cm and 20cm, respectively. Surface 2 is silvered to act as a mirror. A point object is placed at point A on the axis (OA = 60cm) as shown in the figure. If its image is formed at point Q, the distance d between O and Q is _____________ cm. (Rounded off to two decimal places)



Ans: 3.55 to 3.59

Section A: Q.1-Q.10 Carry ONE mark each.



Q7. A plane electromagnetic wave is incident on an interface AB separating two media (refractive indices $n_1 = 1.5$ and $n_2 = 2.0$) at Brewster angle θ_B, as schematically shown in the figure. The angle α (in degrees) between the reflected wave and the refracted wave is:

(A) 120 (B) 116 (C) 90 (D) 74

Ans: (C)

Solution: At Q_B reflected and refracted wave are $\perp^r$ so $\alpha = 90°$

Q8. If the electric field of an electromagnetic wave is given by,

$\vec{E} = (4\hat{x} + 3\hat{y})e^{i(\omega t + ax - 600y)}$ then the value of a is: (all values are in the SI units)

(A) 450 (B) −450 (C) 800 (D) −800

Ans: (A)

Solution: Propagation vector $\vec{k} = a\hat{x} - 600\hat{y}$; Polarization vector $\hat{n} = 4\hat{x} + 3\hat{y}$

Since E.M. waves are transverse in nature $\vec{k}\cdot\hat{n} = 0 \Rightarrow 4a - 600\times 3 = 0 \Rightarrow a = \frac{1800}{4} = 450$

Q9. A vector field is expressed in the cylindrical coordinate system (s, ϕ, z) as, $\vec{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}$

If this field represents an electrostatic field, then the possible values of A and B, respectively, are:

(A) 1 and 0 (B) 0 and 1

(C) −1 and 1 (D) 1 and −1

Ans: (A)

Solution: For $\vec{F}$ to be electrostatic field, $\vec{\nabla}\times\vec{F} = 0$

Thus $\vec{\nabla}\times\vec{F} = \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_s & sA_\phi & A_z \end{vmatrix} = \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{A}{s} & s\times 0 & \frac{B}{s} \end{vmatrix}$ $\qquad \because \vec{F} = \frac{A}{s}\hat{s} + \frac{B}{s}\hat{z}$

$$\Rightarrow \vec{\nabla}\times\vec{F} = \frac{1}{s}\left[\hat{s}(0-0) - s\hat{\phi}\left(-\frac{B}{s^2} - 0\right) + \hat{z}(0-0)\right] \Rightarrow \vec{\nabla}\times\vec{F} = \frac{B}{s^2}\hat{\phi} = 0 \Rightarrow B = 0$$

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Section A: Q.11-Q.30 Carry TWO marks each.

Q22. A 15.7 mW laser beam has a diameter of 4 mm. If the amplitude of the associated magnetic field is expressed as $\frac{A}{\sqrt{\varepsilon_0 c^3}}$, the value of A is:

(ε_0 is the free space permittivity and c is the speed of light)

(A) 50 (B) 35.4 (C) 100 (D) 70.8

Ans: (A)

Solution: Intensity $I=\frac{P}{\pi r^2}=\frac{1}{2}c\in_0 E_0^2=\frac{cB_0^2}{2\mu_0}=\frac{cB_0^2}{2}\times c^2\in_0=c^3\in_0\frac{B_0^2}{2}$

$$\Rightarrow B_0^2=\frac{2P}{\pi r^2\times c^3\in_0}\Rightarrow B_0=\sqrt{\frac{2P}{\pi r^2 c^3\in_0}}=\sqrt{\frac{2\times 15.7\times 10^{-3}}{3.14\times\left(2\times 10^{-3}\right)^2\times c^3\in_0}}$$

$$\Rightarrow B_0=\sqrt{\frac{15.7}{3.14\times 2\times 10^{-3}\times c^3\in_0}}=\frac{\sqrt{2500}}{\sqrt{c^3\in_0}}=\frac{50}{\sqrt{c^3\in_0}}\Rightarrow A=50$$

Q23. The plane $z=0$ separates two linear dielectric media with relative permittivities $\varepsilon_{r1}=4$ and $\varepsilon_{r2}=3$, respectively. There is no free charge at the interface. If the electric field in the medium 1 is $\vec{E}_1=3\hat{x}+2\hat{y}+4\hat{z}$, then the displacement vector $\vec{D}_2$ in medium 2 is:

(ε_0 is the permittivity of free space)

(A) $(3\hat{x}+4\hat{y}+6\hat{z})\varepsilon_0$ (B) $(3\hat{x}+6\hat{y}+8\hat{z})\varepsilon_0$

(C) $(9\hat{x}+6\hat{y}+16\hat{z})\varepsilon_0$ (D) $(4\hat{x}+2\hat{y}+3\hat{z})\varepsilon_0$

Ans: (C)

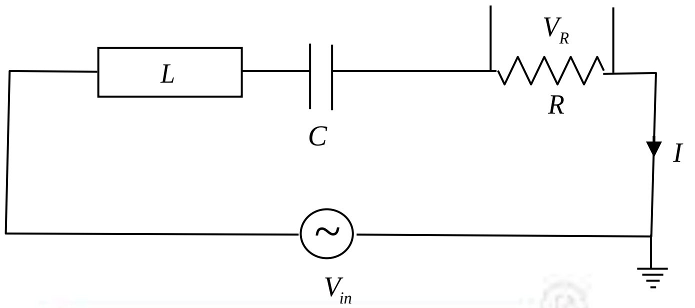
Solution: Interface is xy-plane at $z=0$ and $\sigma_f=0$. Here $\vec{E}_1^{\parallel}=3\hat{x}+2\hat{y}=\vec{E}_2^{\parallel}$, $\vec{E}_1^{\perp}=4\hat{z}$

$$\because\ \vec{D}_1^{\perp}=\vec{D}_2^{\perp}\Rightarrow\in_1\vec{E}_1^{\perp}=\in_2\vec{E}_2^{\perp}\Rightarrow\vec{E}_2^{\perp}=\frac{\in_1}{\in_2}\vec{E}_1^{\perp}=\frac{4\in_0}{3\in_0}\times 4\hat{z}=\frac{16}{3}\hat{z}$$

$$\Rightarrow\vec{E}_2'=\vec{E}_2^{\parallel}+\vec{E}_2^{\perp}=3\hat{x}+2\hat{y}+\frac{16}{3}\hat{z}\Rightarrow\vec{D}_2=\in_2\vec{E}_2=3\in_0\vec{E}_2=\left(9\hat{x}+6\hat{y}+16\hat{z}\right)\in_0$$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q37. For the LCR AC-circuit (resonance frequency ω_0) shown in the figure below, choose the correct statement(s).



