IIT-JAM 2022

Section A

Multiple Choice Questions

Q.1-Q.10 Carry ONE marks each.

Q1. The equation $z^2+\bar{z}^2=4$ in the complex plane (where $\bar{z}$ is the complex conjugate of z) represents

(a) Ellipse (b) Hyperbola (c) Circle of radius 2 (d) Circle of radius 4

Ans.1: (b)

Solution: $z=x+iy$ & $\bar{z}=x-iy$

$z^2+\bar{z}^{2}=4 \Rightarrow (x+iy)^2+(x-iy)^2=4$

$\Rightarrow x^2-y^2=2$ Equation of Hyperbola

Q2. A rocket (S') moves at a speed $\frac{c}{2} m/s$ along the positive x-axis, where c is the speed of light. When it crosses the origin, the clocks attached to the rocket and the one with a stationary observer (S) located at $x=0$ are both set to zero. If S observes an event at (x,t) the same event occurs in the S' frame at

(a) $x'=\frac{2}{\sqrt{3}}\left(x-\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t-\frac{x}{2c}\right)$

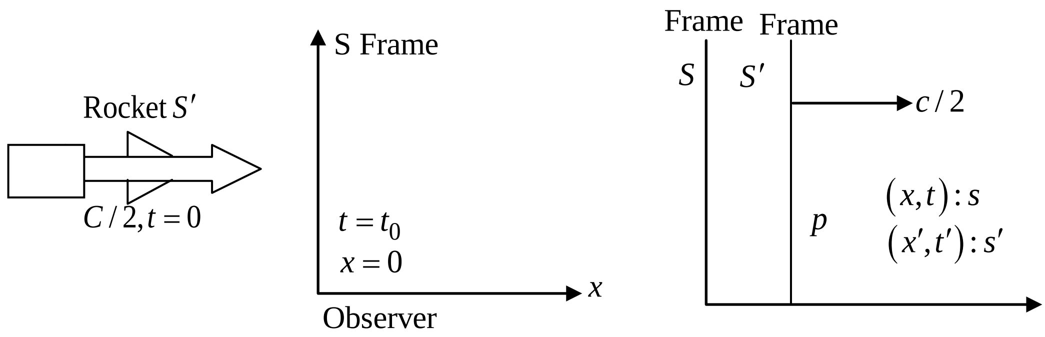
(b) $x'=\frac{2}{\sqrt{3}}\left(x+\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t-\frac{x}{2c}\right)$

(c) $x'=\frac{2}{\sqrt{3}}\left(x-\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t+\frac{x}{2c}\right)$

(d) $x'=\frac{2}{\sqrt{3}}\left(x+\frac{ct}{2}\right)$ and $t'=\frac{2}{\sqrt{3}}\left(t+\frac{x}{2c}\right)$

Ans. 2: (a)

Solution: From the question one Rocket S' moving with speed of $C/2$ and one observer is standing $x=0$. For point P the coordinate at S frame is (x,y) and the coordinate at S' frame is (x',y')



$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{x-\frac{c}{2}t}{\sqrt{1-\frac{c^2}{4c^2}}};\ x' = \left(x-\frac{c}{2}t\right)\Big/\sqrt{1-\frac{1}{4}};\ x' = \frac{2}{\sqrt{3}}[x-ct/2]$$

$$t' = \frac{t-xv/c^2}{\sqrt{1-v^2/c^2}} = \frac{t-\frac{c}{2}x/c^2}{\sqrt{1-c^2/4c^2}};\ t' = \left[t-\frac{x}{2c}\right]\Big/\sqrt{1-\frac{1}{4}};\ t' = \frac{2}{\sqrt{3}}\left[t-\frac{x}{2c}\right]$$

$$x' = \frac{2}{\sqrt{3}}[x-ct/2];\ t' = \frac{2}{\sqrt{3}}\left[t-\frac{x}{2c}\right]$$

So option (a) is correct.

Q3. Consider a classical ideal gas of N molecules equilibrium at temperature T. Each molecule has two energy levels, $-\in$ and $\in$. The mean energy of the gas is

(a) 0 (b) $N\in\tanh\left(\frac{\in}{k_BT}\right)$ (c) $-N\in\tanh\left(\frac{\in}{k_BT}\right)$ (d) $\frac{\in}{2}$

Ans. 3: (c)

Solution: The partition function for a single gas molecule is

$$Z_1 = e^{-\beta\varepsilon} + e^{\beta\varepsilon}$$

Mean energy per particle,

$$\langle E_1\rangle = -\frac{\partial \ln Z}{\partial\beta} = -\frac{-\partial\left[\ln\left(e^{-\beta\varepsilon}+e^{\beta\varepsilon}\right)\right]}{\partial\beta}$$

$$= -\frac{1}{e^{-\beta\varepsilon}+e^{\beta\varepsilon}}\left[\left(e^{-\beta\varepsilon}(-\varepsilon)+e^{\beta\varepsilon}(\varepsilon)\right)\right]$$

$$= -\varepsilon\frac{e^{\beta\varepsilon}-e^{-\beta\varepsilon}}{e^{\beta\varepsilon}+e^{-\beta\varepsilon}}$$

$\therefore$ for a classical system of N-molecules,

the mean energy is

$$U = N\langle E_1\rangle = -N\varepsilon\frac{e^{\beta\varepsilon}+e^{-\beta\varepsilon}}{e^{\beta\varepsilon}+e^{-\beta\varepsilon}} = -N\varepsilon\tanh\left(\frac{\varepsilon}{k_\beta T}\right)$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q4. At a temperature T, let β and k denote the volume expansively and isothermal compressibility of a gas, respectively. Then $\frac{\beta}{k}$ is equal to

(a) $\left(\frac{\partial P}{\partial T}\right)_V$ (b) $\left(\frac{\partial P}{\partial V}\right)_T$ (c) $\left(\frac{\partial T}{\partial P}\right)_V$ (d) $\left(\frac{\partial T}{\partial V}\right)_P$

Ans. 4: (a)

Solution: given

$$\beta=\frac{1}{V}\left(\frac{\partial V}{\partial T}\right) \text{ and } k=\frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\frac{\beta}{k}=\frac{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P}{\frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T}=-\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P V\left(\frac{\partial V}{\partial P}\right)_T=-\left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_P=\left(\frac{\partial P}{\partial T}\right)_V$$

Here, in the last step we made use of reciprocity theorem i.e

$$\left(\frac{\partial x}{\partial y}\right)_z=-\left(\frac{\partial x}{\partial z}\right)_y\left(\frac{\partial z}{\partial y}\right)_x$$

Q5. The resultant of the binary subtraction $1110101-0011110$ is

(a) 1001111 (b) 1010111 (c) 1010011 (d) 1010001

Ans. 5: (b)

Solution: 2s compliment of $0011110 \rightarrow$

$$\begin{array}{r} 1100001 \\ +1 \\ \hline 1100010 \end{array}$$

Thus $1110101-0011110$ is

$$\begin{array}{r} 1110101 \\ 1100010 \\ \hline 11010111 \end{array}$$

Q6. Consider a particle trapped in a three-dimensional potential well such that $U(x,y,z)=0$ for $0\le x\le a, 0\le y\le a, 0\le z\le a$ and $U(x,y,z)=\infty$ everywhere else. The degeneracy of the 5^{th} excited state is

(a) 1 (b) 3 (c) 6 (d) 9

Ans. 6: (c)

Solution: Energy for 3d Potential well is

$$E=\frac{\pi^2\hbar^2}{2m}\left[\frac{n_x^2}{a^2}+\frac{n_y^2}{b^2}+\frac{n_z^2}{c^2}\right]$$

From question $a=b=c=a$ so,

$$E=\frac{\pi^2\hbar^2}{2ma^2}\left[n_x^2+n_y^2+n_z^2\right]$$

Energy State	(n_x,n_y,n_z)	Energy	Degeneracy
Ground State	(1,1,1)	$3\pi^2\hbar^2/2ma^2$	1
1st Excited State	(1,1,2), (1,2,1), (2,1,1)	$6\pi^2\hbar^2/2ma^2$	3
2nd Excited State	(1,2,2), (2,1,2), (2,2,1)	$9\pi^2\hbar^2/2ma^2$	3
3rd Excited State	(1,1,3), (1,3,1), (3,1,1)	$11\pi^2\hbar^2/2ma^2$	3
4th Excited State	(2,2,2)	$12\pi^2\hbar^2/2ma^2$	1
5th Excited State	(1,2,3), (2,1,3), (2,3,1), (3,1,2), (3,2,1), (1,3,2)	$14\pi^2\hbar^2/2ma^2$	6

Q7. A particle of mass m and angular momentum L moves in space where its potential energy is $U(r)=kr^2\,(k>0)$ and r is the radial coordinate. If the particle moves in a circular orbit, then the radius of the orbit is

(a) $\left(\frac{L^2}{mk}\right)^{\frac{1}{4}}$ (b) $\left(\frac{L^2}{2mk}\right)^{\frac{1}{4}}$ (c) $\left(\frac{2L^2}{mk}\right)^{\frac{1}{4}}$ (d) $\left(\frac{4L^2}{mk}\right)^{\frac{1}{4}}$

Ans. 7: (b)

Solution: Let the velocity of the particle is v and angular momentum L.

Method 1:- We have

$$U(r)=kr^2$$

$$\text{Force}\,(F)=-\frac{\partial U(r)}{\partial r}=-2kr$$

We know that centripetal force

$$F=-\frac{mv^2}{r}$$

$$-\frac{mv^2}{r}=-2kr\ ;\ v^2=\frac{2kr^2}{m};\ v=\sqrt{\frac{2kr^2}{m}}$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

angular momentum $L = mvr$

$$L = m\sqrt{\frac{2kr^2}{m}}\cdot r;\quad L^2 = m^2\left(\frac{2kr^2}{m}\right)r^2;\quad L^2 = 2km\,\varepsilon^4$$

$$r^4 = \frac{L^2}{2km};\qquad r = \left(\frac{L^2}{2km}\right)^{1/2}$$ So option (b) is right.

Method:- 2

We know that the effective potential of the system is

$$V_{eff} = \frac{L^2}{2mr^2} + kr^2\ ;\ \frac{dV_{eff}}{d\varepsilon} = \frac{-L^2}{mr^3} + 2kr$$

at $r = r_0$, $\frac{dV_{eff}}{dr} = 0$

$$2kr_0 = \frac{L^2}{2mr_0^3};\ r_0^4 = \frac{L^2}{2mk} \Rightarrow r_0 = \left(\frac{L^2}{2mk}\right)^{1/4}$$

Q8. Consider a two-dimensional force field

$$\vec{F}(x, y) = (5x^2 + ay^2 + bxy)\hat{x} + (4x^2 + 4xy + y^2)\hat{y}$$

If the force field is conservative, then the values of a and b are

(a) $a = 2$ and $b = 4$ (b) $a = 2$ and $b = 8$

(c) $a = 4$ and $b = 2$ (d) $a = 8$ and $b = 2$

Ans. 8: (b)

Solution:

$$\vec{\nabla}\times\vec{F} = 0 \Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 5x^2 + ay^2 + bxy & 4x^2 + 4xy + y^2 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}(8x + 4y - 2ay - bx) = 0 \qquad \Rightarrow 8x + 4y = 2ay + bx$$

$$\Rightarrow (8-b)x + (4-2a)y = 0 \qquad \Rightarrow b = 8,\ a = 2$$

Q9. Consider an electrostatic field $\vec{E}$ in a region of space. Identify the INCORRECT statement.

(a) The work done in moving a charge in a closed path inside the region is zero

(b) The curl of $\vec{E}$ is zero

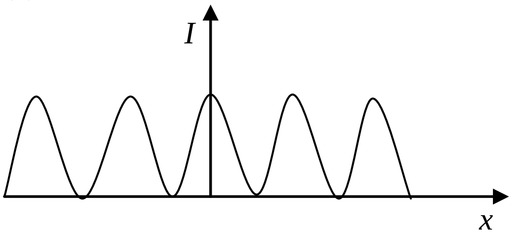
(c) The field can be expressed as the gradient of a scalar potential

(d) The potential difference between any two points in the region is always zero

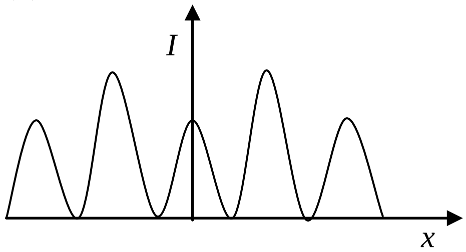
Ans. 9: (d)

Q10. Which one of the following figures correctly depicts the intensity distribution for Fraunhofer diffraction due to a single slit? Here, x denotes the distance from centre of the central fringe and I denotes the intensity.

