CSIR - NET/ JRF (PHYSICS) - June 2024 [Solution]

Full Length Test – 01

Ans. 1: (b)

Solution: We consider one digit, two digits and three digits numbers respectively.

Single digit number: There is only are 6

Two digit numbers:

(a) Number of numbers having 6 at unit place only $=8$

(b) Number of numbers having 6 at decimal place only $=9$

(c) Number of numbers having 6 at both places $=1$

So, the numbers from 10 to 99 contain

$$8+9+2=19 \text{ sixes}$$

Three digit numbers

(a) Number of numbers having 6 at unit place only $=8\times9=72$

(b) Number of numbers having 6 at decimal place only $=8\times9=72$

(c) Number of numbers having 6 at the decimal place only $=9\times9=81$

(d) Number of numbers having 6 at exactly two places $=8+9+9=26$

(e) Number of numbers having 6 at all three places $=1$

So, numbers from 1 to 999 contain

$$1+19+72+72+81+2\times26+3\times1=300 \text{ sixes}$$

Ans. 2: (a)

Solution: Let the price of a mango be x and the price of an orange be y.

From the question

$$5x+10y=40$$

$$\Rightarrow x+2y=8 \qquad \text{(I)}$$

Also, $x=2y$ (II)

From (I) and (II)

$$2y+2y+8\Rightarrow 4y=8\Rightarrow y=2$$

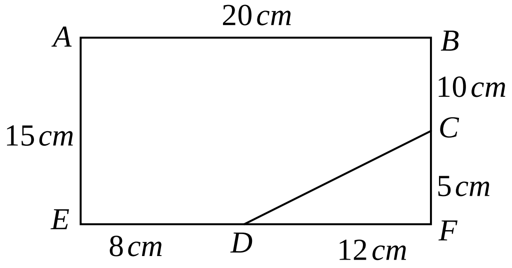
Ans. 3: (b)

Solution: Complete the figure to make a rectangle.

The area of the given shape may be found by subtracting the area of the right triangle from the area of the large rectangle.

Hence required area

$$= 20\times 15 - \frac{1}{2}\times 5\times 12 = 300 = 30 = 270 cm^2$$



Ans. 4: (c)

Solution: We know that

Product of two numbers $= LCM \times HCF$

Here one of the numbers is $2\times 33 = 66$. Let the other number be x, then

$$66\times x = 33\times 264 \Rightarrow x = 132$$

Ans. 5: (d)

Solution: The average sale in 6 months is Rs. 6500. Let the sale in the sixth month be x.

From the question

$$6435 + 6927 + 7230 + 6855 + 6562 + x = 6\times 6500$$

$$\Rightarrow 34009 + x = 39000$$

$$\Rightarrow x = 39000 - 34009 = 4991$$

Ans. 6: (c)

Solution: Forty consecutive integers will have 20 odd and 20 even integers. The sum of 2 chosen integer will be odd, only if

(a) First is even and second is odd OR

(b) First is odd and second is even

Mathematically, the probability is given by

$P(\text{First is even})\times P(\text{Second is odd})+P(\text{First is odd})\times P(\text{Second is even})$

$$=\frac{20}{40}\times\frac{20}{39}+\frac{20}{40}\times\frac{20}{39}$$

$$=\frac{10}{39}+\frac{10}{39}=\frac{20}{39}$$

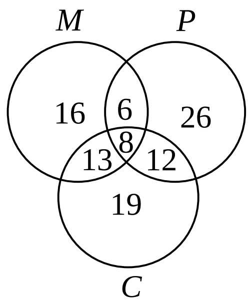
Ans. 7: (a)

Solution: The ratio between total amount of first class fare and total amount of second class fare

$=6\times 1:4\times 30=1:20$

Hence the amount collected from first class passengers $=\frac{1}{21}\times 2100=100$

Ans. 8: (c)



Solution: From the Venn-diagram we see that

Percentage of students passing in math only $=16$

Hence total number of students passing in math only

$=16\%\ \text{of}\ 200=\frac{16}{100}\times 200=32$

Ans. 9: (d)

Solution: Each word will be of the form

$$R------W$$

Since in the formation of English words order matters, hence the remaining 6 places are just the arrangements of remaining 6 letters.

Hence required number of words $=6!=120$

Ans. 10: (d)

Solution: Let the CP of VCR be x.

Then selling price $=x\times\frac{100+20}{100}=\frac{6x}{5}$

After reduction, $CP=x-1000,\ SP=\frac{6x}{5}-1000$

New profit percentage $=20+\frac{5}{3}=\frac{65}{3}$

$$\left(\frac{6x}{5}-1000\right)-(x-1000)=\frac{65}{3}\% \text{ of } (x-1000)$$

$$\Rightarrow \frac{x}{5}=\frac{65}{3}\times\frac{1}{100}(x-1000)$$

$$\Rightarrow \frac{x}{5}=\frac{13x}{60}-\frac{650}{3} \Rightarrow \frac{13x}{60}-\frac{x}{5}=\frac{650}{3}$$

$$\Rightarrow \frac{x}{60}=\frac{650}{3} \Rightarrow x=650\times 20=13000$$

Ans. 11: (c)

Solution: In one hour

Number of pages typed by Ravi $=\frac{32}{6}=\frac{16}{3}$

Number of pages typed by kumar $=\frac{40}{5}=8$

Together, in one hour, the number of pages typed by them $=\frac{16}{3}+8=\frac{16+24}{3}=\frac{40}{3}$

$\frac{40}{3}$ page is typed by them in 1 hour

1 page is typed by them in $\frac{3}{40}$ hour

110 page is typed by them in $\frac{3}{40}\times 110$ hour

$$=\frac{33}{4}hr=8.25hr$$

$$=8 \text{ hour } 15 \text{ minutes}$$

Ans. 12: (b)

Solution: Let Abhay's speed be v_1 and Sameer's speed be v_2. Then

$$\frac{30}{v_1}-\frac{30}{v_2}=2 \qquad \text{(I)}$$

Also, $$\frac{30}{v_2}-\frac{30}{2v_1}=1 \qquad \text{(II)}$$

Adding (I) and (II) gives

$$\frac{30}{v_1}-\frac{30}{2v_1}=3$$

$$\Rightarrow \frac{60-30}{2v_1}=3 \Rightarrow \frac{30}{2v_1}=3$$

$$\Rightarrow \frac{15}{v_1}=3 \Rightarrow v_1=\frac{15}{3}=5km/hr$$

Ans. 13: (b)

Solution: Look for smallest triangles first – there are 12 such triangles. Now, look for triangles which are equal to half the rectangle – there are 12 of them. Besides, there are 4 bigger triangles (spanning across 2 rectangles).

Thus total number of triangles $=12+12+4=28$

Ans. 14: (b)

Solution: Two lines are perpendicular if the product of their slopes $=-1$

The slope of line joining $(1,2)$ and $(2,-2)$

$$m_1=\frac{-2-2}{2-1}=\frac{-4}{1}=-4$$

The slope of line joining $(8,2)$ and $(4,p)$

$$m_2=\frac{p-2}{4-8}=\frac{p-2}{(-4)}$$

$$\Rightarrow m_1m_2=-1 \Rightarrow (-4)\left(\frac{p-2}{-4}\right)=-1$$

$$\Rightarrow p-2=-1 \Rightarrow p=1$$

Ans. 15: (c)

Solution: In the first figure: $4\times5+2^2=20+4=24$

In the second figure: $6\times4+3^2=24+9=33$

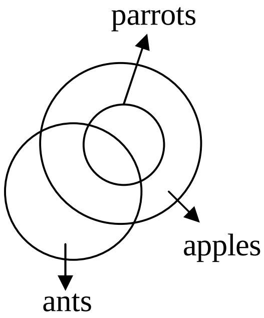
In the third figure: $8\times6+5^2=48+25=73$

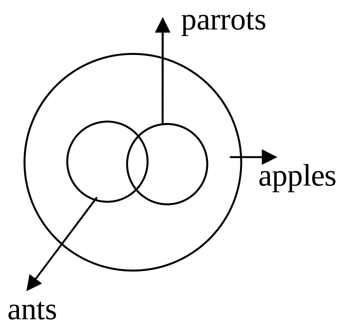
Ans. 16: (c)

Solution: From figures (i), (ii) and (iv) we conclude that $6, 4, 1$ and 2 dots appear adjacent to 3 dots. Clearly, there will be 5 dots on the face opposite the face with 3 dots.