(A) ω_0 depends on the values of L, C, and R

(B) At $\omega = \omega_0$, voltage V_R and current I are in-phase

(C) The amplitude of V_R at $\omega = \omega_0/2$ is independent of R

(D) The amplitude of V_R at $\omega = \omega_0$ is independent of L and C

Ans: (B), (D)

Solution:

In series LCR circuit, resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, so option (A) is incorrect.

At $\omega = \omega_0$: V_R and I are in phase, so option (B) is correct.

At $\omega \neq \omega_0$: V_R depends on R, so option (C) is incorrect

At $\omega = \omega_0$, V_R depends on R and independent of L, C, so option (D) is correct.

Section C: Q.41 - Q.50 Carry ONE mark each.

Q46. A collimated light beam of intensity I_0 is incident normally on an air-dielectric (refractive index 2.0) interface. The intensity of the reflected light is ________ I_0. (Rounded off to two decimal places)

Ans: 0.10 to 0.12

Solution:

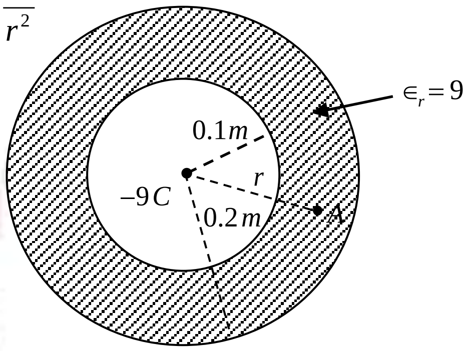
$$R = \frac{I_R}{I_I} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \Rightarrow \frac{I_R}{I_0} = \left(\frac{1-2}{1+2}\right)^2 \Rightarrow I_R = \frac{1}{9} I_0 = 0.11 I_0$$

Since for Air $n_1 = 1$, Dielectric $n_2 = 2$

Q47. A charge of -9C is placed at the center of a concentric spherical shell made of a linear dielectric material (relative permittivity 9) and having inner and outer radii of 0.1m and 0.2m, respectively. The total charge induced on its inner surface is ______C. (Rounded off to two decimal place)

Ans: 7.90 to 8.10

Solution: $\vec{E}=\frac{1}{4\pi(\epsilon_0\epsilon_r)}\frac{q}{r^2}\hat{r}=\frac{1}{4\pi(9\epsilon_0)}\frac{(-9)}{r^2}\hat{r}\Rightarrow\vec{E}=-\frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$



Polarization $\vec{P}=\epsilon_0\chi_e\vec{E}=\epsilon_0(\epsilon_r-1)\vec{E}$

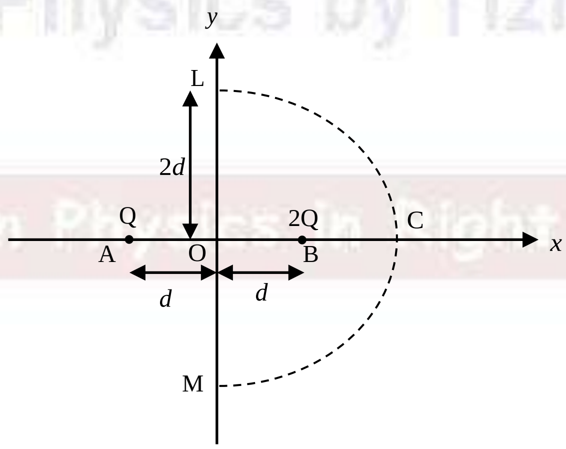
$$\Rightarrow\vec{P}=\epsilon_0(9-1)\times\left[-\frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\right]=-\frac{2}{\pi}\frac{\hat{r}}{r^2}$$

Surface bound charge at $r=0.1$m is

$$\sigma_b=\vec{P}\cdot\hat{n}\left[-\frac{2}{\pi}\frac{\hat{r}}{(0.1)^2}\right]\cdot(-\hat{r})=\frac{2}{\pi\times0.01}=\frac{200}{\pi}\quad\Rightarrow q_{\sigma_b}=\sigma_b\times4\pi r^2=\frac{200}{\pi}\times4\pi(0.01)=8$$

Section C: Q.51-Q60 Carry TWO marks each.

Q59. Two positive charges Q and $2Q$ are kept at points A and B, separated by a distance $2d$, as shown in the figure. MCL is a semicircle of radius $2d$ centered at the origin O. If $Q=2C$ and $d=10$ cm, the value of the line integral $\int_M^L\vec{E}\cdot\vec{dl}$ (where $\vec{E}$ represents electric field) along the path MCL will be ________V.



Ans: 0

Solution: $\int_M^L\vec{E}\cdot\vec{dl}=-[V(L)-V(M)]=V(M)-V(L)=0$

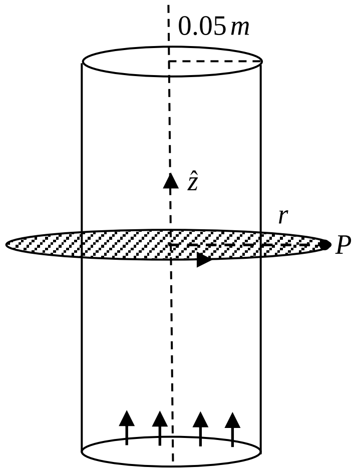
$V(L)=\frac{Q}{AL}+\frac{2Q}{BL}=V(M)\qquad\because\ AL=BL=AM=BM=\frac{1}{\sqrt{d^2+2d^2}}=\frac{1}{\sqrt{5}d}$

Q60. A time dependent magnetic field inside a long solenoid of radius 0.05m is given by $\vec{B}(t) = B_0 \sin \omega t \hat{z}$. If $\omega = 100\,\text{rad/s}$ and $B_0 = 0.98\,\text{Weber/ m}^2$, then the amplitude of the induced electric field at a distance of 0.07m from the axis of the solenoid is ______V/m. (Rounded off to two decimal places)

Ans: 1.71 to 1.75

Solution:

At $r = 0.07\,\text{m}$



$$\oint_{\text{line}} \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}, \; d\vec{a} = r dr d\phi \hat{z}, \; \vec{B}(t) = \begin{cases} B_0 \sin \omega t \hat{z} & inside \\ 0 & outside \end{cases}$$

$$|\vec{E}| \times 2\pi r = -B_0 \omega \cos \omega t \times \pi (0.05)^2$$

$$|\vec{E}| = -\frac{B_0 \omega}{2r} \times 25 \times 10^{-4} \cos \omega t$$

$$\text{Amplitude} = \frac{B_0 \omega \times 25 \times 10^{-4}}{2r} = \frac{0.98 \times 100 \times 25 \times 10^{-4}}{2 \times 0.07} = 7 \times 25 \times 10^{-2} = 1.75$$

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Section A: Q.1-Q.10 Carry ONE mark each. (No Question)

Section A: Q.11-Q.30 Carry TWO marks each.