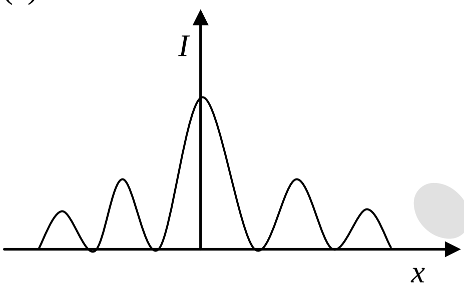
(a)



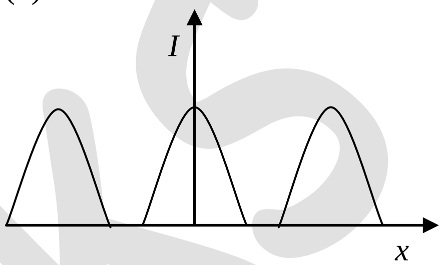
(b)



(c)

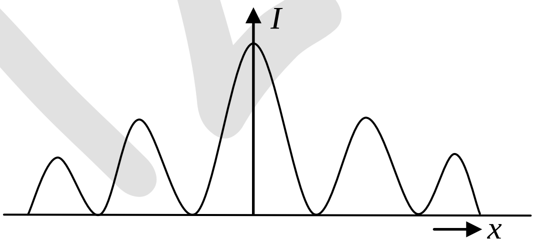


(d)



Ans. 10: (c)

Solution: Resultant intensity $I = A^2\left(\frac{\text{Sin}\,\alpha}{\alpha}\right)^2$



Q.11-Q.30 Carry TWO marks each.

Q11. The function $f(x) = e^{\sin x}$ is expanded as a Taylor series in x, around $x = 0$, in the form $f(x) = \sum_{n=0}^{\infty} a_n x^n$. The value of $a_0 + a_1 + a_2$ is

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 5

Ans. 11: (c)

Solution: $f(x) = e^{\sin x}$

Taylor's series around $x = 0$

$f(x)=\sum_{n=0}^{\infty} a_n x^n$ where $a_n=\dfrac{f^n(0)}{n!} \Rightarrow a_0=f(0)$

$a_1=\dfrac{f'(0)}{1!}, a_2=\dfrac{f''(0)}{2!}$

$f(x)=e^{\sin x} \Rightarrow t(0)=e^{\sin 0}=e^0=1$

$f'(x)=e^{\sin x}\cdot\cos x \;=\cos x\, f(x) \Rightarrow f'(0)=1\times 1=1$

$f''(x)=-\sin x\, f(x)+\cos x\, f'(x) \Rightarrow f''(0)=-0\times 1+1\times 1=1$

$\Rightarrow a_0+a_1+a_2=f(0)+\dfrac{f'(0)}{1!}+\dfrac{f''(0)}{2!}=1+1+\dfrac{1}{2}=\dfrac{5}{2}$

Q12. Consider a unit circle C in the xy plane, centered at the origin. The value of the integral $\oint[(\sin x-y)dx-(\sin y-x)dy]$ over the circle C, traversed anticlockwise, is

(a) 0 (b) 2π (c) 3π (d) 4π

Ans. 12: (b)

Solution: $\phi_c(\sin x-y)dx-(\sin y-x)dy$

Using Green's theorem for a plane

$\phi_c\, \phi dx+\psi\, dy=\iint_s\left(\dfrac{\partial\psi}{\partial x}-\dfrac{\partial\phi}{\partial y}\right)dxdy$

$\phi=\sin x-y \Rightarrow \dfrac{\partial\phi}{\partial y}=-1$

$\psi=-\sin y+x \Rightarrow \dfrac{\partial\psi}{\partial x}=1$

$\Rightarrow \phi_c(\sin x-y)dx-(\sin y-x)dy=\iint_S(1+1)dxdy$

$=2\cdot\iint_S dxdy=2\cdot\pi r^2$

$r=1$ for unit circle $=2\pi$

Q13. The current through a series RL circuit, subjected to a constant $emf\ \varepsilon$. Obeys $L\dfrac{di}{dt}+iR=\varepsilon$. Let $L=1mH, R=1k\Omega$ and $\varepsilon=1V$. The initial condition is $i(0)=0$ at $t=1\mu s$, the current in mA is

(a) $1-2e^{-2}$ (b) $1-2e^{-1}$ (c) $1-e^{-1}$ (d) $2-2e^{-1}$

Ans. 13: (c)

Solution:

$i(t)=\dfrac{E}{R}\left[1-e^{-t/L/R}\right]=\dfrac{1V}{1K\Omega}\left[1-e^{\frac{-1\mu s}{1mA/1k}}\right]=\left[1-e^{-1}\right]$

Q14. An ideal gas in equilibrium at temperature T expands isothermally to twice its initial volume. If $\Delta S, \Delta U$ and ΔF denote the changes in its entropy, internal energy and Helmholtz free energy respectively, then

(a) $\Delta S < 0, \Delta U > 0, \Delta F < 0$
(b) $\Delta S > 0, \Delta U = 0, \Delta F < 0$
(c) $\Delta S < 0, \Delta U = 0, \Delta F > 0$
(d) $\Delta S > 0, \Delta U > 0, \Delta F = 0$

Ans. 14: (b)

Solution: For an Ideal gas undergoing isothermal expression

$$dU = d\theta - dW$$

$$dU = 0$$

$$d\theta = dW = PdV$$

$$\Delta S = \int dS = \int_{v_i}^{v_f} \frac{d\theta}{T} = \int_{v_i}^{v_f} \frac{P}{T} dV = nR \ln\left(\frac{v_f}{v_i}\right) > 0$$

$$\Delta F = -Pdv - SdT$$

$$dT = 0, dV > 0$$

$$\Rightarrow \Delta F < 0$$

$$\therefore \Delta S > 0, \Delta U = 0, \Delta F < 0$$

Q15. In a dilute gas, the number of molecules with free path length $\geq x$ is given by $N(x) = N_0 e^{-x/\lambda}$, where N_0 is the total number of molecules and λ is the mean free path. The fraction of molecules with free path lengths between λ and 2λ is

(a) $\frac{1}{e}$
(b) $\frac{e}{e-1}$
(c) $\frac{e^2}{e-1}$
(d) $\frac{e-1}{e^2}$

Ans. 15: (d)

Solution: The fraction of molecules that do not undergo collisions after path length x is $= e^{-x/\lambda}$, Therefore, the fraction of molecules with free path length between $\lambda \to 2\lambda$ is given by

$$f = e^{-\frac{\lambda}{\lambda}} - e^{-\frac{2\lambda}{\lambda}}$$

$$= e^{-1} - e^{-2} = \frac{1}{e} - \frac{1}{e^2} = \frac{e-1}{e^2}$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q16. Consider a quantum particle trapped in a one-dimensional potential well in the region $[-L/2 < x < L/2]$, with infinitely high barriers at $x=-L/2$ and $x=L/2$. The stationary wave function for the ground state is $\psi(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$. The uncertainties in momentum and position satisfy

(a) $\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x = 0$

(b) $\Delta p = \frac{2\pi\hbar}{L}$ and $0 < \Delta x < \frac{L}{2\sqrt{3}}$

(c) $\Delta p = \frac{\pi\hbar}{L}$ and $\Delta x > \frac{L}{2\sqrt{3}}$

(d) $\Delta p = 0$ and $\Delta x = \frac{L}{2}$

Ans. 16: (b)

Solution: The stationary wave function for the ground state is $\psi(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$

$$\langle x\rangle = \int_{-\infty}^{\infty} x|\psi(x,t)|^2\,dx = \int_{-\infty}^{\infty} x\frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_{-L/2}^{L/2} x\frac{1}{2}\left\{\cos\left(\frac{2\pi x}{L}\right)-1\right\}dx$$

$$\langle x\rangle = 0$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\psi(x,t)|^2\,dx = \int_{-\infty}^{\infty} x^2\frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_{-L/2}^{L/2} x^2\frac{1}{2}\left\{\cos\left(\frac{2\pi x}{L}\right)-1\right\}dx$$

$$\langle x^2\rangle = \frac{1}{L}\int_{-L/2}^{L/2} x^2 * \left\{\cos\left(\frac{2\pi x}{L}\right)-1\right\}dx = \frac{L^2}{12}-\frac{L^2}{2\pi^2}$$

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{L^2}{12}-\frac{L^2}{2\pi^2}-0} = \sqrt{\frac{L^2}{12}-\frac{L^2}{2\pi^2}} = \frac{L}{2\sqrt{3}}\sqrt{\left(1-\frac{6}{\pi^2}\right)} = \frac{L}{2\sqrt{3}}\times(0.62)$$

So $0 < \Delta x < \frac{L}{2\sqrt{3}}$

$$\langle p\rangle = m\frac{d\langle x\rangle}{dt} = 0$$

We know that $\langle E\rangle = \frac{\langle p^2\rangle}{2m}$ and $\langle E\rangle = \frac{n^2\pi^2\hbar^2}{2ma^2}$

Put the $n=1$ and $a=L/2$

$$\langle p^2\rangle = \frac{4\pi^2\hbar^2}{L^2}$$

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{\frac{4\pi^2\hbar^2}{L^2}-0} = \sqrt{\frac{4\pi^2\hbar^2}{L^2}} = \frac{2\pi\hbar}{L}$$

Q17. Consider a particle of mass m moving in a plane with a constant radial speed $\dot{r}$ and a constant angular speed $\dot{\theta}$. The acceleration of the particle in (r,θ) coordinates is

(a) $2r\dot{\theta}^2\hat{r} - \dot{r}\dot{\theta}\hat{\theta}$ (b) $-r\dot{\theta}^2\hat{r} + 2\dot{r}\dot{\theta}\hat{\theta}$ (c) $\ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta}$ (d) $\ddot{r}\theta\hat{r} + r\ddot{\theta}\hat{\theta}$

Ans. 17: (b)

Solution: We have $\dot{\varepsilon} =$ Constant $\dot{\theta} =$ Constant

So $\ddot{\varepsilon} = 0$ $\ddot{\theta} = 0$

We know that $\vec{a} = \left(\ddot{\varepsilon} - \varepsilon\dot{\theta}^2\right)\hat{\varepsilon} + \left(\varepsilon\ddot{\theta} + 2\dot{\varepsilon}\dot{\theta}\right)\dot{\theta}$

$a = -\varepsilon\dot{\theta}^2\hat{\varepsilon} + 2\dot{\varepsilon}\dot{\theta}\hat{\theta}$

So option (b) is right.

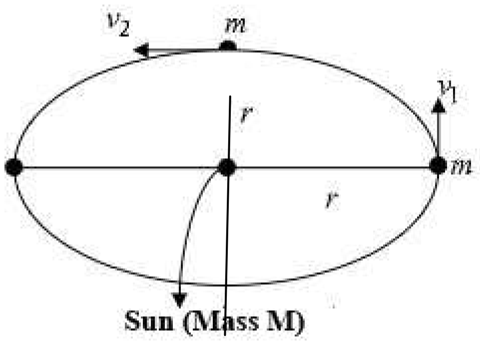
Q18. A planet of mass m moves in an elliptical orbit. Its maximum and minimum distances from the Sun are R and r, respectively. Let G denote the universal gravitational constant, and M the mass of the Sun. Assuming $M >> m$, the angular momentum of the planet with respect to the center of the Sun is

(a) $m\sqrt{\dfrac{2GMRr}{(R+r)}}$ (b) $m\sqrt{\dfrac{GMRr}{2(R+r)}}$ (c) $m\sqrt{\dfrac{GMRr}{(R+r)}}$ (d) $2m\sqrt{\dfrac{2GMRr}{(R+r)}}$

Ans. 18: (a)

Solution: Assume Sun is at the center of the elliptical orbit.

