GATE Physics-2023

Solution

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Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q17. Consider a two dimensional Cartesian coordinate system in which a rank 2 contravariant tensor is represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The coordinate system is rotated anticlockwise by an acute angle θ with the origin fixed. Which one of the following matrices represents the tensor in the new coordinate system?

(A) $\begin{pmatrix} 0 & \cos 2\theta \\ -\sin 2\theta & 0 \end{pmatrix}$ (B) $\begin{pmatrix} \sin 2\theta & \cos 2\theta \\ \cos 2\theta & -\sin 2\theta \end{pmatrix}$

(C) $\begin{pmatrix} \sin 2\theta & -\cos 2\theta \\ \cos 2\theta & \sin 2\theta \end{pmatrix}$ (D) $\begin{pmatrix} \sin 2\theta & 0 \\ 0 & -\cos 2\theta \end{pmatrix}$

Ans. : (b)

Solution:

Determinant of the matrix should be ± 1 and it should be symmetric also.

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q26. A 4×4 matrix M has the property $M^\dagger = -M$ and $M^4 = 1$, where 1 is the 4×4 identity matrix. Which one of the following is the CORRECT set of eigenvalues of the matrix M?

(A) $(1,1,-1,-1)$ (B) $(i,i,-i,-i)$ (C) $(i,i,i,-i)$ (D) $(1,1,-i,-i)$

Ans. : (b)

Solution:

Eigenvalues of a Skew-Hermitian matrix($M^\dagger = -M$) is either zero or pure imaginary and

$$Trace(M) = \sum_i \lambda_i = 0.$$

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3).

Q32. Which of the following options represent(s) linearly independent pair(s) of functions of a real variable x?

(A) e^{ix} and e^{-ix} (B) x and e^x

(C) 2^x and 2^{-3+x} (D) e^{ix} and $\sin x$

Ans.: (a), (b) and (d)

Solution:

For linearly independent pair(s) $\frac{y_2}{y_1} \neq \text{const.}$

(a) $\frac{y_2}{y_1} = \frac{e^{-ix}}{e^{ix}} = e^{-2ix} \neq \text{const.}$

(b) $\frac{y_2}{y_1} = \frac{e^{x}}{x} \neq \text{const.}$

(c) $\frac{y_2}{y_1} = \frac{2^{-3+x}}{2^{x}} = 2^{-3} = \frac{1}{8} = \text{const.}$

(d) $\frac{y_2}{y_1} = \frac{\sin x}{e^{ix}} \neq \text{const.}$

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

Q43. Consider two real functions

$$U(x,y) = xy\left(x^2 - y^2\right), \qquad V(x,y) = ax^4 + by^4 + cx^2y^2 + k$$

where k is a real constant and a, b, c are real coefficients. If $U(x,y) + iV(x,y)$ is analytic, then what is the value of $a \times b \times c$?

(A) $\frac{1}{8}$ (B) $\frac{3}{28}$ (C) $\frac{5}{36}$ (D) $\frac{3}{32}$

Ans.: (d)

Solution:

Since $U(x,y) = xy\left(x^2 - y^2\right)$, $V(x,y) = ax^4 + by^4 + cx^2y^2 + k$

From C-R equation:

$$\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y} \Rightarrow 3x^2y - y^3 = 4by^3 + 2cx^2y \Rightarrow 4b = -1, \quad 2c = 3 \Rightarrow b = -\frac{1}{4}, \quad c = \frac{3}{2}$$

From C-R equation:

$$\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x} \Rightarrow x^3 - 3xy^2 = -4ax^3 - 2cx^2y \Rightarrow -4a = 1, \quad -2c = -3 \Rightarrow a = -\frac{1}{4}, \quad c = \frac{3}{2}$$

Thus $a \times b \times c = \left(-\frac{1}{4}\right) \times \left(-\frac{1}{4}\right) \times \left(\frac{3}{2}\right) = \frac{3}{32}$

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3)

Q56. Consider the complex function $f(z)=\dfrac{z^2 \sin z}{(z-\pi)^4}$

At $z=\pi$, which of the following options is (are) CORRECT?

(A) The order of the pole is 4

(B) The order of the pole is 3

(C) The residue at the pole is $\dfrac{\pi}{6}$

(D) The residue at the pole is $\dfrac{2\pi}{3}$

Ans.: (b)

Solution:

Since $\lim\limits_{z\to\pi}(z-\pi)^3 \dfrac{z^2\sin z}{(z-\pi)^4}=\lim\limits_{z\to\pi}\dfrac{z^2\sin z}{(z-\pi)}=\lim\limits_{z\to\pi}\dfrac{2z\sin z+z^2\cos z}{1}=-\pi^2=\text{constant}$

So $f(z)$ has pole of order 3 at $z=\pi$.

Let $z-\pi=t \Rightarrow f(t)=\dfrac{(t+\pi)^2\sin(t+\pi)}{t^4}=-\dfrac{(t+\pi)^2}{t^4}\sin t$

Let $\pi>t$: $f(t)=-\dfrac{\pi^2}{t^4}\left(1+\dfrac{t}{\pi}\right)^2\sin t=-\dfrac{\pi^2}{t^4}\left(1+\dfrac{2t}{\pi}+\dfrac{2}{2!}\dfrac{t^2}{\pi^2}+\ldots\right)\left(t-\dfrac{t^3}{3!}++\dfrac{t^5}{5!}\ldots\right)$

$$\Rightarrow f(t)=-\left(\frac{\pi^2}{t^4}+\frac{2\pi}{t^3}+\frac{1}{t^2}+\ldots\right)\left(t-\frac{t^3}{3!}+\frac{t^5}{5!}\ldots\right)=-\left(-\frac{\pi^2}{6}+1\right)\frac{1}{t}+\ldots$$

Thus residue at the pole $z=\pi$ is $=-\left(-\dfrac{\pi^2}{6}+1\right)=\dfrac{\pi^2}{6}-1$.

Let $t>\pi$: $f(t)=-\dfrac{t^2}{t^4}\left(1+\dfrac{\pi}{t}\right)^2\sin t=-\dfrac{1}{t^2}\left(1+\dfrac{2\pi}{t}+\dfrac{2}{2!}\dfrac{\pi^2}{t^2}+\ldots\right)\left(t-\dfrac{t^3}{3!}++\dfrac{t^5}{5!}\ldots\right)$

$$\Rightarrow f(t)=-\left(\frac{1}{t^2}+\frac{2\pi}{t}+\frac{\pi^2}{t^4}+\ldots\right)\left(t-\frac{t^3}{3!}+\frac{t^5}{5!}\ldots\right)=-\left(1-\frac{\pi^2}{6}\right)\frac{1}{t}+\ldots$$

Thus residue at the pole $z=\pi$ is $=-\left(1-\dfrac{\pi^2}{6}\right)=\dfrac{\pi^2}{6}-1$.

So only option (b) is correct.

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Q57. Consider the vector field $\vec{V}$ consisting of the velocities of points on a thin horizontal disc of radius $R = 2\,\text{m}$, moving anticlockwise with uniform angular speed $\omega = 2$ rad/sec about an axis passing through its center. If $V = |\vec{V}|$, then which of the following options is(are) CORRECT ? (In the options, $\hat{r}$ and $\hat{\theta}$ are unit vectors corresponding to the plane polar coordinates r and θ).

You may use the fact that in cylindrical coordinates (s, ϕ, z) (s is the distance from the z-axis), the gradient, divergence, curl and Laplacian operators are:

$$\vec{\nabla} f = \frac{\partial f}{\partial s}\hat{S} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{Z}; \qquad \vec{\nabla}\cdot\vec{A} = \frac{1}{s}\frac{\partial}{\partial s}(sA_S) + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{S} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_S}{\partial \phi}\right)\hat{Z};$$

$$\vec{\nabla}^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

(A) $\vec{\nabla} V = 2\hat{r}$

(B) $\vec{\nabla}\cdot\vec{V} = 2$

(C) $\vec{\nabla}\times\vec{V} = 4\hat{Z}$, where $\hat{Z}$ is a unit vector perpendicular to the (r, θ) plane

(D) $\vec{\nabla}^2 V = \frac{4}{3}$ at $r = 1.5\,\text{m}$

Ans.: (a), (c), (d)

Solution:

Since $\vec{v} = \vec{\omega}\times\vec{r} = \omega r\hat{\phi} \Rightarrow |\vec{v}| = v = \omega r$

(a) $\vec{\nabla} v = \frac{\partial}{\partial r}(\omega r)\hat{r} = \omega\hat{r} = 2\hat{r}$

(b) $\vec{\nabla}.\vec{v} = \frac{\partial}{\partial \phi}(\omega r) = 0$

(c) $\vec{\nabla}\times\vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(rv_\phi)\hat{z} = \frac{1}{r}\frac{\partial}{\partial r}(r\times\omega r)\hat{z} = 2\omega\hat{z} = 4\hat{z} \qquad \because \vec{v} = \omega r\hat{\phi}$

(d) $\vec{\nabla}^2 v = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}(r\omega) = \frac{\omega}{r} = \frac{2}{1.5} = \frac{4}{3} \qquad \because v = \omega r$

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3) No Question

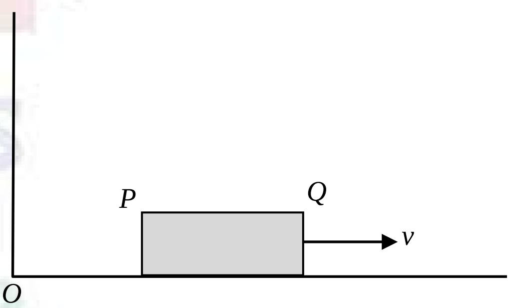
Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3). No Question

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3). No Question

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3). No Question

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

Q40. A rod PQ of proper length L lies along the x-axis and moves towards the positive x-direction with speed $v=\frac{3c}{5}$ with respect to the ground (see figure), where c is the speed of light in vacuum. An observer on the ground measures the positions of P and Q at different times t_P and t_Q respectively in the ground frame, and finds the difference between them to be $\frac{9L}{10}$. What is the value of t_Q-t_P?



(A) $\frac{L}{3c}$

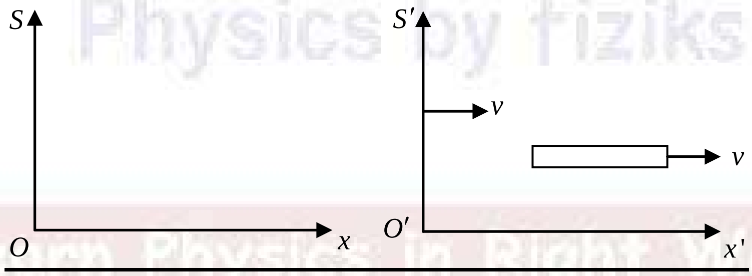
(B) $\frac{L}{5c}$

(C) $\frac{L}{6c}$

(D) $\frac{2L}{3c}$

Ans.: (c)

Solution.:



<u>**In S-frame:**</u> $x_Q-x_P=\frac{9L}{10}$; $\quad t_Q-t_P=??$

<u>**In S'-frame:**</u> $x'_Q-x'_P=L$ (Proper length); $t'_Q-t'_P$ = Not given

(Here positions x'_Q and x'_P may be measured by the observer in S'-frame at different time.

$$x'_Q-x'_P=\frac{x_Q-x_P}{\sqrt{1-\frac{v^2}{c^2}}}-v\frac{t_Q-t_P}{\sqrt{1-v^2/c^2}} \Rightarrow L=\frac{(9L/10)}{\sqrt{1-\frac{9}{25}}}-\frac{3}{5}c\frac{(t_Q-t_P)}{\sqrt{1-\frac{9}{25}}} \Rightarrow \frac{3}{4}c(t_Q-t_P)=\frac{L}{8}$$

$$\Rightarrow t_Q-t_P=\frac{L}{6c}$$

Q41. A symmetric top has principal moments of inertia $I_1 = I_2 = \frac{2\alpha}{3}, I_3 = 2\alpha$ about a set of principal axes 1, 2, 3 respectively, passing through its center of mass, where α is a positive constant. There is no force acting on the body and the angular speed of the body about the 3-axis is $\omega_3 = \frac{1}{8}$ rad/s. With what angular frequency in rad/s does the angular velocity vector $\vec{\omega}_1$ precess about the 3-axis?

(A) 2 (B) 3 (C) 5 (D) 7

Ans.: (a)

Solution.: Angular frequency with which $\vec{\omega}_1$ precess about the 3-axis is $\Omega = \frac{I_3 - I_1}{I_1}\omega_3$.

$$\Rightarrow \Omega = \frac{2\alpha - 2\alpha/3}{2\alpha/3}\omega_3 \Rightarrow \Omega = 2\omega_3$$

Q42. A particle of mass m is free to move on a frictionless horizontal two dimensional (r,θ) plane, and is acted upon by a force $\vec{F} = -\frac{k}{2r^3}\hat{r}$ with k being a positive constant. If p_r and p_θ are the generalized momenta corresponding to r and θ respectively, then what is the value of $\frac{dp_r}{dt}$?

(A) $\frac{p_\theta^2 - 2mk}{2mr^3}$ (B) $\frac{2p_\theta^2 - mk}{mr^3}$

(C) $\frac{p_\theta^2 - 2mk}{mr^3}$ (D) $\frac{2p_\theta^2 - mk}{2mr^3}$

Ans.: (d)

Solution.: Hamiltonian in polar coordinates $H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + V(r)$

From HEM: $\dot{p}_r = -\frac{\partial H}{\partial r} = -\left[-\frac{p_\theta^2}{mr^3} + \frac{\partial V}{\partial r}\right] \Rightarrow \frac{dp_r}{dt} = \frac{p_\theta^2}{mr^3} - \frac{\partial V}{\partial r} = \frac{p_\theta^2}{mr^3} + F(r)$

$$\Rightarrow \frac{dp_r}{dt} = \frac{p_\theta^2}{mr^3} - \frac{k}{2r^3} \quad \Rightarrow \frac{dp_r}{dt} = \frac{2p_\theta^2 - mk}{2mr^3}$$

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3) No Question

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3) No Question

Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3). No Question

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q31. Consider an isolated magnetized sphere of radius R with a uniform magnetization $\vec{M}$ along the positive z-direction, with the north and south poles of the sphere lying on the z-axis. It is given that the magnetic field inside the sphere is $\vec{B} = \frac{2\mu_0}{3}\vec{M}$, where μ_0 is the permeability of vacuum. Which of the following statements is (are) CORRECT?