Q27. A cylinder-piston system contains N atoms of an ideal gas. If t_{avg} is the average time between successive collisions of a given atom with other atoms. If the temperature T of the gas is increased isobarically, then t_{avg} is proportional to:

(A) $\sqrt{T}$ (B) $\frac{1}{\sqrt{T}}$ (C) T (D) $\frac{1}{T}$

Ans: (A)

Solution: $t_{avg}=\frac{1}{n\sigma v}$, $n=\frac{p}{kT}$, $v=\sqrt{\frac{8kT}{\pi m}}$. Thus $t_{avg}=\frac{kT}{p\sigma}\sqrt{\frac{\pi m}{8kT}}\propto\sqrt{T}$ at constant P.

Q28. A gas consists of particles, each having three translational and three rotational degrees of freedom. The ratio of specific heats, C_p/C_v, is:

(C_p and C_v are the specific heats at constant pressure and constant volume, respectively)

(A) 5/3 (B) 7/5 (C) 4/3 (D) 3/2

Ans: (C)

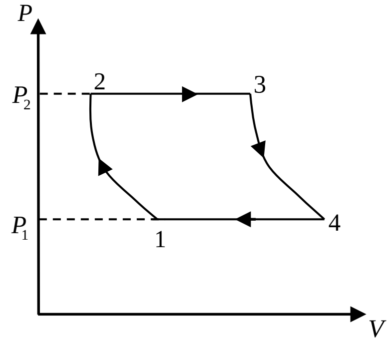
Solution: Total dof $=3+3=6$; $\langle E\rangle=6\times\frac{1}{2}kT=3kT$

$$C_V=\frac{d\langle E\rangle}{dT}=3k=3R \text{ per mole; } C_P=C_V+R=3R+R=4R \quad\Rightarrow\frac{C_P}{C_V}=\frac{4}{3}$$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q38. The $P-V$ diagram of an engine is shown in the figure below. The temperatures at points 1, 2, 3 and 4 are T_1, T_2, T_3 and T_4, respectively. $1\rightarrow 2$ and $3\rightarrow 4$ are adiabatic processes, and $2\rightarrow 3$ and $4\rightarrow 1$ are isobaric processes. Identify the correct statement(s).

[γ is the ratio of specific heats C_p (at constant P) and C_v (at constant V)]



(A) $T_1T_3=T_2T_4$

(B) The efficiency of the engine is $1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

(C) The change in entropy for the entire cycle is zero

(D) $T_1T_2=T_3T_4$

Ans: (a), (b) and (c)

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Solution:

Process $1\rightarrow 2$ and $3\rightarrow 4$ are adiabatic

$$\therefore T_1^{\gamma}P_1^{1-\gamma}=T_2^{\gamma}P_2^{1-\gamma}\Rightarrow \frac{T_1}{T_2}=\left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} \quad ...(i) \quad \text{and} \quad T_4^{\gamma}P_1^{1-\gamma}=T_3^{\gamma}P_2^{1-\gamma}\Rightarrow \frac{T_4}{T_3}=\left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}} \quad ...(ii)$$

(i) & (ii) implies $\frac{T_1}{T_2}=\frac{T_4}{T_3}$, i.e. $T_1T_3=T_2T_4$. $\quad \therefore$ (a) is correct & (d) is incorrect

Further, in a cyclic process, system returns to its initial state (thermodynamic coordinates are same), $\Delta S_{\text{cycle}}=0$. $\quad \therefore$(c) is correct

Calculating the efficiency:

In processes, $1\rightarrow 2$ and $3\rightarrow 4$, $dQ=0$ ($\because$ adiabatic)

Process $2\rightarrow 3$, Isobaric and volume increases, $V\propto T$. $\therefore$ Heat enters (Q_1) during this step.

Process $4\rightarrow 1$, Isobaric and volume reduces, $V\propto T$. $\therefore$ Heat exists (Q_2) during this step.

$$Q_1=nC_P\Delta T=C_P\left(T_3-T_2\right),\ n=1; \qquad Q_2=nC_P\Delta T=C_P\left(T_4-T_1\right).$$

$$\eta=1-\frac{Q_2}{Q_1}=1-\frac{T_4-T_1}{T_3-T_2}$$

Now, from (ii), $T_3=\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}T_4$ and from (i) $T_2=\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}T_1$

$$\therefore \quad \eta=1-\frac{T_4-T_1}{\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}\left(T_4-T_1\right)}=1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$ $\therefore$ (b) is also correct

Q40. One mole of an ideal monoatomic gas, initially at temperature T_0 is expanded from an initial volume V_0 to $2.5V_0$. Which of the following statements is (are) correct?

(R is the ideal gas constant)

(A) When the process is isothermal, the work done is $RT_0\ln 2$

(B) When the process is isothermal, the change in internal energy is zero

(C) When the process is isobaric, the work done is $\frac{3}{2}RT_0$

(D) When the process is isobaric, the change in internal energy is $\frac{9}{2}RT_0$

Ans: (B), (C)

Solution:

Let us check one by one

(a) For an isothermal process

$$W = nRT\ln\frac{V_2}{V_1} = 1\times R\times T_0\ln\left(\frac{2.5V_0}{V_0}\right) = RT_0\ln(2.5). \qquad \therefore \text{(a) is incorrect}$$

(b) For an ideal gas, when temperature does not change $(dT=0)$, implies that $dU=0$. There is no change in internal energy. $\therefore$ (b) is correct.

(c) For an isobaric process, the work done is

$$W = P\int dV = P[2.5V_0 - V_0] = 1.5PV_0 = 1.5RT_0 \qquad \therefore \text{(c) is correct.}$$

(d) When the process is Isobaric, the internal energy change is

$$dU = nC_V dT \;\; = 1\times\frac{3}{2}R[2.5T_0 - T_0] = \frac{3}{2}R\times\frac{3}{2}T_0 = \frac{9}{4}RT_0 \qquad \because V\propto T \qquad \therefore \text{(d) is incorrect.}$$

Section C: Q.41 - Q.50 Carry ONE mark each.

Q50. One mole of an ideal monoatomic gas is heated in a closed container, first from 273K to 303K, and then from 303K to 373K. The net change in the entropy is ______R. (Rounded off to two decimal places) (R is the ideal gas constant)

Ans: 0.44 to 0.48

Solution:

Closed container implies $dV=0$. $\therefore dQ = dU = nC_V dT$, $n=1$

$$dS = \frac{dQ}{T} = \frac{C_V dT}{T} \Rightarrow \Delta S = \int C_V\frac{dT}{T} = C_V\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \Delta S_1 + \Delta S_2 = C_V\ln\frac{303}{273} + C_V\ln\left(\frac{373}{303}\right) = C_V\ln\left(\frac{303}{273}\times\frac{373}{303}\right) = C_V\ln\left(\frac{373}{273}\right)$$

$$\Rightarrow \Delta S = \frac{3}{2}R\ln\left(\frac{373}{273}\right) = \frac{3}{2}R\times 0.3121 = 0.468R \quad \Rightarrow \Delta S \approx 0.47R$$

Note: As for the gas temperature is changing from 273K to 373K. Net entropy change is

$$\Delta S = C_V\ln\frac{T_f}{T_i} = C_V\ln\left(\frac{373}{273}\right)$$

Section C: Q.51-Q60 Carry TWO marks each.

