Physics by fiziks

Learn Physics in Right Way

CSIR NET-JRF Physical Sciences Paper June-2023 Solution

Be Part of Disciplined Learning

PART B

Q1. A jar J1 contains equal number of balls of red, blue and green colours, while another jar J2 contains balls of only red and blue colours, which are also equal in number. The probability of choosing J1 is twice as large as choosing J2. If a ball picked at random from one of the jars turns out to be red, the probability that it came from J1 is

1. $\frac{2}{3}$ 2. $\frac{3}{5}$

3. $\frac{2}{5}$ 4. $\frac{4}{7}$

Ans: (4)

Solution:

$P(J1)$ = Probability of choosing Jar 1 $(J1) = 2/3$

$P(J2)$ = Probability of choosing Jar 2 $(J2) = \frac{1}{3}$

$P(R/J1)$ = Probability of picking a red ball given that it is from Jar $1(J1) = \frac{1}{3}$

$P(R/J2)$ = Probability of picking a red ball given it is from Jar 2 $(J2) = \frac{1}{2}$

$\therefore\ P(J1/R)$ = Probability a ball picked is red and it came from $J1$

Using Bayes' theorem,

$$P(J1/R) = \frac{P(J1)\cdot P(R/J1)}{P(J1)\cdot P(R/J1) + P(J2)\cdot P(R/J2)} = \frac{\frac{2}{3}\cdot\frac{1}{3}}{\frac{2}{3}\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{2}} = \frac{4}{7}$$

Q9. The matrix $M = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ satisfies the equation $M^3 + \alpha M^2 + \beta M + 3 = 0$ if (α, β) are

1. $(-2, 2)$ 2. $(-3, 3)$

3. $(-6, 6)$ 4. $(-4, 4)$

Ans: (3)

Solution:

Characteristic equation $|M-\lambda I|=0 \Rightarrow \begin{vmatrix} 3-\lambda & -1 & 2 \\ -1 & 2-\lambda & 0 \\ 2 & 0 & 1-\lambda \end{vmatrix}=0$

$$\Rightarrow 2\left[0-2(3-\lambda)\right]-0+(1-\lambda)\left[(3-\lambda)(2-\lambda)-1\right]=0$$

$$\Rightarrow -4(3-\lambda)+(1-\lambda)\left[6-3\lambda-2\lambda+\lambda^2-1\right]=0 \quad \Rightarrow -4(3-\lambda)+(1-\lambda)\left[\lambda^2-5\lambda+5\right]=0$$

$$\Rightarrow -12+4\lambda+\left(\lambda^2-5\lambda+5\right)-\lambda^3+5\lambda^2-5\lambda=0 \quad \Rightarrow -\lambda^3+6\lambda^2-6\lambda-7=0$$

$$\Rightarrow \lambda^3-6\lambda^2+6\lambda+7=0 \Rightarrow \alpha=-6,\ \beta=6.$$

Q10. The value of the integral $I=\int_0^\infty e^{-x}x\sin(x)\,dx$ is

1. $\frac{3}{4}$
2. $\frac{2}{3}$
3. $\frac{1}{2}$
4. $\frac{1}{4}$

Ans: (3)

Solution:

$I=\int_0^\infty e^{-x}x\sin(x)\,dx=L[x\sin x]$ with $s=1$.

$$\because L\left[x^n f(x)\right]=(-1)^n\frac{d}{ds}F(s)\Rightarrow L[x\sin x]=-\frac{d}{ds}\left(\frac{1}{s^2+1}\right)=\frac{2s}{\left(s^2+1\right)^2}$$

$$I=\int_0^\infty e^{-x}x\sin(x)\,dx=L[x\sin x]=\frac{2\times 1}{\left(1^2+1\right)^2}=\frac{1}{2} \qquad \because s=1$$

Q17. The locus of the curve $\operatorname{Im}\left(\frac{\pi(z-1)-1}{z-1}\right)=1$ in the complex z-plane is a circle centred at (x_0,y_0) and radius R. The values of (x_0,y_0) and R, respectively, are

1. $\left(1,\frac{1}{2}\right)$ and $\frac{1}{2}$
2. $\left(1,-\frac{1}{2}\right)$ and $\frac{1}{2}$
3. $(1,1)$ and 1
4. $(1,-1)$ and 1

Ans: (1)

Solution:

$$\because \operatorname{Im}\left(\frac{\pi(z-1)-1}{z-1}\right)=1 \Rightarrow \operatorname{Im}\left(\pi-\frac{1}{z-1}\right)=1 \Rightarrow \operatorname{Im}\left(\pi-\frac{1}{x+iy-1}\right)=1$$

$$\Rightarrow \operatorname{Im}\left(\pi-\frac{1}{(x-1)+iy}\right)=1$$

$$\Rightarrow \operatorname{Im}\left(\pi-\frac{(x-1)-iy}{(x-1)^2+y^2}\right)=1 \Rightarrow \frac{y}{(x-1)^2+y^2}=1 \Rightarrow (x-1)^2+y^2=y$$

$$\Rightarrow (x-1)^2+\left(y-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2$$

The values of (x_0, y_0) and R, respectively, are $\left(1,\frac{1}{2}\right)$ and $\frac{1}{2}$.

PART C

Q6. The bisection method is used to find a zero x_0 of the polynomial $f(x) = x^3 - x^2 - 1$. Since $f(1) = -1$, while $f(2) = 3$ the values $a = 1$ and $b = 2$ are chosen as the boundaries of the interval in which the x_0 lies. If the bisection method is iterated three times, the resulting value of x_0 is

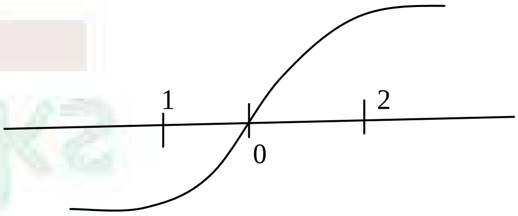
1. $\frac{15}{8}$
2. $\frac{13}{8}$
3. $\frac{11}{8}$
4. $\frac{9}{8}$

Ans: (3)

Solution:

Given $a = 1, b = 2$

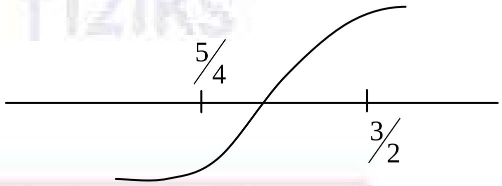
Zero after 1st Iteration $= \frac{a+b}{2} = \frac{1+2}{2} = 3/2$



$\therefore f\left(3/2\right) = \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 - 1 = 0.125(+ve)$

$\therefore [a,b] \leftarrow \left[1, 3/2\right]$

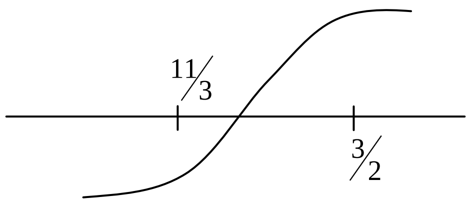
Zero after second iteration: $= \frac{1 + 3/2}{2} = \frac{5}{4}$



$\therefore \quad f\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)^3 - \left(\frac{5}{4}\right)^2 - 1 = -0.609375 \quad (-ve)$

$\therefore [a,b] \leftarrow \left[\frac{5}{4}, \frac{3}{2}\right]$

Zero after third iteration $x_0 = \frac{5/4 + 3/2}{2} = \frac{11}{8}$



$\therefore \quad f\left(\frac{11}{8}\right) = -0.2890 \quad (-ve)$

$\therefore [a,b] \leftarrow \left[\frac{11}{8}, 3/2\right]$

$\therefore$ Zero, x_0 after three iteration is $x_0 = \frac{11}{8}$

Q10. A random variable Y obeys a normal distribution

$$P(Y)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(Y-\mu)^2}{2\sigma^2}\right]$$

The mean value of e^Y is

1. $e^{\mu+\frac{\sigma^2}{2}}$
2. $e^{\mu-\sigma^2}$
3. $e^{\mu+\sigma^2}$
4. $e^{\mu-\frac{\sigma^2}{2}}$

Ans: (1)

Solution:

$$P(Y)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(Y-\mu)^2}{2\sigma^2}\right]$$

Mean value of $e^Y=\int_{-\infty}^{\infty}e^Y\cdot\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(Y-\mu)^2}{2\sigma^2}\right]dY$

Now, Let $\frac{Y-\mu}{\sigma}=X\Rightarrow Y=\mu+\sigma X\quad\Rightarrow dY=\sigma dX$

Mean value of $e^Y=\int_{-\infty}^{\infty}e^{\mu+\sigma X}\cdot\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}X^2}\cdot\sigma dX=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{\left(\mu+\sigma X-X^2/2\right)}dX$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}\left[(X-\sigma)^2-(\sigma^2+2\mu)\right]}dX=\frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}(\sigma^2+2\mu)}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(X-\sigma)^2}dX$$

Also, $\int_{-\infty}^{\infty}e^{-\frac{1}{2}(X-\sigma)^2}dX=\sqrt{2\pi}$

$\therefore$ Mean Value of $e^Y=\frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}(\sigma^2+2\mu)}\sqrt{2\pi}=e^{\left(\frac{\sigma^2}{2}+\mu\right)}=e^{\left(\mu+\sigma^2/2\right)}$

Q21. The matrix $R_{\hat{n}}(\theta)$ represents a rotation by an angle θ about the axis $\hat{n}$. The value of θ and $\hat{n}$ corresponding to the matrix $\begin{pmatrix}-1 & 0 & 0\\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3}\\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3}\end{pmatrix}$, respectively, are

1. $\pi/2$ and $\left(0,-\sqrt{\frac{2}{3}},\frac{1}{\sqrt{3}}\right)$
2. $\pi/2$ and $\left(0,\frac{1}{\sqrt{3}},\sqrt{\frac{2}{3}}\right)$

3. π and $\left(0,-\sqrt{\frac{2}{3}},\frac{1}{\sqrt{3}}\right)$ 4. π and $\left(0,\frac{1}{\sqrt{3}},\sqrt{\frac{2}{3}}\right)$

Ans: (4)

Solution:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}=\begin{pmatrix} 0 \\ \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}.$$ So $\left(0,\frac{1}{\sqrt{3}},\sqrt{\frac{2}{3}}\right)$ is rotation axis.

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}=-\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$ So $\hat{i}\to-\hat{i}$ and rotation angle is π.

Or let check $\left(R_{\hat{n}}\right)^2=R_{\hat{n}}R_{\hat{n}}=\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}=I_3$

So $\left(R_{\hat{n}}\right)^2$ corresponds to 360^0 rotation, thus $R_{\hat{n}}$ corresponds to 180^0.

Q25. The value of the integral $\int_{-\infty}^{\infty} dx 2^{-\frac{|x|}{\pi}}\delta(\sin x)$ where $\delta(x)$ is the Dirac delta function, is

1. 3 2. 0 3. 5 4. 1

Ans: (1)

Solution: $I=\int_{-\infty}^{\infty} 2^{-\frac{|x|}{\pi}}\delta(\sin x)dx$

Let $f(x)=\sin x=0 \Rightarrow x_i=n\pi$ where $n=0,\pm1,\pm2,\pm3....$ and $f'(x)=\cos x$

Thus $\delta(\sin x)=\frac{\delta(x-0)}{|\cos 0|}+\frac{\delta(x-\pi)}{|\cos\pi|}+\frac{\delta(x+\pi)}{|\cos(-\pi)|}+\frac{\delta(x-2\pi)}{|\cos\pi|}+\frac{\delta(x+2\pi)}{|\cos(-\pi)|}+...$

$$\delta(\sin x)=\delta(x-0)+\delta(x-\pi)+\delta(x+\pi)+\delta(x-2\pi)+\delta(x+2\pi)+\ldots$$

$$I=\int_{-\infty}^{\infty} 2^{-\frac{|x|}{\pi}}\left[\delta(x-0)+\delta(x-\pi)+\delta(x+\pi)+\delta(x-2\pi)+\delta(x+2\pi)+\ldots\right]dx$$

$$\Rightarrow I=2^{-0}+2.2^{-1}+2.2^{-2}+2.2^{-3}+\ldots.\Rightarrow I=1+2\left(2^{-1}+2^{-2}+2^{-3}+\ldots.\right)$$

$$\Rightarrow I=1+2\left(\frac{2^{-1}}{1-2^{-1}}\right)=1+2=3\Rightarrow I=\int_{-\infty}^{\infty} 2^{-\frac{|x|}{\pi}}\delta(\sin x)dx=3$$

Q27. If the Bessel function of integer order n is defined as $J_n(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!(n+k)!}\left(\frac{x}{2}\right)^{2k+n}$ then $\frac{d}{dx}\left[x^{-n}J_n(x)\right]$ is

1. $-x^{-(n+1)}J_{n+1}(x)$
2. $-x^{-(n+1)}J_{n-1}(x)$
3. $-x^{-n}J_{n-1}(x)$
4. $-x^{-n}J_{n+1}(x)$

Ans: (4)

Solution: We know that $J_1(x)=-J_0'(x)$ and $J_{-n}(x)=(-1)^n J_n(x)$. Let us verify each option by taking $n=0$. Then $\frac{d}{dx}\left[x^{-n}J_n(x)\right]=\frac{d}{dx}\left[x^{-0}J_0(x)\right]=J_0'(x)$

(1) $-x^{-(n+1)}J_{n+1}(x)$: $\quad -x^{-(0+1)}J_{0+1}(x)=-\frac{1}{x}J_1(x)$

(2) $-x^{-(n+1)}J_{n-1}(x)$: $\quad -x^{-(0+1)}J_{0-1}(x)=-\frac{1}{x}J_{-1}(x)=\frac{1}{x}J_1(x)$

(3) $-x^{-n}J_{n-1}(x)$: $\quad -x^{-0}J_{0-1}(x)=-J_{-1}(x)=J_1(x)$

(4) $-x^{-n}J_{n+1}(x)$: $\quad -x^{-0}J_{0+1}(x)=-J_1(x)$.

Hence option (4) is correct.