(A) The bound volume current density is zero

(B) The bound surface current density has maximum magnitude at the equator, where this magnitude equals $\left|\vec{M}\right|$

(C) The auxiliary field $\vec{H} = -\frac{2}{3}\vec{M}$

(D) Far from the sphere, the magnetic field is due to a dipole of moment $\vec{m}$, where $\frac{\vec{m}}{4\pi R^3} = \frac{B}{2\mu_0}\hat{z}$

Ans.: (a), (b) and (d)

Solution:

, $\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$ and

(a) $\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$ $\quad\because \vec{M} = M\,\hat{z}$

(b) $\vec{K}_b = \vec{M} \times \hat{n} = M\,\hat{z} \times \hat{r} = M\sin\theta\hat{\phi} \Rightarrow \left|\vec{K}_b\right| = M\sin\theta = \left|\vec{M}\right|\sin\theta$ $\quad$ where $M = \left|\vec{M}\right|$

At equator $\theta = \frac{\pi}{2} \Rightarrow \left|\vec{K}_b\right| = \left|\vec{M}\right|\sin\frac{\pi}{2} = \left|\vec{M}\right|$

(c) $\because \vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) \Rightarrow \frac{2\mu_0}{3}\vec{M} = \mu_0\left(\vec{H} + \vec{M}\right) \Rightarrow \vec{H} = -\frac{1}{3}\vec{M}$ $\quad \because \vec{B} = \frac{2\mu_0}{3}\vec{M}$

(d) $\because \vec{M} = \frac{\vec{m}}{\frac{4}{3}\pi R^3} \Rightarrow \frac{\vec{m}}{4\pi R^3} = \frac{M}{3}\hat{z} = \frac{1}{3} \times \frac{3B}{2\mu_0}\hat{z} = \frac{B}{2\mu_0}\hat{z}$

Far from the sphere, the magnetic field is due to a dipole of moment $\vec{m}$.

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3).

Q35. An electric field as a function of radial coordinate r has the form $\vec{E} = \alpha \frac{e^{-r^2}}{r}\hat{r}$, where α is a constant. Assume that dimensions are appropriately taken care of. The electric flux through a sphere of radius $\sqrt{2}$, centered at the origin, is Φ. What is the value of $\frac{\Phi}{2\pi\alpha}$ (rounded off to two decimal places)?

Ans.: 0.36 to 0.40

Solution:

The electric flux $\Phi = \oint_S \vec{E}.d\vec{a} = \int_0^{\pi}\int_0^{2\pi}\left(\alpha\frac{e^{-r^2}}{r}\hat{r}\right).\left(r^2 \sin\theta d\theta d\phi \hat{r}\right)$

$\Rightarrow \Phi = \alpha r e^{-r^2} \times 4\pi = \sqrt{2}\alpha e^{-2} \times 4\pi \quad \Rightarrow \frac{\Phi}{2\pi\alpha} = \frac{2\sqrt{2}}{e^2} = 0.38 \qquad \because r = \sqrt{2}$

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3) No Question

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3) No Question

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3)

Q59. Two independent electrostatic configurations are shown in the figure. Configuration (I) consists of an isolated point charge $q = 1\,\text{C}$, C, and configuration (II) consists of another identical charge surrounded by a thick conducting shell of inner radius $R_1 = 1\,\text{m}$ and outer radius $R_2 = 2\,\text{m}$, with the charge being at the centre of the shell. $W_I = \frac{\epsilon_0}{2}\int E_I^2 dV$ and $W_{II} = \frac{\epsilon_0}{2}\int E_{II}^2 dV$, where E_I and E_{II} are the magnitudes of the electric fields for configurations (I) and (II) respectively, ϵ_0 is the permittivity of vacuum, and the volume integrations are carried out over all space. If $8\pi \epsilon_0 \left|W_I - W_{II}\right| = \frac{1}{2}$, what is the value of the integer n?

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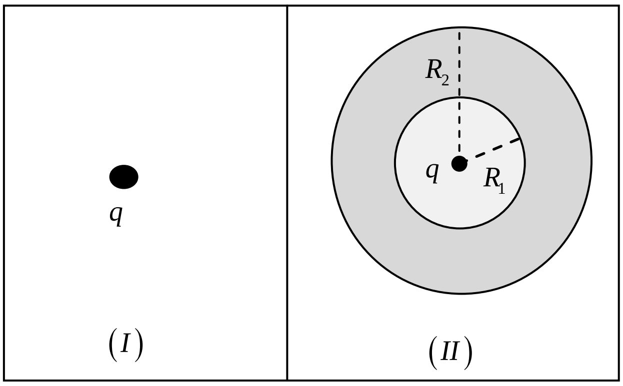
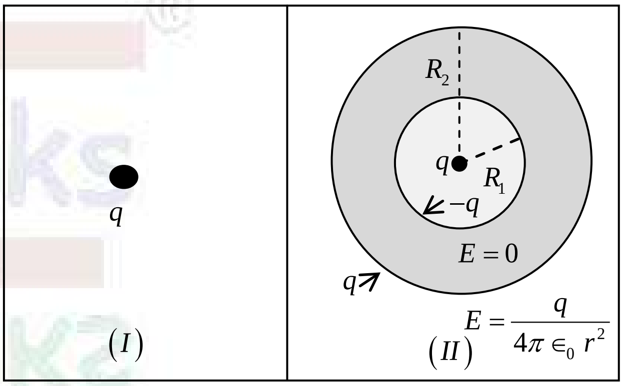


Ans.: 2

Solution:



$$W_I=\frac{\in_0}{2}\int E_I^2 dV\ ;\ W_{II}=\frac{\in_0}{2}\int E_{II}^2 dV$$

$$\Rightarrow W_{II}=W_I+0+\frac{\in_0}{2}\int_{R_2}^{\infty}\left(\frac{q}{4\pi\in_0 r^2}\right)^2\times 4\pi r^2 dr$$

$$\Rightarrow W_{II}=W_I+\frac{q^2}{8\pi\in_0 R_2}\Rightarrow 8\pi\in_0\left|W_I-W_{II}\right|=\frac{q^2}{R_2}$$

$$\Rightarrow 8\pi\in_0\left|W_I-W_{II}\right|=\frac{1^2}{2}=\frac{1}{2}=\frac{1}{n}\qquad\Rightarrow n=2$$

Q61. Consider an electromagnetic wave propagating in the z-direction in vacuum, with the magnetic field given by $\vec{B}=\vec{B}_0 e^{i(kz-\omega t)}$. If $B_0=10^{-8}$ T, the average power passing through a circle of radius 1.0 m placed in the xy plane is P (in Watts). Using $\in_0=10^{-11}\dfrac{c^2}{Nm^2}$, what is the value of $\dfrac{10^3 P}{\pi}$ (rounded off to one decimal place)?

Ans.: 11.5 to 13.7

Solution:

$$\text{Intensity } I=\frac{P}{A}=<u>c=\frac{B_0^2}{2\mu_0}c=\frac{B_0^2 c}{2}\times\varepsilon_0 c^2=\frac{B_0^2 c^3\varepsilon_0}{2}\quad\Rightarrow\frac{P}{\pi(1)^2}=\frac{B_0^2 c^3\varepsilon_0}{2}$$

$$\Rightarrow\frac{10^3 P}{\pi}=\frac{10^3 B_0^2 c^2\varepsilon_0}{2}\Rightarrow\frac{10^3 P}{\pi}=\frac{10^3}{2}\times\left(10^{-8}\right)^2\left(3\times10^8\right)^3\left(10^{-11}\right)$$

$$\Rightarrow\frac{10^3 P}{\pi}=\frac{10^3}{2}\times10^{-16}\times\left(27\times10^{24}\right)\left(10^{-11}\right)=\frac{27}{2}=13.5$$

Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q15. H is the Hamiltonian, $\vec{L}$ the orbital angular momentum and L_z is the z-component of $\vec{L}$. The $1s$ state of the hydrogen atom in the non-relativistic formalism is an eigen function of which one of the following sets of operators?

(A) H, L^2 and L_z (B) $H, \vec{L}, L^2$ and L_z

(C) L^2 and L_z only (D) H and L_z only

Ans. : (b)

Solution.: Let $|n, l, m_l\rangle$ represent the state of the system.

For ground state of H-atom; $n = 1, l = 0, m_l = 0 \quad \therefore \; |\psi_{1s}\rangle = |1,0,0\rangle$

$$\hat{H}|\psi_{1s}\rangle = \hat{H}|1,0,0\rangle = -\frac{13.6}{n^2}|1,0,0\rangle = -13.6\,\text{eV}|1,0,0\rangle$$

$$\hat{L}^2|\psi_{1s}\rangle = \hat{L}^2|1,0,0\rangle = \ell(\ell+1)\hbar^2|1,0,0\rangle = 0\;\hbar^2|1,0,0\rangle$$

$$L_z|\psi_{1s}\rangle = L_z|1,0,0\rangle = m_l\hbar|1,0,0\rangle = 0\;\hbar|1,0,0\rangle$$

$$\left[L_x, L_y\right]\psi_{1s} = i\hbar L_z|\psi_{1s}\rangle = 0; \quad \left[L_x, L_z\right]\psi_{1s} = -i\hbar L_y|\psi_{1s}\rangle = 0 \qquad \because L_{\pm}|\psi_{1s}\rangle = 0$$

$$\left[L_y, L_z\right]\psi_{1s} = i\hbar L_x|\psi_{1s}\rangle = 0$$

Thus $\hat{L}_x, \hat{L}_y$ and $\hat{L}_z$ are commuting operators for ψ_{1s}. Thus ψ_{1s} is an eigenfunction of $H, \vec{L}, L^2$ and L_z.

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q25. An atom with non-zero magnetic moment has an angular momentum of magnitude $\sqrt{12}\hbar$. When a beam of such atoms is passed through a Stern-Gerlach apparatus, how many beams does it split into?

(A) 3 (B) 7 (C) 9 (D) 25

Ans. : (b)

Solution.:

$$\sqrt{\ell|\ell+1\rangle}\hbar = \sqrt{12}\hbar \Rightarrow \sqrt{\ell|\ell+1\rangle} = \sqrt{12} \Rightarrow \ell = 3$$

$$\therefore \; m_\ell = 2\ell+1 = 2\times 3+1 = 7$$

Thus, the beam will split into $(2\ell+1)$ components i.e. 7 component.

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3). No Question

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3) No Question

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3)

Q49. A particle has wavefunction

$$\psi(x,y,z) = Nze^{-\alpha\left(x^2+y^2+z^2\right)}$$

where N is a normalization constant and α is a positive constant. In this state, which one of the following options represents the eigenvalues of L^2 and L_z respectively?

Some values of Y_ℓ^m are: $Y_0^0 = \sqrt{\frac{1}{4\pi}}$, $Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$, $Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$

(A) 0 and 0

(B) $\hbar^2$ and $-\hbar$

(C) $2\hbar^2$ and 0

(D) $\hbar^2$ and $\hbar$

Ans.: (c)

Solution.:

In spherical polar coordinate, $z = r\cos\theta$ and $r^2 = x^2 + y^2 + z^2$

$\because \psi(x,y,z) = Nze^{-\alpha\left(x^2+y^2+z^2\right)}$ $\therefore$ $\psi(r,\theta,\phi) = Nr\cos\theta\, e^{-\alpha r^2}$

Thus $\ell = 1$ and $m_\ell = 0$

The eigenvalue of $L^2 = \ell(\ell+1)\hbar^2 = 2\hbar^2$

The eigenvlaue of $L_z = m_l\hbar = 0$

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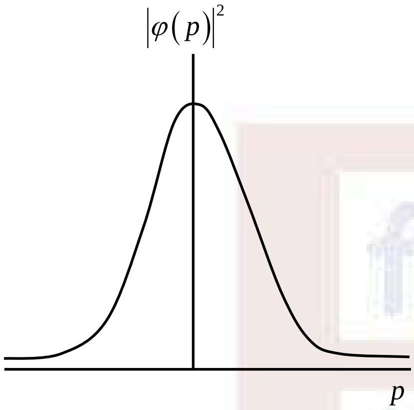
Q50. The wavefunction of a particle in one dimension is given by

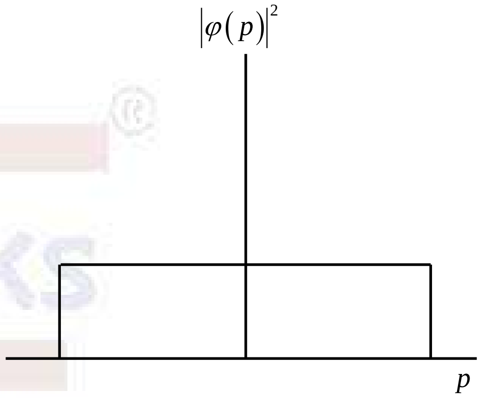
$$\psi(x)=\begin{cases} M, & -a<x<a \\ 0, & \text{otherwise} \end{cases}$$

Here M and a are positive constants. If $\varphi(p)$ is the corresponding momentum space wavefunction, which one of the following plots best represents $|\varphi(p)|^2$?

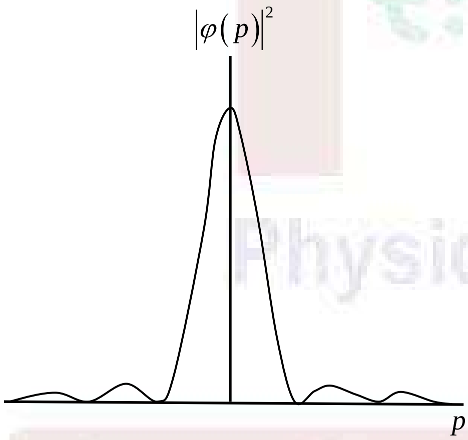
(A)



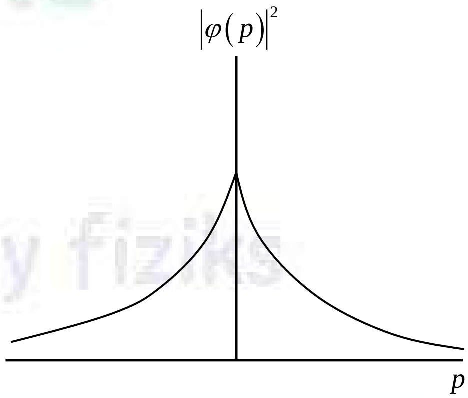
(B)



(C)



(D)



Ans.: (c)

Solution.:

$$\psi(x)=\begin{cases} M, & -a<x<a \\ 0, & \text{otherwise} \end{cases}$$

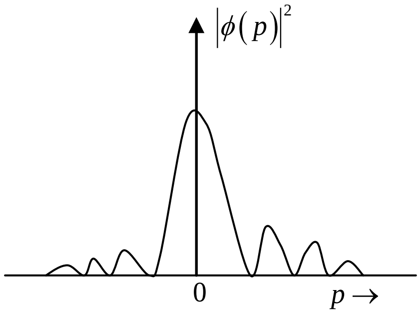
wave function is momentum space is $\phi(p)=\dfrac{1}{\sqrt{2\pi\hbar}}\displaystyle\int_{-\infty}^{\infty}\psi(x)e^{-ipx/\hbar}dx=\dfrac{M}{\sqrt{2\pi\hbar}}\displaystyle\int_{-a}^{a}e^{-ipx/\hbar}dx$

$$\Rightarrow \phi(p)=\frac{M}{\sqrt{2\pi\hbar}}\left[\frac{e^{-ipx/\hbar}}{-ip/\hbar}\right]_{-a}^{a}=\frac{M}{\sqrt{2\pi\hbar}}\left(\frac{e^{ipa/\hbar}-e^{-ipa/\hbar}}{ip/\hbar}\right)=\frac{M}{\sqrt{2\pi\hbar}}2a\left(\frac{e^{ipa/\hbar}-e^{-ipa/\hbar}}{2iap/\hbar}\right)$$

$$\Rightarrow \phi(p) = \frac{2Ma}{\sqrt{2\pi\hbar}} \frac{\sin(pa/\hbar)}{pa/\hbar}$$

$$\Rightarrow |\phi(p)|^2 = \frac{2M^2a^2}{\pi\hbar}\left(\frac{\sin(pa/\hbar)}{pa/\hbar}\right)^2$$

Thus correct option is (C).