(No Question)

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Section A: Q.1-Q.10 Carry ONE mark each.

Q5. The magnitudes of spin magnetic moments of electron, proton and neutron are μ_e, μ_p and μ_n, respectively. Then,

(A) $\mu_e > \mu_p > \mu_n$ (B) $\mu_e = \mu_p > \mu_n$

(C) $\mu_e < \mu_p < \mu_n$ (D) $\mu_e < \mu_p = \mu_n$

Ans: (A)

Solution:

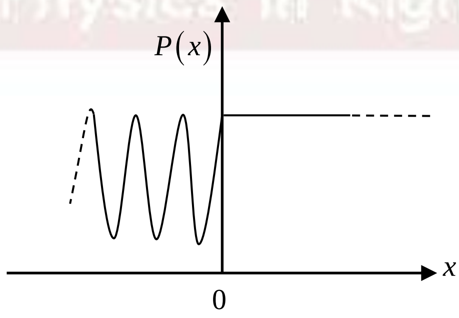
The spin magnetic moment (μ) of a particle is given by the expression:

$$\vec{\mu} = g\frac{q}{2m}\vec{S}$$

where, g is Gyromagnetic ratio, q is Charge of the particle, m is Mass of the particle, and S is Spin angular momentum.

The spin magnetic moments of particles are inversely proportional to their masses. Thus, the electron has the smallest mass, resulting in the largest magnetic moment (μ_e). The proton has a mass much larger than the electron but smaller than the neutron, giving it a magnetic moment (μ_p) smaller than the electron. The neutron has the largest mass among the three particles, resulting in the smallest magnetic moment (μ_n). Based on their masses and the given expression, the order of magnetic moments is: $\mu_e > \mu_p > \mu_n$. Thus, the correct option is (A)

Q6. A particle moving along the x-axis approaches $x = 0$ from $x = -\infty$ with a total energy E. It is subjected to a potential $V(x)$. For time $t \to \infty$, the probability density $P(x)$ of the particle is schematically shown in the figure.



The correct option for the potential $V(x)$ is:

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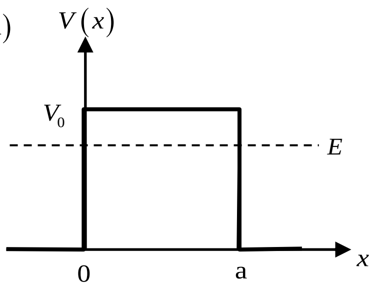
(A)


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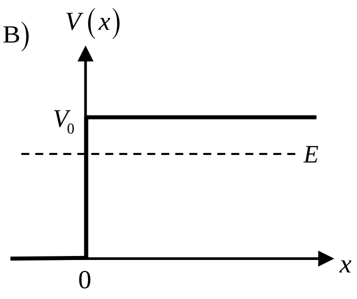

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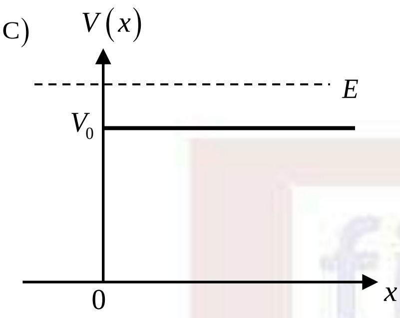

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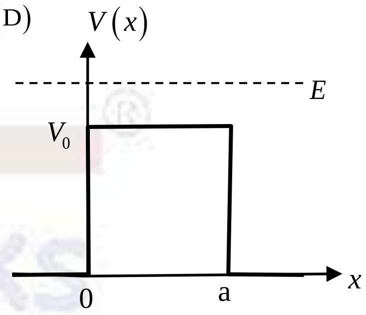

Ans: (C)

Solution:

The probability density $P(x)$ is oscillatory for $x<0$, indicating a classically allowed region where the particle's total energy $E>V(x)$.

The probability density $P(x)$ is constant for $x>0$, indicating the particle exists with uniform probability in that region. This suggests the potential $V(x)$ is constant.

Thus, correct option is (C)

Section A: Q.11-Q.30 Carry TWO marks each.

Q17. The ultraviolet catastrophe in the classical (Rayleigh-Jeans) theory of cavity radiation is attributed to the assumption that

(A) the standing waves of all allowed frequencies in the cavity have the same average energy

(B) the density of the standing waves in the cavity is independent of the shape and size of the cavity

(C) the allowed frequencies of the standing waves inside the cavity have no upper limit

(D) the number of allowed frequencies for the standing waves in a frequency range v to $(v+dv)$ is proportional to v^2

Ans: (A)

Solution:

The Rayleigh-Jeans law describes the spectral energy density of blackbody radiation at a given temperature. The expression for the density of radiation is:

$$u(\nu,T)=\rho(\nu)\times\langle E\rangle=\frac{8\pi\nu^2}{c^3}\times k_BT$$

where, $u(\nu,T)$ is the spectral energy density (energy per unit frequency per unit volume), $\rho(\nu)=\frac{8\pi\nu^2}{c^3}$ is the density of modes (number of standing wave modes per unit frequency per unit volume) and $\langle E\rangle=k_BT$ is the average energy.

This law predicts that as the frequency ν increases, the energy density increases without bound, which leads to the ultraviolet catastrophe. The theory assumes the energy is equally distributed among all modes, leading to an overestimation of energy at high frequencies.

Thus, the correct answer is option (A).

