GATE 2020

Section - GA (General Aptitude)

Q1 – Q5 carry one mark each.

Q1. He is known for his unscrupulous ways. He always sheds ___________ tears to deceive people.

(a) fox's (b) crocodile's

(c) crocodile (d) fox

Ans. : (c)

Solution: 'To shed crocodile tear's means 'to pretend to be sad or to sympathize with someone without really caring about them.

Q2. Jofra Archer, the England fast bowler, is __________ than accurate.

(a) more fast (b) faster (c) less fast (d) more faster

Ans. : (b)

Solution: The use 'less fast' and 'more fast' is incorrect because of using adjective. Similarly more faster is incorrect because faster itself is in comparative degree. The only choice that remains is 'faster'. Hence correct option is (b).

Q3. Select the word that fits the analogy:

Build : Building :: Grow :

(a) Grown (b) Grew (c) Growth (d) Growed

Ans. : (c)

Solution: The verb 'build' finally results in 'building' similarly verb 'grow' finally result in noun 'growth'

Q4. I do not think you know the case well enough to have opinions. Having said that, I agree with your other point.

What does the phrase "having said that" mean in the given text?

(a) as opposed to what I have said

(b) despite what I have said

(c) in addition to what I have said

(d) contrary to what I have said

Ans. : (b)

Solution: 'Having said that' means 'despite what has been mentioned earlier' or 'despite what has been said earlier'. Option (1) and (2) convey the same meaning and they do not fit in the context here.

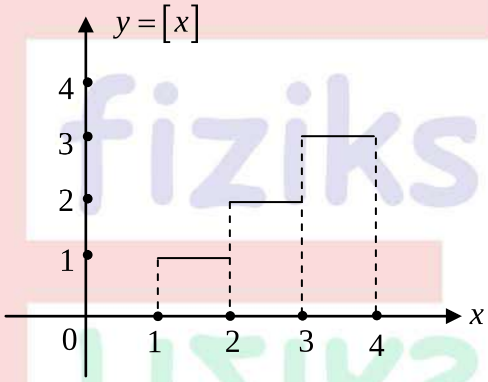
Q5. Define $[x]$ as the greatest integer less than or equal to x, for each $x \in (-\infty, \infty)$. If $y = [x]$, then area under y for $x \in [1,4]$ is

(a) 1 (b) 3 (c) 4 (d) 6

Ans. : (d)

Solution: The groups of $y = [x]$ in the interval $[1,4]$ is shown in the figure. The required area is the sum of areas of all the rectangles.

Required area $= 1+2+3 = 6$



Q6 – Q10 carry two marks each.

Q6. Crowd funding deals with mobilisation of funds for a project from a large number of people, who would be willing to invest smaller amounts through web-based platforms in the project.

Based on the above paragraph, which of the following is correct about crowd funding?

(a) Funds raised through unwilling contributions on web-based platforms

(b) Funds raised through large contributions on web-based platforms

(c) Funds raised through coerced contributions on web-based platforms

(d) Funds raised through voluntary contributions on web-based platforms

Ans. : (d)

Solution: The paragraph states that people are willing to invest through well-based platforms hence correct option is (d)

Q7. P, Q, R and S are to be uniquely coded using α and β. If P is coded as $\alpha\alpha$ and Q as $\alpha\beta$, then R and S, respectively, can be coded as

(a) $\beta\alpha$ and $\alpha\beta$ (b) $\beta\beta$ and $\alpha\alpha$

(c) $\alpha\beta$ and $\beta\beta$ (d) $\beta\alpha$ and $\beta\beta$

Ans. : (d)

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Solution: The code for P is $\alpha\alpha$ and the code for Q is $\alpha\beta$. Now there are only two codes left $\beta\alpha$ and $\beta\beta$. Hence the codes of R and S will be $\beta\alpha$ and $\beta\beta$ (not necessarily in that order).

Q8. The sum of the first n terms in the sequence $8, 88, 888, 8888, ...$ is

(a) $\frac{81}{80}\left(10^n - 1\right) + \frac{9}{8}n$ (b) $\frac{81}{80}\left(10^n - 1\right) - \frac{9}{8}n$

(c) $\frac{80}{81}\left(10^n - 1\right) + \frac{8}{9}n$ (d) $\frac{80}{81}\left(10^n - 1\right) - \frac{8}{9}n$

Ans. : (d)

Solution: We have to find the sum of n term of

$$8, 88, 888, 8888, ...$$

Let $S = 8 + 88 + 888 + 8888 + \cdots$ upto n term

$$\Rightarrow S = 8\left(1 + 11 + 111 + 1111 + \cdots \text{upto } n \text{ terms}\right)$$

$$\Rightarrow S = \frac{8}{9}\left(9 + 99 + 999 + 9999 + \cdots \text{upto } n \text{ terms}\right)$$

$$\Rightarrow S = \frac{8}{9}\left[(10-1) + (100-1) + (1000-1) + \cdots \text{ upto } n \text{ terms}\right]$$

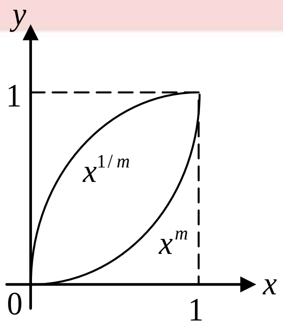
$$\Rightarrow S = \frac{8}{9}\left[(10 + 100 + 1000 + \cdots \text{upto } n \text{ terms}) - n\right]$$

Now, $10 + 100 + 1000 + \cdots$ upto n term $= \frac{10\left(1 - 10^n\right)}{1 - 10} = \frac{10}{9}\left(10^n - 1\right)$

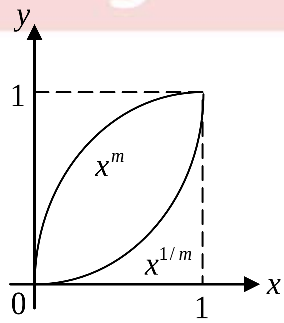
Therefore: $S = \frac{8}{9}\left[\frac{10\left(10^n - 1\right)}{9} - n\right] = \frac{80}{81}\left(10^n - 1\right) - \frac{8n}{9}$

Q9. Select the graph that schematically represents BOTH $y = x^m$ and $y = x^{1/m}$ properly in the interval $0 \le x \le 1$, for integer values of m, where $m > 1$

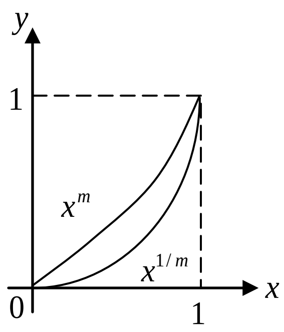
(a)



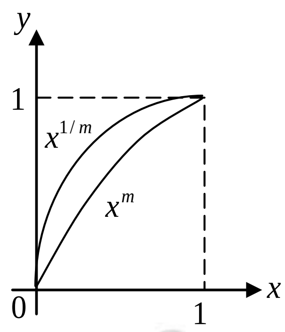
(b)



(c)



(d)



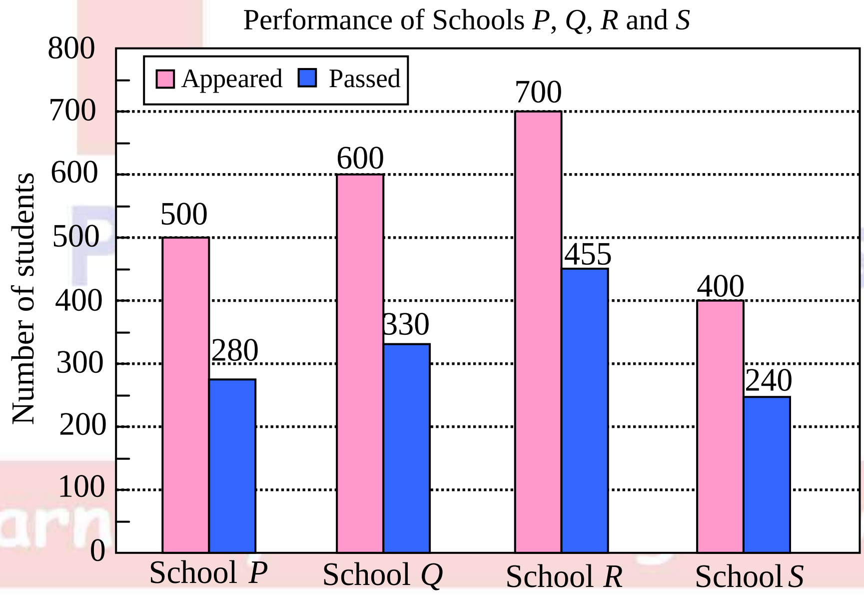
Ans. : (a)

Solution: Since the schematic shape of the graph is independent of the value of m hence we can take any value of m to find our answer.