Q51. Consider a particle in a two-dimensional infinite square well potential of side L, with $0 \le x \le L$ and $0 \le y \le L$. The wavefunction of the particle is zero only along the line $y = \frac{L}{2}$, apart from the boundaries of the well. If the energy of the particle in this state is E, what is the energy of the ground state?

(A) $\frac{1}{4}E$ (B) $\frac{2}{5}E$ (C) $\frac{3}{8}E$ (D) $\frac{1}{2}E$

Ans.: (b)

Solution.:

$$\because E = \left(n_x^2 + n_y^2\right)\frac{\pi^2\hbar^2}{2mL^2} \Rightarrow E_{GND} = (1+1)\frac{\pi^2\hbar^2}{2mL^2} = \frac{\pi^2\hbar^2}{mL^2}$$

$$\psi(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi}{L}x\right)\sin\left(\frac{n_y\pi}{L}y\right)$$

At $y = \frac{L}{2}$; $\psi\left(x, \frac{L}{2}\right) = 0 \Rightarrow \frac{2}{L}\sin\left(\frac{n_x\pi}{L}x\right)\sin\left(\frac{n_y\pi}{2}\right) = 0 \Rightarrow \sin\left(\frac{n_y\pi}{2}\right) = 0 \Rightarrow n_y = 2$

Since wave function is zero only at $y = \frac{L}{2}$, thus $n_x = 1$

So $E = \left(1^1 + 2^2\right)\frac{\pi^2\hbar^2}{2mL^2} = 5\frac{\pi^2\hbar^2}{2mL^2} = \frac{5}{2}E_{GND} \Rightarrow E_{GND} = \frac{2}{5}E$

Q52. Consider two non-identical spin $\frac{1}{2}$ particles labelled 1 and 2 in the spin product state $\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$. The Hamiltonian of the system is

$$H=\frac{4\lambda}{\hbar^2}\vec{S}_1\cdot\vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spin operators of particles 1 and 2, respectively, and λ is a constant with appropriate dimensions. What is the expectation value of H in the above state?

(A) $-\lambda$ (B) -2λ (C) λ (D) 2λ

Ans.: (a)

Solution.:

For $S_1=\frac{1}{2}$, $m_1=\pm\frac{1}{2}$; For $S_2=\frac{1}{2}, m_2=\pm\frac{1}{2}$

There are four possible states

$$|s_1,m_1\rangle|s_2,m_2\rangle=\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle,\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle,\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle,\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$=|\uparrow\uparrow\rangle,|\uparrow\downarrow\rangle,|\downarrow\uparrow\rangle,|\downarrow\downarrow\rangle$$

Since, $s_1=\frac{1}{2},s_2=\frac{1}{2}$, therefore $s=\vec{s}_1+\vec{s}_2=0,1$

The possible states for singlet $(s=0)$ and triplet $(s=1)$ are

For $s=0,m_s=0$ $\quad\therefore|0,0\rangle$

For $s=1,m_s=0,\pm1$ $\therefore|1,1\rangle,|1,0\rangle$ and $|1,-1\rangle$

where $|1,1\rangle=|\uparrow\uparrow\rangle$; $\quad|1,0\rangle=\frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle\right]$

$|1,-1\rangle=|\downarrow\downarrow\rangle$ and $\quad|0,0\rangle=\frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle\right]$

According to the question, we have to calculate expectation value of H in the state $|\uparrow\downarrow\rangle=\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$. This state can be written is linear combination of $|1,0\rangle$ and $|0,0\rangle$ as

$|\uparrow\downarrow\rangle=c_1|1,0\rangle+c_2|0,0\rangle$ $\quad$ where $c_1=\langle 1,0|\uparrow\downarrow\rangle=\frac{1}{\sqrt{2}}\left(\langle\uparrow\downarrow|+\langle\downarrow\uparrow|\right)|\uparrow\downarrow\rangle=\frac{1}{\sqrt{2}}$

and $c_2 = \langle 0,0|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}\left(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right)|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}$

$\therefore \ |\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{2}}|0,0\rangle$

The expectation value of H in the state $|\uparrow\downarrow\rangle$ is

$$\langle\uparrow\downarrow|H|\uparrow\downarrow\rangle = \langle H\rangle = \sum_n E_n P(n) = E_{1,0}P(1,0) + E_{0,0}P(0,0)$$

Given $H = \frac{4\lambda}{\hbar^2}\vec{s}_1\cdot\vec{s}_2 = \frac{4\lambda}{\hbar^2}\left[\frac{s^2 - s_1^2 - s_2^2}{2}\right] = \frac{2\lambda}{\hbar^2}\left[s^2 - s_1^2 - s_2^2\right]$

Now, $H|1,0\rangle = \frac{2\lambda}{\hbar^2}\left(s^2 - s_1^2 - s_2^2\right)|1,0\rangle = \frac{2\lambda}{\hbar^2}\left(2 - \frac{3}{4} - \frac{3}{4}\right)\hbar^2|1,0\rangle$

$H|1,0\rangle = \lambda|1,0\rangle = E_{1,0}|1,0\rangle$ and $H|0,0\rangle = \frac{2\lambda}{\hbar^2}\left(s^2 - s_1^2 - s_2^2\right)|0,0\rangle = \frac{2\lambda}{\hbar^2}\left(0 - \frac{3}{4} - \frac{3}{4}\right)\hbar^2|0,0\rangle$

$H|0,0\rangle = -3\lambda|0,0\rangle = E_{0,0}|0,0\rangle$

Also, $P(1,0) = \frac{|c_1|^2}{\sum |c_i|^2} = \frac{1}{2}$ and $P(0,0) = \frac{|c_2|^2}{\sum |c_i|^2} = \frac{1}{2}$

Therefore $\langle H\rangle = \lambda\times\frac{1}{2} - 3\lambda\times\frac{1}{2} = -\lambda$

Thus, correct option is (A)

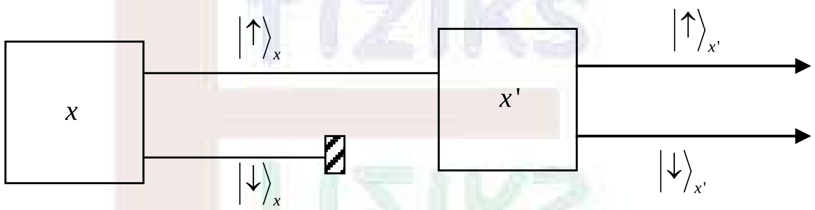
Q53. A spin $\frac{1}{2}$ particle is in a spin up state along the x-axis (with unit vector $\hat{x}$) and is denoted as $\left|\frac{1}{2},\frac{1}{2}\right\rangle_x$. What is the probability of finding the particle to be in a spin up state along the direction $\hat{x}'$, which lies in the xy-plane and makes an angle θ with respect to the positive x-axis, if such a measurement is made?

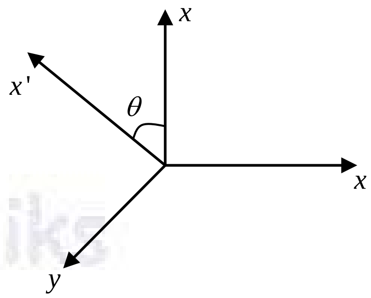
(A) $\frac{1}{2}\cos^2\frac{\theta}{4}$ (B) $\cos^2\frac{\theta}{4}$

(C) $\frac{1}{2}\cos^2\frac{\theta}{2}$ (D) $\cos^2\frac{\theta}{2}$

Ans.: (d)

Solution.:





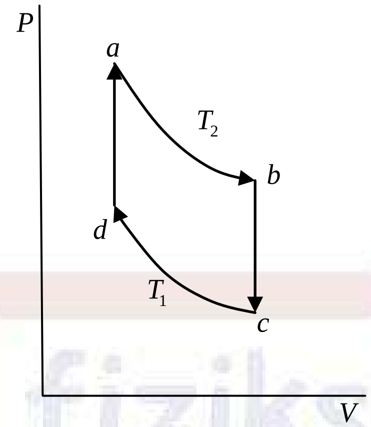
The probability of finding the particle in spin up state in the direction of $\hat{x}'$ is $P\left(|\uparrow\rangle_{x'}\right)=\cos^2\frac{\theta}{2}$

The probability of finding the particle in spin down state in the direction of $\hat{x}'$ is $P\left(|\downarrow\rangle_{x'}\right)=\sin^2\frac{\theta}{2}$

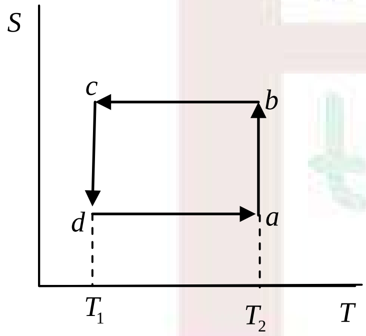
Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3) No Question

Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

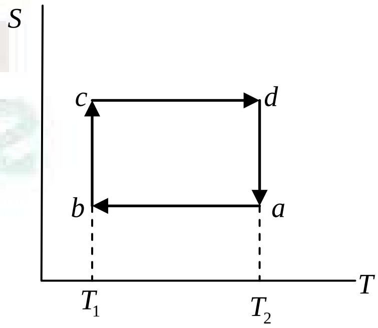
Q11. Which one of the following entropy (S) - temperature (T) diagrams CORRECTLY represents the Carnot cycle $abcda$ shown in the $P-V$ diagram?



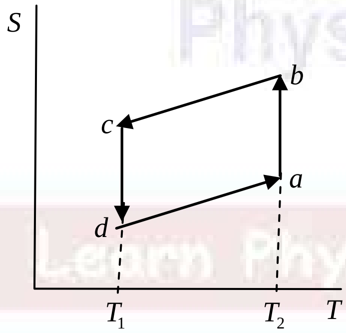
(A)



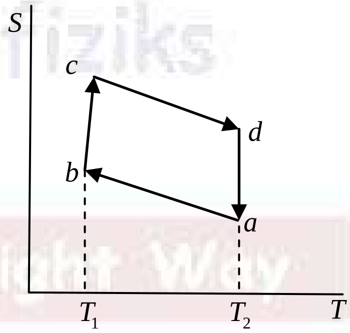
(B)



(C)



(D)



Ans. : (c)

Solution: From *P*-*V* diagram $T_2 > T_1$

Process ab, Isothermal expansion at T_2;

Process bc is Isochoric process, in which pressure reduces (say from P_2 to P_1, $P_2 > P_1$)

Process cd, Isothermal compression at T_1;

Process da is Isochoric process, in which pressure increases (say from P_1 to P_2)

Assuming Ideal gas as working substance, for an Isochoric process, $P \propto T$

Entropy change in an Isochoric process is calculated as below

$$dQ_V = C_V dT$$

$$\Delta S = \int_{T_2}^{T_1} \frac{dQ_V}{T} = C_V \int_{T_2}^{T_1} \frac{dT}{T} = C_V \ln\left(\frac{T_1}{T_2}\right) = C_V \ln\left(\frac{P_1}{P_2}\right)$$

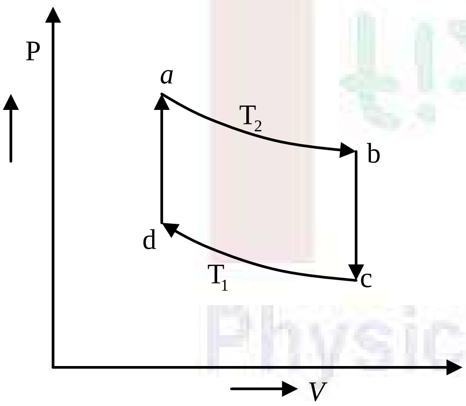
Hence for given process bc, $\Delta S = S_c - S_b = C_V \ln\left(\frac{P_1}{P_2}\right) < 0$

It means entropy at b is larger as compared to entropy at c (i.e. $S_b > S_c$)

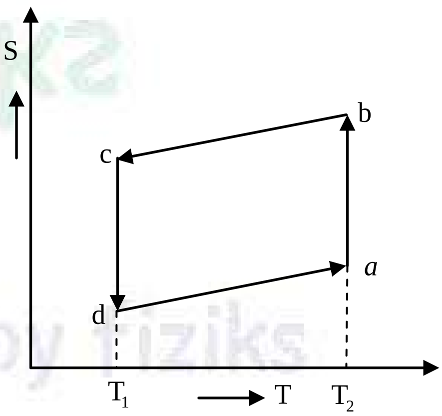
Process ab, Isothermal expansion at T_2.

It is accompanied with addition of Heat system. Hence, in process ab, entropy increases but Temperature is constant at T_2.

Similarly we can check for other processes. Hence correct diagram should be as below.