Q18. Given that the rest mass of electron is $0.511\text{MeV}/\text{c}^2$, the speed (in units of c) of an electron with kinetic energy 5.11MeV is closest to:

(A) 0.996 (B) 0.993 (C) 0.990 (D) 0.998

Ans: (A)

Solution: $E=K+m_0c^2=(10\times0.511)+0.511$

$$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}=11\times0.511\Rightarrow1-\frac{v^2}{c^2}=\frac{1}{(11)^2}=\frac{1}{121}\Rightarrow\frac{v^2}{c^2}=\frac{120}{121}$$

$$\Rightarrow v=\sqrt{\frac{120}{121}}c=0.9958c\cong0.996c$$

Q19. A one-dimensional infinite square-well potential is given by:

$$V(x)=0\text{ for }-\frac{a}{2}<x<+\frac{a}{2}$$
$$=\infty\text{ elsewhere}$$

Let $E_e(x)$ and $\psi_e(x)$ be the ground state energy and the corresponding wave function, respectively, if an electron (e) is trapped in that well. Similarly, let $E_\mu(x)$ and $\psi_\mu(x)$ be the corresponding quantities, if a muon (μ) is trapped in the well. Choose the correct option:

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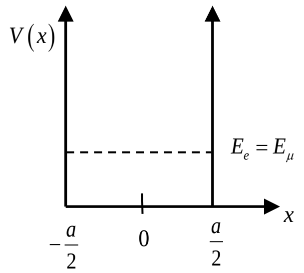
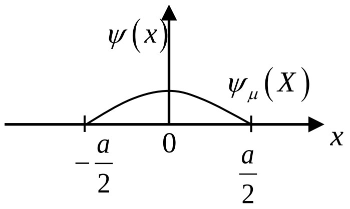
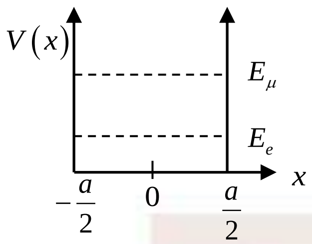
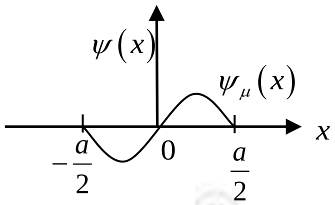
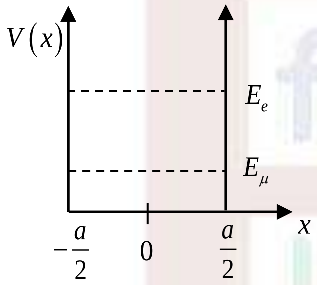
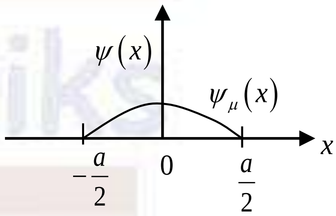
(A) 

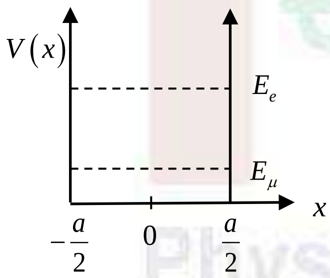
and 

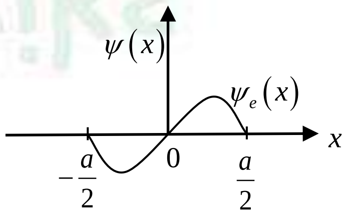
(B) 

and 

(C) 

and 

(D) 

and 

Ans: (C)

Solution:

The normalized ground state wave function of particle in a 1D infinite symmetric square well potential of width a is:

$$\psi(x)=\sqrt{\frac{2}{a}}\cos\left(\frac{\pi x}{a}\right), \quad for\ n=1$$

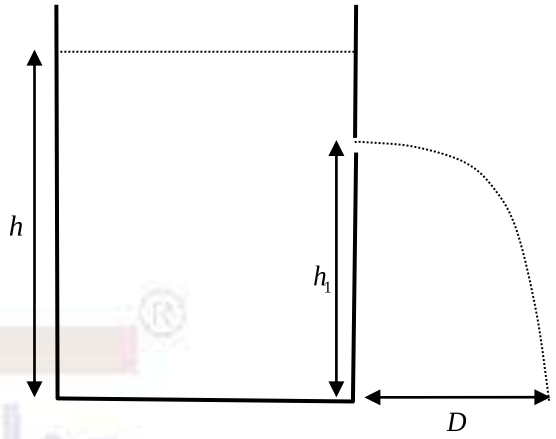
The ground state wavefunction is symmetric about center of the potential well.

The ground state energy is: $E_1=\dfrac{\pi^2\hbar^2}{2ma^2}$

where, m is mass of the particle. Since the **muon** is heavier than the **electron** $(m_\mu > m_e)$, and since energy is inversely proportional to the particle's mass, therefore $E_\mu < E_e$.

Thus, correct option is (C).

Q24. A tank, placed on the ground, is filled with water up to a height h. A small hole is made at a height h_1 such that $h_1 < h$. The water jet emerging from the hole strikes the ground at a horizontal distance D, as shown schematically in the figure. Which of the following statements is correct? (g is the acceleration due to gravity)



(A) Velocity at h_1 is $\sqrt{2gh_1}$

(B) $D = 2(h-h_1)$

(C) D will be maximum when $h_1 = \frac{2}{3}h$

(D) The maximum value of D is h

Ans: (D)

Solution: (A) The velocity of water exiting the hole is: $v = \sqrt{2g(h-h_1)}$.

Thus option (A) is incorrect

(B) The time taken for the water to reach the ground is determined by the vertical motion:

$$t = \sqrt{\frac{2h_1}{g}}$$

The horizontal distance traveled by the water is: $D = v.t = \sqrt{2g(h-h_1)}.\sqrt{\frac{2h_1}{g}} = 2\sqrt{h_1(h-h_1)}$

Thus, the option (B) is incorrect.

(C) To maximize D, take the derivative of D and set it equal to zero

$$\frac{dD}{dh_1} = \frac{2}{2\sqrt{h_1(h-h_1)}}\left[(h-h_1)-h_1\right] = 0 \Rightarrow h - 2h_1 = 0 \Rightarrow \quad h_1 = \frac{h}{2}$$. Thus option (C) incorrect.

(D) Maximum Value of D: At $h_1 = \frac{h}{2}$, substitute into $D = 2\sqrt{h_1(h-h_1)} = 2\sqrt{\frac{h}{2}(h-\frac{h}{2})} = h$

Thus, the option D is correct

Q25. An incompressible fluid is flowing through a vertical pipe (height h and cross-sectional area A_0). A thin mesh, having n circular holes of area A_h, is fixed at the bottom end of the pipe. The speed of the fluid entering the top-end of the pipe is v_0. The volume flow rate from an individual hole of the mesh is given by:(g is the acceleration due to gravity)

(A) $\frac{A_0}{n}\sqrt{v_0^2 + 2gh}$ (B) $\frac{A_0}{n}\sqrt{v_0^2 + gh}$

(C) $n(A_0 - A_h)\sqrt{v_0^2 + 2gh}$ (D) $n(A_0 - A_h)\sqrt{v_0^2 + gh_0}$

Ans: (A)

Solution:

Applying Bernoulli's equation between the top of the pipe and the bottom:

$$P_0+\frac{1}{2}\rho v_0^2+\rho gh=P_1+\frac{1}{2}\rho v_1^2$$

where, P_0 is pressure at the top and P_1 is the pressure at the bottom and both are equal to the atmospheric pressure i.e. $P_0=P_1$, Thus, we get $\frac{1}{2}\rho v_0^2+\rho gh=\frac{1}{2}\rho v_1^2 \Rightarrow v_1^2=v_0^2+2gh$