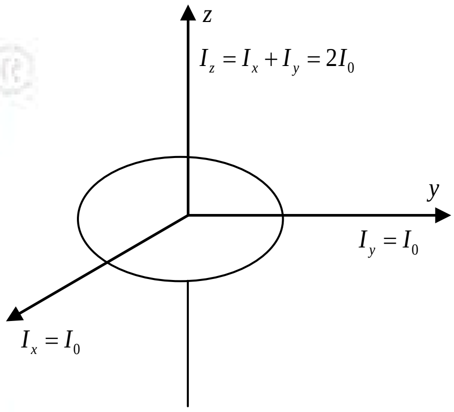
PART B

Q3. A uniform circular disc on the xy-plane with its centre at the origin has a moment of inertia I_0 about the x-axis. If the disc is set in rotation about the origin with an angular velocity $\vec{\omega} = \omega_0\left(\hat{j}+\hat{k}\right)$, the direction of its angular momentum is along

1. $-\hat{i}+\hat{j}+\hat{k}$
2. $-\hat{i}+\hat{j}+2\hat{k}$
3. $\hat{j}+2\hat{k}$
4. $\hat{j}+\hat{k}$

Ans: (3)

Solution.:



Moment of inertia tensor $\overline{\overline{I}} = \begin{pmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & 2I_0 \end{pmatrix}$

$$\vec{L} = \overline{\overline{I}}\,\vec{\omega} = \begin{pmatrix} I_0 & 0 & 0 \\ 0 & I_0 & 0 \\ 0 & 0 & 2I_0 \end{pmatrix}\begin{pmatrix} 0 \\ \omega_0 \\ \omega_0 \end{pmatrix} \qquad \because \vec{\omega} = \begin{pmatrix} 0 \\ \omega_0 \\ \omega_0 \end{pmatrix}$$

$$\Rightarrow \vec{L} = I_0\omega_0\hat{j} + 2I_0\omega_0\hat{k}$$

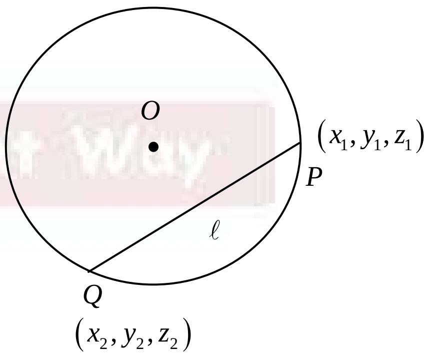
$$\Rightarrow \vec{L} = I_0\omega_0\left(\hat{j}+2\hat{k}\right)$$

Q12. A one-dimensional rigid rod is constrained to move inside a sphere such that its two ends are always in contact with the surface. The number of constraints on the Cartesian coordinates of the endpoints of the rod is

1. 3
2. 5
3. 2
4. 4

Ans: (1)

Solution.:



Equation of constraints for the coordinates of end points P and Q are as follows:

(i) $x_1^2 + y_1^2 + z_1^2 = R^2$

(ii) $x_2^2 + y_2^2 + z_2^2 = R^2$

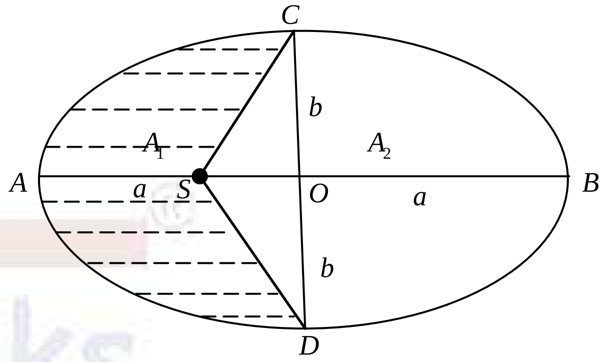
(iii) $\sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2} = \ell$

Q14. The minor axis of Earth's elliptical orbit divides the area within it into halves. The eccentricity of the orbit is 0.0167. The difference in time spent by Earth in the two halves is closest to

1. 3.9 days
2. 4.8 days
3. 12.3 days
4. 0 days

Ans: (1)

Solution.:



As a real velocity remains constant, so

$$\frac{t_1}{t_2}=\frac{A_1}{A_2}=\frac{\frac{1}{2}\pi ab-(\Delta CDS)}{\frac{1}{2}\pi ab+(\Delta CDS)}$$

$$SO=AO-AS=a-r_{\min}=a-a(1-e)=ae$$

$$\Delta CDS=\frac{1}{2}\times SO\times CD=\frac{1}{2}ae(2b)=abe$$

$$\frac{t_1}{t_2}=\frac{\frac{1}{2}\pi ab-abe}{\frac{1}{2}\pi ab+abe}=\frac{1-\frac{2e}{\pi}}{1+\frac{2e}{\pi}}=\frac{1-0.01063}{1+0.01063}\Rightarrow\frac{t_1}{t_2}=0.9789\Rightarrow t_1=0.9789t_2$$

$$\because\frac{2e}{\pi}=0.01063$$

We know that the earth completes one revolution around the earth in 365 days. So, $t_1+t_2=365$

Thus $t_2=184.445$ days and $t_1=180.55$ days.

$\Rightarrow \Delta t=t_2-t_1=3.89$ days

Q22. The Hamiltonian of a two particle system is $H=p_1p_2+q_1q_2$, where q_1 and q_2 are generalized coordinates and p_1 and p_2 are the respective canonical momenta. The Lagrangian of this system is

1. $\dot{q}_1\dot{q}_2+q_1q_2$
2. $-\dot{q}_1\dot{q}_2+q_1q_2$
3. $-\dot{q}_1\dot{q}_2-q_1q_2$
4. $\dot{q}_1\dot{q}_2-q_1q_2$

Ans: (4)

Solution.:

Hamilton's first equation of motion $\dot{q}_1 = \frac{\partial H}{\partial p_1} = p_2$, $\dot{q}_2 = \frac{\partial H}{\partial p_2} = p_1$

The Lagrangian $L = \dot{q}_1 p_1 + \dot{q}_2 p_2 - H = \dot{q}_1\dot{q}_2 + \dot{q}_2\dot{q}_1 - \left(p_1 p_2 + q_1 q_2\right)$ $\quad \because H = p_1 p_2 + q_2 q_2$

$\Rightarrow L = 2\dot{q}_1\dot{q}_2 - \dot{q}_1\dot{q}_2 - q_1 q_2 \quad \Rightarrow L = \dot{q}_1\dot{q}_2 - q_1 q_2$

Q25. The trajectory of a particle moving in a plane is expressed in polar coordinates (r,θ) by the equations $r = r_0 e^{\beta t}$ and $\frac{d\theta}{dt} = \omega$, where the parameters r_0, β and ω are positive. Let v_r and a_r denote the velocity and acceleration, respectively, in the radial direction. For this trajectory

1. $a_r < 0$ at all times irrespective of the values of the parameters
2. $a_r > 0$ at all times irrespective of the values of the parameters
3. $\frac{dv_r}{dt} > 0$ and $a_r > 0$ for all choices of parameters
4. $\frac{dv_r}{dt} > 0$, however, $a_r = 0$ for some choices of parameters

Ans: (4)

Solution.: $\because r = r_0 e^{\beta t}$, $\frac{d\theta}{dt} = \dot{\theta} = \omega$

$\Rightarrow v_r = \dot{r} = r_0 \beta e^{\beta t} \Rightarrow \frac{dv_r}{dt} = \ddot{r} = r_0 \beta^2 e^{\beta t} > 0$

Thus $a_r = \ddot{r} - r\dot{\theta}^2 = r_0 \beta^2 e^{\beta t} - r_0 e^{\beta t}\omega^2 \Rightarrow a_r = r_0\left(\beta^2 - \omega^2\right)e^{\beta t}$

$a_r = 0$ if $\beta^2 = \omega^2$. So, option (d) is correct.

PART C

Q9. For the transformation $x \to X = \frac{\alpha p}{x}, p \to P = \beta x^2$ between conjugate pairs of a coordinate and its momentum, to be canonical, the constants α and β must satisfy

1. $1+\frac{1}{2}\alpha\beta = 0$
2. $1-\frac{1}{2}\alpha\beta = 0$
3. $1+2\alpha\beta = 0$
4. $1-2\alpha\beta = 0$

Ans: (3)

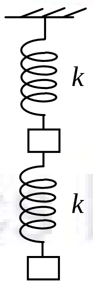
Solution.:

$$X = \frac{\alpha p}{x}, P = \beta x^2$$

For canonical transformation $[X, P]_{x,p} = 1 \Rightarrow \frac{\partial X}{\partial x}\frac{\partial P}{\partial p} - \frac{\partial X}{\partial p}\frac{\partial P}{\partial x} = 1$

$$\Rightarrow \left(-\frac{\alpha p}{x^2}\right)(0) - \left(\frac{\alpha}{x}\right)(2\beta x) = 1 \Rightarrow 2\alpha\beta + 1 = 0$$

Q18. A system of two identical masses connected by identical springs, as shown in the figure, oscillates along the vertical direction.

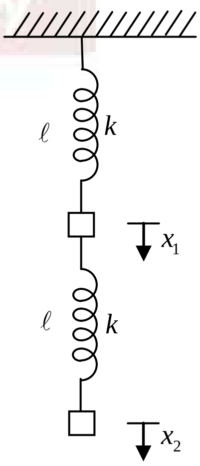


The ratio of the frequencies of the normal modes is

1. $\sqrt{3-\sqrt{5}} : \sqrt{3+\sqrt{5}}$
2. $3-\sqrt{5} : 3+\sqrt{5}$
3. $\sqrt{5-\sqrt{3}} : \sqrt{5+\sqrt{3}}$
4. $5-\sqrt{3} : 5+\sqrt{3}$

Ans: (1)

Solution.:



$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 ; \quad T = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$V = -mg(\ell + x_1) - mg(2\ell + x_1 + x_2) + \frac{1}{2}kx_1^2 + \frac{1}{2}k(x_2 - x_1)^2$$

$$V = -2mgx_1 - mgx_2 - 3mg\ell + \frac{1}{2}k\left[2x_1^2 + x_2^2 - x_1x_2 - x_2x_1\right]$$

$$V = \begin{pmatrix} 2k & -k \\ -k & k \end{pmatrix}$$

Secular Equation

$$\left|V - \omega^2 T\right| = 0 \Rightarrow \begin{vmatrix} 2k - \omega^2 m & -k \\ -k & k - \omega^2 m \end{vmatrix} = 0 \Rightarrow \left(2k - \omega^2 m\right)\left(k - \omega^2 m\right) - k^2 = 0$$

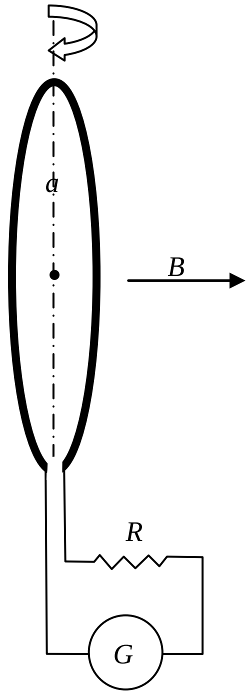
$$\Rightarrow m^2\left(\omega^2\right)^2 - 3km\left(\omega^2\right) + k^2 = 0 \Rightarrow \omega^2 = \frac{3km \pm \sqrt{9k^2m^2 - 4k^2m^2}}{2m^2} \Rightarrow \omega^2 = \frac{3km \pm \sqrt{5}km}{2}$$

$\Rightarrow \omega_1 = \frac{1}{2}km\sqrt{3+\sqrt{5}}$ and $\omega_2 = \frac{1}{2}km\sqrt{3-\sqrt{5}}$. Thus $\omega_2 : \omega_1 = \sqrt{3-\sqrt{5}} : \sqrt{3+\sqrt{5}}$

PART B

Q7. A small circular wire loop of radius a and number of turns N, is oriented with its axis parallel to the direction of the local magnetic field **B**. A resistance R and a galvanometer are connected to the coil, as shown in the figure.

When the coil is flipped (i.e., the direction of its axis is reversed) the galvanometer measures the total charge Q that flows through it. If the induced emf through the coil $E_F = IR$, then Q is



1. $\pi Na^2 B/(2R)$ 2. $\pi Na^2 B/R$

3. $\sqrt{2}\pi Na^2 B/R$ 4. $2\pi Na^2 B/R$

Ans: (4)

Solution.:

Magnetic Flux $\phi = \int_S \vec{B}.d\vec{a}$

So initial flux $\phi = NB\times\pi a^2$ and final flux $\phi = -NB\times\pi a^2$.

So change in flux $\Delta\phi = 2NB\pi a^2$

So induced current $i = \frac{\varepsilon}{R} = \frac{1}{R}\left|\frac{\Delta\phi}{\Delta t}\right| = \frac{\Delta q}{\Delta t} \Rightarrow \Delta q = \frac{\Delta\phi}{R} = \frac{2NB\pi a^2}{R}$

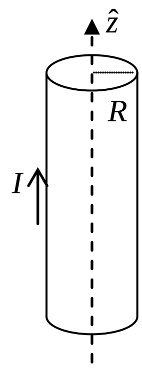
Q11. A long cylindrical wire of radius R and conductivity σ, lying along the z-axis, carries a uniform axial current density I. The Poynting vector on the surface of the wire is (in the following $\hat{\rho}$ and $\hat{\varphi}$ denote the unit vectors along the radial and azimuthal directions respectively)

1. $\frac{I^2R}{2\sigma}\hat{\rho}$ 2. $-\frac{I^2R}{2\sigma}\hat{\rho}$

3. $-\frac{I^2\pi R}{4\sigma}\hat{\varphi}$ 4. $\frac{I^2\pi R}{4\sigma}\hat{\varphi}$

Ans: (2)

Solution.:



$I_1 = IA = \sigma EA \Rightarrow \vec{E} = \frac{I}{\sigma}\hat{z}$

$\vec{B} = \frac{\mu_0 I_1}{2\pi R}\hat{\varphi} = \frac{\mu_0 I\times\pi R^2}{2\pi R}\hat{\varphi} = \frac{\mu_0 IR}{2}\hat{\varphi}$

The Poynting vector

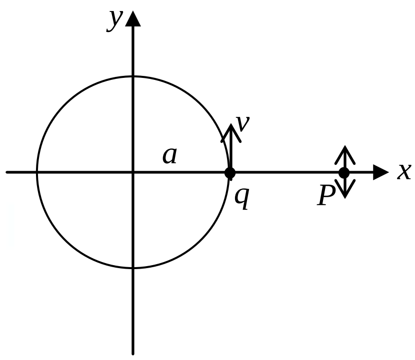
$$\vec{S}=\frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)=\frac{1}{\mu_0}\left(\frac{I}{\sigma}\hat{z}\right)\times\left(\frac{\mu_0 IR}{2}\hat{\varphi}\right)=-\frac{I^2R}{2\sigma}\hat{r}$$

Q20. A charged particle moves uniformly on the xy-plane along a circle of radius a centred at the origin. A detector is put at a distance d on the x-axis to detect the electromagnetic wave radiated by the particle along the x-direction. If $d \gg a$, the wave received by the detector is

1. Unpolarised
2. Circularly polarized with the plane of polarization being the yz-plane
3. Linearly polarized along the y-direction
4. Linearly polarized along the z-direction

Ans: (3)

Solution.:



Since charge particle is accelerating in y-direction and confined in xy-plane so at point P charged particle will be linearly polarized in xy-plane in the y-direction.

Note: Electric field will be confined in a plane containing acceleration ($\vec{\alpha}$) and position vector ($\vec{r}$).

Q23. The electric potential on the boundary of a spherical cavity of radius R, as a function of the polar angle θ, is $V_0\cos^2\frac{\theta}{2}$. The charge density inside the cavity is zero everywhere. The potential at a distance $R/2$ from the centre of the sphere is

1. $\frac{1}{2}V_0\left(1+\frac{1}{2}\cos\theta\right)$
2. $\frac{1}{2}V_0\cos\theta$
3. $\frac{1}{2}V_0\left(1+\frac{1}{2}\sin\theta\right)$
4. $\frac{1}{2}V_0\sin\theta$

Ans: (1)

Solution:

$$\because V(r,\theta)=\sum_{l=0}^{\infty}\left(A_l r^l+\frac{B_l}{r^l+1}\right)P_l(\cos\theta).$$

The potential $V_0(\theta)$ is specified on the surface of a hollow sphere, of radius R. We have to find potential inside the sphere.

In this case $B_l=0$ for all l-otherwise the potential would blow up at the origin.

Thus, $V(r,\theta)=\sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$.