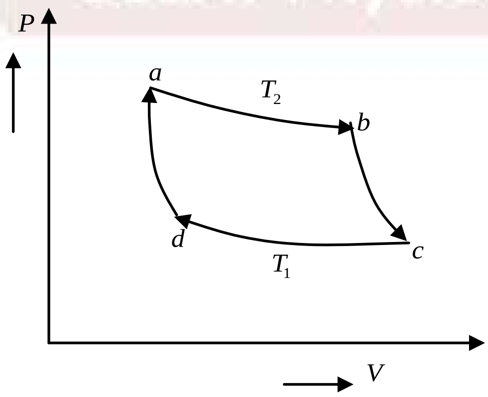


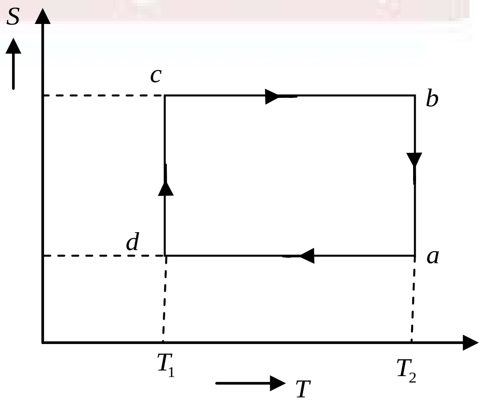
Given diagram



Correct one

NOTE: When bc is adiabatic process





Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q28. Which of the following is (are) the CORRECT option(s) for the Joule-Thomson effect?

(A) It is an isentropic process

(B) It is an isenthalpic process

(C) It can result in cooling as well as heating

(D) For an ideal gas it always results in cooling

Ans. : (b) and (c)

Solution:

Joule-Thomson or Joule-Kelvin effect is also known as porous-plug experiment. In this effect a gas expands from high pressure region (P_1) to low pressure region $(P_2, P_1 > P_2)$. Following conclusions are valid for such as expansion.

(i) Joule-Thomson expansion is an irreversible expansion.

(ii) It is Isenthalpic process, i.e., $U_1 + P_1V_1 = U_2 + P_2V_2$

(iii) For an Ideal gas $\mu_{JK} = \left(\frac{\partial T}{\partial P}\right)_H = 0$

This means, there is no change in temperature of an ideal gas undergoing $J-T(J-K)$ expansion.

(iv) For a real gas, the Joule-Kelvin coefficients, i.e. μ_{JK} could be positive or negative and accordingly cooling or heating may be resulted.

(v) As the gas expands and process is irreversible this process is always accompanied with increase in entropy.

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3). No Question

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

Q38. A simple harmonic oscillator with an angular frequency ω is in thermal equilibrium with a reservoir at absolute temperature T, with $\omega = \frac{2k_B T}{\hbar}$. Which one of the following is the partition function of the system?

(A) $\frac{e}{e^2-1}$ (B) $\frac{e}{e^2+1}$

(C) $\frac{e}{e-1}$ (D) $\frac{e}{e+1}$

Ans.: (a)

Solution.:

For a Harmonic Oscillator, the energy spectrum is $E_n = \left[n+\frac{1}{2}\right]\hbar\omega,\ n = 0,1,2,3,4,....$

Partition function is $Q = \sum_{n=0}^{\infty} e^{-\beta E_n} = \sum_{n=0}^{\infty} e^{-\beta\left(n+\frac{1}{2}\right)\hbar\omega} = e^{\frac{-\beta\hbar\omega}{2}} + e^{\frac{-3\beta\hbar\omega}{2}} + e^{\frac{-5\beta\hbar\omega}{2}} + e^{\frac{-7\beta\hbar\omega}{2}} + ...$

$$\Rightarrow Q = e^{\frac{-\beta\hbar\omega}{2}}\left[1 + e^{-\beta\hbar\omega} + e^{-2\beta\hbar\omega} + e^{-3\beta\hbar\omega} + ...\right] \qquad \Rightarrow Q = \frac{e^{-\frac{\beta\hbar\omega}{2}}}{1-e^{-\beta\hbar\omega}} \qquad ....(1)$$

Given, $\omega = \frac{2k_B T}{\hbar} \Rightarrow \frac{\hbar}{2k_B T} = \frac{1}{\omega}$

$$\therefore\ Q = \frac{e^{-\frac{\hbar\omega}{2k_B T}}}{1-e^{-\frac{\hbar\omega}{k_B T}}} = \frac{e^{-1}}{1-e^{-2}} = \frac{e}{e^2-1}$$

Hence, option (a) is CORRECT.

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3) No Question

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3)

Q63. Consider 6 identical, non-interacting, spin $\frac{1}{2}$ atoms arranged on a crystal lattice at absolute temperature T. The z-component of the magnetic moment of each of these atoms can be $\pm\mu_B$. If P and Q are the probabilities of the net magnetic moment of the solid being $2\mu_B$ and $6\mu_B$ respectively, what is the value of $\frac{P}{Q}$ (integer)?

Ans.: **15**

Solution.:

Total spins = 6, z-component of spin can take $=\pm\mu_B$ two values. Hence, total no of accessible microstates are $\Omega_{\text{Total}} = 2^6 = 64$ ....(1)

Note that all of these microstates are not a meaningful. Meaningful microstates are those which satisfy the condition that net magnetic moment is $2\mu_B$ and $6\mu_B$.

Case-1: Magnetic moment $=2\mu_B$ is obtained when 4 spins point upawrd and 2 spins point downwards as shown below

$$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \downarrow \quad \downarrow$$

No. of microstates which satisfy this condition is

$$\Omega_{2\mu_B} = \frac{6!}{4!2!} = \frac{6\times5\times4!}{4!2} = 15 \qquad ....(2)$$

$$\therefore \; P = \frac{15}{64} \qquad ....(3)$$

Case-2: Magnetic moment $=6\mu_B$ is obtained when all six spins point upward as below

$$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$$

$$\Omega_{6\mu_B} = \frac{6!}{6!} = 1 \text{ hence } Q = \frac{1}{64}$$

$$\therefore \; \frac{P}{Q} = 15$$

Q64. Two identical, non-interacting ${}^4He_2$ atoms are distributed among 4 different non-degenerate energy levels. The probability that they occupy different energy levels is p. Similarly, two ${}^3He_2$ atoms are distributed among 4 different non-degenerate energy levels, and the probability that they occupy different levels is q. What is the value of $\frac{p}{q}$ (rounded off to one decimal place)?

Ans.: **0.6**

Solution.:

${}_2He^4$ Behaves like Boson; ${}_2He^3$ Behaves like Fermion

Case-1: $N=2$, $g=4$ (4 Energy levels)

Total no of ways of distributing 2 Bosons in 4 energy levels is

$$\Omega_{BE}=\frac{(N+g-1)!}{N!(g-1)!}=\frac{(2+4-1)!}{2!(4-1)!}=\frac{5\times4\times3!}{2\times3\,!}=10$$

These are shown below

			00			0		0	0
		00			0		0		0
	00			0			0	0	
00				0	0	0			

$\therefore$ Probability that ${}_2He^4$ atoms occupying different levels $=p=\frac{\text{Favourable outcomes}}{\text{Total outcomes}}=\frac{6}{10}$

Case-II: $N=2, g=4, {}_2He^3$ Fermions, subjected to Pauli Principle

		0		0	0
	0		0		0
0			0	0	
0	0	0			

Probability that ${}_2He^3$ atoms occupy different levels $=q=\frac{\text{Favourable outcomes}}{\text{Total outcomes}}=\frac{6}{6}=1$

$$\therefore\ \frac{p}{q}=\frac{6/10}{1}=0.6$$

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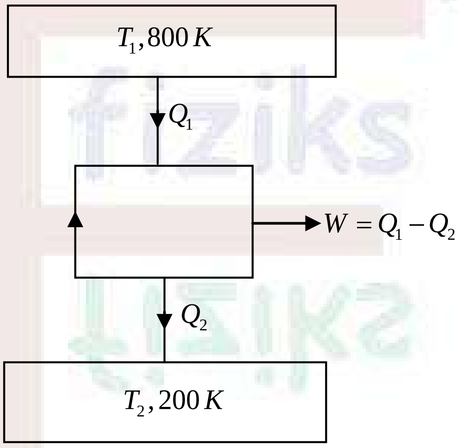
Q65. Two identical bodies kept at temperatures 800 K and 200 K act as the hot and the cold reservoirs of an ideal heat engine, respectively. Assume that their heat capacity (C) in Joules/K is independent of temperature and that they do not undergo any phase change. Then, the maximum work that can be obtained from the heat engine is $n \times C$ Joules. What is the value of n (in integer)?

Ans.: **200**

Solution.:

Let Q_1 be the total Heat extracted from source at $T_1 = 800\,K$.

Let Q_2 be the total Heat rejected to sink at $T_2 = 200\,K$.



Let T be the final common Temperature of both reservoirs and W be the total work done obtained.

Hence, $Q_1 = C[T_1 - T]$ and $Q_2 = C[T - T_2]$

$W = Q_1 - Q_2 = C[T_1 - T] - C[T - T_2] \quad \Rightarrow W = C[T_1 + T_2 - 2T]$ ....(1)

Entropy change of source during complete process $\Delta S_1 = C \ln\left(\frac{T}{T_1}\right)$

For sink $\Delta S_2 = C \ln\left(\frac{T}{T_2}\right)$

Net entropy change of during full cycles $\Delta S = \Delta S_1 + \Delta S_2 = C \ln \frac{T^2}{T_1 T_2} \geq 0$ .....(2)

For W to be maximum (see 1), T has to be minimum.

Hence (2) gives $T = \sqrt{T_1 T_2} = \sqrt{800 \times 200} = 400\,K$

$\therefore \; W = C[800 + 200 - 2 \times 400] = 200\,C$

Hence, $n = 200$

Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q13. Choose the most appropriate matching of the items in **Column 1** with those in **Column 2**.

Column 1	Column 2
(i) PIN diode	P. Voltage regulation
(ii) Tunnel diode	Q. Radio frequency and microwave devices
(iii) Zener diode	R. Optoelectronic detection
(iv) Photo diode	S. Oscillator

(A) (i) - Q; (ii) - S; (iii) - P; (iv) - R

(B) (i) - R; (ii) - Q; (iii) - P; (iv) - S

(C) (i) - R; (ii) - S; (iii) - P; (iv) - Q

(D) (i) - P; (ii) - Q; (iii) - R; (iv) - S

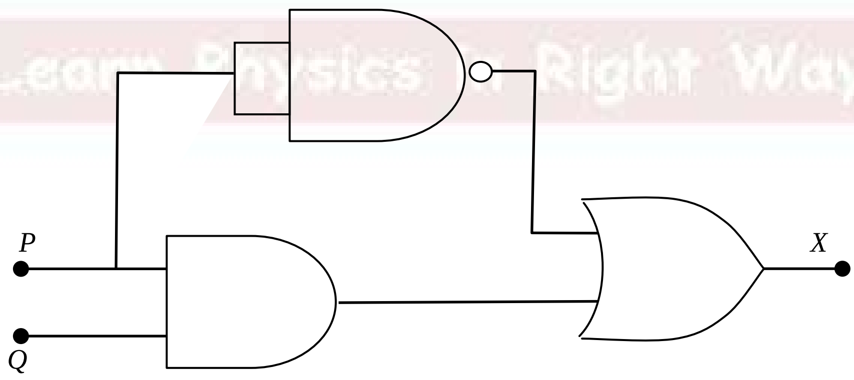
Ans. : (a)

Solution:

Column 1	Column 2
(i) PIN diode	Q. Radio frequency and microwave devices
(ii) Tunnel diode	S. Oscillator
(iii) Zener diode	P. Voltage regulation
(iv) Photo diode	R. Optoelectronic detection

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q24. Which one of the following options is CORRECT for the given logic circuit?



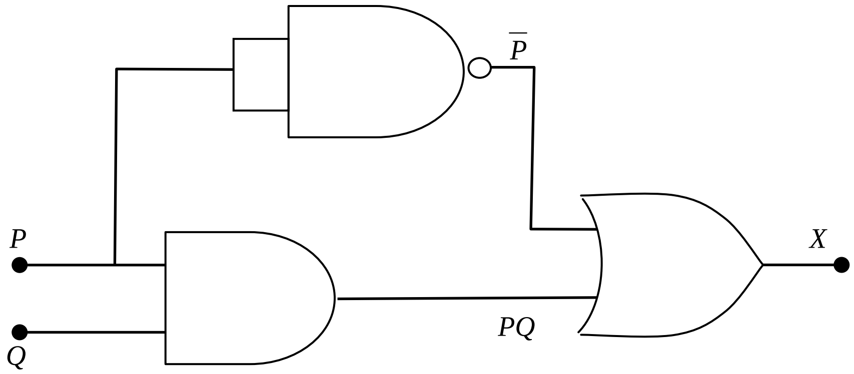
(A) $P = 1, Q = 1; X = 0$ (B) $P = 1, Q = 0; X = 1$

(C) $P = 0, Q = 1; X = 0$ (D) $P = 0, Q = 0; X = 1$

Ans. : (d)

Solution:

$X=\overline{P}+PQ=\overline{P}+\overline{\overline{P}}Q=\overline{P}+Q$.



(a) P = 1, Q = 1; $X=\overline{1}+1=1$

(b) P = 1, Q = 0; $X=\overline{1}+0=0$

(c) P = 0, Q = 1; $X=\overline{0}+1=1$

(d) P = 0, Q = 0; $X=\overline{0}+0=1$

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3).

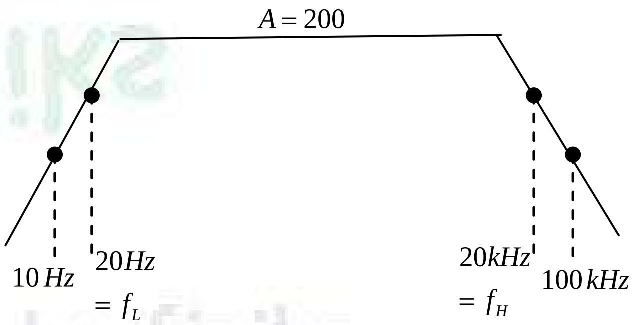
Q34. For a transistor amplifier, the frequency response is such that the mid band voltage gain is 200. The cutoff frequencies are 20 Hz and 20 kHz. What is the ratio (rounded off to two decimal places) of the voltage gain at 10 Hz to that at 100 kHz?