The volume flow rate through an individual hole is: $Q=A_h v_1=\frac{A_0}{n}\sqrt{v_0^2+2gh}$

where, $A_0=nA_h$ total area of the n meshes.Thus, the correct answer is option (A)

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q34. Consider two different Compton scattering experiments, in which X-rays and γ-rays of wavelength (λ) 1.024Å and 0.049Å, respectively, are scattered from stationary free electrons. The scattered wavelength (λ') is measured as a function of the scattering angle (θ). If Compton shift is $\Delta\lambda=\lambda'-\lambda$, then which of the following statement(s) is/are true: $\left(h=6.63\times10^{-34}\,\text{J.s}, m_e=9.11\times10^{-31}\,\text{kg}, c=3\times10^8\,\text{m/s}\right)$

(A) For γ-rays, $\lambda'_{\max}\approx 0.098\text{Å}$

(B) For X-rays, $(\Delta\lambda)_{\max}$ is observed at $\theta=180°$

(C) For X-rays, $(\Delta\lambda)_{\max}\approx 1.049\text{Å}$

(D) For γ-rays, at $\theta=90°, \lambda'\approx 0.049\text{Å}$

Ans: (A), (B)

Solution:

The Compton shift is $\Delta\lambda=\lambda'-\lambda=\frac{h}{m_e c}(1-\cos\theta)=0.02426\text{Å}(1-\cos\theta)$

(A) For γ-rays, the Maximum scattered wavelength is

$$\lambda'_{\max}=\lambda+0.02426\text{Å}(1-\cos\pi)=0.049+2\times0.02426\approx0.098\text{Å}$$

Thus, the option (A) is correct

(B) The maximum shift occurs at $\theta=180^0$.Thus, the option (B) is correct

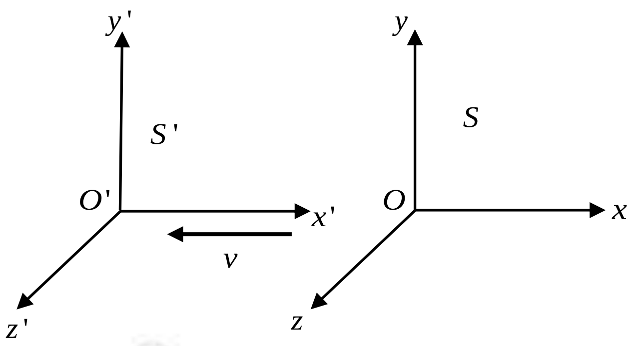
(C) The maximum Compton shift for X-rays is $\Delta\lambda=0.02426\text{Å}(1-\cos\pi)=0.04852\text{Å}$

Thus, the option (C) is not correct

(D) For γ-rays, at $\theta=90°$; $\lambda'=\lambda+0.02426\text{Å}\left(1-\cos90^0\right)=0.049\text{Å}+0.02426\text{Å}=0.07326\text{Å}$

Thus, the option (D) is not correct

Q36. The inertial frame S' is moving away from the inertial frame S with a speed $v = 0.6c$ along the negative x-direction (see figure). The origins O' and O of the frames coincide at $t = t' = 0$. As observed in the frame S', two events occur simultaneously at two points on the x'-axis with a separation of $\Delta x' = 5\text{m}$. If, Δt and Δx are the magnitudes of the time interval and the space interval, respectively, between the events in S, then which of the following statements is(are) correct? $\left(c = 3\times10^8\ \text{m/s}\right)$



(A) $\Delta t = 12.5\,\text{ns}$ (B) $\Delta t = 4.2\,\text{ns}$ (C) $\Delta x = 10.6\,\text{m}$ (D) $\Delta x = 6.25\,\text{m}$

Ans: (A), (D)

Solution:

(i) $x = \dfrac{x' + (-v)t'}{\sqrt{1 - v^2/c^2}} \Rightarrow \Delta x = \dfrac{\Delta x' - v\Delta t'}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{5m - 0}{\sqrt{1 - (0.6)^2}} = 6.25m$

(ii) $t = \dfrac{t' + \dfrac{(-v)}{c^2}x'}{\sqrt{1 - \dfrac{v^2}{c^2}}} \Rightarrow \Delta t = \dfrac{\Delta t' - \dfrac{u}{c^2}\Delta x'}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{0 - \dfrac{0.6c}{c^2}\times 5}{\sqrt{1 - (0.6)^2}} = -\dfrac{0.6\times 5}{3\times10^8\times 0.8} = \dfrac{50}{4}\times 10^{-9}s$

$\Rightarrow \Delta t = 12.5ns$

Section C: Q.41 - Q.50 Carry ONE mark each.

(No Question)

Section C: Q.51-Q60 Carry TWO marks each.

Q54. A stationary nitrogen $\left({}^{14}_{7}N\right)$ nucleus is bombarded with α-particle $\left({}^{4}_{2}He\right)$ and the following nuclear reaction takes place:

$${}^{4}_{2}He + {}^{14}_{7}N \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

Mass: $4.003u$ $14.003u$ $16.999u$ $1.008u$

If the kinetic energies of ${}^{4}_{2}He$ and ${}^{1}_{1}H$ are 5.314 MeV and 4.012 MeV, respectively, then the kinetic energy of ${}^{17}_{8}O$ is ____________ MeV. (Rounded off to one decimal place)

(Masses are given in units of $u = 931.5\,\text{MeV}/\text{c}^2$)

Ans: 0.4

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Solution: ${}_2^4He + {}_7^{14}N \rightarrow {}_8^{17}O + {}_1^1H$

$$Q = \left[M_{He} + M_N - M_O - M_H\right] \times 931.5MeV = \left[4.003 + 14.003 - 16.999 - 1.008\right] \times 931.5MeV$$

$$\Rightarrow Q = -0.9315MeV$$

$$Q = K_O + K_H - K_{He} \Rightarrow K_O = Q - K_H + K_{He} = -0.9315 - 4.012 + 5.314 = 0.3705MeV$$

$$\Rightarrow K_0 = 0.4MeV$$

Q58. An unstable particle created at a point P moves with a constant speed of $0.998c$ until it decays at a point Q. If the lifetime of the particle in its rest frame is 632ns, the distance between points P and Q is ______m. (Rounded off to the nearest integer)

$\left(c = 3 \times 10^8 \text{ m/s}\right)$

Ans: 2992 to 2994

Solution:

$$\tau = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{632 \times 10^{-9}}{\sqrt{1 - (0.998)^2}} = \frac{632 \times 10^{-9}}{0.0632} \Rightarrow \tau = 10^{-5} s$$

$$PQ = v\tau = 0.998 \times 3 \times 10^8 \times 10^{-5} = 2994m$$

Section A: Q.1-Q.10 Carry ONE mark each.