If we take $m=2$, then $y=x^2$ and $y=x^{1/2}=\sqrt{x}$ are the two curves.

We know that $x^2=\sqrt{x}$ at $x=0$ and $x=1$. Also $x^2<\sqrt{x}$ for all x in the interval $(0,1)$.

Q10. The bar graph shows the data of the students who appeared and passed in an examination for four schools P,Q,R and S. The average of success rates (in percentage) of these four school is__________.



(a) 58.5% (b) 58.8% (c) 59.0% (d) 59.3%

Ans. : (c)

Solution: Success rate of school $P=\frac{280}{500}\times 100=56\%$

Success rate of school $Q=\frac{330}{600}\times 100=55\%$

Success rate of school $R = \frac{455}{700} \times 100 = 65\%$

Success rate of school $S = \frac{240}{400} \times 100 = 60\%$

Average of success rate $= \frac{56+55+65+60}{4} = 59\%$

Q1 – Q25 carry one marks each.

Q1. Which one of the following is a solution of $\frac{d^2u(x)}{dx^2} = k^2u(x)$, for k real?

(a) e^{-kx} (b) $\sin kx$ (c) $\cos kx$ (d) $\sinh x$

Ans. : (a)

Solution: $m^2 - k^2 = 0 \Rightarrow m = \pm k \Rightarrow u = c_1e^{kx} + c_2e^{-kx}$

Q2. A real, invertible 3×3 matrix M has eigenvalues $\lambda_i, (i = 1,2,3)$ and the corresponding eigenvectors are $|e_i\rangle, (i = 1,2,3)$ respectively. Which one of the following is correct?

(a) $M|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1,2,3$

(b) $M^{-1}|e_i\rangle = \frac{1}{\lambda_i}|e_i\rangle$, for $i = 1,2,3$

(c) $M^{-1}|e_i\rangle = \lambda_i|e_i\rangle$, for $i = 1,2,3$

(d) The eigenvalues of M and M^{-1} are not related

Ans. : (b)

Solution:

Q3. A quantum particle is subjected to the potential

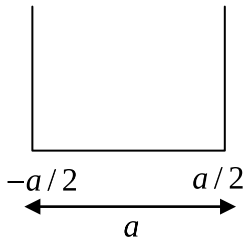
$$V(x) = \begin{cases} \infty, & x \le -\frac{a}{2} \\ 0, & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, & x \ge \frac{a}{2} \end{cases}$$

The ground state wave function of the particle is proportional to

(a) $\sin\left(\frac{\pi x}{2a}\right)$ (b) $\sin\left(\frac{\pi x}{a}\right)$ (c) $\cos\left(\frac{\pi x}{2a}\right)$ (d) $\cos\left(\frac{\pi x}{a}\right)$

Ans. : (d)

Solution: Ground state has even parity so



$$|\psi_1\rangle = \sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$$ Option (d) is correct.

Q4. Let $\hat{a}$ and $\hat{a}^{\dagger}$, respectively denote the lowering and raising operators of a one-dimensional simple harmonic oscillator. Let $|n\rangle$ be the energy eigenstate of the simple harmonic oscillator. Given that $|n\rangle$ is also an eigen state of $\hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}$, the corresponding eigenvalue is

(a) $n(n-1)$ (b) $n(n+1)$ (c) $(n+1)^2$ (d) n^2

Ans. : (a)

Solution: $\hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\hat{a}|n\rangle = \hat{a}^{\dagger}\hat{a}^{\dagger}\hat{a}\sqrt{n}|n-1\rangle = \hat{a}^{\dagger}\hat{a}^{\dagger}\sqrt{n}\sqrt{n-1}|n-2\rangle$

$$= \hat{a}^{\dagger}\sqrt{n}\sqrt{n-1}\sqrt{n-1}|n-1\rangle$$

$$= \sqrt{n}\sqrt{n-1}\sqrt{n-1}\sqrt{n}|n\rangle = n(n-1)|n\rangle$$

Q5. Which one of the following is a universal logic gate?

(a) AND (b) NOT (c) OR (d) NAND

Ans. : (d)

Solution:

Q6. Which one of the following is the correct binary equivalent of the hexadecimal $F6C$?

(a) 0110 1111 1100 (b) 1111 0110 1100

(c) 1100 0110 1111 (d) 0110 1100 0111

Ans. 6: (b)

Solution:

$F \rightarrow (15)_{10} \rightarrow (1111)_2$, $6 \rightarrow (6)_{10} \rightarrow (0110)_2$ and $C \rightarrow (12)_{10} \rightarrow (1100)_2$

Thus $F6C \rightarrow (1111\,0110\,1100)_2$

Q7. The total angular momentum j of the ground state of the ${}^{17}_{8}O$ nucleus is

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

Ans. 7: (d)

Solution: For ${}^{17}_{8}O$: $z = 8$ and $N = 9$

For $N = 9$: $(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^1$

The angular momentum is $I = \frac{5}{2}$. Thus the correct option is (d).

Q8. A particle X is produced in the process $\pi^+ + p \to K^+ + X$ via the strong interaction. If the quark content of the K^+ is $u\bar{s}$, the quark content of X is

(a) $c\bar{s}$ (b) uud (c) uus (d) $u\bar{d}$

Ans. : (c)

Solution: Lets first identify the particle X

$$\pi^+ + p \to K^+ + X$$

	π^+	p	K^+	X
q:	$+1$	$+1$	$+1$	$+1$
spin:	0	$1/2$	0	$1/2$
B:	0	$+1$	0	$+1$
I:	0	$1/2$	1	$1/2$
I_3:	$+1$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$
S:	0	0	$+1$	-1

Thus the particle X is Σ^+. The quark content of Σ^+ is uus. Thus the correct option (c)

Q9. A medium $(\varepsilon_r > 1, \mu_r = 1, \sigma > 0)$ is semi-transparent to an electromagnetic wave when

(a) Conduction current >> Displacement current

(b) Conduction current << Displacement current

(c) Conduction current = Displacement current

(d) Both Conduction current and Displacement current are zero

Ans. 9: (b)

Solution:

Conduction current $J_c = \sigma E = \sigma E_0 \cos\omega t$

Displacement current $J_d = \varepsilon \frac{\partial E}{\partial t} \Rightarrow |J_d| = \omega\varepsilon E_0 \sin\omega t$

For semi-transparent medium i.e for poor conductor $\sigma << \omega\varepsilon$.