(1)



Consider conservation of Energy

$$\frac{1}{2}mv_1^2 - \frac{GMm}{R} = \frac{1}{2}mv_2^2 - \frac{GMm}{r}$$

(1)

Conservation of angular momentum

$$mrv_2 = mv_1R$$

$$v_2 = \left(\frac{R}{\varepsilon}\right)v_1$$

From Eq (1)

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = \frac{GMm}{R} - \frac{GMm}{r}$$

$Sum\left(Mass\,M\right)$

$$\frac{1}{2}mv_1^2 - \frac{1}{2}m\left(\frac{R}{r}\right)^2 v_1^2 = GMm\left[\frac{r-R}{rR}\right]$$

$$\frac{1}{2}mv_1^2\left[\frac{r^2-R^2}{r^2}\right]=GMm\left(\frac{r-R}{rR}\right)$$

$$\frac{1}{2}mv_1^2\left[\frac{(r-R)(r+R)}{r^2}\right]=GMm\left[\frac{r-R}{rR}\right]$$

$$\frac{1}{2}mv_1^2\left(\frac{R+r}{r}\right)=\frac{GMm}{R}$$

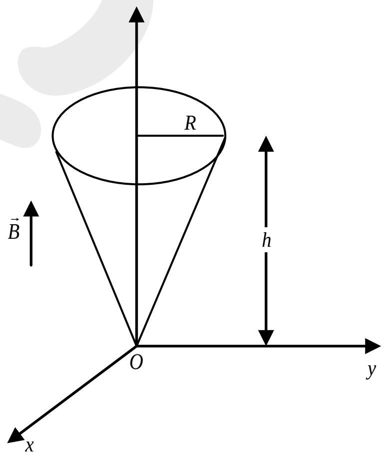
$$\frac{1}{2}mv_1^2=\frac{GMm\,r}{R(r+R)};\quad v_1^2=\frac{2GMr}{R(r+R)};\qquad v_1=\sqrt{\frac{2GMr}{R(r+R)}}$$

$$L=mv_1R$$

$$=m\left(\sqrt{\frac{2GMr}{R(r+R)}}\right)\times R;\qquad L=m\sqrt{\frac{2GMrR}{r+R}}$$

So option (a) is correct.

Q19. Consider a conical region of height h hand base radius R with its vertex at the origin, Let the outward normal to its base be along the positive z-axis, as shown in the figure. A uniform magnetic field, $\vec{B}=B_0\hat{z}$ exists everywhere. Then the magnetic flux through the base (ϕ_b) and that through the curved surface of the cone (ϕ_c) are



(a) $\phi_b=B_0\pi R^2;\phi_c=0$

(b) $\phi_b=-\frac{1}{2}B_0\pi R^2;\phi_c=\frac{1}{2}B_0\pi R^2$

(c) $\phi_b=0;\phi_c=-B_0\pi R^2$

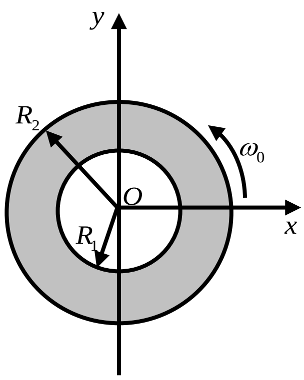
(d) $\phi_b=B_0\pi R^2;\phi_c=-B_0\pi R^2$

Ans. 19: (d)

Solution:

Base $\phi_b=\int_s\vec{B}\cdot d\vec{a}=\int_0^R\int_0^{2\pi}(B_0\hat{z})\cdot(rdrd\phi\hat{z})=B_0\times\frac{R^2}{2}\times2\pi=\pi B_0R^2$

$\Rightarrow\phi_c=-\pi B_0R^2$ Because $\phi_b+\phi_c=0$.



Q20. Consider a thin annular sheet, lying on the xy-plane, with R_1 and R_2 as its inner and outer radii, respectively. If the sheet carries a uniform surface-charge density σ and spins about the origin O with a constant angular velocity $\vec{\omega}=\omega_0\hat{z}$ then, the total current flow on the sheet is

(a) $\dfrac{2\pi\sigma\omega_0\left(R_2^3-R_1^3\right)}{3}$ (b) $\sigma\omega_0\left(R_2^3-R_1^3\right)$

(c) $\dfrac{\pi\sigma\omega_0\left(R_2^3-R_1^3\right)}{3}$ (d) $\dfrac{2\pi\sigma\omega_0\left(R_2-R_1\right)^3}{3}$

Ans. 20: (a)

Solution:

$$dI=\frac{dq}{dt}=\frac{\sigma\times 2\pi r dr}{2\pi/\omega_0}=\sigma\omega_0 r dr \quad \Rightarrow I=\sigma\omega_0\int_{R_1}^{R_2} r dr=\frac{\sigma\omega_0}{2}\left(R_2^2-R_1^2\right)$$

Q21. A radioactive nucleus has a decay constant λ and its radioactive daughter nucleus has a decay constant 10λ. At time $t=0, N_0$. No is the number of parent nuclei and there are no daughter nuclei present. $N_1(t)$ and $N_2(t)$ are the number of parent and daughter nuclei present at time t, respectively. The ratio $N_2(t)/N_1(t)$ is

(a) $\frac{1}{9}\left[1-e^{-9\lambda t}\right]$ (b) $\frac{1}{10}\left[1-e^{-10\lambda t}\right]$ (c) $\left[1-e^{-10\lambda t}\right]$ (d) $\left[1-e^{-9\lambda t}\right]$

Ans. 21: (a)

Solution:

We know that

$$N_1(t)=N_0e^{-\lambda_1 t}$$

$$N_2(t)=\frac{N_0\lambda_1}{\lambda_2-\lambda_1}\left[e^{-\lambda_1 t}-e^{-\lambda_2 t}\right]$$

So $\lambda_1=\lambda$; $\lambda_2=10\lambda$

$$N_1(t)=N_0e^{-\lambda t}$$

$$N_2(t)=\frac{N_0\lambda}{10\lambda-\lambda}\left[e^{-\lambda t}-e^{-10\lambda t}\right]$$

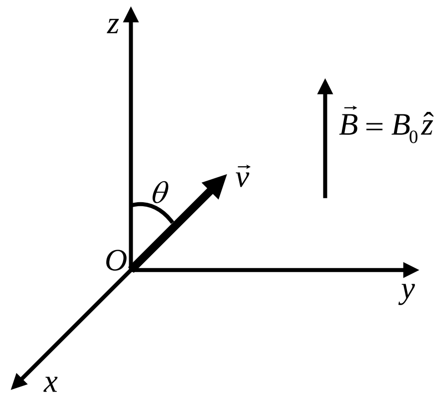
$$\frac{N_1(t)}{N_2(t)}=\frac{N_0e^{-\lambda t}}{\frac{N_0\lambda}{9\lambda}\left[e^{-\lambda t}-e^{-10\lambda t}\right]};\quad \frac{N_2(t)}{N_1(t)}=\frac{1}{9}\frac{\left[e^{-\lambda t}-e^{-10\lambda t}\right]}{e^{-\lambda t}}$$

$$\frac{N_2(t)}{N_1(t)}=\frac{1}{9}\left[e^{-\lambda t+\lambda t}-e^{-10\lambda t+\lambda t}\right];\quad \frac{N_2(t)}{N_1(t)}=\frac{1}{9}\left[1-e^{-9\lambda t}\right]$$

So option (a) is correct.

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q22. A uniform magnetic field $\vec{B} = B_0\hat{z}$, where $B_0 > 0$ exists as shown in the figure. A charged particle of mass m and charge $q(q > 0)$ is released at the origin, in the yz-plane, with a velocity $\vec{v}$ directed at an angle $\theta = 45^\circ$ with respect to the positive z-axis. Ignoring gravity, which one of the following is TRUE.



(a) The initial acceleration $\vec{a} = \dfrac{qvB_0}{\sqrt{2}m}\hat{x}$

(b) The initial acceleration $\vec{a} = \dfrac{qvB_0}{\sqrt{2}m}\hat{y}$

(c) The particle moves in a circular path

(d) The particle continues in a straight. line with constant in a straight line with constant speed

Ans. 22: (a)

Solution:

$$\vec{F} = q\vec{v}\times\vec{B} \Rightarrow m\vec{a} = q\left(\vec{v}\times B_0\hat{z}\right) = q\ {}_0B\sin 45^\circ\,\hat{x} \quad \Rightarrow \vec{a} = \frac{qB_0v}{\sqrt{2}m}\hat{x}$$

Q23. For an ideal intrinsic semiconductor, the Fermi energy at $0\,K$

(a) Lies at the top of the valence band

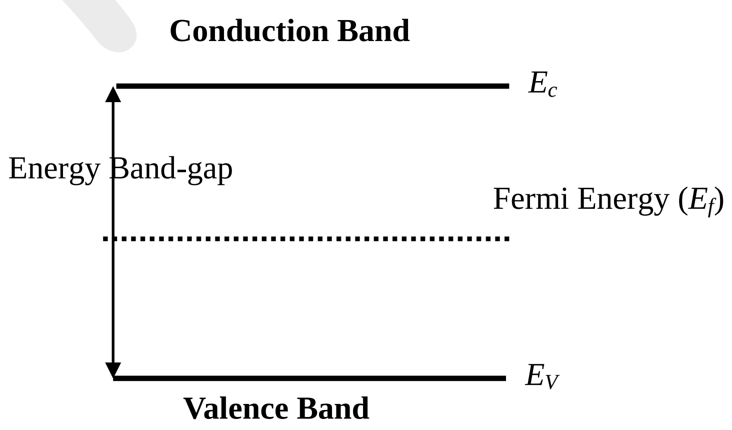
(b) Lies at the bottom of the conduction band

(c) Lies at the center of the band gap

(d) Lies midway between center of the band gap and bottom the of conduction band

Ans. 23: (c)

Solution:



Q24. A circular loop of wire with radius R is centered at the origin of the xy-plane. The magnetic field at a point within the loop is, $\vec{B}(\rho,\phi,z,t)=k\rho^3t^3\hat{z}$, where k is a positive constant of appropriate dimensions. Neglecting the effects of any current induced in the loop, the magnitude of the induced emf in the loop at t it is

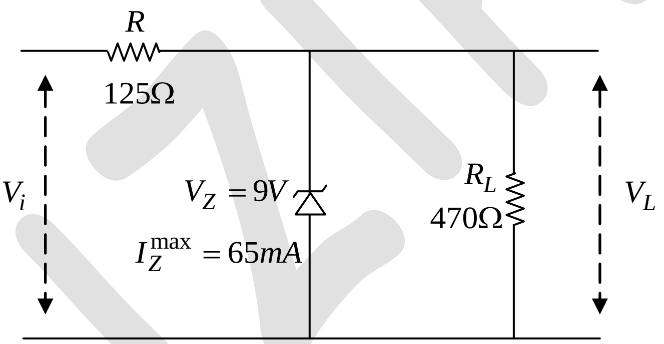
(a) $\frac{6\pi kt^2R^5}{5}$ (b) $\frac{5\pi kt^2R^5}{6}$ (c) $\frac{3\pi kt^2R^5}{2}$ (d) $\frac{\pi kt^2R^5}{2}$

Ans. 24: (a)

Solution: Mag flux $\phi_m=\int_s \vec{B}\cdot d\vec{a}=\int_0^R\int_0^{2\pi}\left(k\rho^3t^3\hat{z}\right)\cdot\left(\rho d\rho d\phi\hat{z}\right)\quad\Rightarrow\phi_m=k\frac{R^5}{5}t^3\times 2\pi$

$$E=-\frac{d\phi_m}{dt}=-\frac{-6\pi kt^2R^5}{5}$$

Q25. For the given circuit, $R=125\,\Omega, R_L=470\,\Omega, V_z=9V$, and $I_z^{max}=65\,mA$. The minimum and maximum value of the input voltage (V_i^{min} and V_i^{max}) for which the Zener diode will be in the 'ON' state are