Ans. 17: (b)

Solutions: All the possibility resulting from the statement is shown in the Venn-diagram

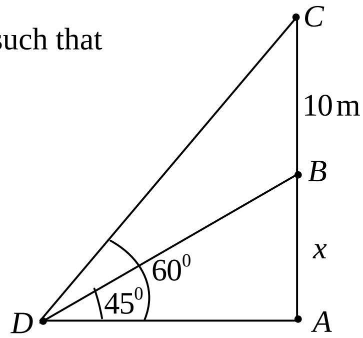




From the two figure we can only conclude that 'some apples are parrots' and not 'All apples are parrots. From the two figure we clearly see that 'some ants are apples'. Hence only conclusions (2) follows.

Ans. 18: (a)

Solution: Let AB be the tower of height x meter and BC be the flag of height $10m$. Let D be the point from where the angles of elevation are 45^0 and 60^0 such that



$$\angle BDA = 45^0 \text{ and } \angle CDA = 60^0$$

In ΔDAB, $\tan 45^0 = \dfrac{AB}{AD} \Rightarrow 1 = \dfrac{x}{AD} \Rightarrow AD = x$

In ΔDAC, $\tan 60^0 = \dfrac{AC}{AD} \Rightarrow \sqrt{3} = \dfrac{10+x}{x}$

$$\Rightarrow \sqrt{3}x = 10 + x \Rightarrow \left(\sqrt{3}-1\right)x = 10 \Rightarrow x = \frac{10}{\sqrt{3}-1} = \frac{10\left(\sqrt{3}+1\right)}{2} = 5\left(\sqrt{3}+1\right)m$$

Ans. 19: (a)

Solution: Each row (as well as each column) contains a figure consisting of a circle and two line segments, a figure consisting of a circle and three line segments and a figure consisting of a circle and four line segments.

Ans. 20: (d)

Solution: Time from $7\,AM$ to $4.15\,PM = 9\,hr\ 15\min = \frac{37}{4}hr$

$3\min 5\sec$ of this clock $= 3\min$ of correct clock

$\Rightarrow \frac{37}{720}$ hours of this clock $= \frac{1}{20}$ hours of correct clock

$\Rightarrow \frac{37}{4}$ hours of this clock $= \frac{1}{20}\times\frac{720}{37}\times\frac{37}{4}$ hours of correct clock

$= 9$ hours of correct clock

Therefore, the correct time is 9 hours after $7\,AM$, i.e, $4\,PM$

Ans. 21: (a)

Solution: $\phi(p) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} dx\, Ae^{-\mu|x|}e^{-ipx}$

$$= \frac{A}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} dx\, e^{-|\mu|x}e^{-i\hbar kx} = \frac{A}{\sqrt{2\pi\hbar}}\left[\int_{-\infty}^{0} dx\, e^{\mu x}e^{-i\hbar kx} + \int_{0}^{\infty} dx\, e^{-\mu x}e^{-i\hbar kx}\right]$$

$$= \frac{A}{\sqrt{2\pi\hbar}}\left[\int_{-\infty}^{0} dx\, e^{(\mu - i\hbar k)x} + \int_{0}^{\infty} dx\, e^{-(\mu + i\hbar k)x}\right] = \frac{A}{\sqrt{2\pi\hbar}}\left\{\frac{1}{\mu - i\hbar k} + \frac{1}{\mu + i\hbar k}\right\}$$

Using $A = \sqrt{\mu}$, we can write

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}}\cdot\frac{2\mu^{3/2}}{\mu^2 + \hbar^2 k^2}$$

If we write $\hbar k = p$ then the answer will be

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}}\cdot\frac{2\mu^{3/2}}{\mu^2 + p^2}$$

Ans. 22: (b)

Solution:

$$V_A = \frac{3Q}{8\pi\in_0 a} - \frac{Q}{4\pi\in_0(2a)} = \frac{Q}{4\pi\in_0 a} \quad \text{and} \quad V_B = \frac{-3Q}{8\pi\in_0 a} + \frac{Q}{4\pi\in_0(2a)} = \frac{-Q}{4\pi\in_0 a}$$

$$\Rightarrow V_B - V_A = -\frac{Q}{2\pi\in_0 a}$$

Ans. 23: (b)

Solution: $L = p_x\dot{x} + p_y\dot{y} - H$, $H = \frac{p_x p_y}{m} - \frac{p_y^2}{2m}$, $\frac{\partial H}{\partial p_x} = \dot{x} \Rightarrow \frac{p_y}{m} = \dot{x} \Rightarrow p_y = m\dot{x}$

$$\frac{\partial H}{\partial p_y} = \dot{y} \Rightarrow \frac{p_x}{m} - \frac{p_y}{m} = p_x = m\dot{x} + m\dot{y}$$

$$L = p_x\dot{x} + p_y\dot{y} - \frac{p_x p_y}{m} + \frac{p_y^2}{2m} \Rightarrow m(\dot{x}+\dot{y})\dot{x} + m\dot{x}\dot{y} - \frac{m(\dot{x}+\dot{y})m\dot{x}}{m} + \frac{(m\dot{x})^2}{2m}$$

$$\Rightarrow L = \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$$

Ans. 24: (b)

Solution: $\eta^2 + \xi^2 = \frac{2mV_0^2}{\hbar^2}$ and $\eta = -\xi \cot \xi$

$$\gamma = \sqrt{\frac{2m(V_0+E)}{\hbar^2}} \qquad k = \sqrt{-\frac{2mE}{\hbar^2}}$$

$$(\gamma a)^2 + (ka)^2 = \frac{2mV_0 a^2}{\hbar^2} \qquad V_0 = \frac{32\hbar^2}{ma^2}$$

$$R = (64)^{1/2} = 8 \qquad \frac{5\pi}{2} < R < \frac{7\pi}{2}$$

Ans. 25: (b)

Solution: $I = \frac{1}{2}MR^2, L = I\omega = \frac{1}{2}MR^2\omega$

Magnetic moment due to disc $\mu = \frac{\pi\sigma\omega R^4}{4}$

Due to cylinder $d\mu = \frac{\pi\omega R^4}{4}(\rho dz)$ $\qquad (\sigma \to \rho dz)$

$$\mu = \frac{\pi\omega R^4}{4}\int_0^L \frac{Q}{\pi R^2 L}dz = \frac{Q\omega R^2}{4}$$

$$\Rightarrow \frac{\mu}{L} = \frac{\frac{Q\omega R^2}{4}}{\frac{1}{2}MR^2\omega} = \frac{Q}{2M}$$

Ans. 26: (b)

Solution: From the Virial theorem. $V = r^{n+1}$ then $\langle T\rangle = \frac{n+1}{2}\langle V\rangle$

$$|V| = r^4 \Rightarrow n+1 = 4 \Rightarrow \langle T\rangle = \frac{4}{2}\langle V\rangle \Rightarrow \langle T\rangle = 2\langle V\rangle$$

$$\langle E\rangle = \langle T\rangle + \langle V\rangle \Rightarrow \langle E\rangle = 3\langle V\rangle$$

Ans. 27: (d)

Solution: $E_n = \left(n+\frac{1}{2}\right)\hbar\omega \qquad n = 1,3,5........$

$$E_g = \left(1+\frac{1}{2}\right)\hbar\omega = \frac{3}{2}\hbar\omega$$

$$E_{1st} = \left(3+\frac{1}{2}\right)\hbar\omega = \frac{7}{2}\hbar\omega$$

$$\langle E\rangle = \frac{2}{3}\times\frac{3}{2}\hbar\omega + \frac{1}{3}\times\frac{7}{2}\hbar\omega \Rightarrow \langle E\rangle = \hbar\omega + \frac{7}{6}\hbar\omega = \frac{13}{6}\hbar\omega$$