Let $\omega t = \frac{\pi}{4} \Rightarrow \frac{J_c}{J_d} = \frac{\sigma E_0}{\omega\varepsilon E_0} = \frac{\sigma}{\omega\varepsilon} << 1 \Rightarrow J_c << J_d$

Q10. A particle is moving in a central force field given by $\vec{F} = -\frac{k}{r^3}\hat{r}$, where $\hat{r}$ is the unit vector pointing away from the center of the field. The potential energy of the particle is given by

(a) $\frac{k}{r^2}$ (b) $\frac{k}{2r^2}$ (c) $-\frac{k}{r^2}$ (d) $-\frac{k}{2r^2}$

Ans. : (d)

Solution: $-\frac{\partial u}{\partial r} = -\frac{k}{r^3} \Rightarrow u = \int \frac{k}{r^3} dr = \frac{kr^{-3+1}}{-3+1} + c$

$$\Rightarrow u = \frac{-k}{2r^2} + c$$

Q11. Choose the correct statement related to the Fermi energy (E_F) and the chemical potential (μ) of a metal

(a) $\mu = E_F$ only at $0K$

(b) $\mu = E_F$ at finite temperature

(c) $\mu < E_F$ at $0K$

(d) $\mu > E_F$ at finite temperature

Ans.: (a)

Solution: In metal the Fermi energy (E_F) is also known as chemical potential (μ) at absolute zero.

Q12. Consider a diatomic molecule formed by identical atoms. If E_V and E_C represent the energy of the vibrational nuclear motion and electronic motion respectively, then in terms of the electronic mass m and nuclear mass M, $\frac{E_V}{E_C}$ is proportional to

(a) $\left(\frac{m}{M}\right)^{1/2}$ (b) $\frac{m}{M}$ (c) $\left(\frac{m}{M}\right)^{3/2}$ (d) $\left(\frac{m}{M}\right)^{2}$

Ans. : (a)

Solution:

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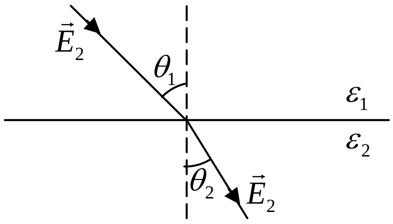
Q13. Which one of the following relations determines the manner in which the electric field lines are refracted across the interface between two dielectric media having dielectric constants ε_1 and ε_2 (see figure)?



(a) $\varepsilon_1 \sin\theta_1 = \varepsilon_2 \sin\theta_2$ (b) $\varepsilon_1 \cos\theta_1 = \varepsilon_2 \cos\theta_2$

(c) $\varepsilon_1 \tan\theta_1 = \varepsilon_2 \tan\theta_2$ (d) $\varepsilon_1 \cot\theta_1 = \varepsilon_2 \cot\theta_2$

Ans. 13: (d)

Solution:

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{E_1^{\|}/E_1^{\perp}}{E_2^{\|}/E_2^{\perp}} = \frac{E_2^{\perp}}{E_1^{\perp}} \qquad \because E_2^{\|} = E_1^{\|}$$

$$\because D_1^{\perp} = D_2^{\perp} \Rightarrow \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \Rightarrow \frac{E_2^{\perp}}{E_1^{\perp}} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\Rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{E_2^{\perp}}{E_1^{\perp}} = \frac{\varepsilon_1}{\varepsilon_2} \Rightarrow \frac{\cot\theta_2}{\cot\theta_1} = \frac{\varepsilon_1}{\varepsilon_2} \Rightarrow \varepsilon_1 \cot\theta_1 = \varepsilon_2 \cot\theta_2$$

Q14. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields respectively, then $\vec{E}\cdot\vec{B}$ is

(a) odd under parity and even under time reversal

(b) even under parity and odd under time reversal

(c) odd under parity and odd under time reversal

(d) even under parity and even under time reversal

Ans. : (c)

Solution: The electric field $E = -\partial V/\partial r$ changes sign under P-operation as r changes its sign and does not under T-operation as r does not

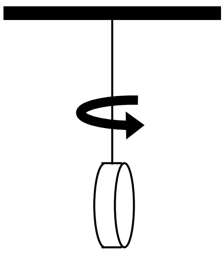
The magnetic field $B = i \times r$ does not changes its sign under P-operation as $i \to -i$ and $r \to -r$ under P-operation but under T-operation changes its sign because $i \to -i$ but $r \to r$.

Thus $\vec{E}\cdot\vec{B}$ changes its sign under P-operation as well as under T-operation. Correct option is (c)

Q15. A small disc is suspended by a fiber such that it is free to rotate about the fiber axis (see figure). For small angular deflections, the Hamiltonian for the disc is given by

$$H = \frac{p_\theta^2}{2I} + \frac{1}{2}\alpha\theta^2$$

where I is the moment of inertia and α is the restoring torque per unit deflection. The disc is subjected to angular deflections (θ) due to thermal collisions from the surrounding gas at temperature T and p_θ is the momentum conjugate to θ. The average and the root-mean-square angular deflection, θ_{avg} and θ_{rms}, respectively are



(a) $\theta_{avg} = 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$

(b) $\theta_{avg} = 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$

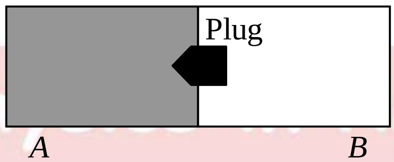
(c) $\theta_{avg} \neq 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{1/2}$

(d) $\theta_{avg} \neq 0$ and $\theta_{rms} = \left(\frac{k_B T}{\alpha}\right)^{3/2}$

Ans. : (b)

Solution:

Q16. As shown in the figure, an ideal gas is confined to chamber A of an insulated container, with vacuum in chamber B. When the plug in the wall separating the chambers A and B is removed, the gas fills both the chambers. Which one of the following statements is true?



(a) The temperature of the gas remains unchanged

(b) Internal energy of the gas decreases

(c) Temperature of the gas decreases as it expands to fill the space in chamber B

(d) Internal energy of the gas increases as its atoms have more space to move around

Ans. : (a)

Solution: Free expansion case, temperature remains unchanged.

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Q17. Particle A with angular momentum $j=\frac{3}{2}$ decays into two particles B and C with angular momenta j_1 and j_2, respectively. If $\left|\frac{3}{2},\frac{3}{2}\right\rangle_A = \alpha\left|1,1\right\rangle_B \otimes \left|\frac{1}{2},\frac{1}{2}\right\rangle_C$, the value of α is_____________

Ans. : 1

Solution:

Q18. Far from the Earth, the Earth's magnetic field can be approximated as due to a bar magnet of magnetic pole strength 4×10^{14} Am. Assume this magnetic field is generated by a current carrying loop encircling the magnetic equator. The current required to do so is about 4×10^{n} A, where n is an integer. The value of n is ______.

(Earth's circumference: $4\times10^{7}\ m$)

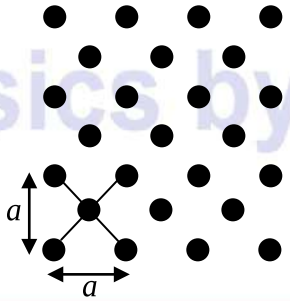
Ans. 18: 7

Solution:

$M = 4\times10^{14}\,Am$ and $2\pi R = 4\times10^{7}\,m$

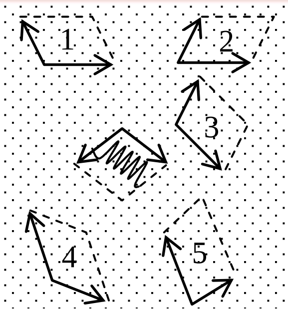
$$M\times L = I\times\pi R^2 \Rightarrow M\times 2R = I\times\pi R^2 \Rightarrow I = \frac{M\times2}{\pi R} = \frac{4\times10^{14}\times2}{2\times10^{7}} = 4\times10^{7}A \Rightarrow n=7$$

Q19. The number of distinct ways the primitive unit cell can be constructed for the two dimensional lattice as shown in the figure is_______.