(a) $V_i^{min}=9.0V$ and $V_i^{max}=11.4V$ (b) $V_i^{min}=9.0V$ and $V_i^{max}=19.5V$

(c) $V_i^{min}=11.4V$ and $V_i^{max}=15.5V$ (d) $V_i^{min}=11.4V$ and $V_i^{max}=19.5V$

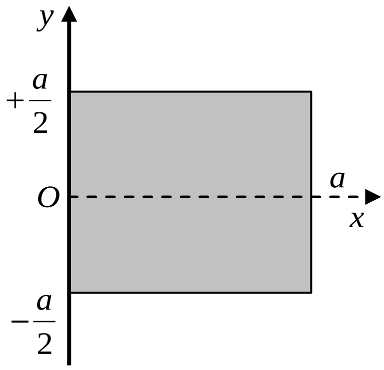
Ans. 25: (d)

$$V_{OC}=\frac{470\Omega}{125+470}\times V_i^{min}=9V\Rightarrow V_i^{min}=9V\times\frac{595}{470}=11.39V$$

$$V_i^{max}=I_{max}\times 125\Omega+9V=\left(I_z^{max}+\frac{9V}{470}\right)\times 125\Omega+9V=\left(65mA+19mA\right)\times 125+9V=19.5V$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q26. A square laminar sheet with side a and mass M, has mass per unit area given by $\sigma(x)=\sigma_0\left[1-\frac{x}{a}\right]$, (see figure). Moment of inertia of the sheet about y-axis is

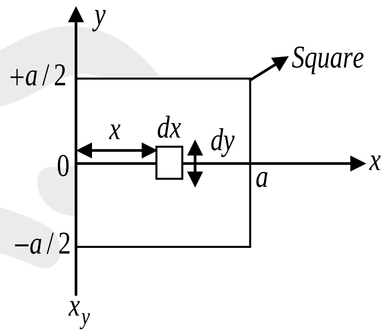


(a) $\frac{Ma^2}{2}$ (b) $\frac{Ma^2}{4}$

(c) $\frac{Ma^2}{6}$ (d) $\frac{Ma^2}{12}$

Ans. 26: (c)

Solution: We have mass per unit area given by



$$\sigma(x)=\sigma_0\left[1-\frac{x}{a}\right]$$

So mass of square linear sheet

$$M=\iint\sigma(x)\,dx\,dy$$

$$M=\int_{-a/2}^{+a/2}\int_0^a\sigma_0\left(1-\frac{x}{a}\right)dx\,dy$$

$$M=\bar{V}_0\int_{-a/2}^{a/2}dy\int_0^a\left(1-\frac{x}{a}\right)dx$$

$$M=\sigma_0(a)\times\left[x-\frac{x^2}{2a}\right]_0^a;\ M=\sigma_0a\times\left[a-\frac{a^2}{2a}-0+0\right]$$

$$M=\sigma_0a\times\left[a-\frac{a}{2}\right]=\sigma_0\frac{a^2}{2};\ M=\frac{\sigma_0a^2}{2}\Rightarrow\sigma_0=\frac{2M}{a^2}$$

The moment of Inertia with y-axis

$$I_{YY}=\int x^2dm$$

$$\sigma(x)=\frac{dm}{dA}$$

$$dx=\sigma(x)\,dA=\sigma(x)\,dxdy$$

$$I_{YY}=\int x^2\sigma(x)\,dx\,dy$$

$$I_{YY}=\int_0^a\int_{-a/2}^{a/2}\sigma_0x^2\left[1-\frac{x}{a}\right]dx\,dy;\ I_{YY}=\int_0^a\sigma_0\left(x^2-\frac{x^3}{a}\right)dx\int_{-a/2}^{a/2}dy$$

$$= \sigma_0 \left[\frac{x^3}{3} - \frac{x^4}{4a}\right]_0^a \times \left[\frac{a}{2} - \left(\frac{-a}{2}\right)\right] = \sigma_0 \left[\frac{a^3}{3} - \frac{a^4}{4a}\right] \times a$$

$$I_{YY} = \sigma_0 \left[\frac{a^3}{3} - \frac{a^3}{4}\right] a;\; I_{YY} = \sigma_0 \left[\frac{a^3}{12}\right] a;\; I_{YY} = \frac{2M}{a^2} \times \frac{a^4}{12}\;\; I_{YY} = \frac{2Ma^2}{12} = \frac{Ma^2}{6}$$

So, Option (c) is Correct.

Q27. A particle is subjected to two simple harmonic motions along the x and y axes, described by $x(t) = a\sin(2\omega t + \pi)$ and $y(t) = 2a\sin(\omega t)$. The resultant motion is given by

(a) $\frac{x^2}{a^2} + \frac{y^2}{4a^2} = 1$ (b) $x^2 + y^2 = 1$

(c) $y^2 = x^2\left(1 - \frac{x^2}{4a^2}\right)$ (d) $x^2 = y^2\left(1 - \frac{y^2}{4a^2}\right)$

Ans. 27: (d)

Solution: From question

$$x(t) = a\sin(2\omega t + \pi) \ldots\ldots\ldots\ldots\ldots\text{..(1)}$$

$$y(t) = 2a\sin\omega t$$

From Eq. (1), we have

$$x(t) = a\left[\sin(2\omega t)\cos\pi + \cos(2\omega t)\sin\pi\right]$$

We know that $\cos\pi = -1, \quad \sin\pi = 0$

$$x(t) = -a\sin 2\omega t = -2a\sin\omega t\cos\omega t$$

$$y(t) = 2a\sin\omega t \Rightarrow \sin\omega t = y/2a$$

So $x^2 + y^2 = 4a^2\sin^2\omega t\cos^2\omega t + 4a^2\sin^2\omega t$

$$x^2 + y^2 == 4a^2\sin^2\omega t\left[\cos^2\omega t + 1\right]$$

$$x^2 + y^2 == 4a^2\sin^2\omega t\left[1 - \sin^2\omega t + 1\right]$$

Put the value of $\sin\omega t = y/2a$

$$x^2 + y^2 == y^2\left[2 - y^2/4a^2\right]$$

$$x^2 = 2y^2 - \frac{y^4}{4a^2} - y^2; \quad x^2 = y^2 - y^4/4a^2; \qquad x^2 = y^2\left[1 - y^2/4a^2\right]$$

$$x^2 = y^2\left[1 - \frac{y^2}{4a^2}\right]$$

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Q28. For a certain thermodynamic system, the internal energy $U = PV$ and P is proportional to T^2. The entropy of the system is s proportional to

(a) UV (b) $\sqrt{\frac{U}{V}}$ (c) $\sqrt{\frac{V}{U}}$ (d) $\sqrt{UV}$

Ans. 28: (d)

Solution:

$U = PV$, $P = KT^2$

$$P = \frac{U}{V}, T = \left(\frac{P}{K}\right)^{1/2} = \left(\frac{U}{KV}\right)^{1/2}$$

$$\Rightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial U}\right) = \left(\frac{KV}{U}\right)^{1/2}$$

$$\Rightarrow dS = (kV)^{1/2} U^{-1/2} dU$$

$$\Rightarrow S_f - S_i = (kV)^{1/2} U^{1/2}$$

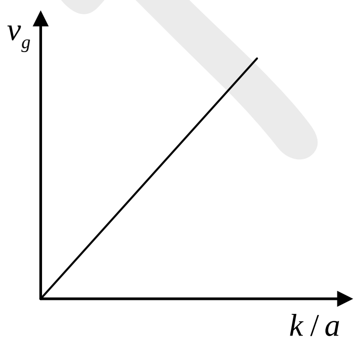
$$\Rightarrow S_f = k'(UV)^{1/2} + S_i$$

$$\Rightarrow S \propto (UV)^{1/2}$$

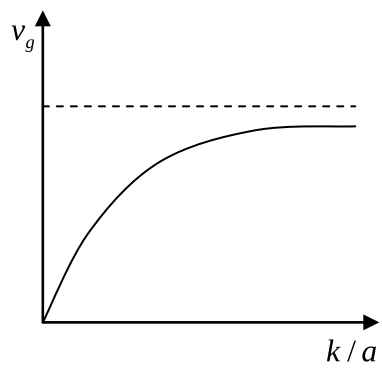
S_i being constant.

Q29. The dispersion relation for certain type of wave is given by $\omega = \sqrt{k^2 + a^2}$, where k is the wave vector and a is a constant. Which one of the following sketches represents v_g, the group velocity?

(a)

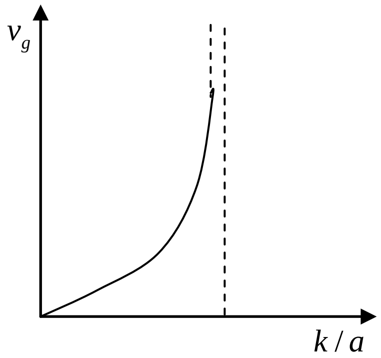


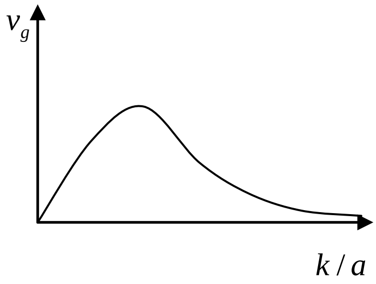
(b)



(c)

(d)



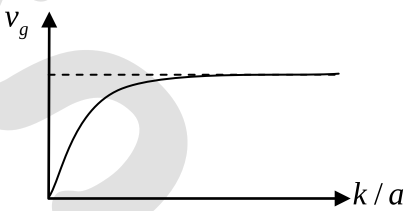


Ans. 29: (b)

Solution: From question, we have

$$\omega = \sqrt{R^2 + a^2}$$

We know that



$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(\sqrt{k^2 + a^2}\right)$$

$$v_g = \frac{d}{dk}\left[k^2 + a^2\right]^{-1/2}$$

$$= \left[k^2 + a^2\right]^{-1/2} 2k$$

$$v_g = \frac{k}{\sqrt{k^2 + a^2}}$$

$$v_g = \frac{k/a}{\sqrt{\left(\frac{k}{a}\right)^2 + 1}}$$

Let $k/a = y$

$$V_g = \frac{y}{\sqrt{y^2 + 1}}$$

Put the value $y = 0 \Rightarrow V_g = 0$

$y = 1 \Rightarrow V_g = 1/\sqrt{2} = 0.70$

$y = 2 \Rightarrow V_g = 2/\sqrt{5} = 0.89$

$y = 3 \Rightarrow V_g = 3/\sqrt{10} = 0.94$

For higher values of y the value of V_g tends to 1.

Q30. Consider a binary number with m digits, where m is an even number. This binary number has alternating $1's$ and $0's$, with digit 1 in the highest place value. The decimal equivalent of this binary number is

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
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(a) $2^m - 1$ (b) $\frac{(2^m - 1)}{3}$ (c) $\frac{(2^{m+1} - 1)}{3}$ (d) $\frac{2}{3}(2^m - 1)$

Ans. 30: (d)

Solution:

$(10)_2 = (2)_{10}$; $(1010)_2 = (10)_{10}$; $(101010)_2 = (42)_{10}$ ....

Decimal equivalent of binary number $= \frac{2}{3}(2^m - 1)$

For $m = 2$; Decimal equivalent $= 2$

For $m = 4$; Decimal equivalent $= 10$

For $m = 6$; Decimal equivalent $= 42$

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q.31 -0.4 Q.40 Carry TWO marks arks each.