Ans.: 2.20 to 2.36

Solution.:

$$A(f)_{10Hz}=\frac{A}{\sqrt{1+\left(f_L/f\right)^2}}$$

$$=\frac{200}{\sqrt{1+\left(\frac{20}{10}\right)^2}}=\frac{200}{\sqrt{5}}=89.44$$

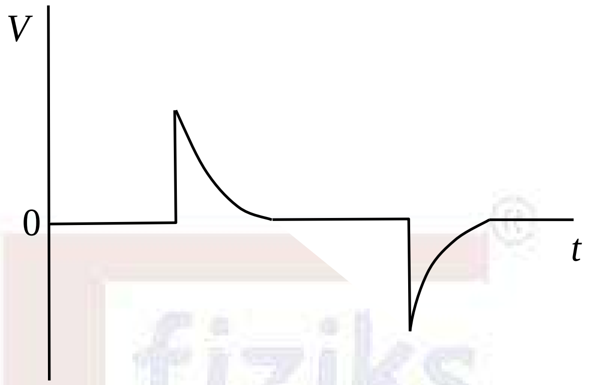


$$A(f)_{100KHz}=\frac{A}{\sqrt{1+\left(f/f_H\right)^2}}=\frac{200}{\sqrt{1+\left(\frac{100}{20}\right)^2}}=39.22$$

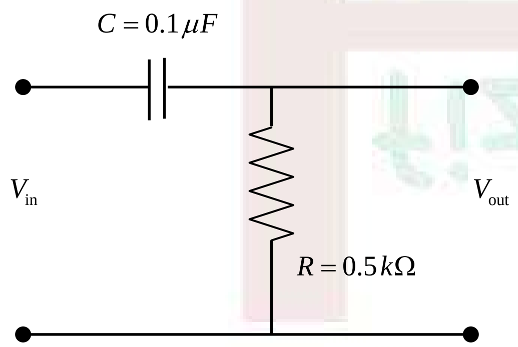
Ratio of voltage gain $=\dfrac{89.44}{39.22}=2.28$

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

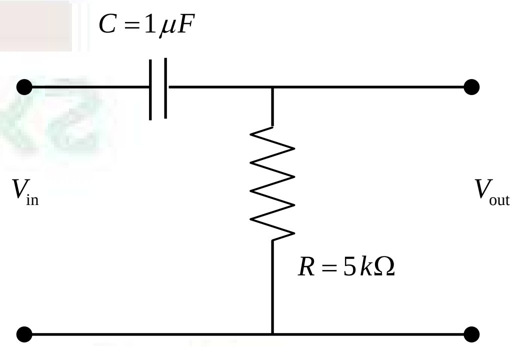
Q37. An input voltage in the form of a square wave of frequency 1 kHz is given to a circuit, which results in the output shown schematically below. Which one of the following options is the CORRECT representation of the circuit?



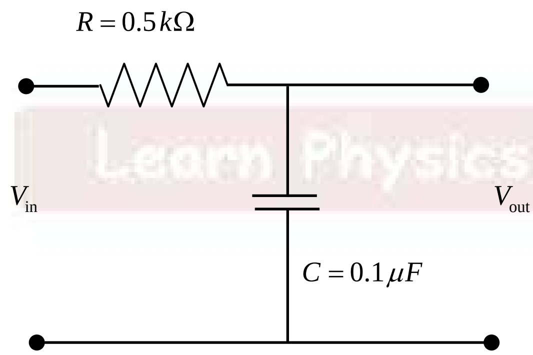
(A)



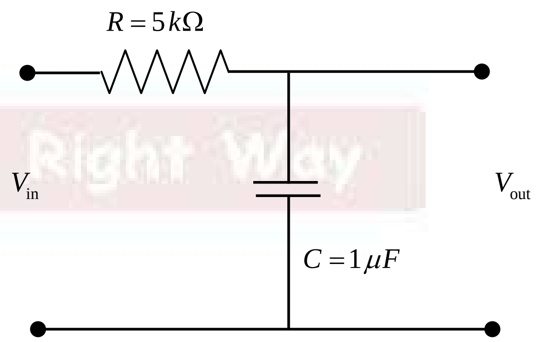
(B)



(C)



(D)



Ans.: (a)

Solution:
The output shown is a differentiated output. So C-R combination will give differentiated output

and $RC << T = \frac{1}{f} = \frac{1}{1000}\sec = 1m\sec$.

So lets check only options (a) and (b).

In option (a):

$RC = 0.5\times10^{3}\times0.1\times10^{-6} \Rightarrow RC = 0.05\ m\sec$ and $RC << T$

Input signal will be differentiated.

In option (b):

$RC = 5\times10^{3}\times1\times10^{-6} \Rightarrow RC = 5\ m\sec$ and $RC >> T$

Input signal will not be differentiated.

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3) No Question

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3) No Question

Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q12. Which one of the following is a dimensionless constant?

(A) Permittivity of free space (B) Permeability of free space

(C) Bohr magneton (D) Fine structure constant

Ans. : (d)

Solution:

(a) Permittivity dimensional formula $\left[M^{-1}L^{-3}T^{4}A^{2}\right]$

(b) Permeability dimensional formula $\left[ML^{2}T^{-2}A^{-2}\right]$

(c) Bohr magneton dimensional formula $\left[M^{0}L^{2}T^{0}A^{1}\right]$

(d) Fine structure constant $\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} \approx \frac{1}{137}$ is a dimensionless constant.

Q14. The atomic number of an atom is 6. What is the spectroscopic notation of its ground state, according to Hund's rules?

(A) ${}^3P_0$ (B) ${}^3P_1$

(C) ${}^3D_3$ (D) ${}^3S_1$

Ans. : (a)

Solution.:

The electronic configuration of the atom with atomic number 6 i.e., carbon is

$1s^2 2s^2 2p^2$ i.e., two equivalent electrons in the $2p$ orbitals

$s_1 = ½,\ s_2 = ½\quad 1_1 = 1, 1_2 = 1$

$S = 0,1;\ L = 0,1,2$

For equivalent electrons under LS coupling scheme, the spectral terms will be

For, $S = 0,\ \ L = 0,\ \ J = 0;\qquad {}^1S_0$

$S = 0,\ \ L = 2,\ \ J = 2;\qquad {}^1D_2$

For, $S = 1,\ \ L = 1,\ \ J = 0,1,2;\qquad {}^3P_{0,1,2}$

Therefore, the ground state spectral term according to Hund's rule is ${}^3P_0$

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3). No Question

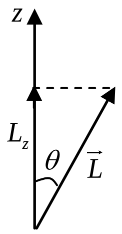
Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3).

Q33. In the vector model of angular momentum applied to atoms, what is the minimum angle in degrees (in integer) made by the orbital angular momentum vector and the positive z-axis for a $2p$ electron?

Ans.: **45**

Solution.:

$$\cos\theta = \frac{L_z}{L} = \frac{m_l \hbar}{\sqrt{l(l+1)}\hbar} \Rightarrow \cos\theta = \frac{m_l}{\sqrt{l(l+1)}}$$



For $2p$ electron, $l = 1$ and $m_\ell = +1, 0, -1$

$$\therefore \cos\theta = +\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \Rightarrow \theta = 45°, 90°, 135°$$

Thus, the smallest angle is 45^0.

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

Q36. It is given that the electronic ground state of a diatomic molecule X_2 has even parity and the nuclear spin of X is 0. Which one of the following is the CORRECT statement with regard to the rotational Raman spectrum (J is the rotational quantum number) of this molecule?

(A) Lines of all J values are present

(B) Lines have alternating intensity in the ratio of 3:1

(C) Lines of only even J values are present

(D) Lines of only odd J values are present

Ans.: (d)

Solution.:

Nuclear spin is given to be 0 and hence the molecule is a boson. The nuclear wavefunction is symmetric under exchange of two nuclei, according to theory for such molecules rotational states are only allowed to have odd values of J.

Q39. Which one of the following options is the most appropriate match between the items given in **Column 1** and **Column 2**?

Column 1	Column 2
(i) Visible light	P. Transition between core energy levels of atoms
(ii) X-rays	Q. Transition between nuclear energy levels
(iii) Gamma rays	R. Pair production
(iv)Thermal neutrons	S. Crystal structure determination
	T. Photoelectric effect

(A) (i) - T; (ii) - P,S,T; (iii) - Q,R; (iv) - S

(B) (i) - P,T; (ii) - S; (iii) - R,S; (iv) - S,T

(C) (i) - T; (ii) - R,S; (iii) - Q,R; (iv) - S

(D) (i) - S,T; (ii) - P,S; (iii) - R,T; (iv) - S

Ans.: (a)

Solution.:

Column 1	Column 2
(i) Visible light	T. Photoelectric effect
(ii) X-rays	P. Transition between core energy levels of atoms S. Crystal structure determination T. Photoelectric effect
(iii) Gamma rays	Q. Transition between nuclear energy levels R. Pair production
(iv)Thermal neutrons	S. Crystal structure determination

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3)

Q54. Different spectral lines of the Balmer series (transitions $n \to 2$, with n being the principal quantum number) fall one at a time on a Young's double slit apparatus. The separation between the slits is d and the screen is placed at a constant distance from the slits. What factor should d be multiplied by to maintain a constant fringe width for various lines, as n takes different allowed values?

(A) $\dfrac{n^2-4}{4n^2}$ (B) $\dfrac{n^2+4}{4n^2}$

(C) $\dfrac{4n^2}{n^2-4}$ (D) $\dfrac{4n^2}{n^2+4}$

Ans.: (c)

Solution.:

The fringe width of the interference pattern for a Young's double slit setup is given by,

$$\beta = \frac{\lambda D}{d}$$

where, d is the separation between the slits and D is the distance of the screen from the slit.

For Balmer's series, wavelength of a Balmer line is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{4}-\frac{1}{n^2}\right) = R\frac{n^2-4}{4n^2} \Rightarrow \lambda = \frac{4n^2}{\left(n^2-4\right)R}$$

where, R is Rydberg's constant

Substituting the value of λ in the expression for fringe width, we have

$$\beta = \frac{\lambda D}{d} = \frac{D}{d}\frac{4n^2}{\left(n^2-4\right)R} = \frac{D}{R}\times\frac{1}{\frac{\left(n^2-4\right)}{4n^2}d}$$

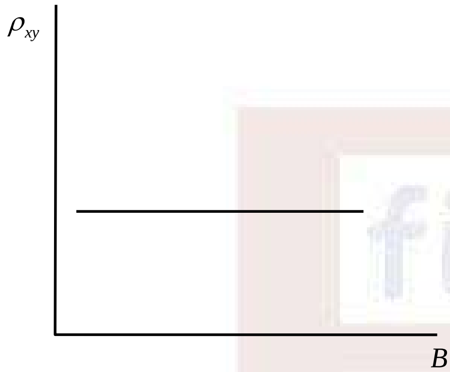
Since, D and R are constant, to have fringe width constant, the variable $'n'$ should not be present in the expression. Hence, the distance between the slits $'d'$ should be multiplied by $\dfrac{4n^2}{\left(n^2-4\right)}$.

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3) No Question

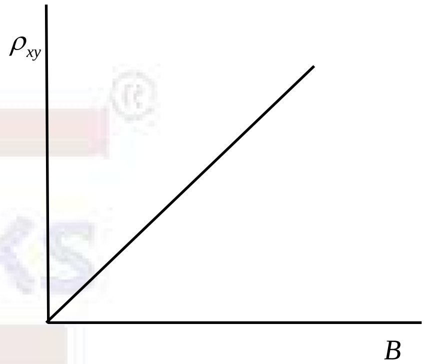
Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q16. The Hall experiment is carried out with a non-magnetic semiconductor. The current I is along the x-axis and the magnetic field B is along the z-axis. Which one of the following is the CORRECT representation of the variation of the magnitude of the Hall resistivity ρ_{xy} as a function of the magnetic field?

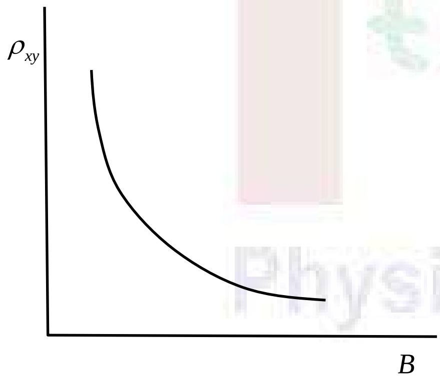
(A)



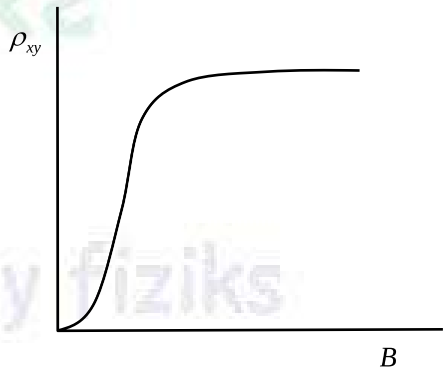
(B)



(C)



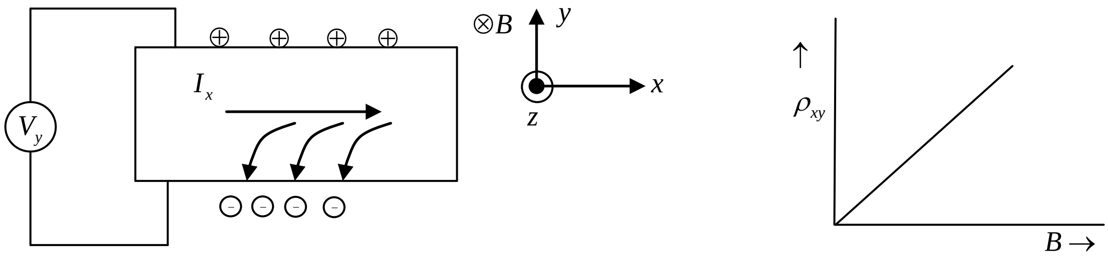
(D)



Ans. : (b)

Solution.:

Hall resistivity $\rho_{xy} \propto \dfrac{V_y}{I_x}$



As B-field increases, number of charges on the lower and upper plate increases consequently hall voltage V_y increases and current I_x decreases. Thus ρ_{xy} increases.

Q18. A compound consists of three ions X, Y and Z. The Z ions are arranged in an FCC arrangement. The X ions occupy $\frac{1}{6}$ of the tetrahedral voids and the Y ions occupy $\frac{1}{3}$ of the octahedral voids. Which one of the following is the CORRECT chemical formula of the compound?

(A) XY_2Z_4 (B) XYZ_3 (C) XYZ_2 (D) XYZ_4

Ans. : (b)

Solution.: In FCC, number of Tetrahedral voids = 8 and number of octahedral voids = 4

Let n_x, n_y & n_z represents effective number of ions at tetrahedral voids, octahedral voids and fcc lallice sites.

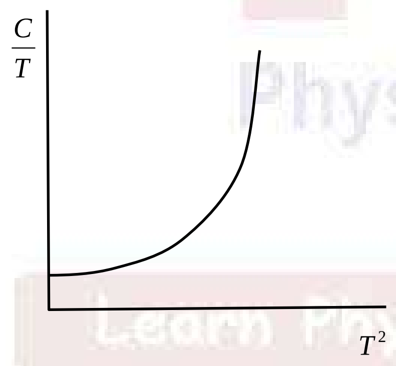
$$\therefore \; n_x = \frac{1}{6}\times 8 = \frac{4}{3}; \quad n_y = \frac{1}{3}\times 4 = \frac{4}{3}; \quad n_z = 4$$

multiply with $\frac{3}{4}$ to make it lowest integer $\therefore \; n_x = 1,\; n_y = 1,\; n_z = 3$

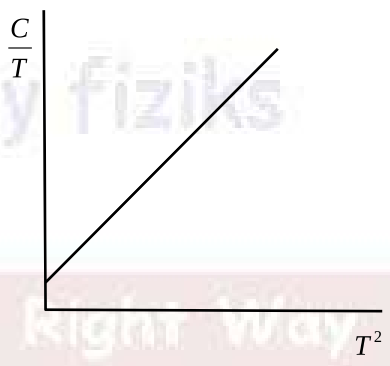
Thus correct chemical formula is XYZ_3

Q19. For a non-magnetic metal, which one of the following graphs best represents the behaviour of $\frac{C}{T}$ vs. T^2, where C is the heat capacity and T is the temperature?