At $r=R$ this must match the specified function $V_0(\theta)$

$$V(R,\theta)=\sum_{l=0}^{\infty} A_l R^l P_1(\cos\theta)=V_0(\theta)$$

$$V(R,\theta)=A_0 R^0 P_0(\cos\theta)+A_1 R^1 P_1(\cos\theta)=\frac{V_0}{2}(1+\cos\theta)$$

$$\because V_0(\theta)=V_0\cos^2(\theta/2)$$

$\Rightarrow A_0+A_1 R\cos\theta=\dfrac{V_0}{2}(1+\cos\theta) \qquad \Rightarrow A_0=V_0/2, A_1=V_0/(2R)$, others A_l's vanish.

Evidently,

Thus $V(r,\theta)=A_0 r^0 P_0(\cos\theta)+A_1 r^1 P_1(\cos\theta)=\dfrac{V_0}{2}+\dfrac{V_0}{2R}r\cos\theta$

$$\Rightarrow V\left(\frac{R}{2},\theta\right)=\frac{V_0}{2}+\frac{V_0}{4R}\cos\theta=\frac{V_0}{2}\left(1+\frac{1}{2}\cos\theta\right)$$

PART C

Q3. The charge density and current of an infinitely long perfectly conducting wire of radius a, which lies along the z-axis, as measured by a static observer are zero and a constant I, respectively. The charge density measured by an observer, who moves at speed $v=\beta c$ parallel to the wire along the direction of the current, is

1. $-\dfrac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$
2. $-\dfrac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$
3. $\dfrac{I\beta}{\pi a^2 c\sqrt{1-\beta^2}}$
4. $\dfrac{I\beta\sqrt{1-\beta^2}}{\pi a^2 c}$

Ans: (1)

Q8. The electric and magnetic fields at a point due to two independent sources are $\vec{E}_1=E(\alpha\hat{i}+\beta\hat{j})$, $\vec{B}_1=B\hat{k}$ and $\vec{E}_2=E\hat{i}$, $\vec{B}_2=-2B\hat{k}$, where α,β,E and B are constants.

If the Poynting vector is along $\hat{i}+\hat{j}$, then

1. $\alpha+\beta+1=0$
2. $\alpha+\beta-1=0$
3. $\alpha+\beta+2=0$
4. $\alpha+\beta-2=0$

Ans: (1)

Solution:

The resultant electric and magnetic fields will be

$$\vec{E}=\vec{E}_1+\vec{E}_2=E\left(\alpha\hat{i}+\beta\hat{j}\right)+E\hat{i}=E\left(\alpha+1\right)\hat{i}+E\beta\hat{j}\,;\qquad \vec{B}=\vec{B}_1+\vec{B}_2=-2B\hat{k}+B\hat{k}=-B\hat{k}$$

The Poynting vector $\vec{S}=\frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right)=\frac{1}{\mu_0}\left[EB\left(\alpha+1\right)\hat{i}+EB\beta\hat{j}\right]\times\left(-B\hat{k}\right)$

$$\Rightarrow \vec{S}=\frac{EB}{\mu_0}\left[-\beta\hat{i}+\left(\alpha+1\right)\hat{j}\right]$$

If the Poynting vector is along $\hat{i}+\hat{j}$, then $-\beta=\alpha+1 \Rightarrow \alpha+\beta+1=0$

Thus $\vec{S}=\frac{EB\left(\alpha+1\right)}{\mu_0}\left[\hat{i}+\hat{j}\right]$.

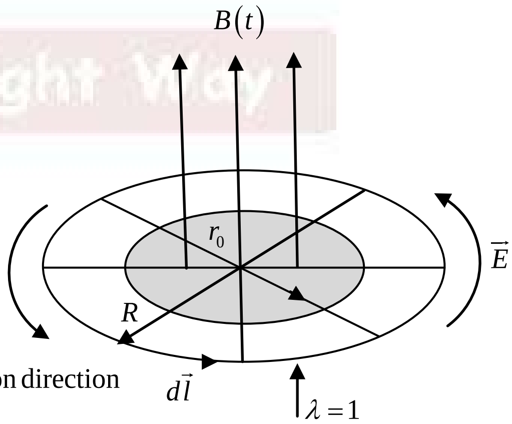
Q12. An infinitely long solenoid of radius r_0 centred at origin which produces a time-dependent magnetic field $\frac{\alpha}{\pi r_0^2}\cos\omega t$ (where α and ω are constants) is placed along the z-axis. A circular loop of radius R, which carries unit line charge density is placed, initially at rest, on the xy-plane with its centre on the z-axis. If $R>r_0$, the magnitude of the angular momentum of the loop is

1. $\alpha R\left(1-\cos\omega t\right)$
2. $\alpha R\sin\omega t$
3. $\frac{1}{2}\alpha R\left(1-\cos 2\omega t\right)$
4. $\frac{1}{2}\alpha R\sin 2\omega t$

Ans: (1)

Solution:

$$\vec{B}\left(t\right)=\frac{\alpha}{\pi r_0^2}\cos\omega t\,\hat{z}$$



The changing magnetic field will induce an electric field, curling around the axis of the ring. This electric field exerts a force on the charges at the rim, and the wheel starts to turn.

From Faraday's laws says

$$\oint\vec{E}\cdot d\vec{l}=-\frac{d\phi}{dt}=-\pi r_0^2\frac{dB}{dt}=-\pi r_0^2\left(-\frac{\alpha\omega}{\pi r_0^2}\sin\omega t\right)=\alpha\omega\sin\omega t$$

Now, the torque on a segment of length $d\vec{l}$ is $\left(\vec{r}\times\vec{F}\right)$, or $R\lambda Edl$. The total torque on the wheel is therefore

$$\tau = R\lambda\oint Edl = R\lambda\alpha\omega\sin\omega t$$

and the angular momentum imparted to the wheel is

$$\int\tau dt = \int_0^t\left(R\lambda\alpha\omega\sin\omega t\right)dt = R\lambda\alpha\omega\left[-\frac{\cos\omega t}{\omega}\right]_0^t = R\alpha\left(1-\cos\omega t\right) \qquad \because \lambda = 1$$

Q29. The angular width θ of a distant star can be measured by the Michelson radiofrequency stellar interferometer (as shown in the figure below).

The distance h between the reflectors M_1 and M_2 (assumed to be much larger than the aperture of the lens), is increased till the interference fringes (at P_0, P on the plane as shown) vanish for the first time. This happens for $h = 3\,\text{m}$ for a star which emits radiowaves of wavelength 2.7 cm. The measured value of θ (in degrees) is closest to

1. 0.63   2. 0.32

3. 0.52   4. 0.26

Ans: (1)

PART B

Q2. The Hamiltonian of a two-dimensional quantum harmonic oscillator is $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + 2m\omega^2 y^2$ where m and ω are positive constants. The degeneracy of the energy level $\frac{27}{2}\hbar\omega$ is

1. 14
2. 13
3. 8
4. 7

Ans: (4)

Solution.:

$$\because H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + 2m\omega^2 y^2 \Rightarrow H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m(2\omega)^2 y^2$$

The Potential is $V(x) = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}m(2\omega)^2 y^2 = \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$

where $\omega_x = \omega$ and $\omega_y = 2\omega$

The energy eigenvalues for this anisotropic harmonic oscillator is

$$E_{n_x n_y} = \left(n_x + \frac{1}{2}\right)\hbar\omega_x + \left(n_y + \frac{1}{2}\right)\hbar\omega_y = \left(n_x + \frac{1}{2}\right)\hbar\omega + \left(n_y + \frac{1}{2}\right)\hbar(2\omega)$$

$$\Rightarrow E_{n_x n_y} = \left(n_x + \frac{1}{2} + 2n_y + 1\right)\hbar\omega = \left(n_x + 2n_y + \frac{3}{2}\right)\hbar\omega$$

Given, $E_{n_x n_y} = \frac{27}{2}\hbar\omega \Rightarrow \left(n_x + 2n_y + \frac{3}{2}\right)\hbar\omega = \frac{27}{2}\hbar\omega \Rightarrow n_x + 2n_y = 12$

The possible combination of n_x and n_y are

$(n_x, n_y) = (0,6),(2,5),(4,4),(6,3),(8,2),(10,1)$ and $(12,0)$

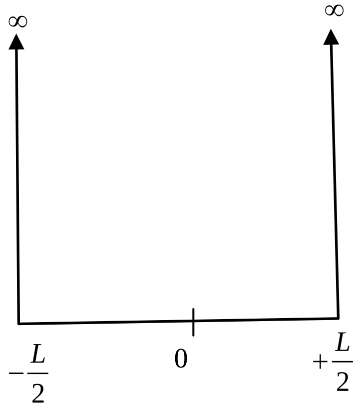
Thus, the degeneracy is 7. Therefore, the correct option is (4).

Q5. A particle in one dimension is in an infinite potential well between $\frac{-L}{2} \le x \le \frac{L}{2}$. For a perturbation $\in \cos\left(\frac{\pi x}{L}\right)$, where $\in$ is a small constant, the change in the energy of the ground state, to first order in $\in$, is

1. $\frac{5\in}{\pi}$
2. $\frac{10\in}{3\pi}$
3. $\frac{8\in}{3\pi}$
4. $\frac{4\in}{\pi}$

Ans: (3)

Solution:



The ground state wave function for symmetric potential well is

$$\psi_1 = \sqrt{\frac{2}{L}} \cos\left(\frac{\pi x}{L}\right)$$

The change in ground state energy $\left(E_g^{'}\right)$ due to the perturbation $H' = \in \cos\left(\frac{\pi x}{L}\right)$ is

$$E_g^{'} = \int_{-\frac{L}{2}}^{+L/2} \psi_1^*(x) H' \psi_1(x)\, dx = \frac{2\in}{L} \int_{-\frac{L}{2}}^{+L} \cos^2\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi x}{L}\right) dx$$

$$\Rightarrow E_g^{'} = \frac{4\in}{L} \int_0^{L/2} \frac{1}{2}\left(1+\cos\left(\frac{2\pi x}{L}\right)\right) \cos\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{2\in}{L}\left[\int_0^{L/2} \cos\left(\frac{\pi x}{L}\right) dx + \int_0^{L/2} \cos\left(\frac{2\pi}{L} x\right) \cos\left(\frac{\pi x}{L}\right) dx\right]$$

$$\Rightarrow E_g^{'} = \frac{2\in}{L}\left[\left.\frac{\sin\left(\frac{\pi x}{L}\right)}{\pi/L}\right|_0^{L/2} + \frac{1}{2}\left[\int_0^{L/2} \cos\left(\frac{3\pi x}{L}\right) dx + \int_0^{L/2} \cos\left(\frac{\pi x}{L}\right) dx\right]\right]$$

$$\Rightarrow E_g^{'} = \frac{2\in}{L}\left\{\frac{L}{\pi}(1-0) + \frac{1}{2}\left[\left.\frac{L}{3\pi}\sin\left(\frac{3\pi x}{L}\right)\right|_0^{L/2} + \left.\frac{L}{\pi}\sin\left(\frac{\pi x}{L}\right)\right|_0^{L/2}\right]\right\}$$

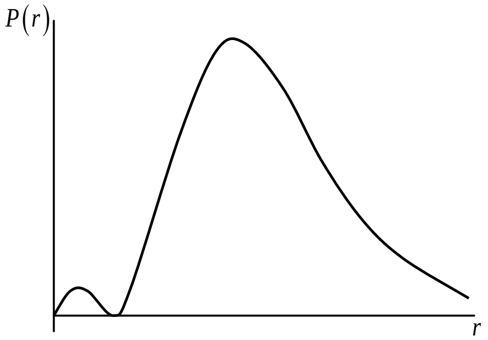
$$\Rightarrow E_g^{'} = \frac{2\in}{L}\left\{\frac{L}{\pi} + \frac{1}{2}\left[\frac{L}{3\pi}(-1-0) + \frac{L}{\pi}(1-0)\right]\right\}$$

$$\Rightarrow E_g^{'} = \frac{2\in}{L}\left(\frac{L}{\pi} + \frac{1}{2}\left[-\frac{L}{3\pi} + \frac{L}{\pi}\right]\right) = 2\in\left(\frac{1}{\pi} + \frac{1}{3\pi}\right) = \frac{8\in}{3\pi} \quad \Rightarrow E_g^{'} = \frac{8\in}{3\pi}$$

Thus, the correct option is (3)

Q8. The radial wavefunction of hydrogen atom with the principal quantum number $n = 2$ and the orbital quantum number $\ell = 0$ is $R_{20} = N\left(1 - \frac{r}{2a}\right) e^{-\frac{r}{2a}}$, where N is the normalization constant. The best schematic representation of the probability density $P(r)$ for the electron to be between r and $r + dr$ is

1.



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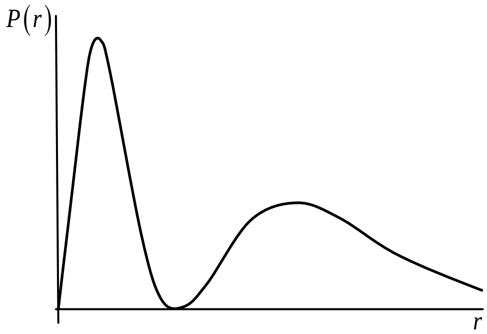
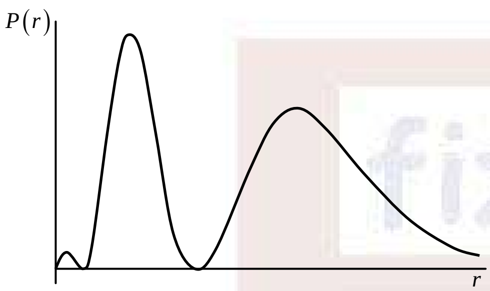
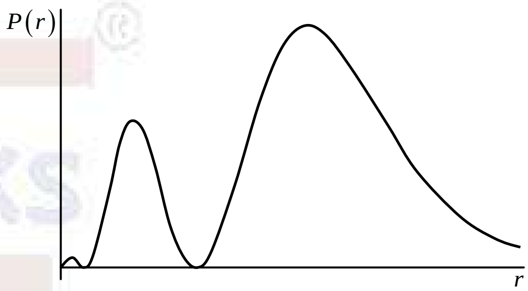


3.



4.



Ans: (1)

Solution:

Number of nodes $= n-1 = 2-1 = 1$; Given $R_{20} = N\left(1-\frac{r}{2a}\right)e^{-r/2a}$

The probability density $P(r) = r^2|R_{20}|^2 = r^2N^2\left(1-\frac{r}{2a}\right)^2 e^{-r/a}$

Thus option (1) is correctly representing the variation of $P(r)$ as a function of r.