Q31. Consider the 2×2 matrix $M = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}$. where $a, b > 0$. Then,

(a) M is a real symmetric matrix

(b) One of the eigenvalues of M is greater than b

(c) One of the eigenvalues of M is negative

(d) Product of eigenvalues M is b

Ans. 31: (a), (b), (c)

Solution: $m = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} = m' \Rightarrow$ Symmetric option (a) is correct

$$|m - \lambda I| = \begin{vmatrix} -\lambda & a \\ a & b - \lambda \end{vmatrix} = 0 \Rightarrow -\lambda(b - \lambda) - a^2 = 0 \Rightarrow \lambda^2 - b\lambda - a^2 = 0$$

$$\Rightarrow \lambda = \frac{b \pm \sqrt{b^2 + 4a^2}}{2}$$

$$a \,\&\, b > 0 \Rightarrow \lambda_1 = \frac{b - \sqrt{b^2 + 4a^2}}{2} > b$$ Option (b) is correct

$$\lambda_2 = \frac{b - \sqrt{b^2 + 4a^2}}{2} \Rightarrow \sqrt{b^2 + 4a^2} < b$$

$= (-)$ ve Option (c) is correct

Q32. In the Compton scattering of electrons, by photons incident with wave length λ,

(a) $\frac{\Delta\lambda}{\lambda}$ is independent of λ

(b) $\frac{\Delta\lambda}{\lambda}$ increases with decreasing λ

(c) there is no change in photon's wave length for all angle of deflection of the photon

(d) $\frac{\Delta\lambda}{\lambda}$ increases with increasing angle of deflection of the photon

Ans. 32: (b), (d)

Solution: We know that

$$\frac{\Delta\lambda}{\lambda}=\frac{\lambda_c[1-\cos\phi]}{\lambda} \qquad (1)$$

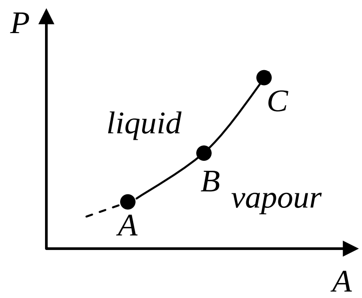
(a) $\frac{\Delta\lambda}{\lambda}$ is dependent of λ. So option (a) is incorrect.

(b) With Eq. (1), $\frac{\Delta\lambda}{\lambda}$ increases with decreasing λ. So option (b) is correct.

(c) If we change the value of deflection angle ϕ. So option (c) is incorrect.

(d) The maximum value of $\cos\phi$ is 1 for deflection angle $\phi=0$. Then if we increase the value of deflection angle $\frac{\Delta\lambda}{\lambda}$ increases. So option (d) is correct.

Q33. The figure shows a section of the phase boundary separating the vapour (1) and liquid (2) states of water in the $P-T$ plane. Here, C is the critical point. μ_1, v_1 and s_1 are the chemical potential, specific volume and specific entropy of the vapour phase respectively, while μ_2, v_2 and s_2 respectively denote the same for the liquid phase. Then



(a) $\mu_1=\mu_2$ along AB

(b) $v_1=v_2$ along AB

(c) $s_1=s_2$ along AB

(d) $v_1=v_2$ at the point C

Ans. 33: (a), (d)

Solution: Along equilibrium line AB

$\mu_1=\mu_2$

Above line $AB, \mu_2<\mu_1$

Below line $AB, \mu_1<\mu_2$

At critical point (and above) the distinction between liquid and vapour phase disappear, as a result $v_1=v_2$ at point C.

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Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q34. A particle is executing simple harmonic motion with time period T. Let x, v and a denote the displacement, velocity and acceleration of the particle, respectively, at time t. Then,

(a) $\frac{aT}{x}$ does not change with time

(b) $(aT + 2\pi v)$ does not change with time

(c) x and v are related by an equation of a straight line

(d) v and a are related by an equation of an ellipse

Ans. 34: (a), (d)

Solution: We know that simple harmonic motion of a particle, the variation of the displacement the velocity and the acieration is sinusoidal with time. So

$$x = A\sin(\omega t + \theta) \qquad (1)$$

$$V = \frac{dx}{dt} = A\omega^2 \cos(\omega t + \theta) \qquad (2)$$

$$a = \frac{dv}{dt} = -A\omega^2 \sin(\omega t + \theta) \qquad (3)$$

Multiply with $\frac{2\pi}{T}$ both sides in Equation (2)

$$\frac{2\pi}{T}V = \frac{2\pi A\omega}{T}\cos^2(\omega t + \theta) \qquad (4)$$

<u>Option (a):-</u>

$$a = -A\omega^2 \sin(\omega t + \theta)$$

$$a = -\omega^2 x \qquad \text{put } \omega = \frac{2\pi}{T}$$

$$a = \frac{-4\pi^2}{T^2}x; \quad \frac{aT}{x} = \frac{-4\pi^2}{T} = \text{Constant}$$

This equation is independent of tine (t). So option (a) is correct.

<u>Option (b):-</u>

$\Rightarrow aT + 2\pi v$

Put the values of a and v from Eq. (2) and (3)

$$aT + 2\pi v = -A\omega^2 T \sin(\omega t + \theta) + 2\pi A\omega \sin(\omega t + \theta)$$

$$aT + 2\pi v = -A\omega^2 T \sin(\omega t + \theta) + 2\pi A\omega \cos(\omega t + \theta)$$

This equation has variation of t. So option (b) is incorrect.

Option (c):-

From Eq. (1) and (2)

$$x+v=A\sin(\omega t+\theta)+A\omega(\cos\omega+\theta)$$

This equation has variation of t. So option (c) is incorrect.

Option (d):-

$$\frac{4\pi^2}{T^2}v^2+a^2=+A^2\omega^4\sin^2(\omega t+\theta)+\frac{4\pi^2A^2\omega^2}{T^2}\cos^2(\omega t+\theta)$$

Putt $\omega=\frac{2\pi}{T}$

$$a^2+\frac{4\pi^2}{T^2}v^2=+A^2\frac{4\pi^2}{T^2}\left[\frac{4\pi^2}{T^2}\sin^2(\omega t+\theta)+\frac{4\pi^2}{T^2}\cos^2(\omega t+\theta)\right]$$

$$=A^2\frac{4\pi^2}{T^2}\times\frac{4\pi^2}{T^2}\left[\sin^2(\omega t+\theta)+\cos^2(\omega t+\theta)\right]$$

$a^2+\frac{4\pi^2}{T^2}v^2=A^2\frac{16\pi^4}{T^4}$; $\quad \frac{a^2}{16\pi^2A^2/T^4}+\frac{v^2}{4\pi^2A^2}=1$ This equation is independent of tine (t). So option (d) is correct.

Q35. A linearly polarized light beam travels from origin to point $A(1,0,0)$. At the point A, the light is reflected by a mirror towards point $B(1,-1,0)$. A second mirror located at point B then reflects the light towards point $C(1,-1,1)$. Let $\hat{n}(x,y,z)$ represent the direction of polarization of light at (x,y,z).

(a) If $\hat{n}(0,0,0)=\hat{y}$, then $\hat{n}(1,-1,1)=\hat{x}$

(b) If $\hat{n}(0,0,0)=\hat{z}$, then $\hat{n}(1,-1,1)=\hat{y}$

(c) If $\hat{n}(0,0,0)=\hat{y}$, then $\hat{n}(1,-1,1)=\hat{y}$

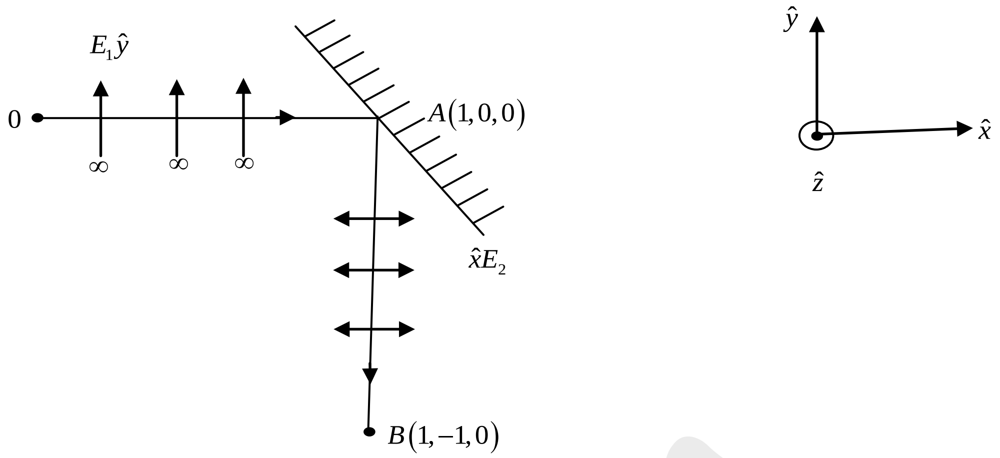
(d) If $\hat{n}(0,0,0)=\hat{z}$, then $\hat{n}(1,-1,1)=\hat{x}$

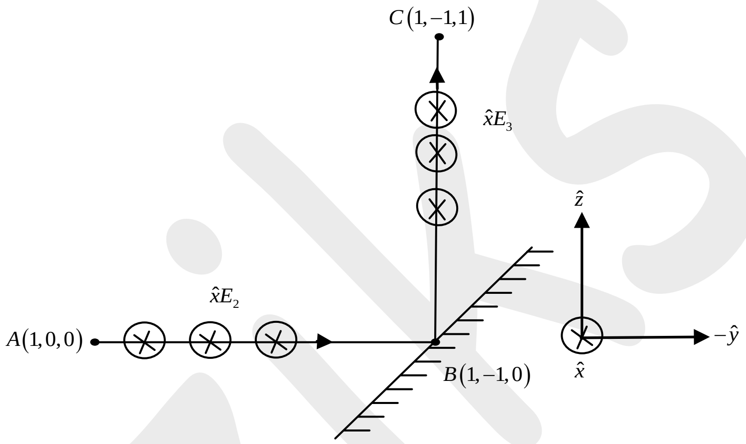
Ans. 35: (a), (b)

Solution: If plane polarized light is incident on a mirror at some oblique angle, then only that component of electric field will propagate after reflection, whose direction is perpendicular to the propagation vector.

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

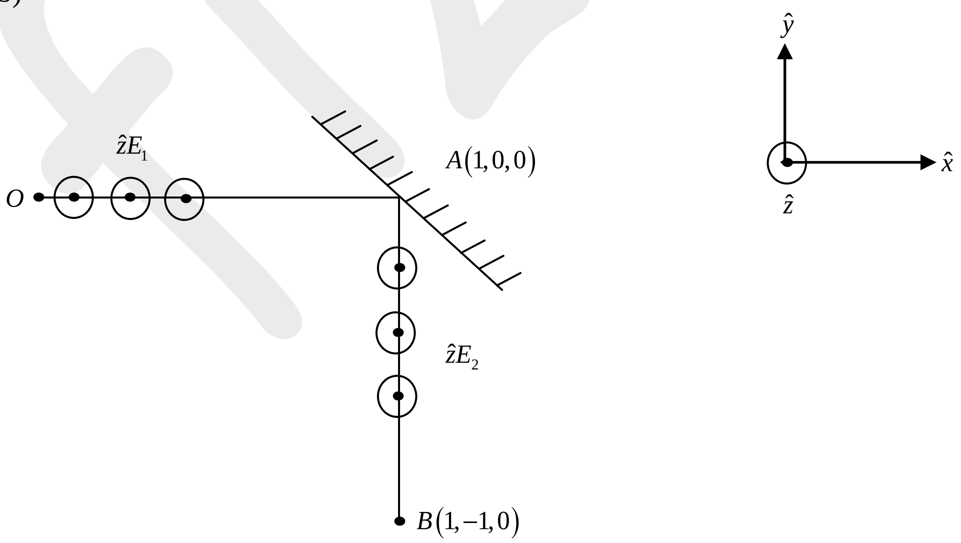
(a)

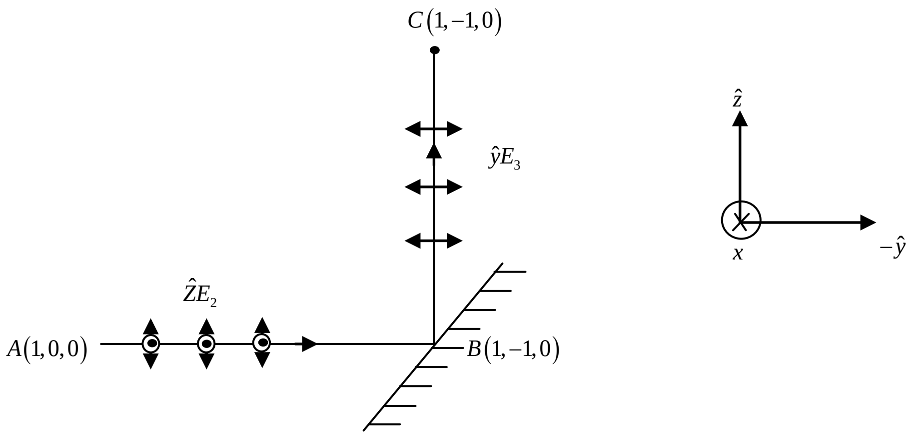




So, $\hat{\eta}(1,-1,1)=\hat{x}$

(b)