Q1. The total number of Na and Cl ions per unit cell of the NaCl crystal is:

(A) 2 (B) 4

(C) 8 (D) 16

Ans: (C)

Solution:

In a NaCl crystal:

- The structure is face-centered cubic (FCC) with Na^+ and Cl^- ions arranged alternately.

In one-unit cell:

- The Na^+ ions contribute 4 ions (from 8 corner atoms contributing 1/8 each and 6 face atoms contributing 1/2 each).

- The Cl^- ions also contribute 4 ions in the same arrangement.

Thus, the total number of ions ($Na^+ + Cl^-$) per unit cell is: 4 + 4 = 8

Thus, correct answer is option (c)

Q2. The sum of three binary numbers, 10110.10, 11010.01, and 10101.11, in decimal system is:

(A) 70.75 (B) 70.25

(C) 70.50 (D) 69.50

Ans: (C)

Solution.:

$$(10110.10)_2 = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 \cdot \left(1\times 2^{-2} + 0\times 2^{-2}\right)$$

$$= (16+0+4+2+0)\cdot\left(\frac{1}{2}+0\right) = 22.5$$

$$(11010.01)_2 = \left(1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0\right)\cdot\left(0\times 2^{-1} + 1\times 2^{-2}\right)$$

$$= (16+8+0+2+0)\cdot\left(0+\frac{1}{4}\right) = 26.25$$

$$(10101.11)_2 = 1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 \cdot \left(1\times 2^{-1} + 1\times 2^{-2}\right)$$

$$= (16+0+4+0+1)\cdot\left(\frac{1}{2}+\frac{1}{4}\right) = 21.75$$

Thus $22.5 + 26.25 + 21.75 = 70.50$

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Section A: Q.11-Q.30 Carry TWO marks each.

Q11. A crystal plane of a lattice intercepts the principal axes $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ at $3a_1, 4a_2$, and $2a_3$, respectively. The Miller indices of the plane are:

(A) (436) (B) (342)

(C) (634) (D) (243)

Ans: (A)

Solution: 1. Intercepts along the crystallographic axes $x = 3a_1, y = 4a_2, z = 2a_3$

2. Divide by lattice parameters: $x = \frac{3a_1}{a_1} = 3, y = \frac{4a_2}{a_2} = 4, z = \frac{2a_3}{a_3} = 2$

3. Take reciprocal: 1/3, 1/4, 1/2

4. Make all integers: 4, 3, 6

Thus, the Miller indices are (4 3 6).

Q12. The number of atoms in the *basis* of a primitive cell of hexagonal closed packed structure is:

(A) 1 (B) 2 (C) 3 (D) 4

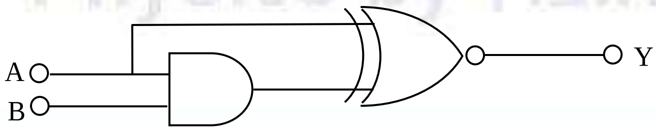
Ans: (B)

Solution:

The HCP lattice is described by a primitive cell with a basis of **2** atoms.

Thus, the correct option is (B).

Q13. Consider the following logic circuit.



The output Y is LOW when:

(A) A is HIGH and B is LOW (B) A is LOW and B is HIGH

(C) Both A and B are LOW (D) Both A and B are HIGH

Ans: (A)

Solution.: $y = \overline{A \oplus AB} = 0$ [Low]

(A) $A = 1$ and $B = 0 \Rightarrow AB = 0 \Rightarrow y = \overline{1 \oplus 0} = 0$

(B) $A = 0$ and $B = 1 \Rightarrow AB = 0 \Rightarrow y = \overline{0 \oplus 0} = 1$

(C) $A = 0$ and $B = 0 \Rightarrow AB = 0 \Rightarrow y = \overline{0 \oplus 0} = 1$

(D) $A = 1$ and $B = 1 \Rightarrow AB = 1 \Rightarrow y = \overline{1 \oplus 1} = 1$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

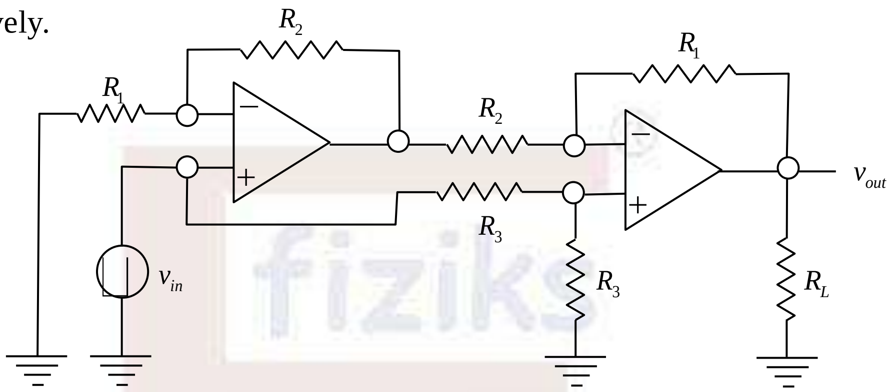
Q31. A pure Si crystal can be converted to an n-type crystal by doping with

(A) P (B) As (C) Sb (D) In

Ans: (A), (B), (C)

Solution.: For n-type crystal Si is doped with pentavalent impurity like P, As, Sb

Q32. In the following OP-AMP circuit, v_{in} and v_{out} represent the input and output signals, respectively.



Choose the correct statement(s):

(A) v_{out} is out-of-phase with v_{in} (B) Gain is unity when $R_1 = R_2$

(C) v_{out} is in-phase with v_{in} (D) v_{out} is zero

Ans: (A), (B)

Solution.: $v_0' = \left(1+\frac{R_2}{R_1}\right)v_{in}$, $v_{out} = -\frac{R_1}{R_2}v_0' + \left(1+\frac{R_1}{R_2}\right)\cdot\frac{R_3}{R_3+R_3}v_{in}$

$$\Rightarrow v_{out} = -\frac{R_1}{R_2}\left(1+\frac{R_2}{R_1}\right)v_{in} + \frac{1}{2}\left(1+\frac{R_1}{R_2}\right)v_{in} \Rightarrow v_{out} = \left[-\frac{R_1}{R_2} - 1 + \frac{1}{2} + \frac{R_1}{2R_2}\right]v_{in} = -\frac{1}{2}\left(1+\frac{R_1}{R_2}\right)v_{in}.$$

So output is out of phase with input. When $R_1 = R_2 \Rightarrow \frac{v_{out}}{v_{in}} = -\frac{1}{2}(1+1) = -1$

Section C: Q.41 - Q.50 Carry ONE mark each.