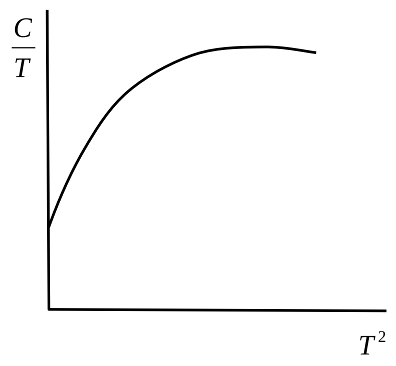
(A)



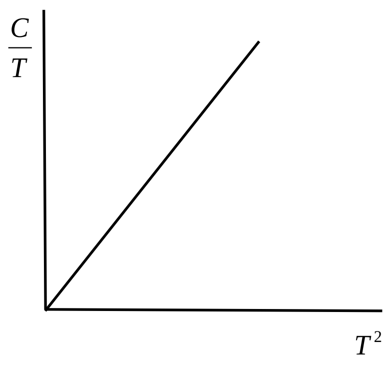
(B)



(C)



(D)

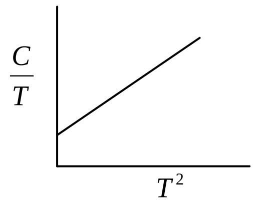


Ans. : (b)

Solution.:

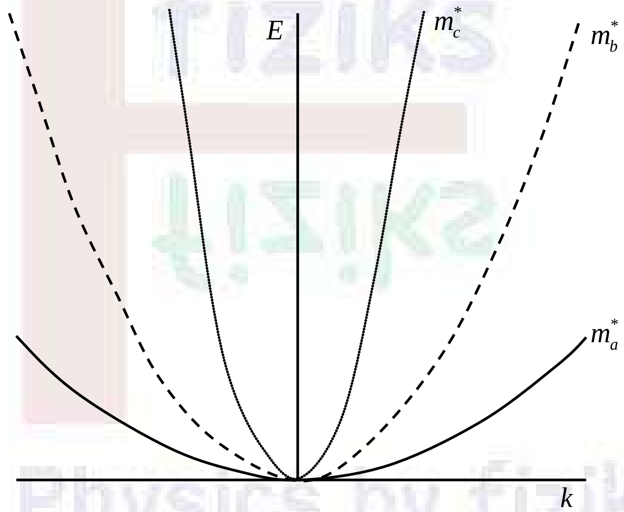
For metal: $C = AT + BT^3$

$$\Rightarrow \frac{C}{T} = A + BT^2$$



Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q20. For non-relativistic electrons in a solid, different energy dispersion relations (with effective masses m_a^*, m_b^*, and m_c^*) are schematically shown in the plots. Which one of the following options is CORRECT?



(A) $m_a^* = m_b^* = m_c^*$ (B) $m_b^* > m_c^* > m_a^*$

(C) $m_c^* > m_b^* > m_a^*$ (D) $m_a^* > m_b^* > m_c^*$

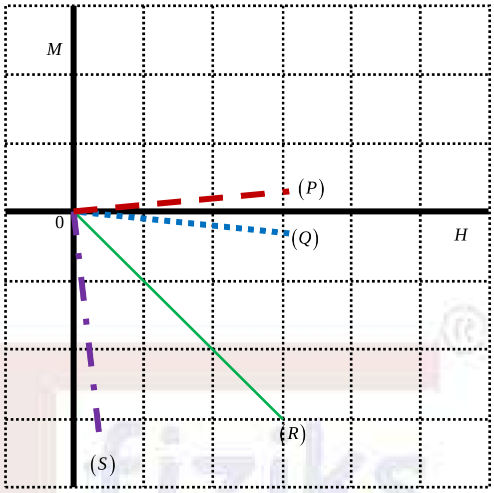
Ans. : (d)

Solution.:

$$m^* = \frac{\hbar^2}{d^2E/dk^2} \text{ and } \left(\frac{d^2E}{dk^2}\right)_c > \left(\frac{d^2E}{dk^2}\right)_b > \left(\frac{d^2E}{dk^2}\right)_a$$

$$\Rightarrow m_a^* > m_b^* > m_c^*$$

Q21. The figure schematically shows the M (magnetization) - H (magnetic field) plots for certain types of materials. Here M and H are plotted in the same scale and units. Which one of the following is the most appropriate combination?



(A) (Q) - Paramagnet; (R) - Type-I Superconductor; (S) - Antiferromagnet

(B) (P) - Paramagnet; (Q) - Diamagnet; (R) - Type-I Superconductor

(C) (P) - Paramagnet; (Q) - Antiferromagnet; (R) - Type-I Superconductor

(D) (P) - Diamagnet; (R) - Paramagnet; (S) - Type-I Superconductor

Ans. : (b)

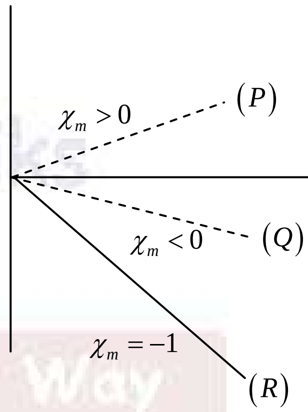
Solution.:

$M = \chi_m H$

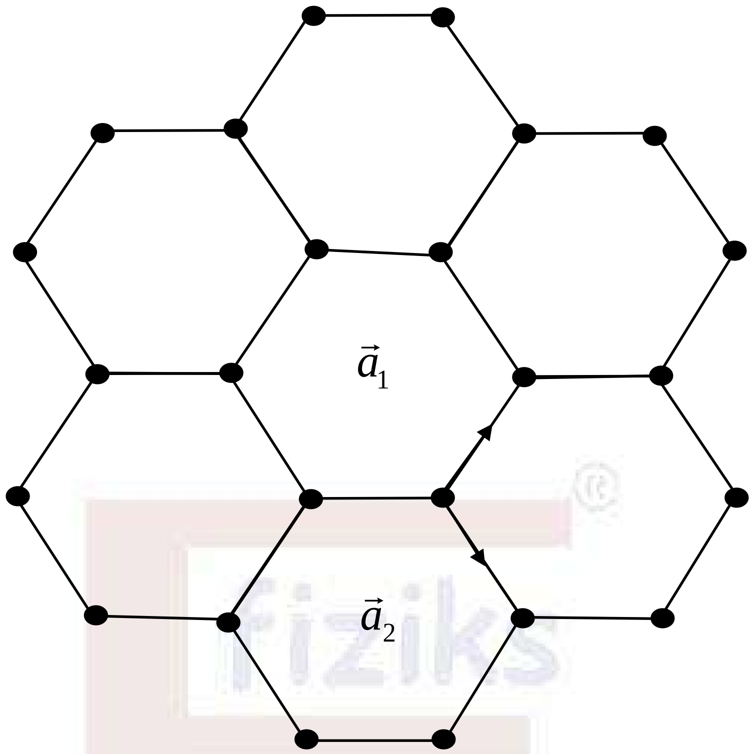
For Paramagnet $\chi_m > 0$

For diamagnet $\chi_m < 0$

For Type-I Superconductor $\chi_m = -1$



Q22. Graphene is a two-dimensional material, in which carbon atoms are arranged in a honeycomb lattice with lattice constant a. As shown in the figure, $\vec{a}_1$ and $\vec{a}_2$ are two lattice vectors. Which one of the following is the area of the first Brillouin zone for this lattice?



(A) $\dfrac{8\pi^2}{3\sqrt{3}a^2}$ (B) $\dfrac{4\pi^2}{3\sqrt{3}a^2}$

(C) $\dfrac{8\pi^2}{\sqrt{3}a^2}$ (D) $\dfrac{4\pi^2}{\sqrt{3}a^2}$

Ans. : (a)

Solution.:

Area of the cell (A) = $6\times\dfrac{\sqrt{3}}{4}a^2 = \dfrac{3\sqrt{3}}{2}a^2$

Area of the 1st Brillouin zone $A^* = \dfrac{(2\pi)^2}{A} = \dfrac{4\pi^2}{\frac{3\sqrt{3}}{2}a^2} = \dfrac{8\pi^2}{3\sqrt{3}a^2}$

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3). No Question

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

Q45. A neutron beam with a wave vector $\vec{k}$ and an energy 20.4meV diffracts from a crystal with an outgoing wave vector $\vec{k}'$. One of the diffraction peaks is observed for the reciprocal lattice vector $\vec{G}$ of magnitude 3.14Å^{-1}. What is the diffraction angle in degrees (rounded off to the nearest integer) that $\vec{k}$ makes with the plane? (Use mass of neutron $=1.67\times10^{-27}$ Kg)

(A) 15 (B) 30 (C) 45 (D) 60

Ans.: (b)

Solution.: $d=\dfrac{2\pi}{|G|}=\dfrac{2\pi}{3.14}A^\circ=2A^\circ$

$$\lambda=\frac{h}{\sqrt{2mE}}=\frac{0.28}{\sqrt{E(eV)}}A^0=\frac{0.28}{\sqrt{20.4\times10^{-3}}}A^0=1.96A^0$$

Bragg's law $2d\sin\theta=\lambda\Rightarrow\sin\theta=\dfrac{\lambda}{2d}=\dfrac{1.9A^\circ}{2\times2A^\circ}=0.5\Rightarrow\theta=30^0$

Q46. In the first Brillouin zone of a rectangular lattice (lattice constants $a=6\text{Å}$ and $b=4\text{Å}$), three incoming phonons with the same wave vector $\left\langle1.2\text{Å}^{-1},0.6\text{Å}^{-1}\right\rangle$ interact to give one phonon. Which one of the following is the CORRECT wave vector of the resulting phonon?

(A) $\left\langle2.56\text{Å}^{-1},0.23\text{Å}^{-1}\right\rangle$ (B) $\left\langle3.60\text{Å}^{-1},1.80\text{Å}^{-1}\right\rangle$

(C) $\left\langle0.48\text{Å}^{-1},0.23\text{Å}^{-1}\right\rangle$ (D) $\left\langle3.60\text{Å}^{-1},0.80\text{Å}^{-1}\right\rangle$

Ans.: (c)

Solution.: $\vec{k}_1+\vec{k}_2+\vec{k}_3=\vec{k}+\vec{G}\Rightarrow\vec{k}=\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{G}$

$\therefore\ k_x=k_{1x}+k_{2x}+k_{3x}-G_x$ and $k_y=k_{1y}+k_{2y}+k_{3y}-G_y$

where $G_x=\dfrac{2\pi}{a}=\dfrac{2\pi}{6A^\circ}=\dfrac{\pi}{3}A^{\circ-1}=1.047A^{\circ-1}$; $G_y=\dfrac{2\pi}{b}=\dfrac{2\pi}{4A^\circ}=\dfrac{\pi}{2}A^{\circ-1}=1.57A^{\circ-1}$

Thus $k_x=1.2+1.2+1.2-1.047=2.56A^{\circ-1}$

and $k_y=0.6+0.6+0.6-1.57=0.23A^{\circ-1}$

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3)

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Q47. For a covalently bonded solid consisting of ions of mass m, the binding potential can be assumed to be given by $U(r) = -\in\left(\frac{r}{r_0}\right)e^{-\frac{r}{r_0}}$, where $\in$ and r_0 are positive constants.

What is the Einstein frequency of the solid in Hz?

(A) $\frac{1}{2\pi}\sqrt{\frac{\in e}{mr_0^2}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{\in}{mer_0^2}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{2\in}{mer_0^2}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{\in e}{2mr_0^2}}$

Ans.: (b)

Solution.:

$$U(r) = -\in\left(\frac{r}{r_0}\right)e^{-r/r_0}$$

At equilibrium $F = -\frac{dU}{dr} = 0 \Rightarrow \frac{dU}{dr} = \frac{-\in}{r_0}\left[e^{-r/r_0} + re^{-r/r_0}\left(-\frac{1}{r_0}\right)\right] = 0 \Rightarrow 1 - \frac{r}{r_0} = 0 \Rightarrow r = r_0$

Now $\frac{d^2U}{dr^2} = \frac{-\in}{r_0}\left[e^{-r/r_0}\left(-\frac{1}{r_0}\right) - \frac{1}{r_0}e^{-r/r_0} - \frac{r}{r_0}e^{-r/r_0}\left(-\frac{1}{r_0}\right)\right] = -\frac{\in}{r_0}\left[-\frac{2}{r_0}e^{-r/r_0} + \frac{r}{r_0^2}e^{-r/r_0}\right]$

at $r = r_0$; $\frac{d^2U}{dr^2} = -\frac{\in}{r_0}\left[-\frac{2}{r_0 e} + \frac{1}{r_0 e}\right] = -\frac{\in}{r_0}\left[-\frac{1}{r_0 e}\right] \Rightarrow \frac{d^2U}{dr^2} = \frac{\in}{r_0^2 e}$

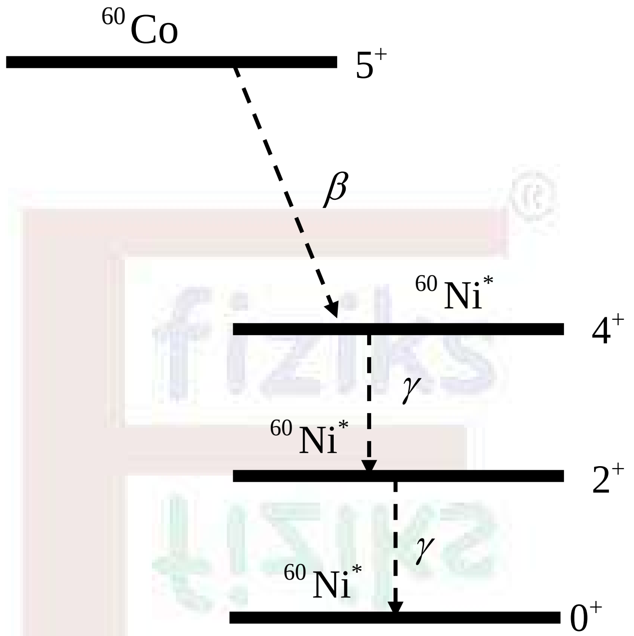
Since, $\frac{d^2U}{dr^2} = m\omega^2 \Rightarrow m\omega^2 = \frac{\in}{r_0^2 e} \Rightarrow \omega = \sqrt{\frac{\in}{mer_0^2}}$ and $\nu = \frac{1}{2\pi}\sqrt{\frac{\in}{mer_0^2}}$

Thus correct option is (B).