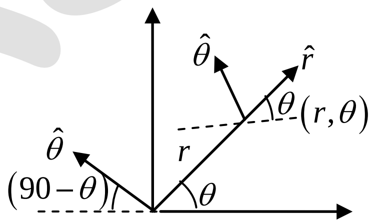


So, $\hat{\eta}(1,-1,1)=\hat{z}$

Q36. Let (r,θ) denote the polar coordinates of a particle moving in a plane. If $\hat{r}$ and $\hat{\theta}$ represent the corresponding unit vectors, then

(a) $\frac{d\hat{r}}{d\theta}=\hat{\theta}$ (b) $\frac{d\hat{r}}{dr}=-\hat{\theta}$ (c) $\frac{d\hat{\theta}}{d\theta}=-\hat{r}$ (d) $\frac{d\hat{\theta}}{dr}=\hat{r}$

Ans. 36: (a), (c)



Solution: $\hat{r}=\cos\theta\hat{i}+\sin\theta\hat{j}$

$\hat{\theta}=-\sin\theta\hat{i}+\cos\theta\hat{j}$

Now $\frac{d\hat{r}}{d\theta}=-\sin\theta\hat{i}+\cos\theta\hat{j}=\hat{\theta}$

option (a) is correct

$\frac{d\hat{\theta}}{d\theta}=-\cos\theta\hat{i}-\sin\theta\hat{j}=-\hat{r}$ option (c) is correct

Q37. The electric field associated with an electromagnetic radiation is given by $E=a(1+\cos\omega_1 t)\cos\omega_2 t$. Which of the following frequencies are present in the field?

(a) ω_1 (b) $\omega_1+\omega_2$ (c) $|\omega_1-\omega_2|$ (d) ω_2

Ans. 37: (b), (c), (d)

Solution:

$$E=a(\cos\omega_2 t+\cos\omega_1 t\cdot\cos\omega_2 t)=a\left[\cos\omega_2 t+2\cos\left(\frac{\omega_1+\omega_2}{2}\right)t\cdot\cos\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

Q38. A string of length L is stretched between two points $x=0$ and $x=L$. The endpoints are rigidly clamped. Which of the following can represent the displacement of the string from the equilibrium position?

(a) $x\cos\left(\frac{\pi x}{L}\right)$ (b) $x\sin\left(\frac{\pi x}{L}\right)$ (c) $x\left(\frac{x}{L}-1\right)$ (d) $x\left(\frac{x}{L}-1\right)^2$

Ans. 38: (b), (c), (d)

Solution: As string is clamped at $x=0$ and $x=L$, so following two condition should be satisfied:

(i) Displacement $y=0$ at $x=0$

(ii) Displacement $y=a$ at $x=L$

Above mentioned conditions are satisfied in case of options (b), (c) and (d).

Q39. The Boolean expression $Y=\overline{PQ}R+Q\overline{R}+\overline{P}QR+PQR$ simplifies to

(a) $\overline{P}R+Q$ (b) $PR+\overline{Q}$ (c) $P+R$ (d) $Q+R$

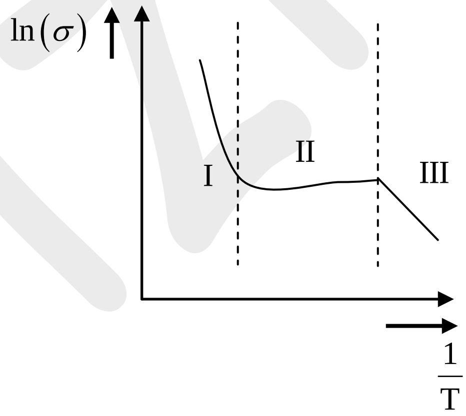
Ans. 39: (d)

Solution:

$$Y=\left(\overline{PQ}+PQ\right)R+Q\left(\overline{R}+\overline{P}R\right)$$

$$Y=R+Q\left(\overline{R}+\overline{\overline{R}}\,\overline{P}\right)=R+Q\left(\overline{R}+\overline{P}\right)=R+\overline{R}Q+\overline{P}Q=R+Q+\overline{P}Q=R+Q$$

Q40. For an n-type silicon, an extrinsic semiconductor, the natural logarithm of normalized conductivity (σ) is plotted as a function of inverse temperature. Temperature interval-I corresponds to the intrinsic regime, interval-II corresponds to saturation regime and interval-III corresponds to the freeze-out regime, respectively. Then



(a) The magnitude of the slope of the curve in the temperature interval-I is proportional to the band gap, E_g

(b) The magnitude of the slope of the curve in the temperature interval-III is proportional to the ionization n energy of the donor, E_d

(c) In the temperature interval-II, the carrier density in the conduction band is equal to the density of donors

(d) In the temperature interval-III, all the donor levels are ionized

Ans. 40: (a), (b), (c)

SECTION – C

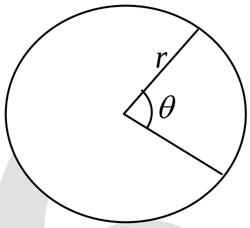
NUMERICAL ANSWER TYPE (NAT)

Q.41 - Q.50 Carry ONE mark each.

Q41. The integral $\iint\left(x^2+y^2\right)dxdy$ over the area of a disk of radius 2 in the xy plane is ____ π.

Ans. 41: 8 to 8

Solution: $\iint\left(x^2+y^2\right)dxdy$



in polar coordinate

$x = r\cos\theta$

$y = r\sin\theta \Rightarrow x^2+y^2 = r^2$

for $dxdy =$ Jacobean

$$J\left(xy;r\theta\right)=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta}\end{vmatrix}=\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}=r\left(\cos^2\theta+\sin^2\theta\right)=r$$

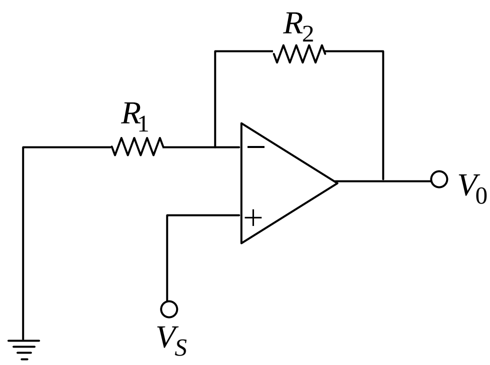
$$\Rightarrow dx\,dy = rdrd\theta$$

$$\Rightarrow \iint\left(x^2+y^2\right)dxdy=\int_0^R\int_0^{2\pi} r^2\cdot rdrd\theta, R=2$$

$$=\left(\frac{r^4}{4}\right)_0^2 2\pi=\frac{2^4}{4}\cdot 2\pi$$

$$=8\pi$$

Q42. For the given operational amplifier circuit $R_1 = 120\,\Omega, R_2 = 1.5\,k\Omega$ and $V_s = 0.6V$, then the output current I_0 is __________ mA.



Ans. 42: 5 to 5

Solution:

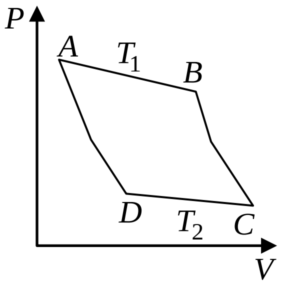
$$V_0=\left(1+\frac{R_2}{R_1}\right)V_S=\left(1+\frac{1.5k}{120\Omega}\right)\times 0.6V=\left(1+\frac{1500}{120}\right)\times 0.6V=8.1V$$

$$I_0=\frac{V_0-V_S}{R_2}=\frac{8.1V-0.6V}{1.5k}=\frac{7.5V}{1.5k}=5\,mA$$

Q43. For an ideal gas, AB and CD are two isothermals at temperatures T_1 and $T_2\left(T_1 > T_2\right)$, respectively. AD and BC represent two adiabatic paths as shown in figure.

Let V_A, V_B, V_C and V_D be the volumes of the gas at A, B, C and D respectively. If

$\frac{v_C}{v_B} = 2$, then $\frac{v_D}{v_A} =$ ______________ .



Ans. 43: 2 to 2

Solution: Point B and C are connected aria adiabatic process

$\therefore T_1 V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$

$$\frac{T_1}{T_2} = \left(\frac{V_C}{V_B}\right)^{\gamma-1} \qquad (1)$$

points $A \& D$ are also connected uia adiabatic process

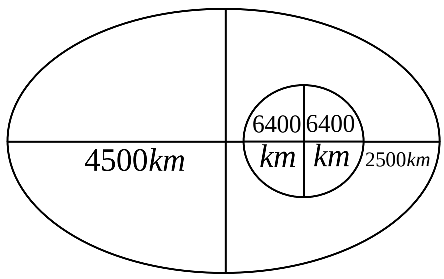
$\therefore T_1 V_A^{\gamma-1} = T_2 V_D^{\gamma-1}$

$$\left(\frac{T_1}{T_2}\right) = \left(\frac{V_D}{V_A}\right)^{\gamma-1} \qquad (2)$$

From Eq. (1) and Eq. (2) implies $\frac{V_C}{V_B} = \frac{V_D}{V_A} = 2$

Q44. A satellite is revolving around the Earth in a closed orbit. The height of the satellite above Earth's surface at perigee and apogee are $2500\,km$ and $4500\,km$, respectively. Consider the radius of the Earth to be $6500\,km$. The eccentricity of the satellite's orbit is _____ (Round off to 1 decimal place).

Ans. 44: 0.1 to 0.1



Solution:

$r_{\max} = 4500 + 6400 = 10400\,km$

$r_{\min} = 2500 + 6400 = 8900\,km$

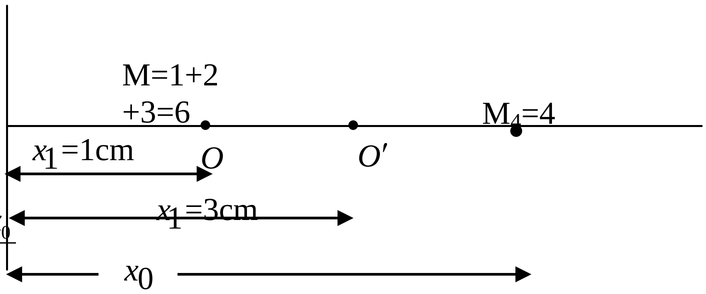
$$e = \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}} = \frac{10900 - 8900}{10900 + 8900}$$

$$e = \frac{2000}{19800} = \frac{10}{98} = \frac{5}{49} = 0.1$$

Q45. Three masses $m_1 = 1, m_2 = 2$ and $m_3 = 3$ are located on the x-axis such that their center of mass is at $x = 1$. Another mass $m_4 = 4$ is placed at x_0, and the new center of mass is at $x = 3$. The value of x_0 is ______ .

Ans. 45: 6 to 6

Solution:



$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{6\times1+4\times x_0}{6+4}$$

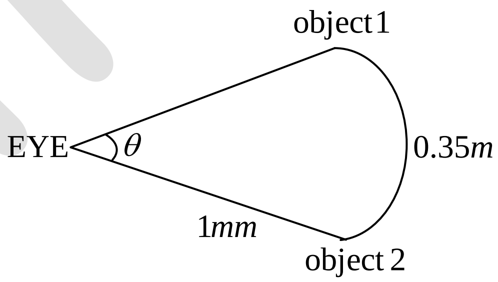
$$3 = \frac{6+4x_0}{10} \Rightarrow 6+4x_0 = 30$$

$$4x_0 = 24 \Rightarrow x_0 = 6cm$$

Q46. A normal human eye can distinguish two objects separated by $0.35m$ when viewed from a distance of $1.0\,km$. The angular resolution of eye is ___________ seconds (Round off to the nearest integer).