Ans.: 5

Solution:



Q20. A hydrogenic atom is subjected to a strong magnetic field. In the absence of spin-orbit coupling, the number of doubly degenerate states created out of the d-level is ________

Ans.: 3

Solution: Number of Zeeman levels in strong field can be found from

$$E = (m_L + 2m_s)\mu_B B$$

For d-level: $L = 2$ and $S = 1/2$

$$M_L = +2 \quad +2 \quad +1 \quad +1 \quad 0 \quad 0 \quad -1 \quad -1 \quad -2 \quad -2$$

$$M_S = +1/2 \quad -1/2 \quad +1/2 \quad -1/2 \quad +1/2 \quad -1/2 \quad +1/2 \quad -1/2 \quad +1/2 \quad -1/2$$

$$M_L + 2m_s = +3 \quad +1 \quad +2 \quad 0 \quad +1 \quad -1 \quad 0 \quad -2 \quad -1 \quad -3$$

$$\therefore E = +3\mu_B B,\ +2\mu_B B,\ +1\mu_B B,\ +1\mu_B B,\ 0,\ 0,\ -1\mu_B B,\ -1\mu_B B,\ -2\mu_B B, -3\mu_B B$$

The number of doubly degenerate states are $+\mu_B B,\ 0,\ -\mu_B B$ only three. Thus correct answer is 3.

Q21. A particle Y undergoes strong decay $Y \rightarrow \pi^- + \pi^-$. The isospin of Y is ________

Ans.: 2

Solution: $Y \rightarrow \pi^- + \pi^-$

I: 2 1 1

In strong interaction, isospin is conserved, thus the isospin of Y is 2.

Q22. For a complex variable z and the contour $c : |z| = 1$ taken in the counter clockwise direction, $\frac{1}{2\pi i}\oint_C \left(z - \frac{2}{z} + \frac{3}{z^2}\right) dz =$ ________

Ans. : -2

Solution: $b_1 = -2$

$$\frac{1}{2\pi i} \times 2\pi i \times -2 = -2$$

Q23. Let p be the momentum conjugate to the generalized coordinate q. If the transformation

$$Q = \sqrt{2q^m} \cos p$$

$$P = \sqrt{2q^m} \sin p$$

is canonical, then $m =$ ________

Ans. : 0.5

Solution: $Q = \sqrt{2q^m} \cos p$, $\quad P = \sqrt{2q^m} \sin p$

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$$\frac{\partial Q}{\partial q}\cdot\frac{\partial P}{\partial p}-\frac{\partial Q}{\partial p}\cdot\frac{\partial P}{\partial q}=1$$

$$\left(\sqrt{2}mq^{m-1}\cos p\right)\left(\sqrt{2}q^{m}\cos p\right)-\left(\sqrt{2}q^{m}\left(-\sin p\right)\cdot\sqrt{2}mq^{m-1}\sin p\right)=1$$

$$2mq^{2m-1}\left(\cos^2 p+\sin^2 p\right)=1$$

$$2mq^{2m-1}=1\Rightarrow 2m-1=0 \text{ or } m=\frac{1}{2}=0.5$$

Q24. A conducting sphere of radius $1m$ is placed in air. The maximum number of electrons that can be put on the sphere to avoid electrical breakdown is about 7×10^n, where n is an integer. The value of n is ______.

Assume:

Breakdown electric field strength in air is $\left|\vec{E}\right|=3\times 10^6\,V/m$

Permittivity of free space $\varepsilon_0=8.85\times 10^{-12}\,F/m$

Electron charge $e=1.60\times 10^{-19}C$

Ans. 24: 14

Solution:

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Ne}{r^2}<3\times 10^6\ V/m \qquad \Rightarrow 9\times 10^9\times N\times\frac{1.6\times 10^{-19}}{(1m)^2}<3\times 10^6$$

$$\Rightarrow N<\frac{10^6}{4.8}\approx 2\times 10^{15}=20\times 10^{14}\Rightarrow n\approx 14$$

Q25. If a particle is moving along a sinusoidal curve, the number of degree of freedom of the particle is ___________

Ans. : 1

Solution: equation of constrain is $y=A\sin x$ and $z=0$

$DOF=3.N-k \quad N=1,k=2$

$3-1-2=1$ So one degree of freedom

Q26 – Q55 carry two marks each.

Q26. The product of eigenvalues of $\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$ is

(a) -1 (b) 1 (c) 0 (d) 2

Ans. : (a)

Solution: $\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = |A| = -1$

Q27. Let $|e_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, |e_2\rangle = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, |e_3\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Let $S = \{|e_1\rangle, |e_2\rangle, |e_3\rangle\}$. Let $\mathbb{R}^3$ denote the three dimensional real vector space. Which one of the following is correct?

(a) S is an orthonormal set

(b) S is a linearly dependent set

(c) S is a basis for $\mathbb{R}^3$

(d) $\sum_{i=1}^{3} |e_i\rangle\langle e_i| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Ans. : (c)

Solution: $\langle e_1 | e_2 \rangle \neq 0$ (i) is false

$$c_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$c_1 + c_2 + c_3 = 0$$

$$c_3 = 0, c_2 = 0, c_1 = 0$$

$$c_2 + c_3 = 0$$

$$c_3 = 0$$

$|e_2\rangle, \langle e_5\rangle$ is limits indeed so $\langle 2|1\rangle$ is correct.

Option (c) is correct.

Q28. $\hat{S}_x$ denotes the spin operator defined $\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Which one of the following is correct?

(a) The eigenstates of spin operator $\hat{S}_x$ are $|\uparrow\rangle_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\downarrow\rangle_x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(b) The eigenstates of spin operator $\hat{S}_x$ are $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) In the spin state $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of $\hat{S}_x$, the probability for obtaining $\left|\uparrow\right\rangle_x$ is $\frac{1}{4}$

(d) In the spin state $\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$, upon the measurement of $\hat{S}_x$, the probability for obtaining $\left|\uparrow\right\rangle_x$ is $\frac{2+\sqrt{3}}{4}$

Ans. : (d)

Solution: $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Eigen value of $\frac{\hbar}{2}, \frac{-\hbar}{2}$ and corresponds eigen state

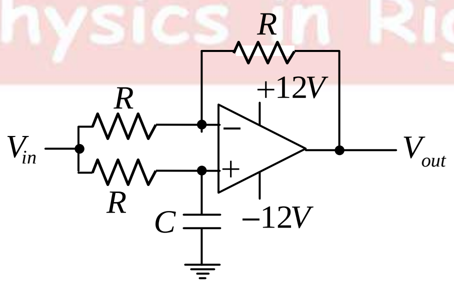
$$\left|\uparrow\right\rangle_x = \left|\phi_1\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \left|\downarrow\right\rangle_x = \left|\phi_2\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ State } \left|\psi\right\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}$$

$$\left\langle \phi_1 | \psi \right\rangle = \frac{\left| \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \right|^2}{\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}} = \left| \frac{1}{2\sqrt{2}}\left(1+\sqrt{3}\right) \right|^2$$

$$= \frac{\left(1+\sqrt{3}\right)^2}{8} = \frac{1+3+2\sqrt{3}}{8} = \frac{4+2\sqrt{3}}{8} = \frac{2+\sqrt{3}}{4}$$

Option (d) is correct.