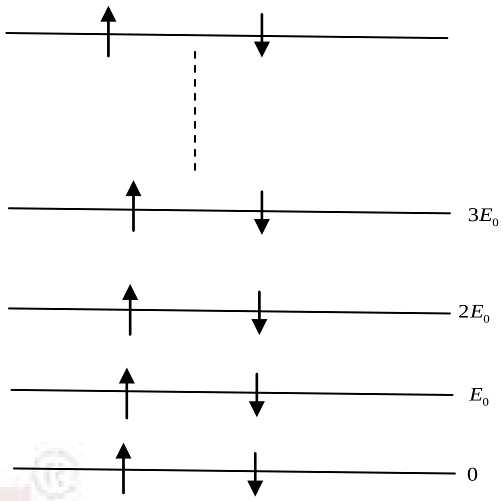
Q18. The energy levels available to each electron in a system of N non-interacting electrons are $E_n = nE_0$, $n = 0,1,2,....$ A magnetic field, which does not affect the energy spectrum, but completely polarizes the electron spins, is applied to the system. The change in the ground state energy of the system is

1. $\frac{1}{2}N^2E_0$
2. N^2E_0
3. $\frac{1}{8}N^2E_0$
4. $\frac{1}{4}N^2E_0$

Ans: (4)

Solution.:

For non-polarized state, two electrons with opposite spin will occupy one state. The energy of the state is given as $E_n = nE_0$, $n = 0,1,2,....$

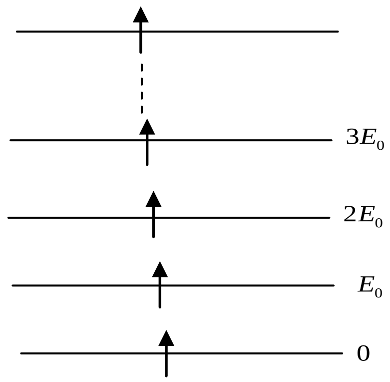


Thus, the total energy of ground state of the system is

$$E_{NP} = 2E_0\left(1+2+3+...+\frac{N}{2}\right)$$

$$\Rightarrow E_{NP} = 2E_0 \cdot \frac{\frac{N}{2}\left(\frac{N}{2}+1\right)}{2} = \frac{E_0}{4}N(N+2)$$

For polarized state, only one electron will occupy one state. Thus, the total energy of ground state of the system is



$$E_P = E_0(1+2+3+...+N) = E_0\frac{N(N+1)}{2}$$

Difference in the ground state energy is

$$\Delta E = E_P - E_{NP} = \frac{E_0}{4}N(N+2) - \frac{E_0}{2}\cdot N(N+1)$$

$$= \frac{E_0}{4}N[N+2-2N-2] = \frac{N^2E_0}{4}$$

Thus, the correct option is (4).

Q19. The value of $\langle L_x^2\rangle$ in the state $|\varphi\rangle$ for which $L^2|\varphi\rangle = 6\hbar^2|\varphi\rangle$ and $L_z|\varphi\rangle = 2\hbar|\varphi\rangle$, is

1. 0 2. $4\hbar^2$ 3. $2\hbar^2$ 4. $\hbar^2$

Ans: (4)

Solution.: $\langle L_x^2\rangle = \frac{\hbar^2}{2}\left[\ell(\ell+1) - m_\ell^2\right]$

Given, $L^2|\phi\rangle = 6\hbar^2|\phi\rangle$, compare it with $L^2|\phi\rangle = \ell(\ell+1)\hbar^2|\phi\rangle \Rightarrow \ell(\ell+1) = 6 \Rightarrow \ell = 2$

also $L_z|\phi\rangle = 2\hbar|\phi\rangle$, compare it with $L_z|\phi\rangle = m_\ell\hbar|\phi\rangle \Rightarrow m_\ell = 2$

Thus $\langle L_x^2\rangle = \frac{\hbar^2}{2}\left[2(1+1) - 2^2\right] = \frac{\hbar^2}{2}[6-4] = \hbar^2$

Correct option is (4).

PART C

Q2. Two distinguishable non-interacting particles, each of mass m are in a one-dimensional infinite square well in the interval $[0,a]$. If x_1 and x_2 are position operators of the two particles, the expectation value $\langle x_1 x_2\rangle$ in the state in which one particle is in the ground state and the other one is in the first excited state, is

1. $\frac{1}{2}a^2$ 2. $\frac{1}{2}\pi^2 a^2$ 3. $\frac{1}{4}a^2$ 4. $\frac{1}{4}\pi^2 a^2$

Ans: (3)

Solution: The net wave function of two non-interacting particles is

$$\psi_T=\psi_1(x_1)\psi_2(x_2)=\sqrt{\frac{2}{a}}\sin\left(\frac{n_1\pi x_1}{a}\right)\cdot\sqrt{\frac{2}{a}}\sin\left(\frac{n_2\pi x_2}{a}\right)$$

Consider first particle in the ground state $(n_1=1)$ and second particle is in the first excited state $(n_2=2)$.

$$\therefore\ \psi_T=\frac{2}{a}\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi}{a}x_2\right)$$

The expectation value $\langle x_1 x_2\rangle$ is

$$\langle x_1x_2\rangle=\int_0^a\int_0^a\psi_T^* x_1x_2\psi_T\,dx_1dx_2=\int_0^a\psi_1^* x_1\psi_1 dx_1\int_0^a\psi_2^* x_2\psi_2 dx_2=\langle x_1\rangle\langle x_2\rangle$$

Since, the expectation value of position in an asymmetric square well of infinite potential of length a is always $\frac{a}{2}$.

$\therefore\ \langle x_1x_2\rangle=\frac{a}{2}\times\frac{a}{2}=\frac{a^2}{4}.$ Thus, the correct option is (3)

Q11. Two operators A and B satisfy the commutation relations $[H,A]=-\hbar\omega B$ and $[H,B]=\hbar\omega A$, where ω constant and H is the Hamiltonian of the system. The expectation value $\langle A\rangle_\psi(t)=\langle\psi|A|\psi\rangle$ in a state $|\psi\rangle$, such that at time $t=0,\langle A\rangle_\psi(0)=0$ and $\langle B\rangle_\psi(0)=i$, is

1. $\sin(\omega t)$ 2. $\sinh(\omega t)$ 3. $\cos(\omega t)$ 4. $\cosh(\omega t)$

Ans: (2)

Solution: According to Ehrenfest theorem $\frac{d\langle A\rangle}{dt}=\frac{1}{i\hbar}\langle[A,H]\rangle+\left\langle\frac{\partial A}{\partial t}\right\rangle$ ....(1)

Since A is time independent, therefore $\frac{\partial A}{\partial t}=0$ and $[H,A]=-\hbar\omega B$

$$\therefore\ \frac{d\langle A\rangle}{dt}=\frac{1}{i\hbar}\langle(+\hbar\omega B)\rangle=-i\omega\langle B\rangle \qquad ....(2)$$

Now, $\frac{d\langle B\rangle}{dt}=\frac{1}{i\hbar}\langle[B,H]\rangle+\left\langle\frac{\partial B}{\partial t}\right\rangle$

Since B is time independent, therefore $\frac{\partial B}{\partial t}=0$ and $\langle H,B\rangle=\hbar\omega A$

$$\therefore\ \frac{d\langle B\rangle}{dt}=\frac{1}{i\hbar}\langle(-i\hbar\omega A)\rangle=i\omega\langle A\rangle \qquad ....(3)$$

Differentiate equation (2), w.r.t. t $\frac{d^2\langle A\rangle}{dt^2}=-i\omega\frac{d\langle B\rangle}{dt}=-i\omega(i\omega)\langle A\rangle=\omega^2\langle A\rangle$

This is a second order linear differential equation; its roots are obtained through auxiliary equation

$D^2-\omega^2\Rightarrow D=\pm\omega$, where $D^2=\frac{d^2}{dt^2}$. The solution is $\langle A\rangle=C_1e^{\omega t}+C_2e^{-\omega t}$ ....(4)

Now at $t=0$, $\langle A\rangle=0$ $\therefore$ $c_1+c_2=0\Rightarrow c_1=-c_2=c$

Thus $\langle A\rangle=c\left[e^{\omega t}-e^{-\omega t}\right]=2c\sinh\omega t$ ....(5)

To find constant $'c'$, use second initial condition that is $\langle B\rangle=i$ at $t=0$.

From equation (1): $\frac{d\langle A\rangle}{dt}=-i\omega\langle B\rangle\Rightarrow\frac{d}{dt}(2c\sinh\omega t)=-i\omega\langle B\rangle$

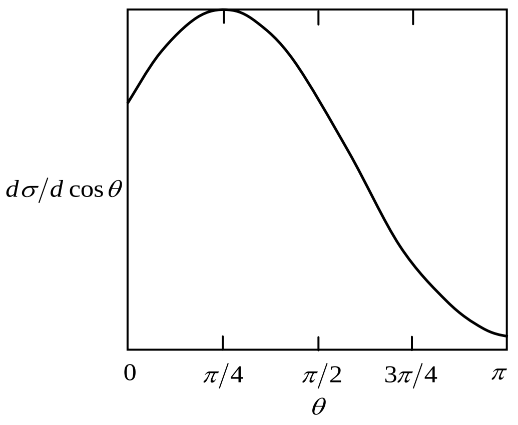
$\Rightarrow 2c(\cosh\omega t)\omega=-i\omega\langle B\rangle$

At $t=0\Rightarrow 2c\omega=-i\omega(i)\Rightarrow c=\frac{1}{2}$. Thus $\langle A\rangle=\sinh\omega t$

Therefore, the correct option is (2)

Q20. The phase shifts of the partial waves in an elastic scattering at energy E are $\delta_0=12^\circ$, $\delta_1=4^\circ$ and $\delta_{\ell\geq 2}\simeq 0^\circ$. The best qualitative depiction of θ-dependence of the differential scattering cross-section $\frac{d\sigma}{d\cos\theta}$ is

1.



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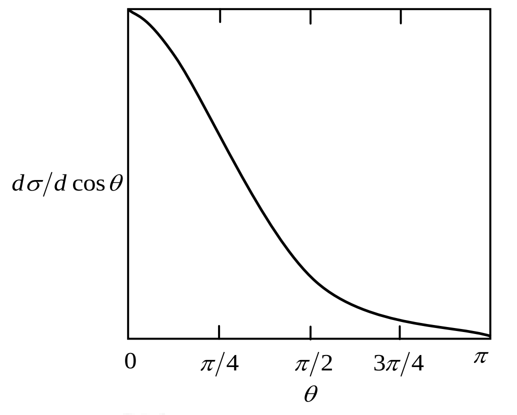

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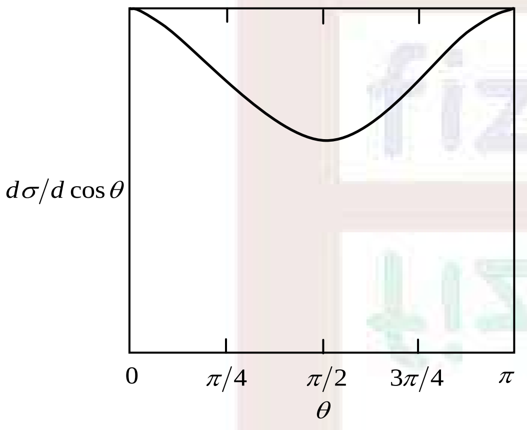

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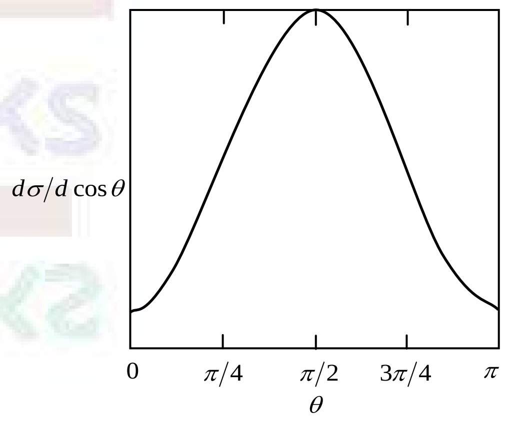


Ans: (2)

Solution:

Scattering amplitude is $f(\theta)=\frac{1}{k}\left[\sum_{\ell}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\right]$

Given, for *s*-wave $(\ell=0)$ $\delta_0=12^\circ$ and for *p*-wave $(\ell=1)$ $\delta_1=4^\circ$ and $\delta_{\ell\ge 2}=0$

$$\therefore\ f(\theta)=\frac{1}{k}\left[e^{i\delta_0}\sin\delta_0 P_0(\cos\theta)+3e^{i\delta_1}\sin\delta_1 P_1(\cos\theta)\right]=\frac{1}{k}\left[e^{i\delta_0}\sin\delta_0+3e^{i\delta_1}\sin\delta_1\cos\theta\right]$$

Now $\frac{d\sigma}{d\Omega}=\frac{d\sigma}{d(\cos\theta)}=|f(\theta)|^2=f^*(\theta)f(\theta)$

$$=\frac{1}{k^2}\left[e^{-i\delta_0}\sin\delta_0+3e^{-i\delta_1}\sin\delta_1\cos\theta\right]\left[e^{i\delta_0}\sin\delta_0+3e^{i\delta_1}\sin\delta_1\cos\theta\right]$$

$$=\frac{1}{k^2}\left[\sin^2\delta_0+3e^{i(\delta_1-\delta_0)}\sin\delta_0\sin\delta_1\cos\theta+3e^{-i(\delta_1-\delta_0)}\sin\delta_0\sin\delta_1\cos\theta+9\sin^2\delta_1\cos^2\theta\right]$$

$$=\frac{1}{k^2}\left[\sin^2\delta_0+3\sin\delta_0\sin\delta_1\cos\theta\left(e^{i(\delta_1-\delta_0)}+e^{-i(\delta_1-\delta_0)}\right)+9\sin^2\delta_1\cos^2\theta\right]$$

$$=\frac{1}{k^2}\left[\sin^2\delta_0+3\sin\delta_0\sin\delta_1\cos\theta\cdot 2\cos(\delta_1-\delta_0)+9\sin^2\delta_1\cos^2\theta\right]$$

$$\frac{d\sigma}{d(\cos\theta)}=\frac{1}{k^2}\left[\sin^2(12)+6\sin(12)\sin(4)\cos(8)\cos\theta+9\sin^2(4)\cos^2\theta\right]$$

$$\therefore\ \frac{d\sigma}{d(\cos\theta)}=\frac{1}{k^2}\left[0.043+0.086\cos\theta+0.043\cos^2\theta\right]$$

At $\theta=0$, $\frac{d\sigma}{d(\cos\theta)}=\frac{0.172}{k^2}$; At $\theta=\pi/2$, $\frac{d\sigma}{d(\cos\theta)}=\frac{0.043}{k^2}$; At $\theta=\pi$; $\frac{d\sigma}{d(\cos\theta)}=0$

Thus, graph (2), correctly represent the variation of $\frac{d\sigma}{d(\cos\theta)}$ as a function of θ.

PART B

Q13. Two energy levels, 0 (non-degenerate) and $\in$ (double degenerate), are available to N non-interacting distinguishable particles. If U is the total energy of the system, for large values of N the entropy of the system is $k_B\left[N\ln N-\left(N-\frac{U}{\in}\right)\ln\left(N-\frac{U}{\in}\right)+X\right]$. In this expression, X is

1. $-\frac{U}{\in}\ln\frac{U}{2\in}$
2. $-\frac{U}{\in}\ln\frac{2U}{\in}$
3. $-\frac{2U}{\in}\ln\frac{2U}{\in}$
4. $-\frac{U}{\in}\ln\frac{U}{\in}$

Ans: (1)

Solution:

$$U=0\times N+\in\times n\Rightarrow n=\left(\frac{U}{\in}\right)$$

$$S=k_B lnW=k_B\left[ln\left(\frac{N!}{(N+n)!n!}\right)+ln\left(\frac{n!}{\frac{n}{2}!\frac{n}{2}!}\right)\right] \qquad \because W=\frac{N!}{(N-n)!n!}\times {}^nC_{n/2}$$

$$S=k_B\left[NlnN-N-(N-n)ln(N-n)+N-nln\left(\frac{n}{2}\right)\right]$$

$$S=k_B\left[NlnN-\left(N-\frac{U}{\in}\right)ln\left(N-\frac{U}{\in}\right)-\frac{U}{\in}ln\left(\frac{U}{2\in}\right)\right]$$

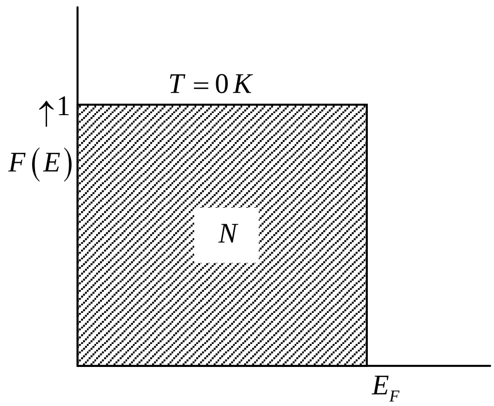
i.e. $X=-\frac{U}{\in}ln\frac{U}{2\in}$

Q21. The single particle energies of a system of N non-interacting fermions of spin s (at $T=0$) are $E_n=n^2E_0$, $n=1,2,3,\ldots$. The ratio $\in_F\left(\frac{3}{2}\right)\Big/\in_F\left(\frac{1}{2}\right)$ of the Fermi energies for fermions of spin 3/2 and spin 1/2, is

1. 1/2
2. 1/4
3. 2
4. 1

Ans: (2)

Solution:



For a general dispersion relation of the type $E\propto k^s$ the density of state $\rho(E)$ in d-dimension is

$\rho(E)\propto(2s+1)E^{\left(\frac{d}{s}-1\right)}$ where $2s+1$ is the spin-degeneracy.