Ans. 46: 71 to 73

Solution: We know that
$\theta \times R =$ are length



$$\theta = \frac{0.35\,m}{1\times10^3\,m} = 3.5\times10^{-4}\,rad$$

$$1\text{rad} = \frac{180^\circ}{\pi};\ 1^\circ = 60';\ 1' = 60''$$

$$\theta = 3.5\times10^{-4}\times\frac{180^\circ}{\pi}\times60'\times60''$$

$$\theta = 718200\times10^{-4}'';\ \theta = 71.82'';\ \theta = 72''$$

Q47. A rod with a proper length of $3m$ moves along x-axis, making an angle of 30° with respect to the x-axis. If its speed is $\frac{c}{2}m/s$, where c is the speed of light, the change in length due to Lorentz contraction is_______ m (Round off to 2 decimal places).

$\left[\text{Use}\, c = 3\times10^8\,m/s\right]$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Ans. 47: 0.29 to 0.31

Solution:

Frame S' is moving with the speed $C/2$ and the length of the rod is $3m$ in the frame of S'. So

$x_B - x_A = \Delta x \quad x_B' - a_A' = \Delta x'$

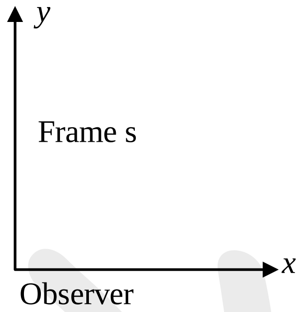
$y_B - y_A = \Delta y \quad y_B' - y_A' = \Delta y'$

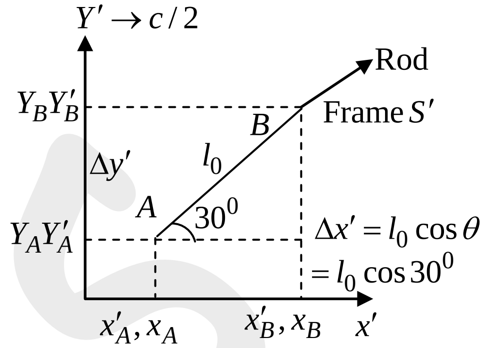
We know that

$$\Delta x = \Delta x' \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Delta y = \Delta y'$$

$$l_0^2 = \Delta x'^2 + \Delta y'^2$$





Moving frame

For observer at S from $l^2 = \Delta x^2 + \Delta y^2$

$$l^2 = \Delta x'^2 \left[1 - \frac{v^2}{c^2}\right] + \Delta y'^2$$

$$= \Delta x'^2 + \Delta y'^2 - \Delta x'^2 \frac{v^2}{c^2}$$

$$= l_0^2 - \Delta x'^2 \frac{v^2}{c^2}$$

$$= l_0^2 - l_0^2 \cos^2 30^\circ \frac{v^2}{c^2}$$

$$l^2 = l_0^2 \left[1 - \left(\frac{\sqrt{3}}{2}\right)^2 \frac{v^2}{c^2}\right]; \quad l^2 = l_0^2 \left[1 - \left(\frac{3}{4}\right) \frac{c^2}{4c^2}\right]; \quad l^2 = l_0^2 \left[1 - \frac{3}{16}\right]; \quad l^2 = l_0^2 \left[\frac{13}{16}\right]$$

$$l = l_0 \left[\frac{13}{16}\right]^{1/2}; \quad l = \frac{3 \times 3.605}{4} = 2.704\, m.$$

Chang in length $= \Delta l = l - l_0 \quad = 3.00 - 2.704 = 0.296$

$\Delta l = 0.30$

Q48. Consider the Bohr model of hydrogen atom. The speed of an electron in the second orbit $(n = 2)$ is __________ $\times 10^6 \, m/s$ (Round off to 2 decimal places).

[Use $h = 6.63 \times 10^{-34} Js, e = 1.6 \times 10^{-19} C, \epsilon_0 = 8.85 \times 10^{-12} C^2 m^2 / N$]

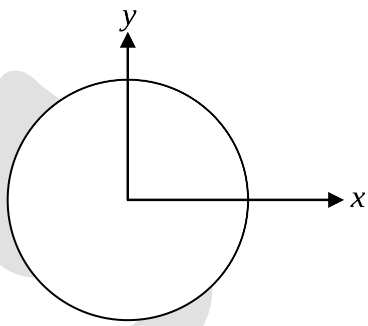
Ans. 48: 1.08 to 1.12

Solution: velocity of nth orbit $v_n = \frac{h}{2\pi m a^2}\left(\frac{1}{n}\right)$

For $n = 2$

$$v_n = \frac{6.63\times10^{-34}}{2\pi\times9.1\times10^{-31}\times(0.52\times10^{-10})}\left(\frac{1}{2}\right) = 1.09\times10^{6}\,m/s$$

Q49. Consider unit circle C in the xy plane with center at the origin. The line integral of the vector field, $\vec{F}(x,y,z) = -2y\hat{x} - 3z\hat{y} + x\hat{z}$, taken anticlockwise over C is _______ π.



Ans. 49: 2 to 2

Solution: $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{i} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -2y & -3z & x \end{vmatrix} = 3\hat{x} - \hat{y} + 2\hat{j}$

Vector field circularly over a plane for line integral of a vector for field using stake's theorem

$$\int_L f.dl = \iint_S (\nabla\times S)\cdot ds$$

$$= \iint\left(3\hat{x} - \hat{y} + 2\hat{j}\right).dxdy\,\hat{j} = \iint 2\,dxdy = 2\iint dxdy = 2\cdot\pi r^2$$ for unit circle $r = 1$

$$= 2\pi$$

Q50. Consider a $p-n$ junction at $T = 300\,K$. The saturation current density at reverse bias is $-20\,\mu A/cm^2$. For this device, a current density of magnitude $10\,\mu A/cm^2$ is realized with a forward bias voltage V_F. The same magnitude of current density can also be realized with a reverse bias voltage, V_R. The value of $|V_F/V_R|$ is____________ (Round off to 2 decimal places).

Ans. 50: 0.57 to 0.61

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q.51-Q60 Carry TWO marks each.

Q51. Consider the second order ordinary differential equation, $y''+4y'+5y=0$. If $y(0)=0$ and $y'(0)=1$, then the value of $y(\pi/2)$ is __________ (Round off to 3 decimal places).

Ans. 51: 0.041 to 0.045

Solution: $y''+4y'+5y=0$

$$\Rightarrow (D^2+4D+5)y=0$$

Auxiliary Equation

$$m^2+4m+5=0 \Rightarrow m=\frac{-4\pm\sqrt{16-20}}{2}$$

$$=-2\pm i$$

$$\Rightarrow y(x)=e^{-2x}(A\sin x+B\cos x)$$

$$x=0 \quad y=0 \Rightarrow 0=e^0(A\times\sin\theta+B\cos 0)$$

$$\Rightarrow B=0$$

$$y'(x)=A\left(\cos x\, e^{-2x}-2e^{-2x}\sin x\right)$$

$$y'(0)=A(1-0)=1 \Rightarrow A=1$$

$$\Rightarrow y(x)=\sin(x)e^{-2x}$$

$$\Rightarrow y\left(\frac{\pi}{2}\right)=\sin\frac{\pi}{2}\cdot e^{-2\cdot\frac{\pi}{2}}=e^{-\pi}=0.0415$$

Q52. A box contains a mixture of two different ideal monoatomic, 1 and 2, in equilibrium at temperature T. Both gases are present in equal proportions. The atomic mass for gas 1 is m, while the same for gas 2 is $2m$. If the rms speed of a gas molecule selected at random is $v_{rms}=x\sqrt{\frac{k_BT}{m}}$ then x is ____________ (Round off to 2 decimal places).

Ans. 52: 1.49 to 1.51

Solution:

$$\left.\begin{aligned} V_{rms_1}&=\sqrt{\frac{3K_BT}{m}} \\ V_{rms_2}&=\sqrt{\frac{3K_BT}{2m}} \end{aligned}\right]$$

$$V_{avg} = \frac{V_{rms_1} + V_{rms_2}}{2} = \frac{\left(\sqrt{3} + \sqrt{\frac{3}{2}}\right)}{2}\sqrt{\frac{K_B T}{m}}$$

$$= \frac{\sqrt{6} + \sqrt{3}}{2\sqrt{2}}\sqrt{\frac{K_B T}{m}}$$

$$= \frac{2.44949 + 1.73205}{2.82842}\sqrt{\frac{K_B T}{m}}$$

$$= 1.48\sqrt{\frac{K_B T}{m}}$$

$\therefore\ x = 1.48$

NOTE: The value obtained does not match with given range i.e $1.49\text{-}1.51$.

Q53. A hot body with constant heat capacity $800\,J/K$ at temperature $925\,K$ is dropped gently into a vessel containing $1\,kg$ of water at temperature $300\,K$ and the combined system is allowed to reach equilibrium. The change in the total entropy ΔS is J/K (Round off to 1 decimal place).

[Take the specific heat capacity of water to be $4200\,J/kg\,K$. Neglect any loss of heat to the vessel and air and change in the volume of water.]

Ans. 53: 537.5 to 537.7

Solution:

$C_1 = 800\,J/K,\ T_1 = 925\,K$

$m_\omega = 1\,kg, T_2 = 300\,K$

$C_\omega = 4200\,J/kg\,K$

Let T_f be the final temperature

Heat lost $=$ Heat gained

$$c_1\left(925 - T_f\right) = 1kg \times c_\omega\left(T_f - 300\right)$$

$$800\left(925 - T_f\right) = 1 \times 4200\left(T_f - 300\right)$$

$$7400 - 8T_f = 42T_f - 12600$$

$$50T_f = 7400 + 12600 = 20,000$$

$$T_f = 400\,K$$

Entropy change of block

$\Delta S_1 = C_1 \ln\left(\frac{T_f}{T_1}\right) = 800\ln\left(\frac{400}{925}\right)$

Entropy change of water

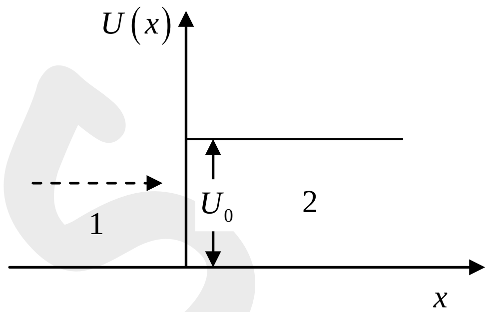
$\Delta S_2 = mc_\omega \ln\left(\frac{T_f}{T_2}\right) = 1\times 4200\ln\left(\frac{400}{300}\right)$

$\Delta S = \Delta S_1 + \Delta S_2$

$= -670.663352 + 1208.2647$

$\Delta S = 537.6$

Q54. Consider an electron with mass m and energy E moving along the x-axis towards a finite step potential of height U_0 as shown in the figure. In region $1(x<0)$, the momentum of the electron is $p_1 = \sqrt{2mE}$. The reflection coefficient at the barrier is given by $R = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2$, where p_2 is the momentum in region 2. If, in the limit $E \gg U_0, R \approx \frac{U_0^2}{nE^2}$, then the integer n is ________.



Ans. 54: 16 to 16

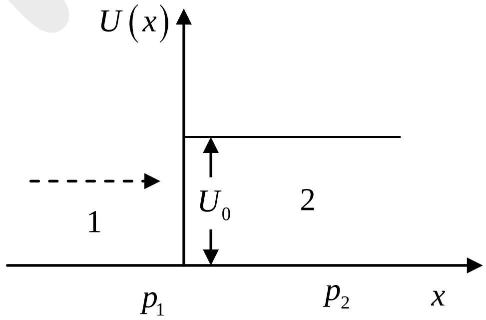
Solution:

In region -1, the momentum of electron is

$p_1 = \sqrt{2mE}$

As, step potential in second region so the momentum of electron is

$p_2 = \sqrt{2m(E-U_0)}$



The reflection coefficient at the barrier is

$R = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2$

$$R = \left(\frac{\sqrt{2mE} - \sqrt{2m(E-U_0)}}{\sqrt{2mE} + \sqrt{2m(E-U_0)}}\right)^2 = \sqrt{2mE}\left(\frac{1-\sqrt{1-\frac{U_0}{E}}}{1+\sqrt{1-\frac{U_0}{E}}}\right)^2$$

Form question $E \gg U_0, U_0/E \ll 1$

$$R = \left(\frac{1-\left(1-\frac{U_0}{2E}\right)}{1+\left(1-\frac{U_0}{E}\right)}\right)^2 = \left(\frac{\frac{U_0}{2E}}{2-\frac{U_0}{2E}}\right)^2 = \left(\frac{U_0}{4E}\right)^2 = \frac{U_0^2}{16E}$$

$n = 16$

Q55. A current density for a fluid flow is given by, $\vec{J}(x,y,z,t)=\frac{8e^{t}}{\left(1+x^2+y^2+z^2\right)}\hat{x}$.