Q29. The input voltage (V_{in}) to the circuit shown in the figure is $2\cos(100t)V$. The output voltage (V_{out}) is $2\cos\left(100t - \frac{\pi}{2}\right)V$. If $R = 1k\Omega$, the value of C (in μF) is



(a) 0.1 (b) 1 (c) 10 (d) 100

Ans. 29: (c)

Solution:

$$V_{out}=-\frac{R}{R}V_{in}+\left(1+\frac{R}{R}\right)\left(\frac{X_C}{R+X_C}\right)V_{in}\Rightarrow V_{out}=-V_{in}+2\left(\frac{1/j\omega C}{R+1/j\omega C}\right)V_{in}$$

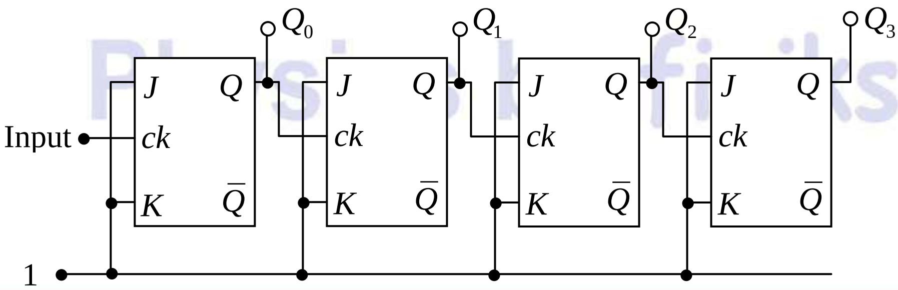
$$\Rightarrow\frac{V_{out}}{V_{in}}=-1+2\left(\frac{1}{j\omega CR+1}\right)=\left(\frac{1-j\omega CR}{1+j\omega CR}\right)$$

$$\Rightarrow\frac{V_{out}}{V_{in}}=\left(\frac{\sqrt{1+(\omega CR)^2}}{\sqrt{1+(\omega CR)^2}}\right)\frac{e^{-j\theta}}{e^{j\theta}}=e^{-j2\theta}\quad\text{where }\theta=\tan^{-1}(\omega RC)$$

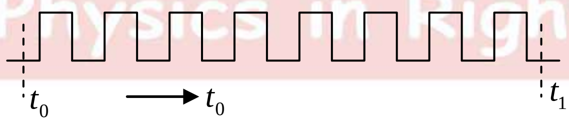
Thus $\phi=-2\theta=-2\tan^{-1}(\omega RC)\Rightarrow-\frac{\pi}{2}=-2\tan^{-1}\left(100\times1\times10^3\times C\right)$

$$\Rightarrow10^5\times C=\tan^{-1}\left(\frac{\pi}{4}\right)\Rightarrow C=\frac{1}{10^5}F=10\,\mu F$$

Q30. Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the Q output to the clock input of the following flip-flop (see the figure). The input signal to the counter is a series of square pulses and the change of state is triggered by the falling edge. At time $t=t_0$ the outputs are in logic low state $(Q_0=Q_1=Q_2=Q_3=0)$. Then at $t=t_1$, the logic state of the outputs is



4-bit ripple counter



Input signal

(a) $Q_0=1,Q_1=0,Q_2=0$ and $Q_3=0$

(b) $Q_0=0,Q_1=0,Q_2=0$ and $Q_3=1$

(c) $Q_0=1,Q_1=0,Q_2=1$ and $Q_3=0$

(d) $Q_0=0,Q_1=1,Q_2=1$ and $Q_3=1$

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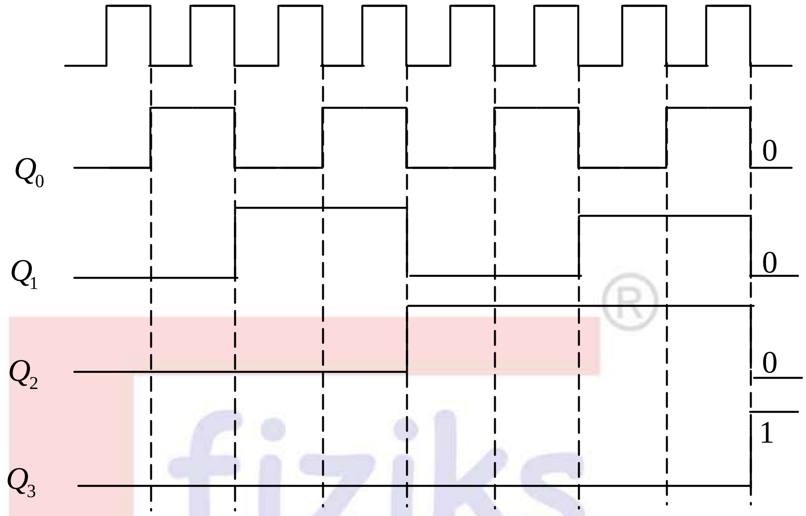
Ans. 30: (b)

Solution:



Q31. Consider the Lagrangian $L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$, where a, b and c are constants. If p_x and p_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is

(a) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$

(b) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$

(c) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$

(d) $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$

Ans. : (a)

Solution: $L = a\dot{x}^2 + b\dot{y}^2 + cxy$

$$\frac{\partial L}{\partial \dot{x}} = p_x = 2a\dot{x} \Rightarrow \dot{x} = \frac{p_x}{2a} \quad \text{and} \quad \frac{\partial L}{\partial \dot{y}} = p_y = 2a\dot{y} \Rightarrow \dot{y} = \frac{p_y}{2a}$$

$$H = p_x\dot{x} + p_y\dot{y} - L \Rightarrow H = 2a\dot{x}^2 + 2b\dot{y}^2 - \left(a\dot{x}^2 + b\dot{y}^2 + cxy\right)$$

$$\Rightarrow H = a\dot{x}^2 + b\dot{y}^2 - cxy = \frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$$

Q32. Which one of the following matrices does NOT represent a proper rotation in a plane?

(a) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

(d) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

Ans. : (d)

Solution: Rotational matrix is orthogonal matrix

(1) $\sin^2\theta-\left(-\cos^2\theta\right)=1$

(2) $\sin^2\theta-\left(\cos^2\theta\right)=1$

(3) $-\sin^2\theta-\left(-\cos^2\theta\right)=\cos^2\theta-\sin^2\theta=\cos^2\theta$

(4) $\cos^2\theta-\left(-\sin^2\theta\right)=1$

Q32. Which one of the following matrices does NOT represent a proper rotation in a plane?

(a) $\begin{pmatrix}-\sin\theta & \cos\theta\\ -\cos\theta & -\sin\theta\end{pmatrix}$ (b) $\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$

(c) $\begin{pmatrix}\sin\theta & \cos\theta\\ -\cos\theta & \sin\theta\end{pmatrix}$ (d) $\begin{pmatrix}-\sin\theta & \cos\theta\\ -\cos\theta & \sin\theta\end{pmatrix}$

Ans. : (d)

Solution: Rotational matrix is orthogonal matrix

(1) $\sin^2\theta-\left(-\cos^2\theta\right)=1$

(2) $\sin^2\theta-\left(\cos^2\theta\right)=1$

(3) $-\sin^2\theta-\left(-\cos^2\theta\right)=\cos^2\theta-\sin^2\theta=\cos 2\theta$

(4) $\cos^2\theta-\left(-\sin^2\theta\right)=1$

Q33. A uniform magnetic field $\vec{B}=B_0\hat{y}$ exists in an internal frame K. A perfect conducting sphere moves with a constant velocity $\vec{v}=v_0\hat{x}$ with respect to this inertial frame. The rest frame of the sphere is K' (see figure). The electric and magnetic fields in K and K' are related as

$$\left.\begin{array}{ll}\vec{E}'_{\parallel}=\vec{E}_{\parallel} & \vec{E}'_{\perp}=\gamma\left(\vec{E}_{\perp}+\vec{v}\times\vec{B}\right)\\ \vec{B}'_{\parallel}=\vec{B}_{\parallel} & \vec{B}'_{\perp}=\gamma\left(\vec{B}_{\perp}-\dfrac{\vec{v}}{c^2}\times\vec{E}\right)\end{array}\right\},\gamma=\frac{1}{\sqrt{1-(v/c)^2}}$$

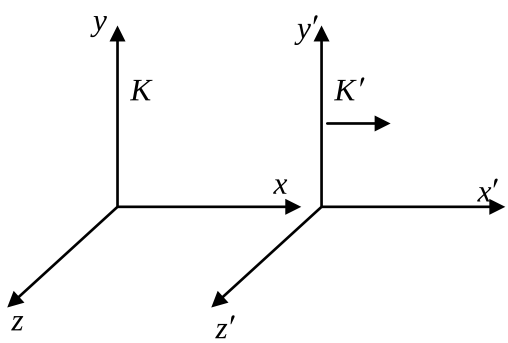
The induced surface charge density on the sphere (to the lowest order in v/c) in the frame K' is