The given energy $E_n = n^2 E_0\,, n = 1, 2, 3, \ldots$ indicates that the system is a one-dimensional i.e. $d = 1$

The dispersion relation of fermions is $E = \frac{\hbar^2 k^2}{2m}$ therefore, $s = 2$; Thus,

$\rho(E) \propto (2s+1) E^{-\frac{1}{2}}$.

At $T = 0\,K$, the total number of fermions is $N = \int_0^{E_F} \rho(E)\, dE$

$$N \propto (2s+1) \int_0^{E_F} E^{-\frac{1}{2}} dE \Rightarrow N \propto (2s+1) E_F^{\frac{1}{2}} \Rightarrow E_F \propto \frac{N^2}{(2s+1)^2}$$

$$\therefore \frac{E_F\left(\frac{3}{2}\right)}{E_F\left(\frac{1}{2}\right)} = \frac{\dfrac{N^2}{\left(2\times\frac{3}{2}+1\right)^2}}{\dfrac{N^2}{\left(2\times\frac{1}{2}+1\right)^2}} = \frac{4}{16} = \frac{1}{4}$$

Thus correct option is (2)

PART C

Q1. Two electrons in thermal equilibrium at temperature $T = k_B/\beta$ can occupy two sites. The energy of the configuration in which they occupy the different sites is $J\vec{S}_1 \cdot \vec{S}_2$ (where $J > 0$ is a constant and $\vec{S}$ denotes the spin of an electron), while it is U if they are at the same site. If $U = 10\,J$, the probability for the system to be in the first excited state is

1. $e^{-3\beta J/4}\big/\left(3e^{\beta J/4} + e^{-3\beta J/4} + 2e^{-1\ \beta J0}\right)$
2. $3e^{-\beta J/4}\big/\left(3e^{-\beta J/4} + e^{3\beta J/4} + 2e^{-10\beta J}\right)$
3. $e^{-\beta J/4}\big/\left(2e^{-\beta J/4} + 3e^{3\beta J/4} + 2e^{-10\beta J}\right)$
4. $3e^{-3\beta J/4}\big/\left(2e^{\beta J/4} + 3e^{-3\beta J/4} + 2e^{-1\ \beta J0}\right)$

Ans.: (2)

Solution:

$$U = J\vec{S}_1 \cdot \vec{S}_2 = \frac{J}{2}\left[s(s+1) - s_1(s_1+1) - s_2(s_2+1)\right]$$

For triplet $s = 1, s_1 = \frac{1}{2}$, $s_2 = \frac{1}{2}$, (degeneracy $= 2s+1 = 3$); $\quad U = \frac{J}{2}\left[2 - \frac{3}{2}\right] = \frac{J}{4}$

For Singlet $s = 0$, $s_1 = \frac{1}{2}$, $s_2 = \frac{1}{2}$ (degeneracy $= 2s+1 = 1$); $\quad U = -\frac{3}{4}J$

So for $s = 0$ we get ground state and for $s = 1$ first excited state

The partition function $\quad Z = \sum g_i e^{-\beta E_i} = 3e^{-\beta J/4} + e^{+3\beta J/4} + \underbrace{2e^{-10\beta J}}_{\text{for both at same side } U = 10J}$

The probability of excited state will be $P\left(3e^{-\beta J/4}\right)=\dfrac{3}{3e^{-\beta J/4}+e^{3\beta J/4}+2e^{-10\beta J}}$

Q14. A layer of ice has formed on a very deep lake. The temperature of water, as well as that of ice at the ice-water interface, are 0°C, whereas the temperature of the air above is -10°C. The thickness $L(t)$ of the ice increases with time t. Assuming that all physical properties of air and ice are independent of temperature, $L(t)\sim L_0t^{\alpha}$ for large t. The value of α is

1. 1/4 2. 1/3 3. 1/2 4. 1

Ans: (3)

Solution:

$$Q=\frac{KA\left(Q_A-Q_1\right)t}{x}\quad\Rightarrow\frac{d\left(mL\right)}{dt}=\frac{KA\Delta Q}{x}\quad\Rightarrow\frac{A\rho_W\,dxL}{dt}=\frac{KA\Delta Q}{x}$$

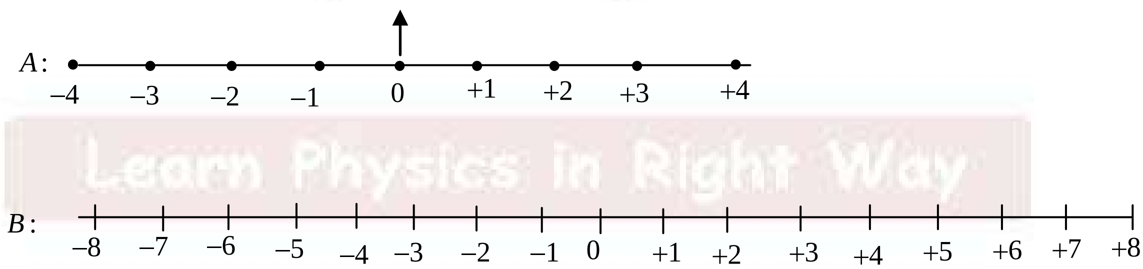
$$\int_0^l x\,dx=\frac{kA\Delta Q}{A\rho_W L_0}\int_0^t dt\quad\Rightarrow\left[\frac{x^2}{2}\right]_0^l=\frac{kA\Delta Q}{A\rho_W L_0}t\quad\Rightarrow\left(\frac{l^2}{2}\right)=\frac{kA\Delta Q}{A\rho_W L_0}t\Rightarrow l=l_0t^{1/2}\quad\Rightarrow\alpha=\frac{1}{2}$$

Q19. Two random walkers A and B walk on a one-dimensional lattice. The length of each step taken by A is one, while the same for B is two, however, both move towards right or left with equal probability. If they start at the same point, the probability that they meet after 4 steps, is

1. $\dfrac{9}{64}$ 2. $\dfrac{5}{32}$ 3. $\dfrac{11}{64}$ 4. $\dfrac{3}{16}$

Ans: (3)

Solution.:



Possible positions after 4 steps: For $A:-4,-2,0,2,4$ and For $B:-8,-4,0,4,8$

So, only points where they may meet after 4 steps are: $-4,0,4$.

Start point:

No. of ways in which after taking 4 steps = 6

$\therefore$ Probability of being at the starting position after 4 steps (for A) $=\dfrac{6}{16}=\dfrac{3}{8}$

Also, the probability of being at start position for B after 4 steps will be same i.e., $\dfrac{3}{8}$.

At position 4

For A: Probability of being at 4 or -4 $=\frac{1}{16}$

For B: $\frac{4}{16}=\frac{1}{4}$

$\therefore$ Probability of A and B to meet after 4 steps at start position $=\frac{3}{8}\cdot\frac{3}{8}=\frac{9}{64}$

At point 4: $\frac{1}{16}\cdot\frac{1}{4}=\frac{1}{64}$;

At point -4: $\frac{1}{16}\cdot\frac{1}{4}=\frac{1}{64}$

$\therefore$ The probability that they meet, i.e., at either point -4, or 0 or +4 is $=\frac{9}{64}+\frac{1}{64}+\frac{1}{64}=\frac{1}{64}$

Q22. In a one-dimensional system of N spins, the allowed values of each spin are $\sigma_i=\{1,2,3,\ldots,q\}$, where $q\geq 2$ is an integer. The energy of the system is

$$-J\sum_{i=1}^{N}\delta_{\sigma_i,\sigma_{i+1}}$$

where $J>0$ is a constant. If periodic boundary conditions are imposed, the number of ground states of the system is

1. q
2. Nq
3. q^N
4. 1

Ans: (2)

Solution:

The energy of the system is $H=-J\sum_{i=1}^{N}\delta_{\sigma_i,\sigma_{i+1}}$ where $\sigma_i=\{1,2,3,\ldots,q\}$, where $q\geq 2$ is an integer and $J>0$ is a constant.

Using Ising Model $\sigma_i,\sigma_{i+1}=\pm 1$ (possible interaction)

Total possible arrangement for two particles system

$$\uparrow\uparrow \quad \downarrow\downarrow \quad \uparrow\downarrow \quad \downarrow\uparrow$$

Ground state is the member of same spin in a state i.e. out of 4 arrangements ground state spin

State is equal to 2. So $q=2$.

Thus for N particle system $2N$ ground state is possible.

the number of ground states of the system is Nq.

PART B

Q4. A DC motor is used to lift a mass M to a height h from the ground. The electric energy delivered to the motor is VIt, where V is the applied voltage, I is the current and t the time for which the motor runs. The efficiency e of the motor is the ratio between the work done by the motor and the energy delivered to it. If $M = 2.00 \pm 0.02\,\text{kg}$, $h = 1.00 \pm 0.01\,\text{m}$, $V = 10.0 \pm 0.1\,\text{V}$, $I = 2.00 \pm 0.02\,\text{A}$ and $t = 300 \pm 15\,\text{s}$, then the fractional error $|\delta e/e|$ in the efficiency of the motor is closest to

1. 0.05
2. 0.09
3. 0.12
4. 0.15

Ans: (1)

Solution:

Work done in lifting object of a mass M to a height h from the ground is $W = Mgh$

where g is acceleration due to gravity.

The efficiency e of the motor is the ratio between the work done by the motor and the energy

delivered to it i.e $e = \dfrac{W}{E} = \dfrac{Mgh}{VIt}$.

Error in the resulting efficiency is

$$(\delta e)^2 = \left(\frac{\partial e}{\partial M}\right)^2 (\delta M)^2 + \left(\frac{\partial e}{\partial h}\right)^2 (\delta h)^2 + \left(\frac{\partial e}{\partial V}\right)^2 (\delta V)^2 + \left(\frac{\partial e}{\partial I}\right)^2 (\delta I)^2 + \left(\frac{\partial e}{\partial t}\right)^2 (\delta t)^2$$

$$\Rightarrow (\delta e)^2 = \left(\frac{gh}{VIt}\right)^2 (\delta M)^2 + \left(\frac{Mg}{VIt}\right)^2 (\delta h)^2 + \left(-\frac{Mgh}{V^2 It}\right)^2 (\delta V)^2 + \left(-\frac{Mgh}{VI^2 t}\right)^2 (\delta I)^2 + \left(-\frac{Mgh}{VIt^2}\right)^2 (\delta t)^2$$

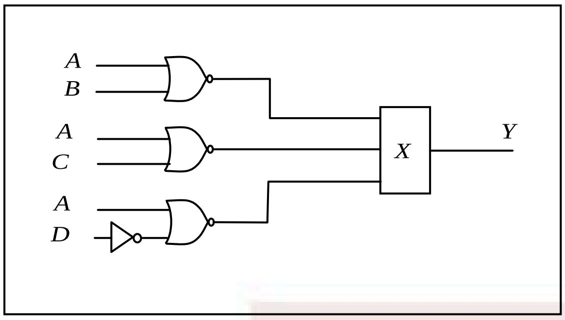
$$\Rightarrow \left(\frac{\delta e}{e}\right)^2 = \left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta h}{h}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta t}{t}\right)^2$$

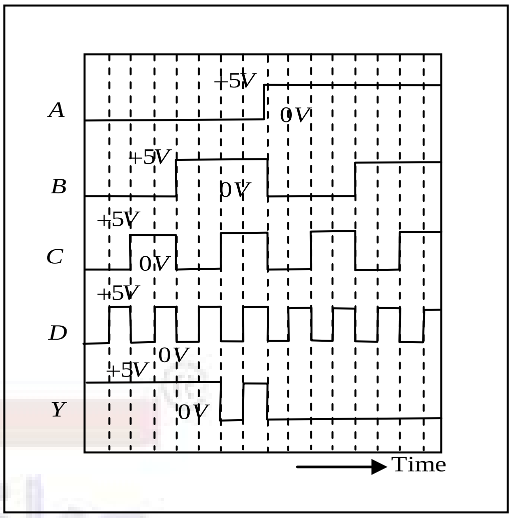
$$\Rightarrow \left|\frac{\delta e}{e}\right| = \sqrt{\left(\frac{0.02}{2}\right)^2 + \left(\frac{0.01}{1}\right)^2 + \left(\frac{0.1}{10}\right)^2 + \left(\frac{0.02}{2}\right)^2 + \left(\frac{15}{300}\right)^2}$$

$$\Rightarrow \left|\frac{\delta e}{e}\right| = \sqrt{(0.01)^2 + (0.01)^2 + (0.01)^2 + (0.01)^2 + (0.05)^2}$$

$$\Rightarrow \left|\frac{\delta e}{e}\right| = \sqrt{10^{-4} + 10^{-4} + 10^{-4} + 10^{-4} + 25\times 10^{-4}} \Rightarrow \left|\frac{\delta e}{e}\right| = \sqrt{29\times 10^{-4}} = 0.054 = 0.05$$

Q6. For the given logic circuit, the input waveforms A, B, C and D are shown as a function of time.