Ans. 28: (c)

Solution: Monopole moment $= -\frac{q}{2} - \frac{q}{2} + q = 0$

$$\vec{p} = -\frac{q}{2}\times(-d\hat{y}) - \frac{q}{2}(d\hat{y}) + q(d\hat{z}) \Rightarrow \vec{p} = qd\hat{z}$$

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{qd\cos\theta}{r^2}$$

$$V(r,\theta) = 9\times10^9\times\frac{10^{-6}\times10^{-3}\times\cos 60^0}{(10)^2} = 9\times10^9\times\frac{10^{-9}}{2\times100} = 0.045 \text{ Volts} = 45\ m\text{V}$$

Ans. 29: (b)

Solution: For either Si or Ge; $\frac{dV}{dT} \simeq -2.5mV/^0C$

In order to maintain a constant values of I

$$T_2 - T_1 = 40 - 20 = 20^0C$$

Change in V_D, $-2.5\times 20mV = -50mV$

Therefore, $V_D = 700 - 50 = 650 \simeq 660mV$

Ans. 30: (c)

Solution: $\int_{-\pi/2}^{\pi/2} \cos x\,\delta(\sin x)\,dx$

$\sin x = 0$ at $x = n\pi, \frac{d}{dx}(\sin x) = \cos x$

Hence, $\delta(\sin x) = \frac{\sum \delta(x - n\pi)}{|\cos n\pi|}$.

$\Rightarrow \delta(\sin x) = (x) + \delta(x-\pi) + \delta(x+\pi) + \delta(x-2\pi) + \delta(x+2\pi) + \delta(x-3\pi) + \delta(x+3\pi) + \cdots$

Hence, $\int_{-\pi/2}^{\pi/2} \cos x\,\delta(\sin x)\,dx = \int_{-\pi/2}^{\pi/2} \cos x\,\delta(x)\,dx = \cos 0 = 1$

$\int_{-2\pi}^{2\pi} e^x \sin x\,\delta'(x-\pi)\,dx$

$\frac{d}{dx}\left[e^x \sin x\right] = e^x \sin x + e^x \cos x$

At, $x = \pi, \frac{d}{dx}\left(e^x \sin x\right) = e^\pi \sin\pi + e^\pi \cos\pi = e^\pi$

Thus, $\int_{-\pi/2}^{\pi/2} \cos x\,\delta(\sin x)\,dx + \int_{-2\pi}^{2\pi} e^x \sin x\,\delta'(x-\pi)\,dx = 1 + (-1)\left(-e^\pi\right) = 1 + e^\pi$.

Where we have used $\int_{-\infty}^{\infty} \phi(x)\,\delta'(x-a)\,dx = -\frac{d}{dx}\{\phi(x)\}\Big|_{x=a}$

Ans. 31: (d)

Solution: $\left(\frac{\partial V}{\partial x}\right) = 0 \Rightarrow \frac{c\left(a^2 - x^2\right)}{\left(x^2 + a^2\right)^2} = 0 \Rightarrow x_1 = -a, x_2 = a$

$\left(\frac{\partial^2 V}{\partial x^2}\right) \Rightarrow \frac{2cx\left(x^2 - 3a^2\right)}{\left(x^2 + a^2\right)^3}$ at $x_1 = -a \Rightarrow \left(\frac{\partial^2 V}{\partial x^2}\right) = \frac{c}{2a^3} > 0$ x_1 is stable equilibrium

$\left(\frac{\partial^2 V}{\partial x^2}\right) \Rightarrow \frac{2cx\left(x^2 - 3a^2\right)}{\left(x^2 + a^2\right)^3}$ at $x_1 = a \Rightarrow \left(\frac{\partial^2 V}{\partial x^2}\right) = -\frac{c}{2a^3} < 0$ x_2 is unstable equilibrium

Ans. 32: (d)

Ans. 33: (d)

Solution: $I_C = \beta I_B + (1+\beta) I_{CO}$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

$$\therefore\ I_C = 49\times 20 + 50\times 0.6 = 980 + 30 = 1010\mu A \ \Rightarrow I_C = 1.01 mA$$

Ans. 34: (c)

Solution: $J = \oint P dx$

$$J \propto (2mE)^{1/2}\left(\frac{E}{a}\right)^{1/6} \Rightarrow E \propto J^{3/2} \text{ and } \nu = \frac{dE}{dJ} \propto J^{1/2} \Rightarrow \nu \propto E^{1/3}$$

Ans. 35: (a)

Solution: $\varepsilon = -\frac{d\phi}{dt} = -\frac{AdB}{dt},\ I = \frac{\varepsilon}{R} = -\frac{d\phi}{dt}\frac{1}{R}$

$$\Rightarrow -\frac{d\phi}{dt} = -\pi r^2 \frac{d}{dt}\left(B_0 e^{-t/t_0}\right) = \pi r^2 B_0 e^{-t}\ (t_0 = 1)$$

$$Q = \int_0^\infty I(t)\,dt = \int_0^\infty \frac{\pi r^2}{R} B_0 e^{-t} dt = \frac{\pi r^2 B_0}{R}\left|\frac{e^{-t}}{-1}\right|_0^\infty = 3.14\times\left(2\times 10^{-2}\right)^2 \times 0.01 = 6.28\mu C$$

Ans. 36: (a)

Solution: $\mathrm{U} = \mathrm{K_B T^2}\frac{\partial \ln z}{\partial \mathrm{T}},\quad \mathrm{C_V} = \left(\frac{\partial \mathrm{U}}{\partial \mathrm{T}}\right)_{\mathrm{V}} \Rightarrow \mathrm{C_V} \propto \mathrm{T}^3$.

Ans. 37: (b)

Solution: The wronskian is $w(t) = ce^{-\int \sin t\,dt}$

$$\Rightarrow w(t) = ce^{\cos t}$$

$$w(0) = e \Rightarrow e = ce \Rightarrow c = 1$$

Thus $= w(t) = e^{\cos t}$

$$\Rightarrow w(2\pi) = e$$

Ans. 38: (d)

Solution: A monatomic gas is a collection of indistinquishable particle. Assuming that the electronic partition function is unity (i.e., the ground electronic energy level is nondegenerate), only translational degrees of freedom remain to be evaluated, resulting in

$$S=\frac{U}{T}+k\ln Q$$

$$=\frac{1}{T}\left(\frac{3}{2}NkT\right)+k\ln\frac{q_{trans}^{N}}{N!}=\frac{3}{2}Nk+Nk\ln q_{trans}-k\left(N\ln N-N\right)$$

$$=\frac{5}{2}Nk+Nk\ln q_{trans}-Nk\ln N=\frac{5}{2}Nk+Nk\ln\frac{V}{\Lambda^3}-Nk\ln N$$

$$=\frac{5}{2}Nk+Nk\ln V-Nk\ln\Lambda^3-Nk\ln N=\frac{5}{2}Nk+Nk\ln V-Nk\ln\left(\frac{h^2}{2\pi mkT}\right)^{3/2}-Nk\ln N$$

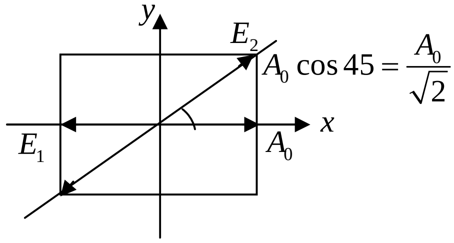
$$=\frac{5}{2}Nk+Nk\ln V+\frac{3}{2}Nk\ln T-Nk\ln\left(\frac{N^{2/3}h^2}{2\pi mk}\right)^{3/2}$$

$$=\frac{5}{2}nR+nR\ln T+\frac{3}{2}nR\ln T-nR\ln\left(\frac{n^{2/3}N_A^{2/3}h^2}{2\pi mk}\right)^{3/2}$$