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3) No Question

Q.11 – Q.19 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q23. A ^{60}Co nucleus emits a β-particle and is converted to $^{60}Ni^{*}$ with $J^{P}=4^{+}$, which in turn decays to the ^{60}Ni ground state with $J^{P}=0^{+}$ by emitting two photons in succession, as shown in the figure. Which one of the following statements is CORRECT?



(A) $4^{+}\to 2^{+}$ is an electric octupole transition

(B) $4^{+}\to 2^{+}$ is a magnetic quadrupole transition

(C) $2^{+}\to 0^{+}$ is an electric quadrupole transition

(D) $2^{+}\to 0^{+}$ is a magnetic quadrupole transition

Ans. : (c)

Solution.:

$4^{+}\to 2^{+}$

$\left|\vec{J}_i-\vec{J}_f\right|\le L\le\left|\vec{J}_i+\vec{J}_f\right|$

$\Delta\pi$ = No change; $L=2,3,4,5,6$

So, $4^{+}\to 2^{+}$ transition will be of electric quadrupole $(E2)$ type.

$2^{+}\to 0^{+}$ $(\Delta\pi=\text{No change}, L=2)$

So, $2^{+}\to 0^{+}$ transition will be of electric quadrupole $(E2)$ type.

Q27. The Ξ^{0*} particle is a member of the Baryon decuplet with isospin state $|I, I_3\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle$ and strangeness quantum number -2. In the quark model, which one of the following is the flavour part of the Ξ^{0*} wavefunction?

(A) $\frac{1}{\sqrt{2}}(uss - ssu)$ (B) $\frac{1}{\sqrt{3}}(uss + sus + ssu)$

(C) $\frac{1}{\sqrt{2}}(uss + ssu)$ (D) $\frac{1}{\sqrt{3}}(uss - sus + ssu)$

Ans. : (b)

Solution.:

Corresponding to different colons (B, R, G) $\Xi^{\circ *}$ wave function will be (flavour part)

$$\psi_{\Xi^{0*}} = \frac{1}{\sqrt{3}}(uss + sus + ssu)$$

$BRG \quad BRG \quad BRG$

Isospin $\vec{I} = \frac{\vec{1}}{2} + \vec{0} + \vec{0}; \quad I_z = \frac{1}{2} + 0 + 0$

Charge $Q = +\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0; \quad B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1; \quad S = 0 - 1 - 1 = -2$

Q.20 – Q.28 Multiple Select Question (MSQ), carry ONE mark each (for each wrong answer: –1/3).

Q29. The deuteron is a bound state of a neutron and a proton. Which of the following statements is(are) CORRECT?

(A) The deuteron has a finite value of electric quadrupole moment due to non-spherical electronic charge distribution

(B) The magnetic moment of the deuteron is equal to the sum of the magnetic moments of the neutron and the proton

(C) The deuteron state is an admixture of $^3S_1$ and $^3D_1$ states

(D) The deuteron state is an admixture of $^3S_1$ and $^3P_1$ states

Ans. : (a), (b) and (c)

Solution.: (a) Finite value of electric quadrupole moment is due to $^3D_1$.

(b) Magnetic moment of the deuteron is due to the orbital motion of proton and spin motion of proton and neutron both.

(c) The ground state of deuteron is $\psi_d = a\psi_1\left(^3S_1\right) + b\psi_2\left(^3D_1\right)$.

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Q30. The Geiger-Muller counter is a device to detect α, β and γ radiations. It is a cylindrical tube filled with monatomic gases like argon, and polyatomic gases such as ethyl alcohol. The inner electrode is along the axis of the cylindrical tube and the outer electrode is the tube. Which of the following statements is (are) CORRECT?

(A) Argon is used so that ambient light coming from the surroundings do not produce any signal in the detector

(B) Ethyl alcohol is used as a quenching gas

(C) The electric field strength decreases from the axis to the edge of the tube and the direction of the field is radially outward

(D) The electric field increases from the axis to the edge of the tube and the field direction is radially inward

Ans.: (a), (b) and (c)

Solution.:

The electric field $E = \dfrac{V}{r \ln \dfrac{b}{a}}$

The electric field strength decreases from the axis to the edge of the tube and the direction of the field is radially outward from a node wire to chamber.

Q.29 – Q.35 Numerical Answer Type (NAT), carry ONE mark each (for each wrong answer: –1/3).

Q36-Q46 Multiple Choice Question (MCQ), carry Two marks each (for each wrong answer: –2/3)

Q44. Young's double slit experiment is performed using a beam of C_{60} (fullerene) molecules, each molecule being made up of 60 carbon atoms. When the slit separation is 50 nm, fringes are formed on a screen kept at a distance of 1 m from the slits. Now, the experiment is repeated with C_{70} molecules with a slit separation of 92.5 nm. The kinetic energies of both the beams are the same. The position of the 4^{th} bright fringe for C_{60} will correspond to the n^{th} bright fringe for C_{70}. What is the value of n (rounded off to the nearest integer)?

(A) 5 (B) 6 (C) 7 (D) 8

Ans.: (d)

Solution.:

YDSE with C_{60} beam

$$\lambda_1 = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2\times(60\times 12\,amu)K}}\;;\quad \beta_1 = \frac{D\lambda_1}{d_1} = \frac{1m}{50\,nm}\frac{h}{\sqrt{2\times(60\times 12\,amu)K}}$$

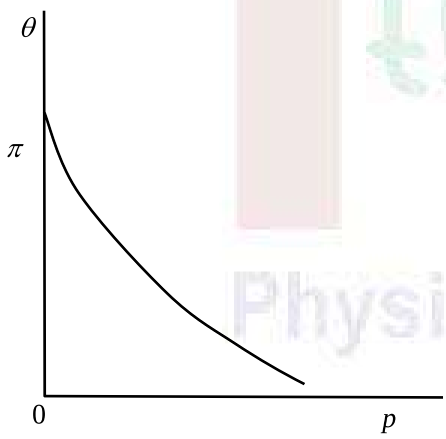
YDSE with C_{70} beam

$$\lambda_2 = \frac{h}{\sqrt{2\times(70\times 12\,amu)K}}\;;\quad \beta_2 = \frac{D\lambda_2}{d_2} = \frac{1m}{92.5\,nm}\frac{h}{\sqrt{2\times(70\times 12\,amu)K}}$$

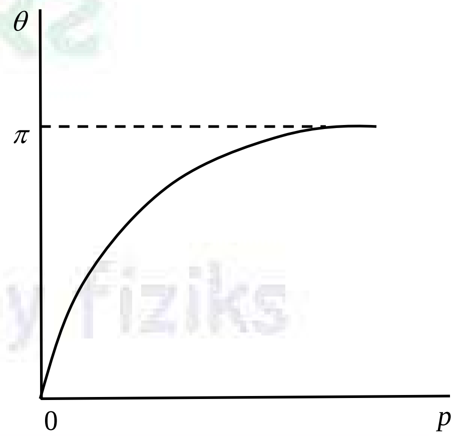
Given $n\beta_2 = 4\beta_1 \Rightarrow \frac{n}{92.5\sqrt{70}} = \frac{4}{50\sqrt{60}} \Rightarrow n = 7.99 \approx 8$

Q48. In a hadronic interaction, π^0's are produced with different momenta, and they immediately decay into two photons with an opening angle θ between them. Assuming that all these decays occur in one plane, which one of the following figures depicts the behaviour of θ as a function of the π^0 momentum p?

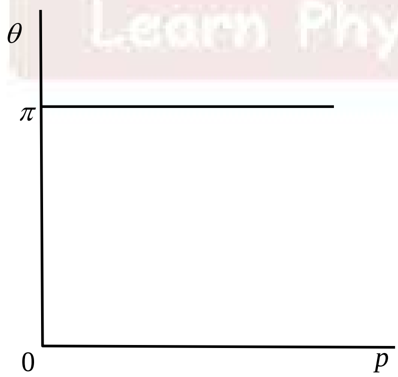
(A)



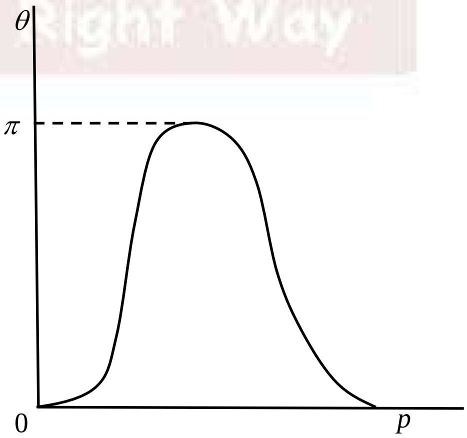
(B)



(C)

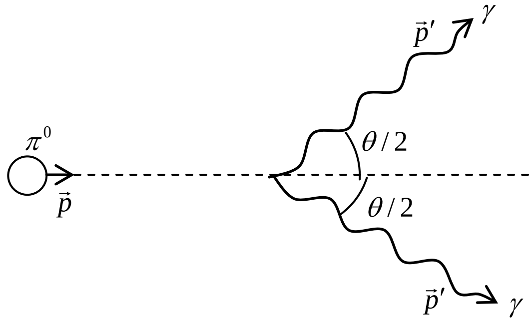


(D)



Ans.: (a)

Solution.:



From conservation of linear momentum $p = 2p'\cos\frac{\theta}{2}$ ....(1)

From conservation of energy

$\sqrt{p^2c^2 + m_\pi^2c^4} = 2p'c \Rightarrow p^2c^2 + m_\pi^2c^4 = 4p'^2c^2$ ....(2)

From (1) we can write $p^2c^2 = 4p'^2c^2\cos^2\frac{\theta}{2}$...............(3)

Divide eq. (2) by eq. (3): $1 + \frac{m_\pi^2c^4}{p^2c^2} = \sec^2\frac{\theta}{2} = 1 + \tan^2\frac{\theta}{2} \quad \Rightarrow p = m_\pi c\cot\frac{\theta}{2}$

When $\theta \to 0$, $p \to \infty$ and when $\theta \to \pi$, $p \to 0$. So option (a) is correct.

Q47-Q58 Multiple Select Question (MSQ), carry Two marks each (for each wrong answer: –2/3)

Q55. Under parity and time reversal transformations, which of the following statements is (are) TRUE about the electric dipole moment $\vec{p}$ and the magnetic dipole moment $\vec{\mu}$?

(A) $\vec{p}$ is odd under parity and $\vec{\mu}$ is odd under time reversal

(B) $\vec{p}$ is odd under parity and $\vec{\mu}$ is even under time reversal

(C) $\vec{p}$ is even under parity and $\vec{\mu}$ is odd under time reversal

(D) $\vec{p}$ is even under parity and $\vec{\mu}$ is even under time reversal

Ans.: (a)

Solution.: $P: \vec{p} \to -\vec{p}$ (odd) because electric dipole moment $\vec{p} = \sum q_i\vec{r}_i$; $\quad P: \vec{r}_i \to -\vec{r}_i$

$T: \vec{\mu} \to -\vec{\mu}$ (odd) because magnetic dipole moment $\vec{\mu} = I\vec{A} = \frac{dq}{dt}\vec{A}$; $\quad T: I \to -I$

Q58. A slow moving π^- particle is captured by a deuteron (d) and this reaction produces two neutrons (n) in the final state, i.e., $\pi^- + d \to n + n$, Neutron and deuteron have even intrinsic parities, whereas π^- has odd intrinsic parity. L and S are the orbital and spin angular momenta, respectively of the system of two neutrons. Which of the following statements regarding the final two-neutron state is (are) CORRECT?

(A) It has odd parity (B) $L+S$ is odd

(C) $L=1, S=1$ (D) $L=2, S=0$

Ans.: (a), (c)

Solution.: $\pi^- + d \to n + n$

Conservation of Parity $\pi_\pi \pi_d = \pi_n \pi_n (-1)^L$

$$(-)(+) = (+)(+)(-1)^L \quad \text{Thus } \boxed{L = \text{odd} = 1,3,5,.....}$$

So, final two neutron state has $(+)(+)(-1)^{odd} = -ve$ or odd parity. So, option (a) is correct.

Spin conservation

$$\pi^- + d \to n + n$$

$$\vec{0} + \vec{1} \quad \frac{\vec{1}}{2} \quad \frac{\vec{1}}{2}$$

So, spin of final two neutron state $\boxed{S = 0,1}$ option (c) is correct and option (d) is wrong. $L+S$ may be odd as well as even.

Q59-Q65, Numerical Answer Type (NAT), carry Two marks each (for each wrong answer: –2/3)

Q60. In pion nucleon scattering, the pion and nucleon can combine to form a short lived bound state called the Δ particle$(\pi + N \to \Delta)$. The masses of the pion, nucleon and the Δ particle are 140 MeV/c^2, 938 MeV/c^2 and 1230 MeV/c^2, respectively. In the lab frame, where the nucleon is at rest, what is the minimum energy (in MeV/c^2, rounded off to one decimal place) of the pion to produce the Δ particle?

Ans.: 314.7

Solution.:

$$\pi + N \to \Delta$$

$$Q = [M_\pi + M_N - M_\Delta]c^2 = [140 + 938 - 1230]\frac{MeV}{c^2} \times c^2 \Rightarrow Q = -152 MeV$$

The minimum kinetic energy of the pion

$$K_\pi = -Q\left(1+\frac{M_\pi}{M_N}\right) = -(-152)\left(1+\frac{140}{938}\right) = 174.68 MeV \Rightarrow K_\pi \approx 174.7 MeV$$

Thus $E_\pi = K_\pi + M_\pi c^2 = 174.7 + 140 = 314.7 MeV$

Q62. An α-particle is emitted from the decay of Americium (Am) at rest, i.e., ${}^{241}_{94}Am \rightarrow {}^{237}_{92}U + \alpha$. The rest masses of ${}^{241}_{94}Am$, ${}^{237}_{92}U$ and α are 224.544 GeV/c^2, 220.811 GeV/c^2 and 3.728 GeV/c^2 respectively. What is the kinetic energy (in MeV/c^2, rounded off to two decimal places) of the α-particle?