(a) maximum along z'

(b) maximum along y'

(c) maximum along x'

(d) uniform over the sphere



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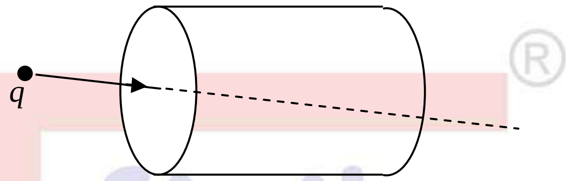
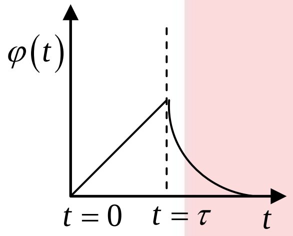
Ans. : (a)

Solution:

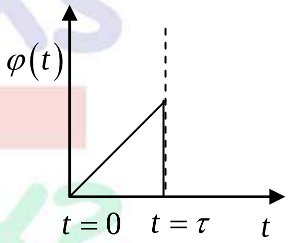
Q34. A charge q moving with uniform speed enters a cylindrical region in free space at $t=0$ and exits the region at $t=\tau$ (see figure). Which one of the following options best describes the time dependence of the total electric flux $\varphi(t)$, through the entire surface of the cylinder?



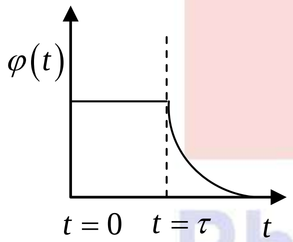
(a)



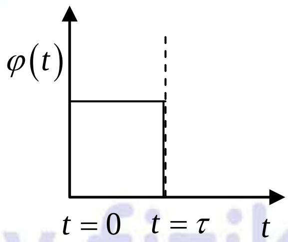
(b)



(c)



(d)



Ans. 34: (d)

Solution: Flux through the closed surface $=\dfrac{Q_{enc}}{\varepsilon_0}$=constant when charge is inside otherwise zero.

Q35. Consider a one-dimensional non-magnetic crystal with one atom per unit cell. Assume that the valence electrons (i) do not interact with each other and (ii) interact weakly with the ions. If n is the number of valence electrons per unit cell, then at $0\ K$,

(a) the crystal is metallic for any value of n

(b) the crystal is non-metallic for any value of n

(c) the crystal is metallic for even values of n

(d) the crystal is metallic for odd values of n

Ans.: (d)

Solution: The conduction band is partially filled for odd value of n and hence behaves as a metal. The band is totally filled for even value of n and known as non-metallic.

Thus correct option is (d)

Q36. According to the Fermi gas model of nucleus, the nucleons move in a spherical volume of radius $R\left(=R_0 A^{\frac{1}{3}}\right.$, where A is the mass number and R_0 is an empirical constant with the dimensions of length). The Fermi energy of the nucleus E_F is proportional to

(a) R_0^2 (b) $\frac{1}{R_0}$ (c) $\frac{1}{R_0^2}$ (d) $\frac{1}{R_0^3}$

Ans.: (c)

Solution: Fermi energy $E_F = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3}$

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(R_0 A^{1/3}\right)^3 = \frac{4\pi}{3}R_0^3 A$$

$$\therefore E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{\frac{4\pi}{3}R_0^3 A}\right)^{2/3} = \frac{\hbar^2}{2m}\left(\frac{9\pi N}{4A}\cdot\frac{1}{R_0^3}\right)^{2/3} \Rightarrow E_F \propto \frac{1}{R_0^2}$$

Thus correct option is (c)

Q37. Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed optical branches (n) and acoustic branches (m) due to the lattice vibrations are

(a) $(n,m)=(2,4)$ (b) $(n,m)=(3,3)$

(c) $(n,m)=(4,2)$ (d) $(n,m)=(1,5)$

Ans. : (c)

Solution: For p-atoms per basis : Total degree of freedom $=2p$

Number of acoustical branches $(n)=2$

Number of optical branches $(n)=2p-2$

For $p=3$, $m=2$ and $n=2\times 3-2=4$

$\therefore (n,m)=(4,2)$ Thus correct option is (c).

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Q38. The internal energy U of a system is given by $U(S,V)=\lambda V^{-2/3}S^2$, where λ is a constant of appropriate dimensions; V and S denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system?

(a) $\frac{PV^{1/3}}{T^2}=\text{constant}$ (b) $\frac{PV}{T^{1/3}}=\text{constant}$

(c) $\frac{P}{V^{1/3}T}=\text{constant}$ (d) $\frac{PV^{2/3}}{T}=\text{constant}$

Ans. : (a)

Solution: $dU=TdS-PdV$

$$\left(\frac{\partial U}{\partial S}\right)_V=T,\ \left(\frac{\partial U}{\partial V}\right)_S=-P$$

$$2\lambda V^{-2/3}S=T \text{ and } -\frac{2}{3}\lambda V^{-5/3}S^2=-P$$

$$\Rightarrow\frac{PV}{TS}=\text{ constant} \qquad \Rightarrow\frac{PV}{T\left(TV^{2/3}\right)}=\text{ constant}$$

$$\Rightarrow\frac{PV^{1/3}}{T^2}=\text{constant}$$

Q39. The potential energy of a particle of mass m is given by

$$U(x)=a\sin\left(k^2x-\pi/2\right),\quad a>0,\ k^2>0$$

The angular frequency of small oscillations of the particle about $x=0$ is

(a) $k^2\sqrt{\frac{2a}{m}}$ (b) $k^2\sqrt{\frac{a}{m}}$ (c) $k^2\sqrt{\frac{a}{2m}}$ (d) $2k^2\sqrt{\frac{a}{m}}$

Ans. : (b)

Solution: $U(x)=a\sin\left(k^2x-\pi/2\right),\quad a>0,\ k^2>0$

$$\Rightarrow U(x)=-a\cos^2 k^2x=-a\left[1-\frac{k^4x^2}{\lfloor 2}+\cdots\right]$$

$$\Rightarrow F=-\frac{\partial U}{\partial x}=-ak^4x$$

$$\Rightarrow\omega^2=\frac{ak^4}{m}\Rightarrow\omega=k^2\sqrt{\frac{a}{m}}$$

Q40. The radial wave function of a particle in a central potential is give by $R(r) = A\frac{r}{a}\exp\left(-\frac{r}{2a}\right)$, where A is the normalization constant and a is positive constant of suitable dimensions. If γa is the most probable distance of the particle from the force center, the value of γ is ___________

Ans. : 4

Solution: $R(r) = A\frac{r}{a}\exp\left(-\frac{r}{2a}\right)$

Radial probability derivative $p(r) = r^2|R|^2 \quad = \frac{r^4}{a^2}\exp-(r/a)$

For must portable distance $\frac{dp}{dr} = 0$

$$\frac{4r^3}{a^2}e^{-r/a_\omega} + \frac{r^4}{a^2}e^{-r/a}\frac{-1}{a} = 0$$

$$\frac{r^3e^{-r/a_0}}{a^2}\left[4 - \frac{r}{a}\right] = 0 \Rightarrow r = 4a = \gamma a \Rightarrow \gamma = 4$$

Q41. A free particle of mass M is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the fifth excited state of the particle is _________

Ans. : 6

Solution: Energy eigen value for particle in cubical $= \left(n_x^2 + n_y^2 + n_z^2\right)E_0$ where $E_0 = \frac{\pi^2\hbar^2}{2ma^2}$.