The final line of Equation is a version of the Sackur-Tetrode equation, which can be written in the more compact form:

$$S=nR\ln\left[\frac{e^{5/2}V}{\Lambda^3N}\right]=nR\ln\left[\frac{RTe^{5/2}}{\Lambda^3N_AP}\right]\text{ where }\Lambda^3=\left(\frac{h^2}{2\pi mkT}\right)^{3/2}$$

Ans. 39: (c)



Solution: $\vec{E}_1=\hat{x}A_0e^{i\omega t}$; $\quad \vec{E}_2=\frac{A_0}{\sqrt{2}}\frac{(\hat{x}+\hat{y})}{\sqrt{2}}e^{i\omega t+i\delta}$

$$I=\left(\vec{E}_1+\vec{E}_2\right)\cdot\left(\vec{E}_1^*+\vec{E}_2^*\right)$$

$$\Rightarrow I=\left|\vec{E}_1\right|^2+\left|\vec{E}_2\right|^2+\vec{E}_1\cdot\vec{E}_2^*+\vec{E}_2\cdot\vec{E}_1^*$$

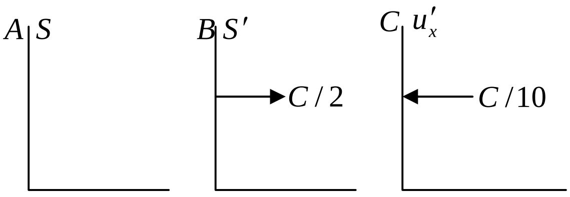
$$=A_0^2+\frac{A_0^2}{4}+(1+1)+\frac{A_0^2}{2}e^{-i\delta}+\frac{A_0^2}{2}e^{i\delta}\Rightarrow I=A_0^2+\frac{A_0^2}{2}+\frac{A_0^2}{2}\frac{e^{i\delta}+e^{-i\delta}}{2}=\frac{3A_0^2}{2}+A_0^2\cos\delta$$

$$I_{\max}=\frac{5A_0^2}{2},I_{\min}=\frac{A_0^2}{2}\Rightarrow\frac{I_{\max}}{I_{\min}}=5$$

Ans. 40: (d)

Solution: $v = \frac{c}{2}, \quad u'_x = -\frac{c}{10}$

$$u_x = \frac{u'_x + v}{1 + \frac{u'v_x}{c^2}} = \frac{8c}{19}$$



Ans. 41: (a)

Solution: The trial of interested consists of 50 separate experiments; therefore, $n = 50$. Considering the case of 25 successful experiments where $j = 25$. The probability (P_{25}) is

$$P_{25} = C(n, j)(P_E)^j (1 - P_E)^{n-j}$$

$$= C(50, 25)(P_E)^{25}(1 - P_E)^{25}$$

$$= \left(\frac{50!}{(25!)(25!)}\right)\left(\frac{1}{2}\right)^{25}\left(\frac{1}{2}\right)^{25} = (1.26 \times 10^{14})(8.88 \times 10^{-16}) = 0.11$$

For the case of 10 successful experiments, $j = 10$ such that

$$P_{10} = C(n, j)(P_E)^j (1 - P_E)^{n-j}$$

$$= C(50, 10)(P_E)^{10}(1 - P_E)^{40} = \left(\frac{50!}{(10!)(40!)}\right)\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{40}$$

Ans. 42: (b)

Solution: Let $F(s) = \ln\frac{s-b}{s-a} = \ln(s-a) - \ln(s-b)$ differentiating $F(s)$ with s gives

$$F' = \frac{1}{s-a} - \frac{1}{s-b}$$

But from the relation

$L\{t\, f(t)\} = -F'(s)$ we obtain

$$L\{t\, f(t)\} = -\frac{1}{s-a} + \frac{1}{s-b} \Rightarrow t\, f(t) = -e^{at} + e^{bt} \Rightarrow f(t) = \frac{e^{bt} - e^{at}}{t}$$

From the result, $L\left\{\frac{f(t)}{t}\right\}=\int_s^\infty F(s)\,ds$

we obtain, $L^{-1}\left\{\int_s^\infty \frac{1}{(s-a)}ds\right\}=\frac{e^{at}}{t}$

Thus the inverse Laplace transform of $H(s)$ is

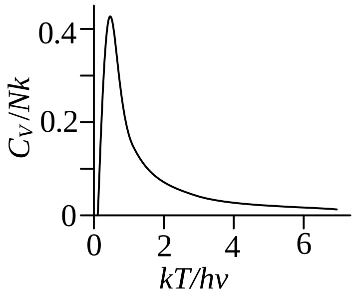
$$L^{-1}\{H(s)\}=\frac{e^{bt}-e^{at}}{t}+\frac{e^{at}}{t}=\frac{e^{bt}}{t}$$

Ans. 43: (c)

Solution: The partition function was determined to be $q=1+e^{-\beta hv}$. The corresponding average energy calculated using the partition function is $U=Nhv/\left(e^{\beta hv}+1\right)$. Given the functional form of the average energy, the heat capacity is most easily determined by taking the derivative with respect to β as follows:

$$C_v=-k\beta^2\left(\frac{\partial U}{\partial \beta}\right)_V=-Nk\beta^2\left(\frac{\partial}{\partial \beta}hv\left(e^{\beta hv}+1\right)^{-1}\right)_V=\frac{Nk\beta^2(hv)^2 e\beta hv}{\left(e^{\beta hv}+1\right)^2}$$

The functional form of the heat capacity is rather complex and is plotted in figure given below. As observed previously, limiting behavior is observed at both low and high temperatures. At the lowest temperature, C_V is zero and then increases to a maximum value after which further increases in temperature result in a decrease in the heat capacity. This behavior is reminiscent of the evolution in energy as a function of



Ans. 44: (c)

Solution: We know that the determinant of a skew-symmetric matrix of *odd* order is 0. For a skew-symmetric A, A^n is symmetric matrix according as n is *even* or *odd*.

For the calculation of eigenvalues

$$\begin{vmatrix} 0-\lambda & 1 & 1 \\ -1 & 0-\lambda & 1 \\ -1 & -1 & 0-\lambda \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 & 1 \\ -1 & -\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda\left(\lambda^2+1\right)-\left(\lambda+1\right)+\left(1-\lambda\right)=0 \Rightarrow -\lambda^3-\lambda-\lambda-1+1-\lambda=0$$

$$\Rightarrow -\lambda^3-3\lambda=0 \Rightarrow \lambda\left(\lambda^2+3\right)=0$$

Thus, $\lambda=0$ and $\lambda\ \lambda=\pm\sqrt{3}\,i$ where $i^2=-1$.

$$\left(A^T\right)^2-A^2=\left(-A\right)^2-A^2=A^2-A^2=0$$

Thus, $\left(A^T\right)^2-A^2$ is a null matrix.