To obtain the output Y as shown in the figure, the logic gate X should be

1. an AND gate 2. an OR gate

3. a NAND gate 4. a NOR gate

Ans: (2)

Solution:

(a) Let the logic gate X be an an AND gate

$$Y=\overline{(A+B)}.\overline{(A+C)}.\overline{\left(A+\overline{D}\right)}=\left(\overline{A}\overline{B}\right).\left(\overline{A}\overline{C}\right).\left(\overline{A}D\right)\Rightarrow Y=\overline{A}\overline{B}\overline{C}D$$

If $ABCD=0000$ then $Y=0$ (From given output $Y=1$)

If $ABCD=1111$ then $Y=0$ (From given output $Y=0$)

(b) Let the logic gate X be an an OR gate

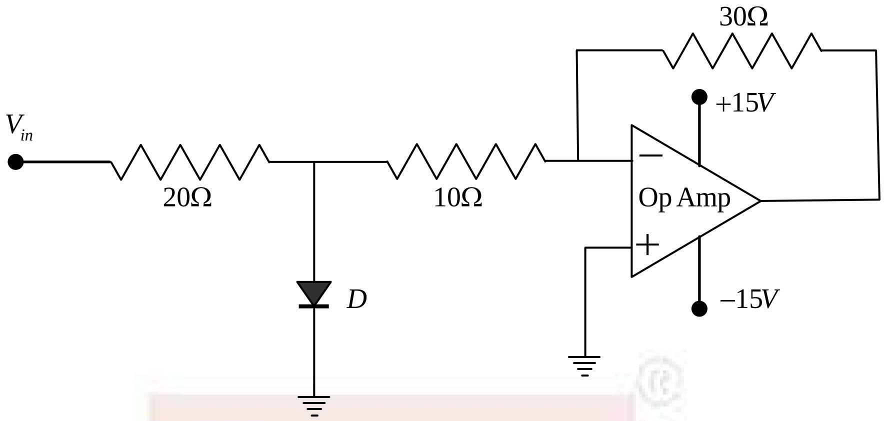
$$Y=\overline{(A+B)}+\overline{(A+C)}+\overline{\left(A+\overline{D}\right)}=\left(\overline{A}\overline{B}\right)+\left(\overline{A}\overline{C}\right)+\left(\overline{A}D\right)\Rightarrow Y=\overline{A}\left(\overline{B}+\overline{C}+D\right)$$

If $ABCD=0000$ then $Y=1$ (From given output $Y=1$)

If $ABCD=1111$ then $Y=0$ (From given output $Y=0$)

So option (b) is correct.

Q15. In the circuit below, there is a voltage drop of 0.7 V across the diode D in forward bias, while no current flows through it in reverse bias.



If V_{in} is a sinusoidal signal of frequency 50 Hz with an RMS value of 1 V, the maximum current that flows through the diode is closest to

1. 1 A
2. 0.14 A
3. 0 A
4. 0.07 A

Ans: (3)

Solution:

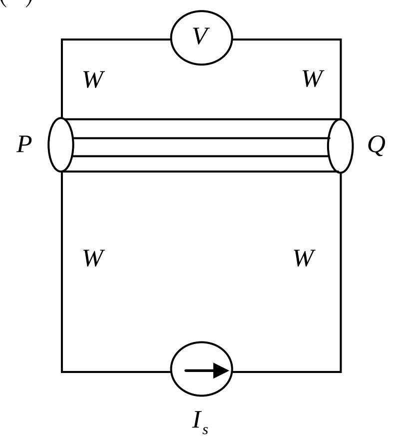
$V_{rms} = 1V \Rightarrow V_p = \sqrt{2}V_{rms} = 1.41V$

Voltage drop across diode $V_D = \dfrac{10}{10+20} \times 1.41V = 0.47V$

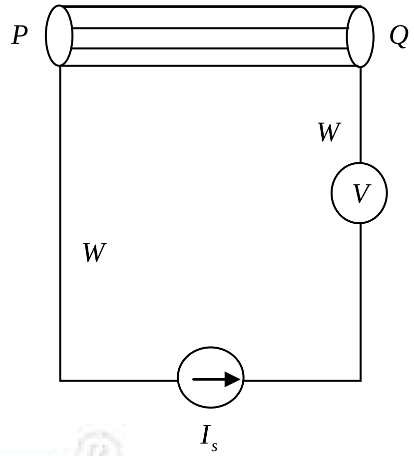
Since voltage drop across diode $V_D = 0.47V$ is less than $0.7V$ so diode will not be ON. Hence diode will be zero.

Q24. A circuit needs to be designed to measure the resistance R of the a cylinder PQ to the best possible accuracy, using an ammeter A, a voltmeter V, a battery E and a current source I_s (all assumed to be ideal). The value of R is known to be approximately $10\,\Omega$, and the resistance W of each of the connecting wires is close to $10\,\Omega$. If the current from the current source and voltage from the battery are known exactly, which of the following circuits

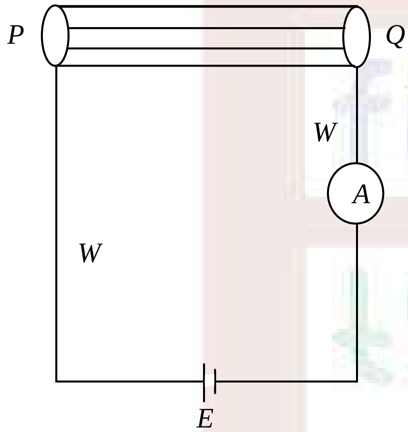
(a)



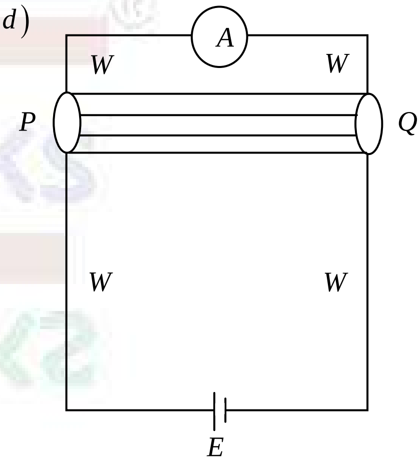
(b)



(c)



(d)



1. (b) 2. (a) 3. (d) 4. (c)

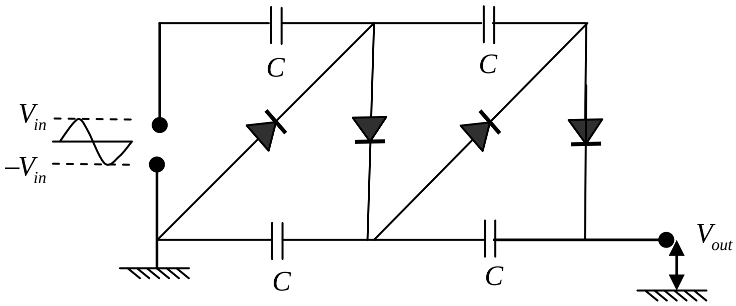
Ans: (2)

Solution:

Option (a) is correct since voltmeter must be in parallel to PQ and then $R = \frac{V}{I}$.

PART C

Q16. In the circuit shown below, four silicon diodes and four capacitors are connected to a sinusoidal voltage source of amplitude $V_{\text{in}} > 0.7V$ and frequency 1 kHz. If the knee voltage for each of the diodes is 0.7 V and the resistances of the capacitors are negligible, the DC output voltage V_{out} after 2 seconds of starting the voltage source is closest to

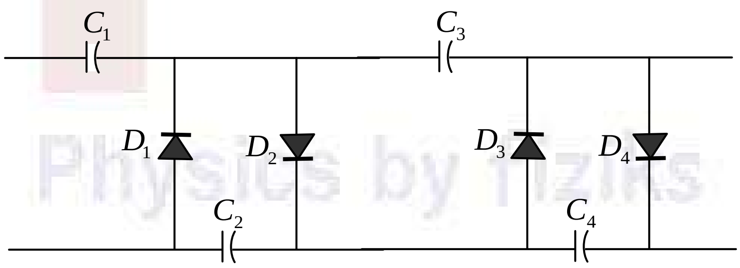


1. $4V_{\text{in}} - 0.7V$
2. $4V_{\text{in}} - 2.8V$
3. $V_{\text{in}} - 0.7V$
4. $V_{\text{in}} - 2.8V$

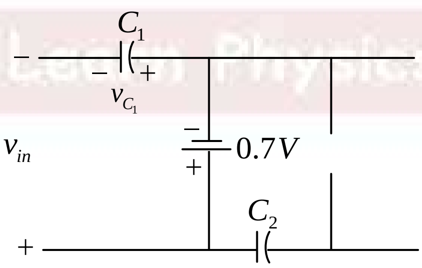
Ans: (2)

Solution:

Given circuit is two voltage multiplier circuit in series.

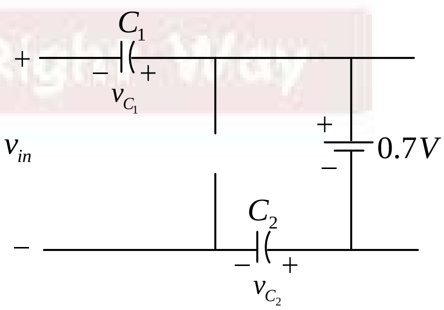


Consider stage-1 first during negative half cycle and then during positive half cycle



$+v_{\text{in}} - v_{C_1} - 0.7V = 0$

$\Rightarrow v_{C_1} = +v_{\text{in}} - 0.7V$



$-v_{\text{in}} - v_{C_1} + 0.7V + v_{C_2} = 0$

$+v_{C_2} = v_{\text{in}} + v_{C_1} - 0.7V \Rightarrow v_{C_2} = 2v_{\text{in}} - 1.4V$

Thus $v_{\text{out}} = v_{C_4} = 2v_{C_2} = 4v_{\text{in}} - 2.8V$

Q28. A train of impulses of frequency 500 Hz, in which the temporal width of each spike is negligible compared to its period, is used to sample a sinusoidal input signal of frequency 100 Hz. The sampled output is

1. Discrete with the spacing between the peaks being the same as the time period of the sampling signal
2. A sinusoidal wave with the same time period as the sampling signal
3. Discrete with the spacing between the peaks being the same as the time period of the input signal
4. A sinusoidal wave with the same time period as the input signal

Ans: (1)

Solution:

According to Shannon sampling theorem, sampling frequency is equal to five times frequency of input system.

The sampled output consists of a series of discrete values representing the amplitude of the input signal at specific time interval.

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

PART C

Q4. Electrons polarized along the x-direction are in a magnetic field $B_1\hat{i} + B_2\left(\hat{j}\cos\omega t + \hat{k}\sin\omega t\right)$, where $B_1 \gg B_2$ and ω are positive constants. The value of $\hbar\omega$ for which the polarization-flip process is a resonant one, is

1. $2\mu_B|B_2|$ 2. $\mu_B|B_1|$

3. $\mu_B|B_2|$ 4. $2\mu_B|B_1|$

Ans: (4)

Solution:

The external magnetic field is $\vec{B} = B_1\hat{i} + B_2\left(\hat{j}\cos\omega t + \hat{k}\sin\omega t\right)$, where $B_1 \gg B_2$ and ω are positive constants.

$$\left|\vec{B}\right|^2 = |B_1|^2 + |B_2|^2\left(\cos^2\omega t + \sin^2\omega t\right) = |B_1|^2 + |B_2|^2 \approx |B_1|^2 \quad \Rightarrow \left|\vec{B}\right| = |B_1| \qquad \because B_1 \gg B_2$$

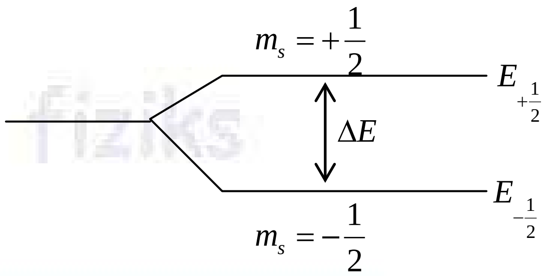
The interaction Hamiltonian in external field is

$$H = -\vec{\mu}_s.\vec{B} = -\left|\vec{\mu}_{s_x}\right|\left|\vec{B}\right| = g\frac{e}{2m_e}S_x\left|\vec{B}\right| \qquad \text{where } \vec{\mu}_s = -g\frac{e}{2m_e}\vec{S} \text{ and } g = 2.$$

The interaction energy is

$$E = \langle H\rangle = g\frac{e}{2m_e}m_s\hbar\left|\vec{B}\right| = (gm_s)\mu_B|B_1|$$

For $m_s = +\frac{1}{2}$; $E_{+1/2} = \left(2\times\frac{1}{2}\right)\mu_B|B_1| = \mu_B|B_1|$



For $m_s = -\frac{1}{2}$;

$$E_{-1/2} = 2\times\left(-\frac{1}{2}\right)\mu_B|B_1| = -\mu_B|B_1|$$

Thus $\Delta E = E_{+1/2} - E_{-1/2} = 2\mu_B|B_1| \Rightarrow \hbar\omega = 2\mu_B|B_1|$

Q15. The electron cloud (of the outermost electrons) of an ensemble of atoms of atomic number Z is described by a continuous charge density $\rho(r)$ that adjusts itself so that the electrons at the Fermi level has zero energy. If $V(r)$ is the local electrostatic potential, then $\rho(r)$ is

1. $\frac{e}{3\pi^2\hbar^3}\left[2m_e eV(r)\right]^{3/2}$ 2. $\frac{Ze}{3\pi^2\hbar^3}\left[2m_e eV(r)\right]^{3/2}$

3. $\frac{e}{3\pi^2\hbar^3}\left[Zm_eeV(r)\right]^{3/2}$ 4. $\frac{e}{3\pi^2\hbar^3}\left[m_eeV(r)\right]^{3/2}$

Ans.:(a)

Solution: $g(p)dp=\frac{V}{h^3}4\pi p^2dp$ where

$$p^2=2mk\Rightarrow \cancel{2}pdp=\cancel{2}mdk\Rightarrow dp=\frac{mdk}{\sqrt{2mk}}=\sqrt{\frac{m}{2k}}dk$$

$$g(p)dp=\frac{V}{h^3}4\pi 2mk\sqrt{\frac{m}{2k}}dk=\frac{2\pi V}{h^3}(2m)^{3/2}\sqrt{k}\,dk$$

A factor of 2 is introduced due to spin degeneracy so $g(p)dp=\frac{4\pi V}{h^3}(2m)^{3/2}\sqrt{k}\,dk$

Charge density $\rho=ne=\frac{N}{V}e=\frac{4\pi e}{h^3}\int_0^{K_F}(2m)^{3/2}\sqrt{k}\,dk$ $\quad\because N=\int g(p)dp$

$$K_F=E_F-U=E_F-(-eV)=E_F+eV=0+eV=eV$$

Here $U=$Potential energy; E_F = Fermi Energy

$$\rho=\frac{4\pi e}{h^3}\int_0^{k_F}(2m)^{3/2}\sqrt{k}\,dk=\frac{e(2m)^{3/2}}{\hbar^3 2\pi^2}\left[\frac{k^{3/2}}{3/2}\right]_0^{k_F}=\frac{e}{3\pi^2\hbar^3}\left[2mK_F\right]^{3/2}$$

$$\Rightarrow \rho(r)=\frac{e}{3\pi^2\hbar^3}\left[2meV(r)\right]^{3/2}$$

Q23. The red line of wavelength 644 nm in the emission spectrum of Cd corresponds to a transition from the ${}^1D_2$ level to the ${}^1P_1$ level. In the presence of a weak magnetic field, this spectral line will split into (ignore hyperfine structure)

1. 9 lines 2. 6 lines

3. 3 lines 4. 2 lines

Ans: (3)

Solution:

$${}^1D_2\rightarrow{}^1P_1$$

In the weak magnetic field, this spectral line will split into 3 components corresponding to normal Zeeman effect.

Q24. Let the separation of the frequencies of the first Stokes and the first anti-Stokes lines in the pure rotational Raman Spectrum of the H_2 molecule be $\Delta\nu(H_2)$, while the corresponding quantity for D_2 is $\Delta\nu(D_2)$. The ratio $\Delta\nu(H_2)/\Delta\nu(D_2)$ is

1. 0.6 2. 1.2 3. 1 4. 2

Ans: (4)

Solution.:

The separation between the first stokes line and first anti-stokes line in a pure Raman Spectrum is given by $12B$, where B is the rotational constant and depends on the molecule.