Q41. Consider a pn junction diode which has 10^{23} acceptor-atoms/ m^3 in the p-side and 10^{22} donor-atoms/ m^3 in the n-side. If the depletion width in the p-side is 0.16μm, then the value of depletion width in the n-side will be __ μm. (Round off to one decimal place)

Ans: 1.6

Solution.: $N_A = 10^{23}\ \text{atom/cm}^3$, $N_D = 10^{22}\ \text{atom/cm}^3$, $x_p = 0.16\,\mu m$, $x_n = ?$

$$\because\ x_p N_A = x_n N_D \Rightarrow x_n = x_p\frac{N_A}{N_D} = (0.16\mu m)\times\frac{10^{23}}{10^{22}} \Rightarrow x_n = 1.6\,\mu m$$

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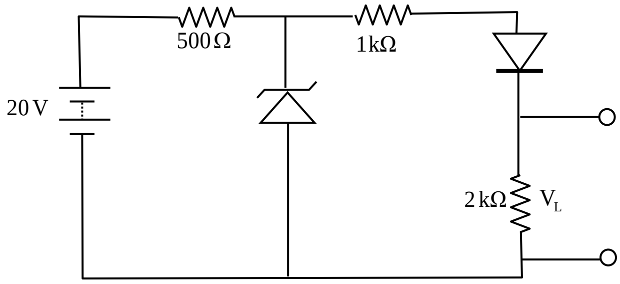
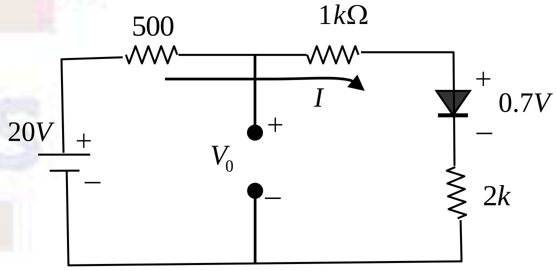
Q48. A Zener diode (rating 10V, 2W) and a normal diode (turn-on voltage 0.7 V) are connected in a circuit as shown in the figure. The voltage drop V_L across the $2k\Omega$ resistance is __________V. (Rounded off to one decimal place)



Ans: 6.2

Solution.: Open circuit voltage across Zener:



$-20 + I\times 0.5 + I\times 1 + 0.7 + I\times 2 = 0 \Rightarrow 3.5I = 19.3$

$\Rightarrow I = \frac{19.3}{3.5} mA$ so $V_0 = 20V - 0.5\times I$

$\Rightarrow V_0 = 20V - 0.5\times\frac{19.3}{3.5} = 20V - 2.76V = 17.24V > V_z = 10V$

So zener is ON [Breakdown region]: $V_L = (V_z - 0.7V)\frac{2}{1+2} = 9.3\times\frac{2}{3} = 6.2V$

Q49. The Fermi energy of a system is 5.5 eV. At 500 K, the energy of a level for which the probability of occupancy is 0.2, is____________eV. (Rounded off to two decimal places) (Boltzmann constant $k_B = 8.62\times 10^{-5}$ eV/K)

Ans: 5.55 to 5.57

Solution: $f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{k_BT}\right)} = 0.2 \Rightarrow 1+\exp\left(\frac{E-E_F}{k_BT}\right) = \frac{10}{2} = 5 \Rightarrow \exp\left(\frac{E-E_F}{k_BT}\right) = 4$

$\Rightarrow \frac{E-E_F}{k_BT} = \ln 4 = 2\ln 2 \Rightarrow E = E_F + 2k_BT\ln 2 \Rightarrow E = 5.5eV + 2\times 8.62\times 10^{-5}\times 500\times 0.693$

$\Rightarrow E = 5.5eV + 0.0597eV = 5.5eV + 0.06eV = 5.56eV$

Section C: Q.51-Q60 Carry TWO marks each.

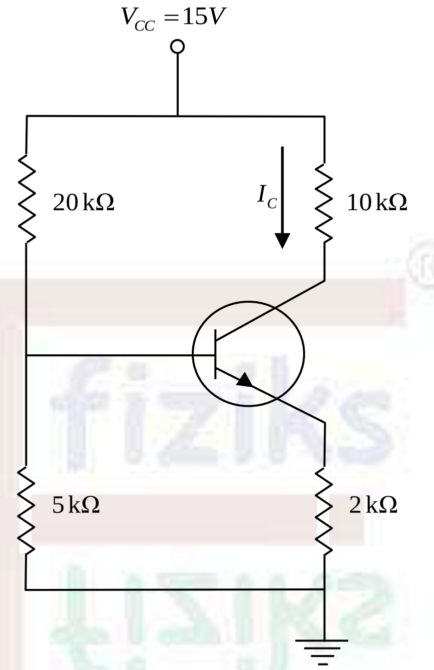
Q51. For a simple cubic crystal, the smallest inter-planar spacing d that can be determined from its second order of diffraction using monochromatic X-rays of wavelength 1.32Å is _________Å . (Round off to two decimal places)

Ans: 1.32

Solution: Bragg's Law is $2d\sin\theta = n\lambda$

For smallest d, $\sin\theta = 1$, thus $d = \dfrac{n\lambda}{2} = \dfrac{2\times 1.32\text{Å}}{2} = 1.32\text{Å}$

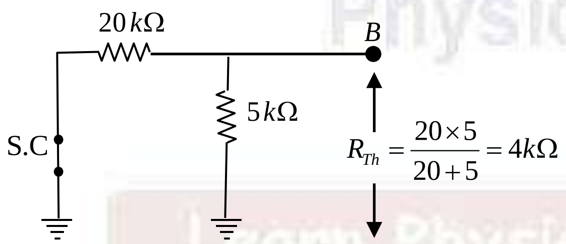
Q52. A transistor $\left(\beta = 100, \text{V}_{\text{BE}} = 0.7V\right)$ is connected as shown in the circuit below.

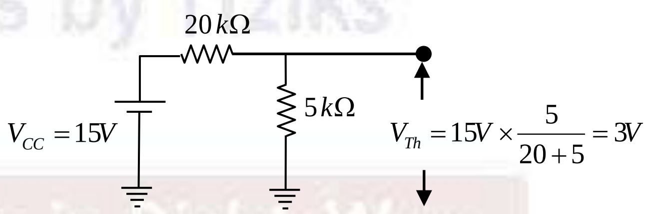


The current I_C will be _____________mA. (Rounded off to two decimal places)

Ans: 1.10 to 1.15

Solution.:





K.V.L. in loop-I:

$$-3V + 4\times I_B + 0.7V + (100+1)I_B \times 2 = 0$$

$$\Rightarrow I_B = \frac{3-0.7}{4+101\times 2} = 0.0111\text{mA}$$

$$\Rightarrow I_c = \beta I_B = 100\times 0.0111\text{mA} = 1.11\text{mA}$$