Ans. 45: (c)

Solution: Since $J=1$ and $Q_n=0$ So $Q_{n+1}=1$

As even if $K=0, Q_{n+1}=1$ (Set)

And if $K=1, Q_{n+1}=\overline{Q_n}=1$ (Toggle)

Ans. 46: (d)

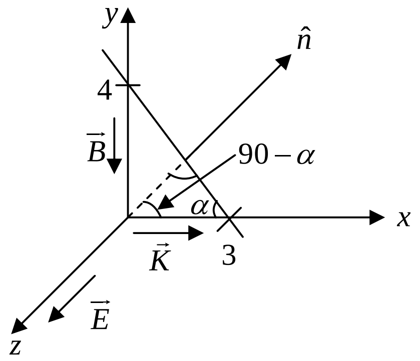
Solution: $E_{n_x,n_y,n_z}=\dfrac{n_x^2\pi^2\hbar^2}{2ma^2}+\dfrac{n_y^2\pi^2\hbar^2}{2ma^2}+\left(n_z+\dfrac{1}{2}\right)\hbar 2\omega$

where $n_x=1,2,3......$, $n_y=1,2,3......$, $n_z=1,3,5......$

So ground state is $E_{1,1,1}=\dfrac{\pi^2\hbar^2}{ma^2}+\dfrac{6}{2}\hbar\omega=\dfrac{\pi^2\hbar^2}{ma^2}+3\hbar\omega$

Ans. 47: (a)

Solution: $4x+3y=0 \Rightarrow \frac{x}{3}+\frac{y}{4}=0$



If $\vec{E}=E_0\cos k(x-ct)\hat{z}\ V/m$ where $E_0=1\,V/m$

$$\vec{B}=-\frac{E_0}{c}\cos(kx-kct)\hat{y}$$

$$\vec{S}=\frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)=\frac{1}{\mu_0}\left(E_0\times\frac{E_0}{c}\cos^2\theta\right)\hat{x}\Rightarrow\left\langle\vec{S}\right\rangle=\frac{E_0^2}{2\mu_0 c}\hat{x}$$

$$I=\left\langle\vec{S}\right\rangle.\hat{n}=\frac{E_0^2}{2\mu_0 c}\cos(90-\alpha)=\frac{E_0^2}{2\mu_0 c}\sin\alpha=\frac{2}{5}c\varepsilon_0E_0^2$$

$\because \tan\alpha=\frac{4}{3}\Rightarrow\sin\alpha=\frac{4}{5}$

$$\Rightarrow I=\frac{2}{5}c\varepsilon_0E_0^2=\frac{2}{5}\left(3\times10^8\right)\left(8.86\times10^{-12}\right)(1)^2=10.6\times10^{-4}V/m\approx1.1\ mV/meter$$

Ans. 48: (b)

Solution: $\frac{V_o}{V_i}=\frac{-R_2}{\left(R_1+\frac{1}{Cs}\right)}=\frac{-R_2Cs}{R_1Cs+1}$

$$\Rightarrow\left|\frac{V_o}{V_i}\right|=\frac{R_2C\omega}{\sqrt{1+R_1^2C^2\omega^2}}=\frac{R_2C\omega}{R_1C\omega\sqrt{1/R_1^2C^2\omega^2+1}}=\frac{R_2/R_1}{\sqrt{\frac{\omega_c^2}{\omega^2}+1}}$$

It is the transfer function of high pass filter with cutoff frequency $\rightarrow\omega_c=\frac{1}{R_1C}rad/\sec$

Ans. 49: (c)

Solution: A: $\tau^-+\tau^+\rightarrow K^-+K^+$

$q:-1\ \ +1 \qquad -1\ \ +1$: Conserved

Spin: $\frac{1}{2}\ \ \frac{1}{2} \qquad 0 \qquad 0$: Conserved

$L_\tau:\ +1\ -1 \qquad 0 \qquad 0$: Conserved

$I:\ \ 0\ \ 0 \qquad \frac{1}{2}\ \ \frac{1}{2}$: Conserved/Not conserved

$I_3: \quad 0 \quad 0 \quad -\frac{1}{2} \quad +\frac{1}{2}$: Conserved

$S: \quad 0 \quad 0 \quad -1 \quad +1$: Conserved

This interaction can't be strong due to presence of Lepton. Therefore it is electromagnetic interaction.

B. $\overline{K^0} + p \rightarrow \pi^+ + \Lambda^0$

$q: -1 \quad +1 \quad 0 \quad 0$: Conserved

Spin: $0 \quad \frac{1}{2} \quad 0 \quad \frac{1}{2}$: Conserved

$B: \quad 0 \quad +1 \quad 0 \quad +1$: Conserved

$I: \quad \frac{1}{2} \quad \frac{1}{2} \quad 1 \quad 0$: Conserved

$I_3: +\frac{1}{2} \quad +\frac{1}{2} \quad +1 \quad 0$: Conserved

$S: -1 \quad 0 \quad 0 \quad -1$: Conserved

This is strong interaction

C: $K^+ \rightarrow \pi^+ + \pi^0$

$q: \quad +1 \quad +1 \quad 0$: Conserved

Spin: $0 \quad 0 \quad 0$: Conserved

$I: \quad \frac{1}{2} \quad 1 \quad 1$: Conserved

$I_3: +\frac{1}{2} \quad +1 \quad 0$: Conserved

$S: \quad +1 \quad 0 \quad 0$: Conserved

This is a weak interaction

Ans. 50: (c)

Solution: If $V_0 << \frac{\hbar^2}{ma^2}$, one can treat $H' = V_0 \cos\left(\frac{2\pi x}{a}\right)$ as a perturbation imposed on the free motion of a particle. For the ground state, the eigenvalue and eigenfunction of the free particle are respectively $\left(n = 0, -1; \text{i.e.}, ka = \pi, -\pi\right)$

$$E^{(0)}_{\binom{0}{-1}}=\frac{\hbar^2\pi^2}{2ma^2},\quad \psi_1^{(0)}(x)=\frac{1}{\sqrt{a}}e^{i\pi x/a},\quad \psi_{-1}^{(0)}(x)=\frac{1}{\sqrt{a}}e^{-i\pi x/a}$$

Let, $\frac{2\pi}{a}=\beta$ and consider $\langle m|H'|n\rangle$. We have

$$\langle -1|H'|-1\rangle=\langle 0|H'|0\rangle=\frac{V_0}{a}\int_0^a\cos\left(\frac{2\pi x}{a}\right)dx=0$$

$$\langle -1|H'|0\rangle=\langle 0|H'|-1\rangle=\frac{V_0}{2a}\int_0^a e^{\pm i\beta x}\left(e^{i\beta x}+e^{-i\beta x}\right)dx=\frac{V_0}{2}$$

Hence for ground state, $H'=\begin{pmatrix}0 & \frac{V_0}{2}\\ \frac{V_0}{2} & 0\end{pmatrix}$,

and the secular equation for first order perturbation is

$$\begin{vmatrix}E^{(1)} & \frac{V_0}{2}\\ \frac{V_0}{2} & E^{(1)}\end{vmatrix}=0,$$

giving $E^{(1)}=\pm\frac{V_0}{2}$.

Thus the ground state energy level splits into two levels

$$E_1=\frac{\hbar^2\pi^2}{2ma^2}-\frac{V_0}{2},\qquad E_2=\frac{\hbar^2\pi^2}{2ma^2}+\frac{V_0}{2}$$

These are the lowest energy eigenvalues of the system.

Ans. 51: (d)

Solution: $\eta=\frac{\pi Pa^4}{8lv}=kPa^4$

$$\sigma_\eta^2=\left(\frac{\partial\eta}{\partial P}\right)^2\sigma_p^2+\left(\frac{\partial\eta}{\partial a}\right)^2\sigma_a^2$$

$$=\left(a^4\right)^2\sigma_P^2+\left(4Pa^3\right)^2\sigma_a^2$$

$$\Rightarrow\left(\frac{\sigma_\eta}{\eta}\times 100\right)^2=\left(\frac{\sigma_P}{P}\times 100\right)^2+16\left(\frac{\sigma_a}{a}\times 100\right)^2$$

$$=(2)^2+16(4)^2=4+256=260$$

$$\Rightarrow \frac{\sigma_\eta}{\eta}\times 100=16\cdot 12\%$$

Ans. 52: (d)

Solution: $u_x=\dfrac{u'_x+v}{1+\dfrac{u'_x v}{c^2}}$ and $t=\dfrac{t'+\dfrac{v}{c^2}x'}{\sqrt{1-\dfrac{v^2}{c^2}}}$ so $dt=\dfrac{dt'+\dfrac{v}{c^2}dx'}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$\Rightarrow du_x=\frac{du'_x}{1+\frac{u'_x v}{c^2}}-\frac{u'_x+v}{\left(1+\frac{u'_x v}{c^2}\right)^2}.\frac{vdu'_x}{c^2}\Rightarrow du_x=\frac{du'_x}{\left(1+\frac{u'_x v}{c^2}\right)^2}\left(\left(1+\frac{u'_x v}{c^2}\right)-(u'_x+v)\frac{v}{c^2}\right)=\frac{du'_x\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{u'_x v}{c^2}\right)^2}$$

$$dt=\frac{dt'+\frac{v}{c^2}dx'}{\sqrt{1-\frac{v^2}{c^2}}}\quad a_x=\frac{du_x}{dt}=\frac{du'_x}{dt'}\frac{\left(1-\frac{v^2}{c^2}\right)^{3/2}}{\left(1+\frac{u'_x v}{c^2}\right)^3}=a_x=a'_x\frac{\left(1-\frac{v^2}{c^2}\right)^{3/2}}{\left(1+\frac{u'_x v}{c^2}\right)^3}$$