Ans.: 4.90 to 4.94

Solution.:

$${}^{241}_{94}Am \rightarrow {}^{237}_{92}U + \alpha$$

$$Q_\alpha = \left[M_{Am} - M_U - M_\alpha\right]c^2 = \left[224.544 - 220.811 - 3.728\right]\frac{GeV}{c^2} \times c^2$$

$$Q_\alpha = 0.005\, GeV = 5\, MeV$$

$$K_\alpha = \frac{A-4}{4}Q_\alpha = \frac{241-4}{241} \times 5\, MeV = 4.917\, MeV$$

$$\boxed{K_\alpha = 4.92\, MeV}$$

Section -GA (General Aptitude)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q1. "You are delaying the completion of the task. Send _______ contributions at the earliest."

(A) you are (B) your

(C) you're (D) yore

Ans. : (b)

Q2. References: ______ : : Guidelines : Implement

(By word meaning)

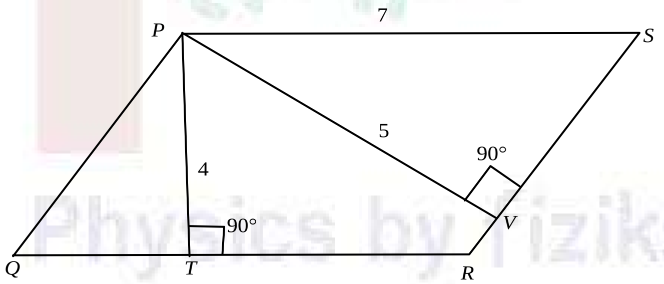
(A) Sight (B) Site

(C) Cite (D) Plagiarise

Ans. : (c)

Solution: Explanation: As guidelines are to be implemented. So are references cited.

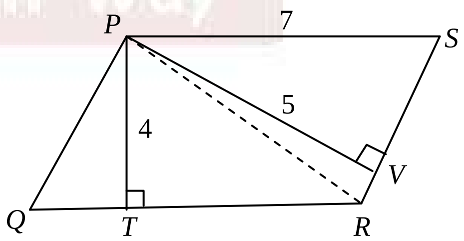
Q3. In the given figure, PQRS is a parallelogram with PS = 7 cm, PT = 4 cm and PV = 5 cm. What is the length of RS in cm? (The diagram is representative.)



(A) $\frac{20}{7}$ (B) $\frac{28}{5}$ (C) $\frac{9}{2}$ (D) $\frac{35}{4}$

Ans. : (b)

Solution:



Construction: Connect vertices P and R.

$\therefore$ Area of $\square PQRS = 2\times \text{area}\left(\Delta PQR\right)$

$$= 2\times\frac{1}{2}\times QR\times PT = 2\times\frac{1}{2}\times 7\times 4 = 28\,\text{cm}^2 \text{....(i)} \left(\because QR = PS\right)$$

Also, Area of $\square PQRS = 2\times \text{area}\left(\Delta PSR\right) = 2\times\frac{1}{2}\times PV = RS\times 5$ ....(ii)

$\therefore$ (i) = (ii) $\Rightarrow 28 = RS\times 5 \quad \Rightarrow RS = \frac{28}{5}$ cm

Q4. In 2022, June Huh was awarded the Fields medal, which is the highest prize in Mathematics. When he was younger, he was also a poet. He did not win any medals in the International Mathematics Olympiads. He dropped out of college.

Based only on the above information, which one of the following statements can be logically inferred with *certainty*?

(A) Every Fields medalist has won a medal in an International Mathematics Olympiad.

(B) Everyone who has dropped out of college has won the Fields medal.

(C) All Fields medalists are part-time poets.

(D) Some Fields medalists have dropped out of college.

Ans. : (d)

Solution:

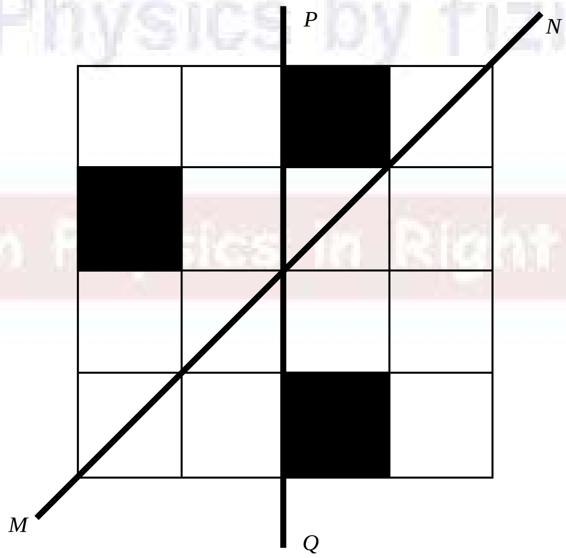
Given passage does not talk about every fields medalist, hence choice (a) is incorrect.

Also, passage does not generalise the case of drop-outs, hence choice (b) is again not correct. Passage has no mention of "past-time", hence it is extraneous. So, choice (C) is incorrect.

What we could say with certainly is the choice (d).

Q5. A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line.

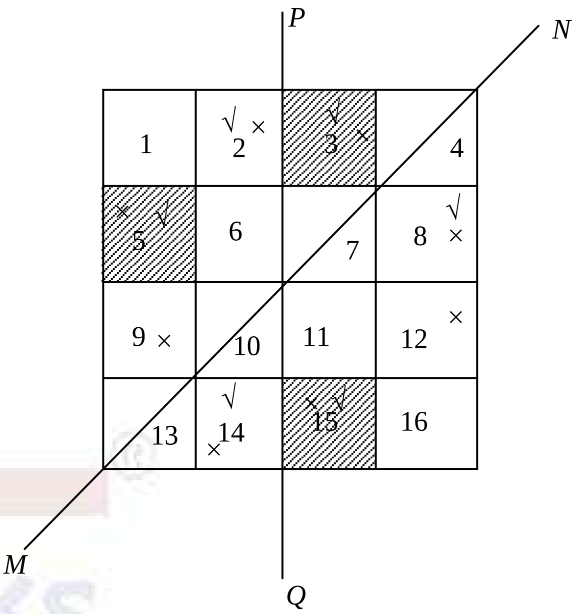
The given figure consists of 16 unit squares arranged as shown. In addition to the three black squares, what is the minimum number of squares that must be coloured black, such that both PQ and MN form lines of symmetry? (The figure is representative)



(A) 3 (B) 4 (C) 5 (D) 6

Ans. : (c)

Solution:



PQ : Taking PQ a line of symmetry.

Corresponding to (3), (2) will go black

Corresponding to (5), (8) will go black

Corresponding to (15), (14) will go black

MN: a line of symmetry

Corresponding to (3), (8) will go black

Corresponding (2), (12) will go black

Corresponding (5), (15) will go black, which is already black.

Corresponding (14), (9) will go black.

So, in total there will be 8 black squares.

So, minimum 5 squares must be coloured.

Q. 6 – Q. 10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: –2/3).

Q6. Human beings are one among many creatures that inhabit an imagined world. In this imagined world, some creatures are cruel. If in this imagined world, it is given that the statement "Some human beings are not cruel creatures" is FALSE, then which of the following set of statement(s) can be logically inferred with *certainty*?

(i) All human beings are cruel creatures.

(ii) Some human beings are cruel creatures.

(iii) Some creatures that are cruel are human beings.

(iv) No human beings are cruel creatures.

(A) only (i) (B) only (iii) and (iv)

(C) only (i) and (ii) (D) (i), (ii) and (iii)

Ans. : (d)

Solution:

Given: "Some human beings are not cruel creatures" is FALSE.

a FALSE statement implies the negation of statement : "At least one human being is not cruel creatures." That is, "All human beings are cruel creature is correct inference.

And if "All human beings are cruel creature is True, then "Some human beings are cruel creatures" is also correct.

And so is, "Some creatures that are cruel are human beings"

Hence, (i), (ii) & (iii) are correct statements.

Q7. To construct a wall, sand and cement are mixed in the ratio of 3:1. The cost of sand and that of cement are in the ratio of 1:2.

If the total cost of sand and cement to construct the wall is 1000 rupees, then what is the cost (in rupees) of cement used?

(A) 400 (B) 600 (C) 800 (D) 200

Ans. : (a)

Solution:

Let Sand $=3x$ kg; Cement $=x$ kg

Also, cost of sand $=y$ rupees per kg. and cement $=2y$ rupees per kg.

$\therefore$ Total cost of sand $=3xy$ rupees

$\therefore$ Total cost of cement $=2xy$ rupees

Given, Total cost = 1000 rupees. $\therefore\ 3xy+2xy=1000$ or, $xy=200$

$\therefore$ Cost of cement used $=2xy=2\times 200=400$ rupees.

Q8. The World Bank has declared that it does not plan to offer new financing to Sri Lanka, which is battling its worst economic crisis in decades, until the country has an adequate macroeconomic policy framework in place. In a statement, the World Bank said Sri Lanka needed to adopt structural reforms that focus on economic stabilisation and tackle the root causes of its crisis. The latter has starved it of foreign exchange and led to shortages of food, fuel, and medicines. The bank is repurposing resources under existing loans to help alleviate shortages of essential items such as medicine, cooking gas, fertiliser, meals for children, and cash for vulnerable households.

Based only on the above passage, which one of the following statements can be inferred with *certainty*?

(A) According to the World Bank, the root cause of Sri Lanka's economic crisis is that it does not have enough foreign exchange.

(B) The World Bank has stated that it will advise the Sri Lankan government about how to tackle the root causes of its economic crisis.

(C) According to the World Bank, Sri Lanka does not yet have an adequate macroeconomic policy framework.

(D) The World Bank has stated that it will provide Sri Lanka with additional funds for essentials such as food, fuel, and medicines.

Ans. : (c)

Solution:

Choice (a) is incorrect inference because nowhere in passage it is stated that Sri Lanka does not have foreign exchange and it is the root cause of crisis according to world bank.

Choice (b) is incorrect because passage does not mention the advise by world bank to Sri Lanka to tackle root causes of economic crisis.

Choice (d) is not correct inference because world bank does not straightly says that it will meet essential such as food, fuel and medicines.

Only choice (c) is correct inference based on passage.

Q9. The coefficient of x^4 in the polynomial $(x-1)^3(x-2)^3$ is equal to ____________.

(A) 33 (B) -3

(C) 30 (D) 21

Ans. : (a)

Solution:

Coefficient of x^4 in $(x-1)^3(x-2)^3$

Coefficient of x^4 in

$$\left(x^3-3x^2+3x-1\right)\left(x^3-6x^2+12x-8\right)$$

$+12$ 3

$(-3)(-6)=18$

Collecting the coefficients of x^4: $1\cdot 12+(-3)(-6)+3\cdot 1=33$

Q10. Which one of the following shapes can be used to tile (completely cover by repeating) a flat plane, extending to infinity in all directions, without leaving any empty spaces in between them? The copies of the shape used to tile are identical and are not allowed to overlap.

(A) circle (B) regular octagon

(C) regular pentagon (D) rhombus

Ans. : (d)

Solution:

Among all regular polygons only equilateral triangle, the square, and the hexagon cover a plane without leaving gaps. Hence correct choice is (d) rhombus.

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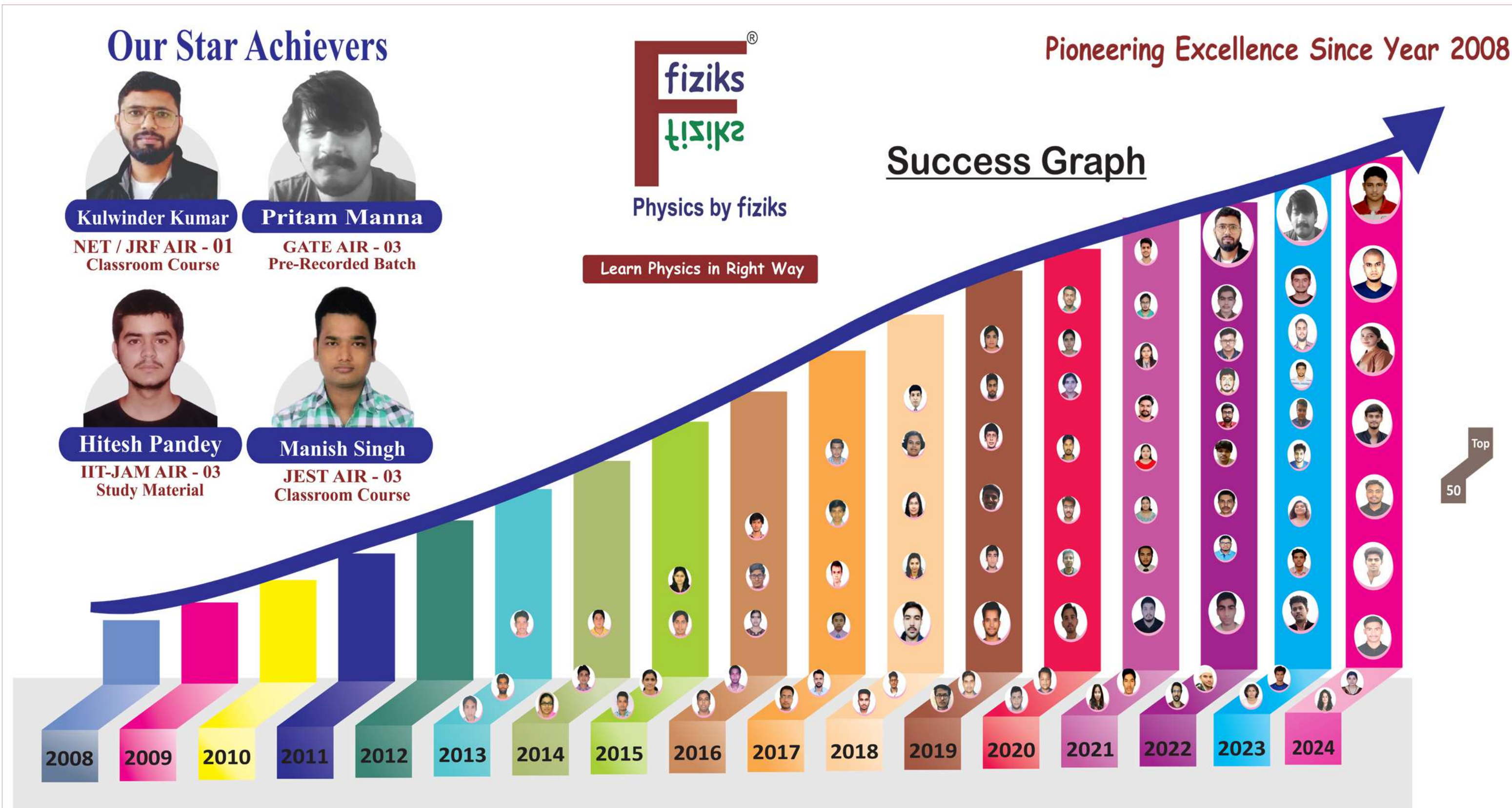

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