Ground state $E_{1,1,1} = 3E_0$

First $E_{2,1,1} = E_{1,2,1} = E_{1,1,2} = 6E_0$

Second Excited state $E_{2,2,1} = E_{2,1,2} = E_{1,2,2} = 9E_0$

Third Excited state $E_{3,1,1} = E_{1,3,1} = E_{1,1,3} = 11E_0$

Fourth Excited state $E_{2,2,2} = 12E_0$ non-degenerate

Fifth Excited state $E_{1,2,3} = E_{1,3,2} = E_{2,1,3} = E_{2,3,1} = E_{3,1,2} = E_{3,2,1} = 14E_0$

So fifth excited state has 6 fold degeneracy.

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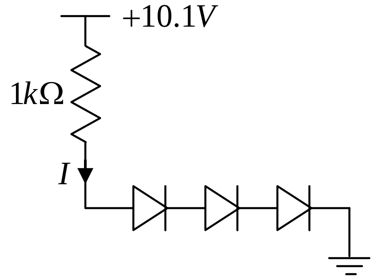
Q42. Consider the circuit given in the figure. Let the forward voltage drop across each diode be $0.7V$. The current I (in mA) through the resistor is ___________.



Ans. 42: 8

Solution:

Current $I = \dfrac{10.1V - 3\times 0.7V}{1k\Omega} = 8\,mA$

Q43. Let u^{μ} denote the 4-velocity of a relativistic particle whose square $u^{\mu}u_{\mu} = 1$. If $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita tensor then the value of $\varepsilon_{\mu\nu\rho\sigma}u^{\mu}u^{\nu}u^{\rho}u^{\sigma}$ is ________.

Ans. : 0

Solution:

Q44. Consider a simple cubic monoatomic Bravais lattice which has a basis with vectors $\vec{r}_1 = 0, \vec{r}_2 = \dfrac{a}{4}(\hat{x}+\hat{y}+\hat{z})$, a is the lattice parameter. The Bragg reflection is observed due to the change in the wave vector between the incident and the scattered beam as given by $\vec{K} = n_1\vec{G}_1 + n_2\vec{G}_2 + n_3\vec{G}_3$, where $\vec{G}_1, \vec{G}_2$ and $\vec{G}_3$ are primitive reciprocal lattice vectors. For $n_1 = 3, n_2 = 3$ and $n_3 = 2$, the geometrical structure factor is __________

Ans. : 2

Solution: Geometric structure factor $S = \sum_{N=1}^{2} e^{2\pi i(n_1x_N + n_2y_N + n_3z_n)} = 1 + e^{2\pi i\left(\frac{3}{4}+\frac{3}{4}+\frac{2}{4}\right)}$

$$= 1 + e^{2\pi i\left(\frac{8}{4}\right)} = 1 + e^{4\pi i} = 1 + 1 = 2$$

$$\therefore S = 2$$

Q45. A plane electromagnetic wave of wavelength λ is incident on a circular loop of conducting wire. The loop radius is $a\,(a << \lambda)$. The angle (in degrees), made by the Poynting vector with the normal to the plane of the loop to generate a maximum induced electrical signal, is __________

Ans. : -270 or -90 or 90 or 270

Q46. An electron in a hydrogen atom is in the state $n=3, l=2, m=-2$. Let $\hat{L}_y$ denote the y-component of the orbital angular momentum operator. If $\left(\Delta\hat{L}_y\right)^2 = \alpha\hbar^2$, the value of α is____________

Ans. : 1

Solution: $\left(\Delta L_y\right) = \sqrt{\left\langle L_y^2\right\rangle - \left\langle L_y\right\rangle^2}$

$\left\langle L_y\right\rangle = 0$

$L_y^2 = \frac{\hbar^2}{2}\left(l\left(l+1\right)-m^2\right) \quad l=2\ m-2$

$= \frac{\hbar^2}{2}\left(2\left(3\right)-4\right) = \frac{\hbar^2}{2}2\hbar^2$

$\left(\Delta L_y\right) = \alpha\hbar^2 \quad \left(\Delta L_y\right)^2 = 1.\hbar^2 \quad \alpha = 1$

Q47. A sinusoidal voltage of the form $V\left(t\right) = V_0\cos\left(\omega t\right)$ is applied across a parallel plate capacitor placed in vacuum. Ignoring the edge effects, the induced emf within the region between the capacitor plates can be expressed as a power series in ω. The lowest non-vanishing exponent in ω is ___________

Ans. 47: 2

Solution:

Induced e.m.f $\varepsilon = -\frac{d\phi}{dt} = -\frac{AdB}{dt}$

$$\oint \vec{B}.d\vec{l} = \mu_0 I_{enc} + \mu_0\varepsilon_0\int_S \frac{\partial\vec{E}}{\partial t}.d\vec{a}$$

Consider an amperian loop of radius $r\left(r<R\right)$, then $I_{enc}=0$ and since

$$E(t) = \frac{V(t)}{d} = \frac{V_0\cos\omega t}{d}$$

Thus $\left|\vec{B}\right|\times 2\pi r = \mu_0\varepsilon_0\times\left(-\frac{V_0\omega\sin\omega t}{d}\right)\times\pi r^2 \quad \Rightarrow \left|\vec{B}\right| \propto \omega\sin\omega t$

$$\Rightarrow \varepsilon \propto \frac{dB}{dt} \propto \omega^2\cos\omega t \propto \omega^2\left(1-\frac{\omega^2t^2}{2}+...\right)$$

The lowest non-vanishing exponent in ω is $n=2$.

Q48. If $x=\sum_{k=1}^{\infty} a_k \sin kx$, for $-\pi \le x \le \pi$, the value of a_2 is_______

Ans. : -1

Solution: $x\sin kx = a_k \sin^2 kx$

$$\int_{-\pi}^{\pi} x\sin kx dx = a_k \int_{-\pi}^{\pi} \sin^2 kx dx = 2a_k \int_0^{\pi} \sin^2 kx dx = \frac{2}{k} a_k \int_0^{k\pi} \sin^2 \theta d\theta = \frac{2}{k} a_k \frac{k\pi}{2} = a_k \pi$$

$$a_2 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin 2x dx = \frac{1}{\pi}(-\pi) = -1$$

Q49. Let $f_n(x) = \begin{cases} 0, & x < -\frac{1}{2n} \\ n, & -\frac{1}{2n} < x < \frac{1}{2n} \\ 0, & \frac{1}{2n} < x \end{cases}$

The value of $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)\sin x dx$ is______.

Ans. : 0

Solution: $\lim_{n\to\infty} \int_{-1/m}^{1/m} x\sin x\, dx = -n\left[\cos\frac{1}{m} - \cos\frac{-1}{m}\right] = 0$

Q50. Consider the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}'$ where

$\hat{H}_0 = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{pmatrix}$ and $\hat{H}'$ is the time independent perturbation given by

$\hat{H}' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}$, where $k>0$. If, the maximum energy eigenvalues of $\hat{H}$ is $3eV$ corresponding to $E = 2eV$, the value of k (rounded off to three decimal places) in eV is______________ check the question

Ans. : 1

Solution: $H_0 = \begin{bmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & E \end{bmatrix}$ $H' = \begin{pmatrix} 0 & k & 0 \\ k & 0 & k \\ 0 & k & 0 \end{pmatrix}$

Eigen value of H'

$$\begin{pmatrix} -\lambda & k & 0 \\ k & -\lambda & k \\ 0 & x & -\lambda \end{pmatrix} = 0 \Rightarrow -\lambda\left(\lambda^2 - k^2\right) - k\left(-\lambda k\right) = 0 \Rightarrow -\lambda\left(\lambda^2 - k^2\right) + \lambda k^2 = 0$$