Ans. 53: (b)

Solution: $\Delta\lambda=\dfrac{\lambda_0^2}{c}\cdot\dfrac{eB}{4\pi m}=\dfrac{\left(4878\times 10^{-10}m\right)}{3\times 10^8 m/s}\times\dfrac{1\cdot 6\times 10^{-19}\times 1}{4\pi\times 9\cdot 1\times 10^{-31}}$

$$=1\cdot 11\times 10^{-11}m=0\cdot 11\overset{0}{\text{A}}$$

Ans. 54: (a)

Solution:

Q_1	Q_0	$D_1(Q_0)$	$D_0(\bar{Q}_1)$
0	0	0	1
0	1	1	1
1	1	1	0
1	0	0	0

Ans. 55: (b)

Solution: $V(x) = -b\delta\left(x - \frac{a}{2}\right);\quad b > 0$ and $\psi(x) = \left\{A\sin\frac{\pi x}{a};\qquad 0 < x < a\right.$

Normalized $\psi = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$

$$\langle T\rangle = \int_0^a \psi^* \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi\, dx = \frac{\pi^2\hbar^2}{2ma^2}$$

$$\langle V\rangle = \int_0^a \psi^*\left\{-b\delta\left(x - \frac{a}{2}\right)\right\}\psi\, dx = \frac{2}{a}(-b) = -\frac{2b}{a}$$

$$\langle E\rangle = \frac{\pi^2\hbar^2}{2ma^2} - \frac{2b}{a}$$

$$\Rightarrow \frac{\partial\langle E\rangle}{\partial a} = \frac{-2\pi^2\hbar^2}{2ma^3} + \frac{2b}{a^2} = 0 \Rightarrow \frac{-\pi^2\hbar^2}{ma} + 2b = 0 \Rightarrow a = \frac{\pi^2\hbar^2}{2mb}$$

Put the value of a in equation: $\langle E\rangle = \frac{\pi^2\hbar^2}{2ma^2} - \frac{2b}{a} = \frac{\pi^2\hbar^2(2mb)^2}{2m(\pi^2\hbar^2)^2} - \frac{2b(2mb)}{(\pi^2\hbar^2)} = -\frac{2mb^2}{\pi^2\hbar^2}$

Ans. 56: (a)

Solution: $B = \frac{E}{c}$

$$|\vec{S}| = \frac{1}{\mu_0}E.B = \frac{E^2}{\mu_0 c} = \frac{E_0^2\omega^4}{2\mu_0 c}\frac{\sin^2\theta}{r^2}\cos^2\left[\omega\left(t - \frac{r}{c}\right)\right]$$

$$\langle|\vec{S}|\rangle = \frac{1}{2}\frac{E_0^2\omega^4}{2\mu_0 c}\frac{\sin^2\theta}{r^2} \qquad \because \left\langle\cos^2\left[\omega\left(t - \frac{r}{c}\right)\right]\right\rangle = \frac{1}{2}$$

$$P = \oint_S \langle|\vec{S}|\rangle.d\vec{a} = \frac{E_0^2\omega^4}{4\mu_0 c}\int_0^{\pi}\int_0^{2\pi}\frac{\sin^2\theta}{r^2}r^2\sin\theta\, d\theta\, d\phi$$

$$P = \frac{E_0^2\omega^4}{4\mu_0 c}\times\frac{4}{3}\times 2\pi = \frac{2\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$$

Ans. 57: (b)

Solution: Parity does not change in transition from $\left(\frac{9}{5}\right)^+ \rightarrow \left(\frac{7}{2}\right)^+$

Now, $I_i = \frac{9}{2};\ I_i = \frac{7}{2}$

$\Delta I = 1, 2, 3, ...8$

The most probable transition is $\Delta I = 1$ and without change in parity. Therefore, the multiple radiation is $M1$.

Ans. 58: (d)

Solution: $\int_0^{\infty} \frac{\ln x^{-2}}{\left(x^2+1\right)^2} dx = -2\int_0^{\infty} \frac{\ln x}{\left(x^2+1\right)^2} dx = -2\int_0^{\infty} \frac{\ln z}{\left(z^2+1\right)^2} dz$

Let us consider new function $f(z) = \left(\frac{\ln z}{z^2+1}\right)^2$, then $I = \int_0^{\infty} \left(\frac{\ln z}{z^2+1}\right)^2 dz$

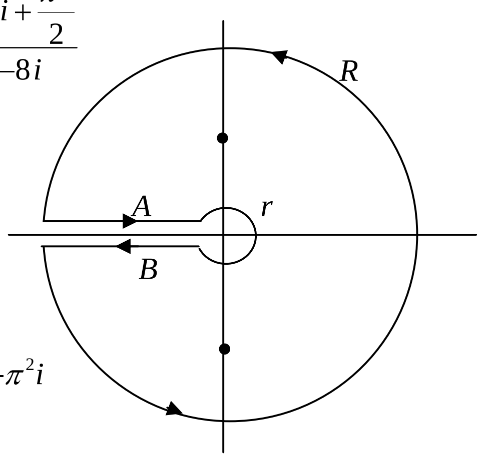
Pole at $z = \pm i$ is simple pole of second order.

Residue at $z = i$ is

$$= \frac{d}{dz}(z-i)^2 \frac{(\ln z)^2}{(z-i)^2 (z+i)^2} = \frac{d}{dz}\frac{(\ln z)^2}{(z+i)^2}$$

$$= \frac{(z+i)^2\, 2(\ln z).\frac{1}{z} - (\ln z)^2 .2(z+i)}{(z+i)^4} = \frac{(z+i)2\ln(z)\frac{1}{z} - (\ln z)^2 .2}{(z+i)^3}$$

$$= \frac{(2i)2\times\frac{1}{i}\ln i - (\ln i)^2 \cdot 2}{(2i)^3} = \frac{4\frac{i\pi}{2} - \left(\frac{i\pi}{2}\right)^2 \times 2}{-8i} = \frac{2\pi i + \frac{\pi^2}{2}}{-8i}$$



$$\Rightarrow \text{Res}\big|_{z=i} = \frac{-\pi}{4} + \frac{\pi^2}{16} i$$

Similarly at $z = -i$; $\text{Res}\big|_{z=-i} = \frac{-\pi}{4} - \frac{\pi^2}{16} i$

$$I = \int_0^{\infty} \left(\frac{\ln z}{z^2+1}\right)^2 dz = 2\pi i\left(\frac{-\pi}{4} + \frac{\pi^2}{16} i - \frac{\pi}{4} - \frac{\pi^2}{16} i\right) = -\pi^2 i$$

$$-\pi^2 i = \left(\int_R \int_A \int_B \int_r\right) f(z)\,dz = \left(\int_A \int_B\right) f(z)\,dz;$$