For $H_2 : B(H) = \frac{h}{8\pi^2 \mu_H r^2 c}$ and For $D_2 : B(D) = \frac{h}{8\pi^2 \mu_D r^2 c}$

$\mu_H = \frac{m_P m_P}{m_P + m_P} = \frac{m_P}{2}$ and $\mu_D = \frac{2m_P 2m_P}{2m_P + 2m_P} = m_P$

Hence, $\frac{\Delta\nu(H_2)}{\Delta\nu(D_2)} = \frac{12B(H)}{12B(D)} = \frac{\mu_D}{\mu_H} = 2$

PART B

Q16. The dispersion relation of a gas of non-interacting bosons in two dimensions is $E(k)=c\sqrt{|k|}$, where c is a positive constant. At low temperatures, the leading dependence of the specific heat on temperature T, is

1. T^4
2. T^3
3. T^2
4. $T^{3/2}$

Ans: (1)

Solution: For a general dispersion relation of the kind $E = Ak^s$

The specific heat in *d*-dimension is $C_V \propto T^{d/s}$

Given is, $E = c(|k|)^{1/2}$. Thus $s = 1/2$ and $d = 2$

$\therefore\ C_V \propto T^{2/\frac{1}{2}} \Rightarrow C_V \propto T^4$

Therefore, the correct option is (1)

PART C

Q7. The Hall coefficient R_H of a sample can be determined from the measured Hall voltage $V_H = \frac{1}{d}R_H BI + RI$, where d is the thickness of the sample, B is the applied magnetic field, I is the current passing through the sample and R is an unwanted offset resistance. A lock-in detection technique is used by keeping I constant with the applied magnetic field being modulated as $B = B_0 \sin\Omega t$, where B_0 is the amplitude of the magnetic field and Ω is frequency of the reference signal. The measured V_H is

1. $B_0\left(\frac{R_H I}{d}\right)$
2. $\frac{B_0}{\sqrt{2}}\left(\frac{R_H I}{d}\right)$
3. $\frac{I}{\sqrt{2}}\left(\frac{R_H B_0}{d}+R\right)$
4. $I\left(\frac{R_H B_0}{d}+R\right)$

Ans: (2)

Solution: In the lock-in detection technique, the signal is passed through low pass *RC* filter and output is *RMS* value.

Given, $V_H = \frac{1}{d}R_H BI + RI = \frac{1}{d}R_H IB_0 \sin\Omega t + RI$

The output is $V_H = \frac{B_0}{\sqrt{2}}\left(\frac{R_H I}{d}\right)$

The unwanted signal RI is filtered out. Thus the correct option is (2).

Q17. The dispersion relation of electrons in three dimensions is $\in(k)=\hbar v_F k$, where v_F the Fermi velocity is. If at low temperatures $(T \ll T_F)$ the Fermi energy $\in_F$ depends on the number density n as $\in_F(n) \sim n^{\alpha}$, the value of α is

1. 1/3
2. 2/3
3. 1
4. 3/5

Ans: (1)

Solution:

For a general dispersion relation of the kind $E = Ak^s$

The Fermi energy (E_F) depends on n is d-dimension as $E_F \propto n^{s/d}$

Given $E=\hbar v_F k$, therefore, $s=1$ and $d=3$.

Thus $E_F \propto n^{1/3}$

The correct option is (1)

Q30. A lattice A consists of all points in three-dimensional space with coordinates (n_x, n_y, n_z) where n_x, n_y and n_z are integers with $n_x+n_y+n_z$ being odd integers. In another lattice B, $n_x+n_y+n_z$ are even integers. The lattices A and B are

1. both BCC
2. both FCC
3. BCC and FCC, respectively
4. FCC and BCC, respectively

Ans: (2)

Solution:

Given, for lattice A: $n_x+n_y+n_z=$ odd; For lattice B: $n_x+n_y+n_z=$ even

The condition for present planes in BCC & FCC is

In BCC: $n_x+n_y+n_z=$ even

In FCC: n_x, n_y, n_z can be either purely odd or purely even.

$\therefore n_x+n_y+n_z=$ odd, for odd n_x, n_y, n_z and $n_x+n_y+n_z=$ even, for even n_x, n_y, n_z

Thus, FCC satisfies the condition of lattice A and B both. The correct option is (2).

PART C

Q5. The nucleus of ^{40}K (of spin-parity 4^+ in the ground state) is unstable and decays to ^{40}Ar. The mass difference between these two nuclei is $\Delta Mc^2 = 1504.4\,\text{keV}$. The nucleus ^{40}Ar has an excited state at 1460.8 keV with spin-parity 2^+. The most probable decay mode of ^{40}K is by

1. a β^+-decay to the 2^+ state of ^{40}Ar
2. an electron capture to the 2^+ state of ^{40}Ar
3. an electron capture to the ground state of ^{40}Ar
4. a β^+-decay to the ground state of ^{40}Ar

Ans: (2)

Solution:

(a) $^{40}K \to {}^{40}Ar + \beta^+ + \nu_e$

$$Q = \left[M_K^{\text{nucl}} - M_{Ar}^{\text{nucl}} - m_e\right]c^2 = \Delta M^{\text{nucl}}c^2 - m_e c^2 = 1504.4 - 510 = 994.4\,keV < 1460.8\,keV$$

In this case, sufficient energy is not available for *Ar* nuclei to move to the excited state 2^+.

(b) Electron capture: $^{40}K + e^- \to {}^{40}Ar + \nu$

$$Q = \left[M_K^{\text{nucl}} + m_e - M_{Ar}^{\text{nucl}}\right]c^2 = \Delta M^{\text{nucl}}c^2 + m_e c^2 = 1504.4 + 510$$

$$Q = 2014.4\,keV > 1460.8\,keV$$

In this case, sufficient energy is available for the excitation of *Ar* nuclei from ground state to 2^+ excited state.

Q13. The energy (in keV) and spin-parity values $E\left(J^p\right)$ of the low lying excited states of a nucleus of mass number $A = 152$ are $122\left(2^+\right), 366\left(4^+\right), 707\left(6^+\right)$ and $1125\left(8^+\right)$. It may be inferred that these energy levels correspond to a

1. Rotational spectrum of a deformed nucleus
2. Rotational spectrum of a spherically symmetric nucleus
3. Vibrational spectrum of a deformed nucleus
4. Vibrational spectrum of a spherically symmetric nucleus

Ans: (1)

Solution.:

(1) Rotational energy levels have energies in KeV range, while vibrational energy levels have energies in MeV range.

(2) Nuclei with $A = 152$ has significant quadruple moment which indicates non spherical distribution of charge

(3) The non-spherical or deformed nucleus rotating about appropriate axis, give rise to quantized rotational energy levels.

In the light of these facts, one can conclude that option (a) is correct.

Q26. A neutral particle X^0 is produced in $\pi^- + p \to X^0 + n$ by s-wave scattering. The branching ratio of the decay of X^0 to $2\gamma, 3\pi$ and 2π are 0.38, 0.30 and less than 10^{-3}, respectively. The quantum numbers J^{CP} of X^0 are

1. 0^{-+} 2. 0^{+-}
3. 1^{-+} 4. 1^{+-}

Ans: (2)

Solution.:

Spin calculation

(i) $\pi^- + p \to X^0 + n$ (Strong interaction)

$$\vec{0} + \frac{\vec{1}}{2} : \left(0, \vec{1}\right) + \frac{\vec{1}}{2}$$

(ii) $X^0 \to \gamma + \gamma$

$$\left(\vec{0}, \vec{1}, \vec{2}\right) \to \vec{1} + \vec{1}$$

(iii) $X^0 \to \pi^+ + \pi^0 + \pi^-$

$$\left(\vec{0}\right) : \vec{0} + \vec{0} + \vec{0}$$

(iv) $X^0 \to \pi^+ + \pi^-$

$$\vec{0} : \vec{0} + \vec{0}$$

So, best option for spin is $\vec{0}$

Parity Calculation

$$\begin{array}{lcccc} & \pi^- + & p \to & X^0 + & n \\ \hat{P}: & - & + & \pi_X & + \end{array}$$

As above mentioned equation is strong interaction in nature, so parity will be conserved.

$$(-)(+) = \pi_{X^0}(+) \Rightarrow \pi_{X^0} = -ve$$

Charge conjugation of neutral X^0 particle is

$$C_{X^0} = (-1)^{\ell + s} = (-1)^{0+0} = +ve$$

So, quantum number $J^{CP} = 0^{+-}$ for X^0 particle.

PART A

Q1. Twenty litres of rainwater having a 2.0 $\mu mol/L$ concentration of sulfate ions is mixed with forty litres water having $4.0\,\mu mol/L$ sulfate ions. If 50% of the total water evaporated, what would be sulfate concentration in the remaining water

1. 3 $\mu mol/L$ 2. 3.3 $\mu mol/L$
3. 4 $\mu mol/L$ 4. 6.7 $\mu mol/L$

Ans: (4)

Solution:

Total sulfate ions in 20 Lts. of rainwater $= 2.0\mu\,mol/L \times 20\,L = 40\,\mu mol$

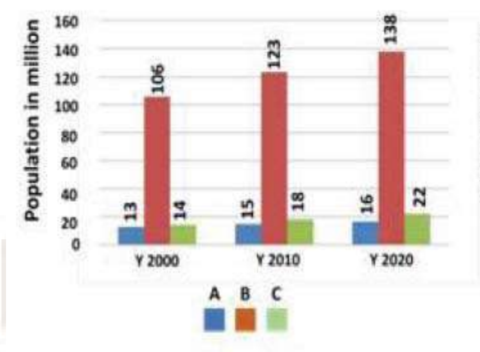
and 40 Lts. of rainwater $= 4.0\mu mol/L \times 40 = 160\,\mu mol$

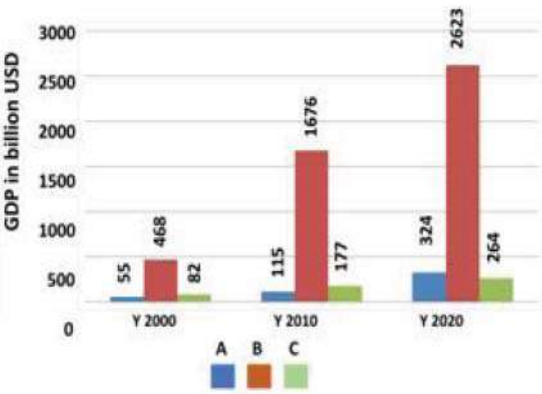
Total rainwater after evaporation $= (20+40)\times\frac{50}{100} = 30\,L$

Total sulfate ions in $30\,L = (40+160)\,\mu mol = 200\,\mu mol$

Sulfate concentration $= \frac{200\mu mol}{30\,L} = 6.66\mu mol/L \approx 6.7\mu mol/L$

Q2. The populations and gross domestic products (GDP) in billion USD of three countries A, B and C in the years 2000, 2010 and 2020 are shown in the two figures below.





The decreasing order of per capita GDP of these countries in the year 2020 is

1. A, B, C 2. A, C, B
3. B, C, A 4. C, A, B

Ans: (1)

Solution:

In 2020:

	Population	GDP
A:	16	324
B:	138	2623
C:	22	264

Per capita GDP:

$A:\dfrac{324}{16}=20.25;$ $B:\dfrac{2623}{138}=19.01;$ $C:\dfrac{264}{22}=12.00$

$\therefore$ Decreasing order: A, B, C

Correct choice: (1)

Q3. In a buffet, 4 curries **A**, **B**, **C** and **D** were served. A guest was to eat any one or more than one curry, but not the combinations having **C** and **D** together. The number of options available for the guest were

1. 3
2. 7
3. 11
4. 15

Ans: (3)

Solution:

The guest can eat one curry in 4 ways.

Two curry in: Possible ways: AB, AC, AD, BC, BD, CD $\rightarrow$ not allowed

$\therefore$ Total no. of ways = 5 ways

Three curries in: Possible ways: ABC, ABD

$\therefore$ Total no. of ways = 2

ACD $\rightarrow$ not allowed, BCD $\rightarrow$ not allowed

Four curries in: Possible way: ABCD$\rightarrow$ not allowed

$\therefore$ Total no. of ways = 4+5+2 = 11 ways.

Q4. Three friends having a ball each stand at the three corners of a triangle. Each of them throws her ball independently at random to one of the others, once. The probability of no two friends throwing balls at each other is

1. 1/4
2. 1/8
3. 1/3
4. 1/2

Ans: (1)

Solution:

Suppose there are three friends: A, B and C.

Then each can throw ball in 2 ways.

$\therefore$ Total no. of ways of throwing balls $= 2\times 2\times 2 = 8$ ways

Scenario 1:

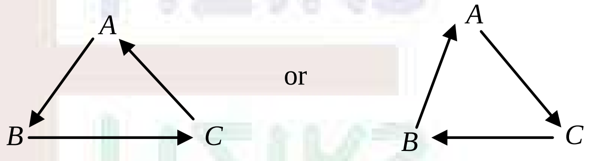
A can throw ball either to B or C. Suppose A throws to B then B cannot throw to A and can throw to C. C can only throw to A.

$\therefore$ No. of ways = 1

Now suppose A throws to C, so C cannot throw to A and can throw to B and B can throw to A.

$\therefore$ No. of ways = 1

Depiction:



$\therefore$ Total favorable outcomes $= 2$

$$\therefore \text{Required Probability } = \frac{\text{Favorable outcome}}{\text{Total outcomes}} = \frac{2}{8} = \frac{1}{4}$$

Q5. What is the largest number of father-son pairs that can exist in a group of four men?

1. 3 2. 2

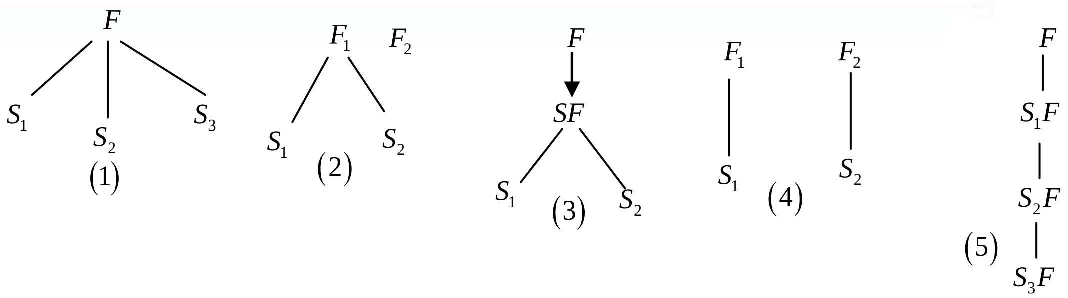
3. 4 4. 6

Ans: (1)

Solution:

Largest no. of father-son pairs that can exist in a group of 4 men:

Possible Scenario:



Hence largest number of father-son pairs that can exist in a group of four men=3

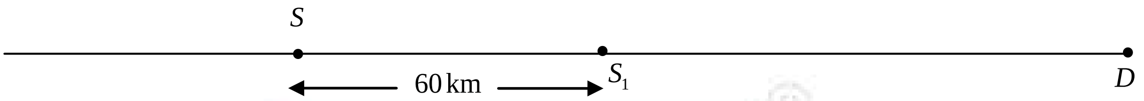
Q6. At a spot **S** en-route, the speed of a bus was reduced by 20% resulting in a delay of 45 minutes. Instead, if the speed were reduced at 60 km after **S**, it would have been delayed by 30 minutes. The original speed, in km/h, was

1. 90 2. 80

3. 70 4. 60

Ans: (4)

Solution:



Case 1: Speed reduced by 20% causing 45 min delay in reaching at D.