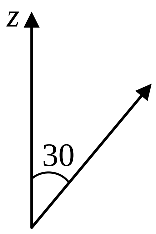
$$-\lambda^3 + \lambda k^2 + \lambda k^2 = 0 \Rightarrow -\lambda^3 + 2\lambda k^2 = 0 \Rightarrow \lambda\left(-\lambda^2 + k^2\right) = 0 \qquad \lambda = 0 \quad \lambda = k, \ \lambda = -k$$

$E_1 = E - k$, $E_2 = E + 0$, $E_3 = E + k$ where $E = 2eV$ and $E_3 = 3eV$

$$E_3 = E + k \Rightarrow k = 3eV - 2eV = 1eV$$

Q51. A hydrogen atom is in an orbital angular momentum state $|l, m = l\rangle$. If $\vec{L}$ lies on a cone which makes a half angle 30^0 with respect to the z-axis, the value of l is __________

Ans. : 3



Solution: $\cos 30 = \dfrac{m}{\sqrt{l(l+1)}}$ $\qquad m = l$

$$\frac{\sqrt{3}}{2} = \frac{l}{\sqrt{l(l+1)}} = \sqrt{3}\left(\sqrt{l(l+1)}\right) = 2l$$

$$3\left(l^2 + l\right) = 4l^2 \Rightarrow 3l = l^2 \Rightarrow l = 3$$

Q52. In the center of mass frame, two protons each having energy $7000\,\text{GeV}$, collide to produce protons and anti-protons. The maximum number of anti-protons produced is___________

(Assume the proton mass to be $1GeV/c^2$)

Ans. : 6999

Solution: Assuming that protons and anti-protons are produced at rest with mass $1GeV/c^2$

$p + p \rightarrow p + p + n$-number of protons $+n$-number of anti protons

$E: 7000 + 7000 = 1 + 1 + 6999 \qquad\qquad +6999$

Q53. Consider a gas of hydrogen atoms in the atmosphere of the Sun where the temperature is $5800\,K$. If a sample from this atmosphere contains 6.023×10^{23} of hydrogen atoms in the ground state, the number of hydrogen atoms in the first excited state is approximately 8×10^n, where n is an integer. The value of n is __________.

(Boltzmann constant: $8.617 \times 10^{-5}\,eV/K$)

Ans. : 14

Solution: $\dfrac{N_1}{N_0} = e^{-\Delta E/kT}$; $\qquad \Delta E = \left(\dfrac{13\cdot 6}{12} - \dfrac{13\cdot 6}{2^2}\right)eV = \left(13\cdot 6 - 3\cdot 4\right) = 10\cdot 2eV$

$$\therefore \frac{\Delta E}{kT} = \frac{10.2eV}{8.617\times10^{-5} ev/k\times5800k} = 20.41$$

Thus $\frac{N_1}{N_0} = e^{-20\cdot41} \Rightarrow N_1 = 6\cdot023\times10^{23}\times1\cdot37\times10^{-9} \Rightarrow N_1 = 8.25\times10^{14} \qquad \Rightarrow n = 14$

Q54. For a gas of non-interacting particles, the probability that a particle has a speed v in the internal v to $v+dv$ is given by

$$f(v)dv = 4\pi v^2 dv\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT}$$

If E is the energy of a particle, then the maximum in the corresponding energy distribution in units of E/k_BT occurs at _________ (rounded off to one decimal place).

Ans. : 0.5

Solution: $E_P = \frac{1}{2}k_BT \Rightarrow \frac{\frac{1}{2}k_BT}{k_BT} = 0.5$

Q55. The Planck's energy density distribution is given by $u(\omega) = \frac{\hbar\omega^3}{\pi^2c^3\left(e^{\hbar\omega/k_BT}-1\right)}$. At long wavelengths, the energy density of photons in thermal equilibrium with a cavity at temperature T varies as T^α , where α is ___________

Ans. : 1

Solution: $u(\omega) = \frac{\hbar\omega^3}{\pi^2c^3\left(1+\frac{\hbar\omega}{k_BT}+...-1\right)}$

At long wavelengths λ, $\omega = \frac{2\pi c}{\lambda}$ is very small.

$u(\omega) \propto T^1$

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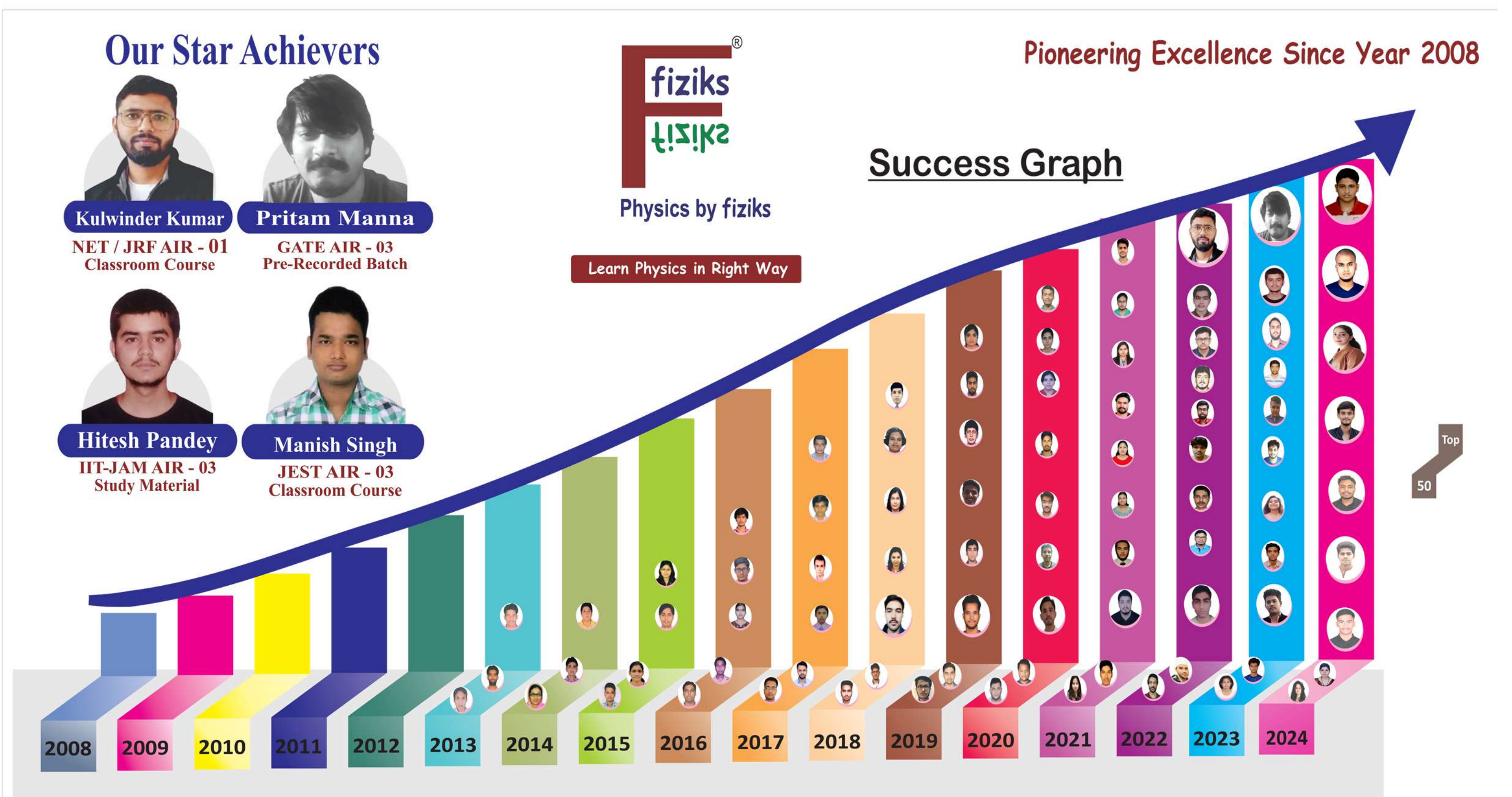

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