$\int_A \int_B$ vanish

Along path A; $z = -x + i\varepsilon$ and along path B; $z = -x - i\varepsilon$

Thus $-\pi^2 i = \left(\iint_{A\,B}\right) f(z)\,dz = -\int_{\infty}^{0}\left[\frac{\ln(-x+i\varepsilon)}{(-x+i\varepsilon)^2+1}\right]dx - \int_{0}^{\infty}\left[\frac{\ln(-x-i\varepsilon)}{(-x-i\varepsilon)^2+1}\right]dx$

$$\Rightarrow -\pi^2 i = \int_0^\infty\left[\frac{\ln(-x+i\varepsilon)}{(-x+i\varepsilon)^2+1}\right]^2 dx - \int_0^\infty\left[\frac{\ln(-x-i\varepsilon)}{(-x-i\varepsilon)^2+1}\right]^2 dx$$

$$\Rightarrow -\pi^2 i = \int_0^\infty\left[\frac{\ln(x)+i\pi}{1+x^2}\right]^2 dx - \int_0^\infty\left[\frac{\ln(x)-i\pi}{1+x^2}\right]^2 dx;\quad \varepsilon\to 0$$

$$\Rightarrow -\pi^2 i = \int_0^\infty \frac{(\ln(x)+i\pi)^2-(\ln(x)-i\pi)^2}{(1+x^2)^2}dx = 4\pi i\int_0^\infty\frac{\ln x}{(x^2+1)^2}$$

$$\Rightarrow \int_0^\infty\frac{\ln x}{(x^2+1)^2} = \frac{-i\pi^2}{4\pi i} = \frac{-\pi}{4} \Rightarrow -2\int_0^\infty\frac{\ln x}{(x^2+1)^2} = \frac{i\pi^2}{4\pi i} = \frac{\pi}{2}$$

Ans. 59: (d)

Solution: $f(\theta) = -\frac{2m}{\hbar^2 k}\int_0^\infty V(r)r\sin kr\,dr \quad \Rightarrow D(\theta)\propto |f(\theta)|^2$

$$f(\theta) = -\frac{2m}{\hbar^2 k}\int_0^\infty \beta\frac{e^{-\mu r}}{r}r\sin kr\,dr$$

$$f(\theta) = -\frac{2m\beta}{\hbar^2 k}\int_0^\infty \frac{e^{-\mu r}}{r}r\left(\frac{e^{ikr}-e^{-ikr}}{2i}\right)dr$$

$$= -\frac{2m\beta}{\hbar^2 2ik}\left(\int_0^\infty e^{-r(\mu-ik)}dr - \int_0^\infty e^{-r(\mu+ikr)}dr\right) = -\frac{2m\beta}{\hbar^2 2ik}\left[\frac{\mu+ik-\mu+ik}{\mu^2+k^2}\right] = -\frac{2m\beta}{\hbar^2}\frac{2ik}{2ik}(\mu^2+k^2)^{-1}$$

$$f(\theta) = -\frac{2m}{\hbar^2}\frac{\beta}{(\mu^2+k^2)},\qquad D(\theta) = \left(\frac{2m\beta}{\hbar^2(\mu^2+k^2)}\right)^2 \qquad \text{where } k=\sqrt{\frac{2mE}{\hbar^2}}\left(\sin\frac{\theta}{2}\right)$$

$$D(\theta) = \left(\frac{2m\beta}{\left(\mu^2\hbar^2+2mE\sin^2\frac{\theta}{2}\right)}\right)^2$$

Ans. 60: (d)

Solution: The partition function takes the form $Z_N = (1/N!)\left(Z_{1(tr)}\right)^N \left(Z_{1(mag)}\right)^N$, where $Z_{1(tr)} = V/\lambda_T^3$. Each atom has magnetic energy $E(s) = -(1/2)s\mu B$, where $s = \pm 1$. The magnetic partition function for a single atom is $Z_{1(mag)} = \Sigma_{s=\pm 1} e^{-s\beta\mu B/2} = 2\cosh\left[(\beta\mu B)/2\right]$. The partition function for the gas is $Z_N = (1/N!)\left(2V/\lambda_T^3\right)^N \cosh^N\left[(\beta\mu B)/2\right]$.

The magnetization is given by $M = -(\partial\Phi/\partial B)_{T,N}$, where Φ is the free energy this system (the translational part is like a Helmholtz free energy, a function of T, V, and N and the magnetic part is like Gibbs free energy, a function of T, B and N). The free energy of the combined system doesn't have a name so we call it Φ. Then $\Phi = -k_B T \ln Z_N$ $M = -(\partial\Phi/\partial B)_{T,N} = (1/2)N\mu \tanh\left[(\beta\mu B)/2\right]$.

Ans. 61: (b)

Solution: $a = 3cm, b = 1.5cm$ a

$$f_{c,\,mn} = \frac{\upsilon_0}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\therefore \quad \lambda_{c,\,mn} = \frac{\upsilon_0}{f_{c,\,mn}}$$

$$\therefore \quad \lambda_{c,\,10} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = 2a = 2\times 3 = 6cm$$

and $\lambda_{c,20} = \frac{2\ \ a}{2} = 3cm$

Ans. 62: (c)

Solution: ${}^{49}_{24}Cr : Z = 24,\ N = 25$

For $N = 25$

$(1s_{1/2})^2(1p_{1/2})^4(1p_{1/2})^2(1d_{3/2})^6(2s_{1/2})^2(1d_{3/2})^4(1f_{7/2})^5$

$\Rightarrow j=\dfrac{7}{2}$ and $l=3$. Therefore parity is $=(-1)^3=-1$

Thus spin and parity of $^{49}_{24}Cr\ =\left(\dfrac{7}{2}\right)^-$

$^{49}_{25}Mn$: $Z=25,\ N=24$

For $Z=25$: $(1s_{1/2})^2(1p_{1/2})^4(1p_{1/2})^2(1d_{5/2})^6(2s_{1/2})^2(1d_{3/2})^4(1f_{7/2})^5$

$\Rightarrow j=7/2$ and $l=3$

Thus spin and parity of $^{49}_{25}Mn=\left(\dfrac{7}{2}\right)^-$

Ans. 63: (b)

Solution: $\dfrac{\partial F_2}{\partial q}=p\Rightarrow \exp(\gamma t)P=p$, $\Rightarrow \dfrac{\partial F_2}{\partial P}=Q\Rightarrow \exp(\gamma t)q=Q\Rightarrow q=Q\exp(-\gamma t)$

$$K=H(Q,P,t)+\frac{\partial F_2}{\partial t}\Rightarrow K=\frac{P^2}{2m}+\frac{1}{2}m\omega^2Q^2+\gamma QP$$

$$\dot{P}=-\frac{\partial K}{\partial Q}=-m\omega^2Q-\gamma P$$

$$\dot{Q}=\frac{\partial K}{\partial P}=\frac{P}{m}+\gamma Q\Rightarrow \ddot{Q}+\left(\omega^2-\gamma^2\right)Q=0$$

$Q=A\sin\sqrt{(\omega^2-\gamma^2)}t$ will satisfy boundary condition $Q(t=0)=0$

Ans. 64: (b)

Solution: Given $\omega\propto k\Rightarrow \omega=\upsilon_s k$

Now, $g(\omega)d\omega=2\left(\dfrac{L}{2\pi}\right)^2\cdot 2\pi k dk=\dfrac{L^2}{\pi}\cdot\dfrac{\omega}{\upsilon_s}\cdot\dfrac{d\omega}{\upsilon_s}$

Also, $N=\displaystyle\int_0^{\omega_D}g(\omega)d\omega=\frac{L^2}{\pi\upsilon_{s^2}}\int_0^{\omega_D}\omega d\omega=\frac{L^2}{\pi\upsilon_{s^2}}\cdot\frac{\omega_D^2}{2}$

$$\Rightarrow \omega_D=\left(2\pi\upsilon_{s^2}\cdot\frac{N}{L^2}\right)^{1/2}=\upsilon_s(2\pi n)^{1/2}\Rightarrow 2\pi\upsilon_D=\upsilon_s(2\pi n)^{1/2}\Rightarrow \upsilon_D=\upsilon_s\left(\frac{2}{2\pi}\right)^{1/2}$$

Ans. 65: (a)