At time $t=0$, the mass density $\rho(x,y,z,0)=1$.

Using the equation of continuity, $\rho(1,1,1,1)$ is found to be _______ (Round off to 2 decimal places).

Ans. 55: 2.70 to 2.74

Q56. The work done in moving a $-5\,\mu C$ charge in an electric field $\vec{E}=\left(8r\sin\theta\hat{r}+4r\cos\theta\hat{\theta}\right)V/m$, from a point $A(r,\theta)=\left(10,\frac{\pi}{6}\right)$ to a point $B(r,\theta)=\left(10,\frac{\pi}{2}\right)$, is ________ mJ.

Ans. 56: 1 to 1

Solution:

$r=10,\theta:{\pi}/{6}\rightarrow{\pi}/{2},dr=0$

$$W=\int\vec{F}\cdot d\vec{l}=\int q\vec{E}\cdot d\vec{l}=-5\times10^{-6}\int\left[8r\sin\theta\hat{r}+4r\cos\theta\hat{\theta}\right]\cdot\left(dr\hat{r}+rd\theta\hat{\theta}\right)$$

$$=-5\times10^{-6}\int_{\pi/6}^{\pi/2}\left[4r^2\cos\theta d\theta\right]=-5\times10^{-6}\times4\times(10)^2\left[-\sin\theta\right]_{\pi/6}^{\pi/2}$$

$$=-20\times10^{-4}\left[-\sin\frac{\pi}{2}+\sin\frac{\pi}{6}\right]=-20\times10^{-4}\left(-1+\frac{1}{2}\right)=1mJ$$

Q57. A pipe of $1m$ length is closed at one end. The air column in the pipe resonates at its frequency of $400\,Hz$. The number of nodes in the sound wave formed in the pipe is _______.

[Speed of sound $=320\,m/s$]

Ans. 57: 5

Solution: From question

Length of pipe is $1m$

$v=f=400\,H_2$

$v=320\,m/s$

$n=??$

We know that

$$f = \frac{nv}{4l}$$

$$400/s = \frac{n \times 320 m/s}{2 \times 2 \times 1m}$$

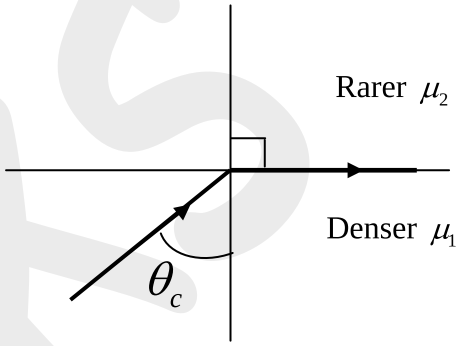
$$n = \frac{1600}{320 \times 2}$$

$$n = \frac{160}{32} = 5$$

Q58. The critical angle of a crystal is 30°. Its Brewster angle is ________ degree (Round off to the nearest integer).

Ans. 58: 63 to 63

Solution: $\theta = 30^\circ$



$$\frac{1}{\sin\theta} = \frac{\mu_2}{\mu_1}$$

where $\mu_2 \to$ Refractive index of rarer

$\mu_1 \to$ Refractive index of denser

$$\frac{\mu_2}{\mu_1} \frac{1}{\sin 30^\circ} = \frac{1}{1/2}$$

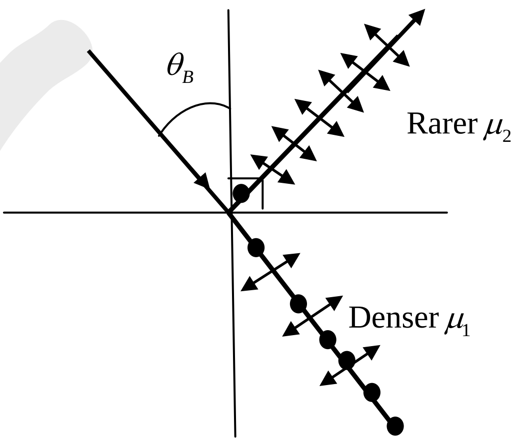
$$\frac{\mu_2}{\mu_1} = 2$$

$$\tan\theta_\beta = \frac{\mu_2}{\mu_1} = 2$$

$$\theta_\beta = \tan^{-1}(2)$$

$$\theta_\beta = 63.20$$

$$\theta_\beta \approx 63$$

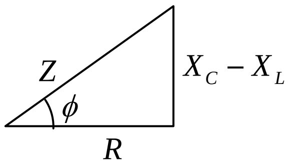


Q59. In an LCR series circuit, a non-inductive resistor of $150\,\Omega$, a coil of $0.2\,H$ inductance and negligible resistance, and a $30\,\mu F$ capacitor are connected across an ac power source of $220V, 50\,Hz$. The power loss across the resistor is _______ W (Round off to 2 decimal places).

Ans. 59: 297 to 299

Solution:

$$P = V_{rms} I_{rms} \cos\phi = I_{rms}^2 \times R = \left(\frac{V_{rms}}{2}\right)^2 \times R$$



$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times\frac{22}{7}\times 50\times 30\times 10^{-6}} = \frac{7\times 10^4}{44\times 15} = 106.1\Omega$$

$$X_L = 2\pi fL = 2\times\frac{22}{7}\times 50\times 0.2H = 62.9\Omega$$

$$Z = \sqrt{R^2 + \left(X_C - X_L\right)^2} = \sqrt{(150)^2 + (106.1-62.9)^2} = \sqrt{22500+1870} = \sqrt{24370} = 156.11\Omega$$

$$P = \left(\frac{220}{156.11}\right)^2 \times 150 = 1.988\times 150 = 298.22W$$

Q60. A charge q is uniformly distributed over the volume of a dielectric sphere of radius a. If the dielectric constant $\in_r = 2$, then the ratio of the electrostatic energy stored inside the sphere to that stored outside is ________ (Round off to 1 decimal place).

Ans. 60: 0.1 to 0.1

Solution:

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{qr}{a^3}\hat{r} \quad (r<a) \quad \text{and} \quad \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad (r>a)$$

For $r<a$; Electrostatic energy

$$W_1 = \frac{\varepsilon_0\varepsilon_r}{2}\int_0^a \left|\vec{E}_1\right|^2 4\pi r^2 dr = \frac{2\varepsilon_0}{2}\int_0^a \left|\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{qr}{a^3}\right|^2 4\pi r^2 dr$$

$$W_1 = \frac{q^2}{16\pi\varepsilon_0 a^6}\int_0^a r^4 dr = \frac{q^2}{16\pi\varepsilon_0 a^6}\frac{a^5}{5} = \frac{q^2}{80\pi\varepsilon_0 a}$$

For $r>a$; Electrostatic energy $W_2 = \frac{\varepsilon_0}{2}\int_a^\infty \left|\vec{E}_1\right|^2 4\pi r^2 dr = \frac{\varepsilon_0}{2}\int_a^\infty \left|\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right|^2 4\pi r^2 dr$

$$W_2 = \frac{q^2}{8\pi\varepsilon_0}\int_a^\infty \frac{1}{r^2}dr = \frac{q^2}{8\pi\varepsilon_0 a}$$

Thus $\frac{W_1}{W_2} = \frac{q^2}{80\pi\varepsilon_0 a}\times\frac{8\pi\varepsilon_0 a}{q^2} = \frac{1}{10} = 0.1$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

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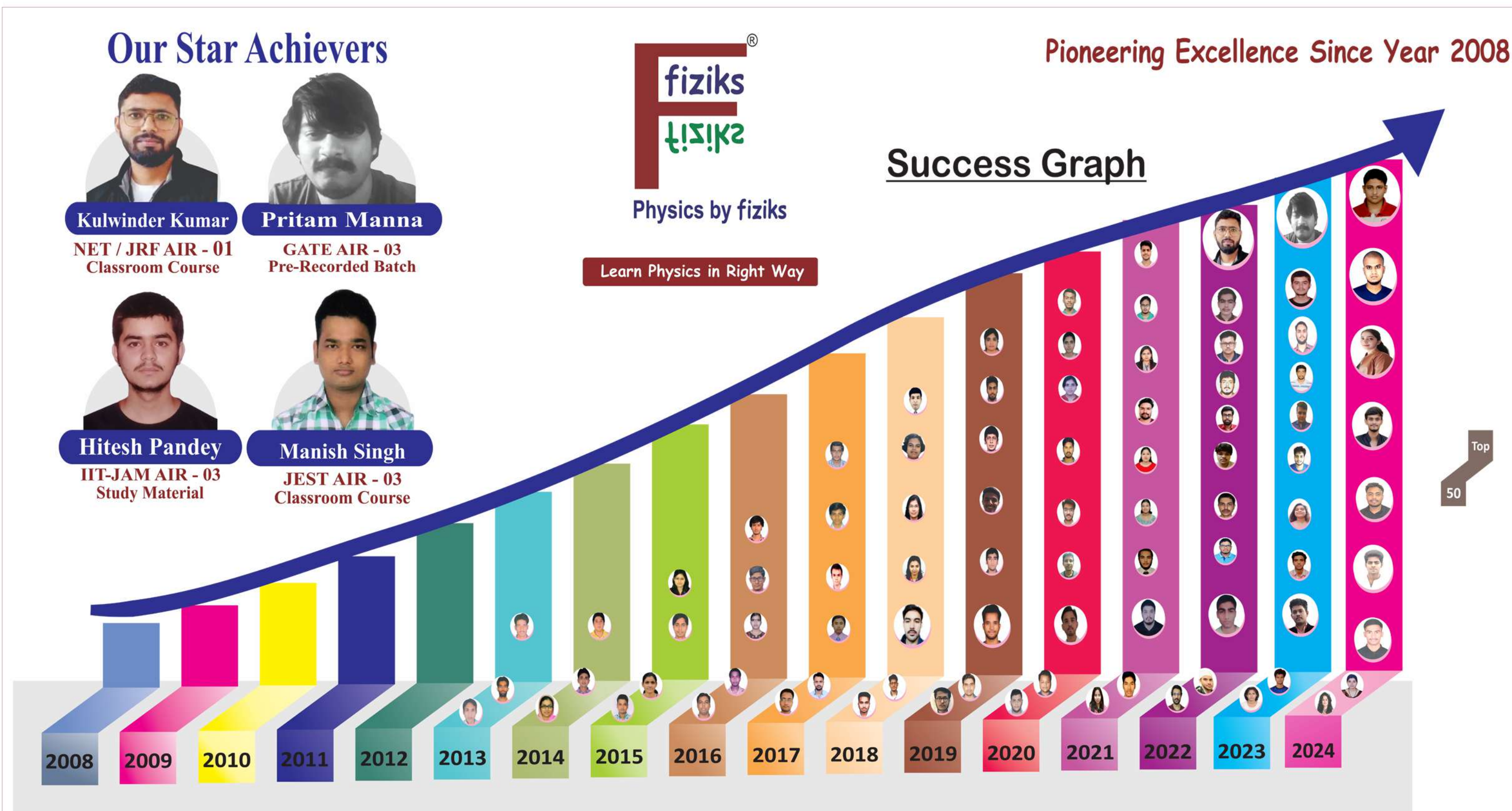

011-26865455, +9871145498

Head Office: House No. 40-D, Ground Floor, Jia Sarai Near IIT-Delhi, Hauz Khas, New Delhi-110016