Solution: $\left(\frac{d^2}{dx^2}+k^2\right)\psi(x)=g(x)$

$$\left(\frac{d^2}{dx^2}k^2\right)\psi=0$$

$$u_1=e^{-ikx}, u_2=e^{ikx}$$

$$G(x,x')=\begin{cases} A\ y_1(x')y_2(x) & x>x' \\ A\ y_2(x')y_1(x) & x<x' \end{cases}$$

$$P(x)=1$$

$$A=\frac{1}{y_2'(x)y_1(x)-y_1'(x)y_2(x)}=\frac{1}{ik+ik}=\frac{-i}{2k}$$

$$G(x,x')=\frac{-i}{2k}\exp\left(i|x-x'|\right)$$

Ans. 66: (a)

Solution: $\frac{A_{21}}{B_{21}}=\frac{8\pi h v^3}{c^3}$

$$\tau=\frac{1}{A_{21}}=\frac{c^3}{8\pi h v^2 B_{21}}=\frac{\lambda^3}{8\pi h B_{21}}$$

$$=\frac{\left(4000\times10^{-10}\right)^3}{8\pi\left(6\cdot625\times10^{-34}\right)\left(6\cdot5\times10^{19}\right)}=\frac{6\cdot4\times10^{-20}}{1\cdot08\times10^{-12}}=5\cdot93\times10^{10^{-8}}\sec=59\cdot3n\sec$$

Ans. 67: (c)

Solution: $\frac{d^2 f}{dx^2}-\frac{2x}{\left(1-x^2\right)}\frac{df}{dx}+\frac{n(n+1)}{\left(1-x^2\right)}f=0$

Regular singular points -1, +1, ∞

Ans. 68: (c)

Solution: $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-x_e\left(n+\frac{1}{2}\right)^2\hbar\omega$

$$\left.\frac{dE_n}{dn}\right|_{n=n_{\max}} = 0 \Rightarrow \hbar\omega - 2x_e\left(n_{\max}+\frac{1}{2}\right)\hbar\omega = 0$$

$$\Rightarrow 1 = 2x_e\left(n_{\max}+\frac{1}{2}\right) \Rightarrow n_{\max} = \frac{1}{2x_e} - \frac{1}{2} \cong \frac{1}{2x_e} = \frac{1}{2\times 0\cdot 004} = 125$$

Ans. 69: (d)

Solution: $\rho' = \dfrac{Q}{L}, J_x' = 0$, $\quad J_x = \dfrac{J_x' + v\rho'}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{v\dfrac{Q}{L}}{\sqrt{1-\dfrac{v^2}{c^2}}} = \dfrac{vQ}{L\sqrt{1-\dfrac{v^2}{c^2}}}$

Ans. 70: (b)

Solution: Because this is an isothermal process, $\Delta U = 0$ and $q_{reversible} = -w$. From,

$$q_{reversible} = -w = nRT\int_{V_i}^{V_f}\frac{dV}{V} = nRT\ln\frac{V_f}{V_i}$$

$$= 1.00\,mol \times 8.314\,mol^{-1}K^{-1} \times 300\,K \times \ln\frac{10.0\,L}{25.0\,L} = -2.285\times 10^3\,J$$

The entropy change of the system is given by

$$\Delta S = \int\frac{dq_{reversible}}{T} = \frac{q_{reversible}}{T} = \frac{-2.285\times 10^3\,J}{300\,K} = -7.62\,JK^{-1}$$

The entropy change of the surroundings is given by

$$\Delta S_{surroundings} = \frac{q_{surroundings}}{T} = \frac{q_{system}}{T} = \frac{2.285\times 10^3\,J}{300\,K} = 7.62\,JK^{-1}$$

The total change in the entropy is given by

$$\Delta S_{total} = \Delta S + \Delta S_{surroundings} = -7.62\,JK^{-1} + 7362\,JK^{-1} = 0$$

Because the process is reversible, there is no direction of spontaneous change and, therefore, $\Delta S_{total} = 0$.

Ans. 71: (a)

Solution: **(i)** $h = 0.5$; The value of x and y are tabulated below:

x	y
0.0	1.0000
0.5	0.6667
1.0	0.5000

Simpson's 1/3 rule

$$I = \int_{x_0}^{x_n} y dx = \frac{h}{3}\left[y_0 + 4\left(y_1 + y_3 + ... + y_{n-1}\right) + 2\left(y_2 + y_4 + ... + y_{n-2}\right) + y_n\right]$$

$$\Rightarrow I = \frac{0.5}{3}\left[1.0000 + 4\left(0.6667\right) + 0.5\right] = 0.6945$$

Ans. 72: (b)

Solution: The cyclotron frequency is $\omega_c = \frac{eB}{m}$

Where m^* is effective mass defined as $m^* = \frac{\hbar^2}{d^2E/dk^2}$

Since, $E = E_0 - 2A\alpha\left(1 - \frac{k^2a^2}{2}\right)e^{-\alpha a}$

$$\Rightarrow \frac{dE}{dk} = 2A\alpha\left(ka^2\right)e^{-\alpha a}$$

$$\frac{d^2E}{dk^2} = 2A\alpha a^2 e^{-\alpha a} \Rightarrow m^* = \frac{\hbar^2}{2A\alpha a^2 e^{-\alpha a}}$$

$$\therefore \omega_c = \frac{eB}{m^*} = \frac{eB}{\hbar^2 / 2A\alpha a^2 e^{-\alpha a}} = \frac{eB}{\hbar^2}\left(2A\alpha a^2 e^{-\alpha a}\right)$$

$$\therefore A = \frac{\hbar^2 \omega_c}{2eB\alpha a^2 e^{-\alpha a}}$$

Ans. 73: (a)

Solution: The number of k state in range k to $k + dk$:

In $2D$, it is given by $g\left(k\right)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k dk$

Since, dispersion relation is $E = |P|v = \hbar kv$

$$g(E)dE = 2\times\left(\frac{L}{2\pi}\right)^2 2\pi\frac{EdE}{(\hbar v)^2} = \frac{L^2}{\pi\hbar^2 v^2}$$

The number of electron at $T = 0^0 K$ is

$$N = \int_0^{E_F} g(E)d(E) = \frac{L^2}{\pi\hbar^2 v^2}\int_0^{E_F} EdE = \frac{L^2}{2\pi\hbar^2 v^2}E_F^2 \Rightarrow 2\pi\hbar^2 v^2 . \frac{N}{L^2} = E_F^2$$

$$E_F^2 = 2\pi\hbar^2 v^2 \rho \qquad \left(\rho = \frac{N}{L^2}\right)$$

The average energy at $T = 0K$ is

$$E_{av} = \frac{\int_0^{E_F} E\cdot g(E)dE}{N} = \frac{L^2}{N\pi\hbar^2 v^2}\int_0^{E_F} E^2 dE = \frac{L^2 E_F^3}{3N\pi\hbar^2 v^2}$$

$$E_{av} = \frac{L^2}{3N\pi\hbar^2 v^2}\times 2\pi\hbar^2 v^2 \rho\sqrt{2\pi\hbar^2 v^2}\,\rho^{1/2} = \frac{2L^2}{3N}\sqrt{2\pi}\hbar v\rho^{3/2}$$

$$\frac{E}{L^2} = \frac{NE_{av}}{L^2} = \frac{2}{3}\sqrt{2\pi}\hbar v\rho^{3/2} \qquad \Rightarrow \frac{E}{L^2} \propto \rho^{3/2}$$

Ans. 74: (a)

Solution: In KCl crystal, all mixed (hkl) are absent and out of all unmixed (hkl), all odd unmixed are also absent.

Ans. 75: (a)

Solution: From conservation of energy $\frac{mv_0^2}{2} = \frac{mv^2}{2} - \frac{GMm}{R}$

If b is impact parameter then from conservation of momentum $mbv_0 = mRv$

$$b = r\sqrt{1+\frac{2GM}{v_0^2 R}}$$

$$D(\theta) = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

$$\sigma = \int D(\theta)\sin\theta d\theta d\phi = \int\frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|\sin\theta d\theta d\phi = \int b\ dbd\phi = \pi b^2$$

$$\sigma = \pi R^2\left(1+\frac{2GM}{v_0^2 R}\right)$$