Case 2: Speed reduced by 20% at S_1, 60 km from S.

Let distance between S and $D = d$

(I) Let original speed $= v$; Time taken to reach $D = \frac{d}{v}$

Speed, now, at $S = \frac{4v}{5}$. $\therefore$ Time taken with new speed $\frac{d}{\frac{4}{5}v}$

$$\therefore \frac{d}{\frac{4}{5}v} = \frac{d}{v} + \frac{45}{60} \Rightarrow \frac{5d}{4v} - \frac{d}{v} = \frac{45}{60} \Rightarrow \frac{d}{4v} = \frac{3}{4} \Rightarrow d = 3v$$

(II) Now, bus reduces its speed by 20% at S_1, 60 km, from S.

$$\text{Time taken } = \frac{60}{v} + \frac{d-60}{\frac{4}{5}v} \Rightarrow \frac{60}{v} + \frac{d-60}{\frac{4}{5}v} = \frac{d}{v} + \frac{30}{60} \Rightarrow \frac{60}{v} + \frac{3v-60}{\frac{4}{5}v} = \frac{3v}{v} + \frac{30}{60}$$

$\because d = 3v$

$$\Rightarrow \frac{60}{v} + \frac{15}{4} - \frac{75}{v} = 3 + \frac{1}{2} \Rightarrow -\frac{15}{v} = \frac{7}{2} - \frac{15}{4} \Rightarrow -\frac{15}{v} = -\frac{1}{4} \qquad \Rightarrow v = 60\,\text{km/h}$$

Q7. If the sound of its thunder is heard 1s after a lightning was observed, how far away (in m) was the source of thunder/lightning from the observer (given, speed of sound $= x\ \text{ms}^{-1}$, speed of light $= y\,\text{ms}^{-1}$)?

1. x^2/y 2. $xy/(y-x)$

3. $xy/(x-y)$ 4. y^2/x

Ans: (2)

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

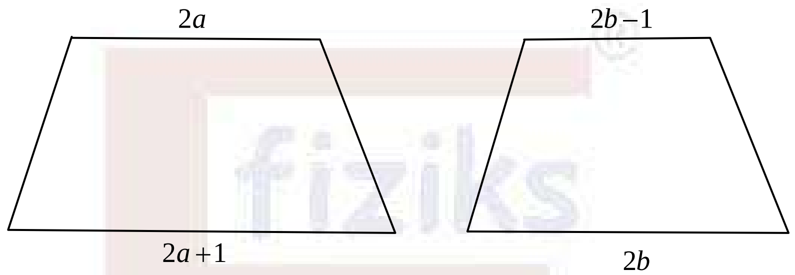
Solution:

Time taken by sound to reach $=\dfrac{d}{x}$, Time taken up light $=\dfrac{d}{y}$

where $d=$distance of observer from source.

Given, $\dfrac{d}{x}=\dfrac{d}{y}+1 \Rightarrow d\left(\dfrac{1}{x}-\dfrac{1}{y}\right)=1 \Rightarrow d=\dfrac{xy}{y-x}$

Q8. If two trapeziums of the same height, as shown below, can be joined to form a parallelogram of area $2(a+b)$, then the height of the parallelogram will be

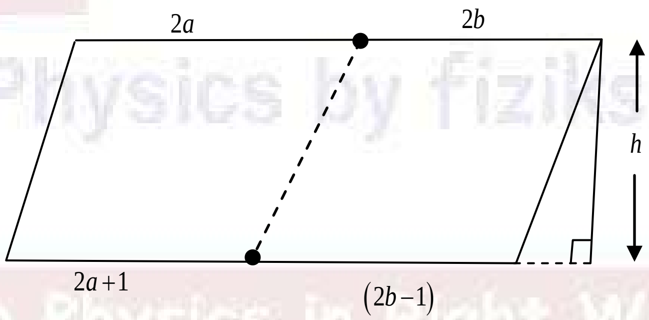


1. 4
2. 1
3. 1/2
4. 2

Ans: (2)

Solution:

Height of two trapezium $=h$ (say)



(Possible way of joining)

Area of parallelogram = Sum of areas of two trapeziums

$$\therefore\ 2(a+b)=\frac{1}{2}\times h(2a+2a+1)+\frac{1}{2}\times h(2b+2b-1)=\frac{1}{2}\times h\{2a+2b+2a+1+2b-1\}$$

$$=\frac{1}{2}\times h\{4a+4b\}=h\cdot 2(a+b) \quad \Rightarrow h=\frac{2(a+b)}{2(a+b)}=1$$

Q9. Three fair cubical dice are thrown, independently. What is the probability that all the dice read the same?

1. 1/6
2. 1/36
3. 1/216
4. 13/216

Ans: (2)

Solution:

All the three dice will read (favorable) either $(1,1,1)$ or $(2,2,2)$ or $(3,3,3)$ or $(4,4,4)$ or $(5,5,5)$ or $(6,6,6)$

$\therefore$ Probability of any one of occurrence $=\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{216}$

$P\{(1,1,1)\text{or}(2,2,2)\text{or}(3,3,3)\text{or}(4,4,4)\text{or}(5,5,5)\text{or}(6,6,6)\}$

$P\{(1,1,1)+(2,2,2)+\ldots+P(6,6,6)\}=\frac{1}{216}+\frac{1}{216}+\ldots+\frac{1}{216}=\frac{6}{216}=\frac{1}{36}$

Q10. Consider two datasets A and B, each with 3 observations, such that both the datasets have the same median. Which of the following MUST be true?

1. Sum of the observations in A = Sum of the observations in B.
2. Median of the squares of the observations in A = Median of the squares of the observations in B.
3. The median of the combined dataset = median of A + median of B.
4. The median of the combined dataset = median of A.

Ans: (4)

Solution:

Choice (1): Suppose $A=\{3,5,8\}\rightarrow$ Median = 5; $B=\{2,5,7\}\rightarrow$ Median = 5

Sum of observations in A is not equal to that of B. Hence (1) is not correct choice.

Choice (2): Suppose $A=\{-13,5,8\}$, Median = 5; $B=\{-4,5,7\}$, Median = 7

Median of squares of $A:\{169,25,64\}$ is 64.

Median of squares of $B:\{16,25,49\}$ is 25.

We see median not equal. Hence choice (2) is also incorrect.

Choice (3): Also incorrect.

Suppose $A=\{3,5,7\}$ Median = 5; $B=\{2,5,8\}$ Median = 5; $C=A+B\Rightarrow\{2,3,5,5,7,8\}$

Median $=\frac{5+5}{2}=5\neq$ Median of A + Median of B.

Choice (4):

From choice (3), we see that when observation from two datasets with same median are combined. There is double entry of median with combined observations even in numbers. Hence, resultant median is always average of middle two, i.e., the same medians

$\therefore$ The median of combined set = Median of A = Median of B

$\therefore$ Choice (4) is correct.

Q11. Price of an item is increased by 20% of its cost price and is then sold at 10% discount for Rs. 2160. What is its cost price?

1. 1680
2. 1700
3. 1980
4. 2000

Ans: (4)

Solution:

Let cost price $= x$; New price $= x\times\left(1+\frac{20}{100}\right)=\frac{120}{100}x$

After discount, selling price $=\frac{120}{100}x\times\frac{90}{100}$

Given, $\frac{120}{100}x\times\frac{90}{100}=2160 \Rightarrow \frac{108}{100}x=2160 \Rightarrow x=2000$

Q12. A 50 litre mixture of paint is made of green, blue, and red colours in the ratio 5:3:2. If another 10 litre of red colour is added to the mixture, what will be the new ratio?

1. 5:2:4
2. 4:3:2
3. 2:3:5
4. 5:3:4

Ans: (4)

Solution:

Quantity of green color $=\frac{5}{5+3+2}\times 50=25$ Lit.

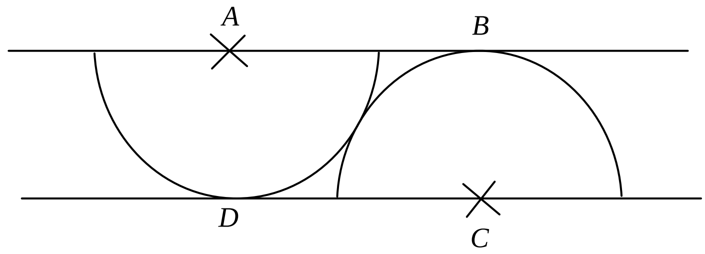
Blue color $=\frac{3}{5+3+2}\times 50=15$ Lit.

Red color $=\frac{2}{5+3+2}\times 50=10$ Lit.

$\therefore$ Now total quantity of red color $=10+10=20$ Lit.

New ratio $=\frac{25}{60}:\frac{15}{60}:\frac{20}{60}=25:15:20 =5:3:4$

Q13. Two semicircles of same radii centred at A and C, touching each other, are placed between two parallel lines, as shown in the figure. The angle BAC is

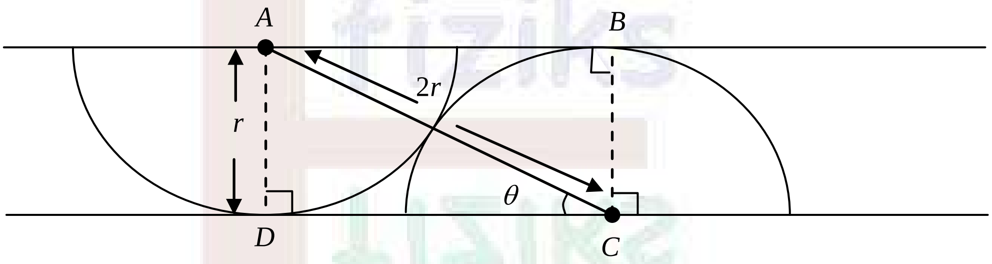


1. 30° 2. 35°

3. 45° 4. 60°

Ans: (1)

Solution:



From the figure it is clear $\angle BAC = \angle DCA$

$\therefore\ \sin\theta = \frac{AD}{AC} = \frac{r}{2r} = \frac{1}{2} \quad \therefore\ \theta = 30^\circ$ (From ΔADC)

Q14. A building has windows of sizes 2, 3 and 4 feet and their respective numbers are inversely proportional to their sizes. If the total number of windows is 26, then how many windows are there of the largest size?

1. 4 2. 6

3. 12 4. 9

Ans: (2)

Solution:

No. of windows of size $2 \propto \frac{1}{2}$; No. of windows of size $3 \propto \frac{1}{3}$; No. of windows of size $4 \propto \frac{1}{4}$

$\therefore$ No. of windows of size 2, 3 and 4 is $\frac{k}{2}, \frac{k}{3}, \frac{k}{4}$ where $k =$ proportionality constant.

$\therefore$ Given, $\frac{k}{2} + \frac{k}{3} + \frac{k}{4} = 26$ or, $\frac{6k + 4k + 3k}{12} = 26$ or, $k = 24$

$\therefore$ No. of windows of largest size $(=4)$ is $\frac{k}{4}=\frac{24}{4}=6$

Q15. Three consecutive integers a,b,c add to 15. Then the value of

$$(a-2)^2+(b-2)^2+(c-2)^2$$

would be

1. 25 2. 27 3. 29 4. 31

Ans: (3)

Solution:

Given $a+b+c=15$ where a,b,c are consecutive integers.

i.e., $2b=a+c$ or, $a+b+c=a+c+b=2b+b=15$ (given) $\therefore\ b=5$

$\therefore\ a=4, b=5$ and $c=6$

$\therefore$ Value of $(a-2)^2+(b-2)^2+(c-2)^2=2^2+3^2+4^2=4+9+16=29$

Q16. Persons A and B have 73 secrets each. On some day, exactly one of them discloses his secret to the other. For each secret A discloses to B in a given day, B discloses two secrets to A on the next day. For each secret B discloses to A in a given day, A discloses four secrets to B on the next day. The one who starts, starts by disclosing exactly one secret. What is the smallest possible number of days it takes for B to disclose all his secrets?

1. 5 2. 6 3. 7 4. 8

Ans: (1)

Solution:

$$A \xrightarrow{1(\text{1st day})} B, \quad B \xrightarrow{2(\text{next day})} A \qquad\qquad B \xrightarrow{1(\text{1st day})} A, \quad A \xrightarrow{4(\text{next day})} B$$

Suppose B starts disclosing the secret.

		Remaining secret with B
day 1:	$B \xrightarrow{1} A$	72(73-1)
day 2:	$A \xrightarrow{4} B$	72
day 3:	$B \xrightarrow{8} A$	64(72-8)
day 4:	$A \xrightarrow{32} B$	64
day 5:	$B \xrightarrow{64} A$	0(64-68)

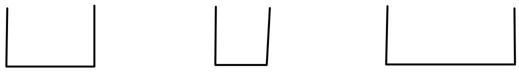
$\therefore$ Smallest possible no. of days it will take for B to disclose all his secrets = 5 days.

Q17. Given only one full 3 litre bottle and two empty ones of capacities 1 litre and 4 litres, all ungraduated, the minimum number of pouring required to ensure 1 litre in each bottle is

1. 2   2. 3   3. 4   4. 5

Ans: (2)

Solution:



	$3L$ (1)	$1L$ (2)	$4L$ (3)	
Start:	3	0	0	
1st:	2	1	0	(Pour from bottle 1 to 2)
2nd:	2	0	1	(From 2 to 3)
3rd:	1	1	1	(From 1 to 2)

$\therefore$ Required minimum no. of pouring = 3

Q18. Sum of all the internal angles of a regular octagon is ____________ degrees.

1. 360   2. 1080

3. 1260   4. 900

Ans: (2)

Solution:

Sum of all internal angles of regular polygon of side n is $(2n-4)\pi/2$.

Here $n=8$; $\therefore$ Sum of angles (internal) $=(2\times 8-4)\pi/2 \; = 6\pi \; = 6\times 180 = 1080^\circ$.

Q19. Which of the numbers $A=162^3+327^3$ and $B=612^3-123^3$ is divisible by 489?

1. Both A and B   2. A but not B

3. B but not A   4. Neither A nor B

Ans: (1)

Solution:

a^3+b^3 is always divisible by $(a+b)$ and a^3-b^3 is always divisible by $(a-b)$.

$\therefore$ 162^3+327^3 will be divisible by $(162+327)$ that is 489.

612^3-123^3 will be divisible by $(612-123)$ that is 489.

$\therefore$ Correct choice is (1).

Q20. When a student in Section A who scored 100 marks in a subject is exchanged for a student in Section B who scored 0 marks, the average marks of the Section A falls by 4, while that of Section B increases by 5. Which of the following statements is true?

1. A has the same strength as B
2. A has 5 more students than B
3. B has 5 more students than A
4. The relative strengths of the classes cannot be assessed from the data

Ans: (2)

Solution:

Let no. of students in $A = x$ and average $= a$

Also, no. of students in $B = y$ and average score $= b$

$\therefore$ Score of section $A = ax$

New Score $= ax - 100 + 0 = ax - 100$

New average $= \dfrac{ax-100}{x} = a - 4$ (given) ....(i)

Also, Score of section B $= by$

New score after exchange $= by + 100$

New average $= \dfrac{by+100}{y} = b + 5$ (given) ....(ii)

From equation (i): $x = 25$ (no. of students in Section A)

From equation (ii): $y = 20$ (no. of students in Section B)

$\therefore$ No. of students in section A is 5 more then B.

$\therefore$ Correct choice is (2).

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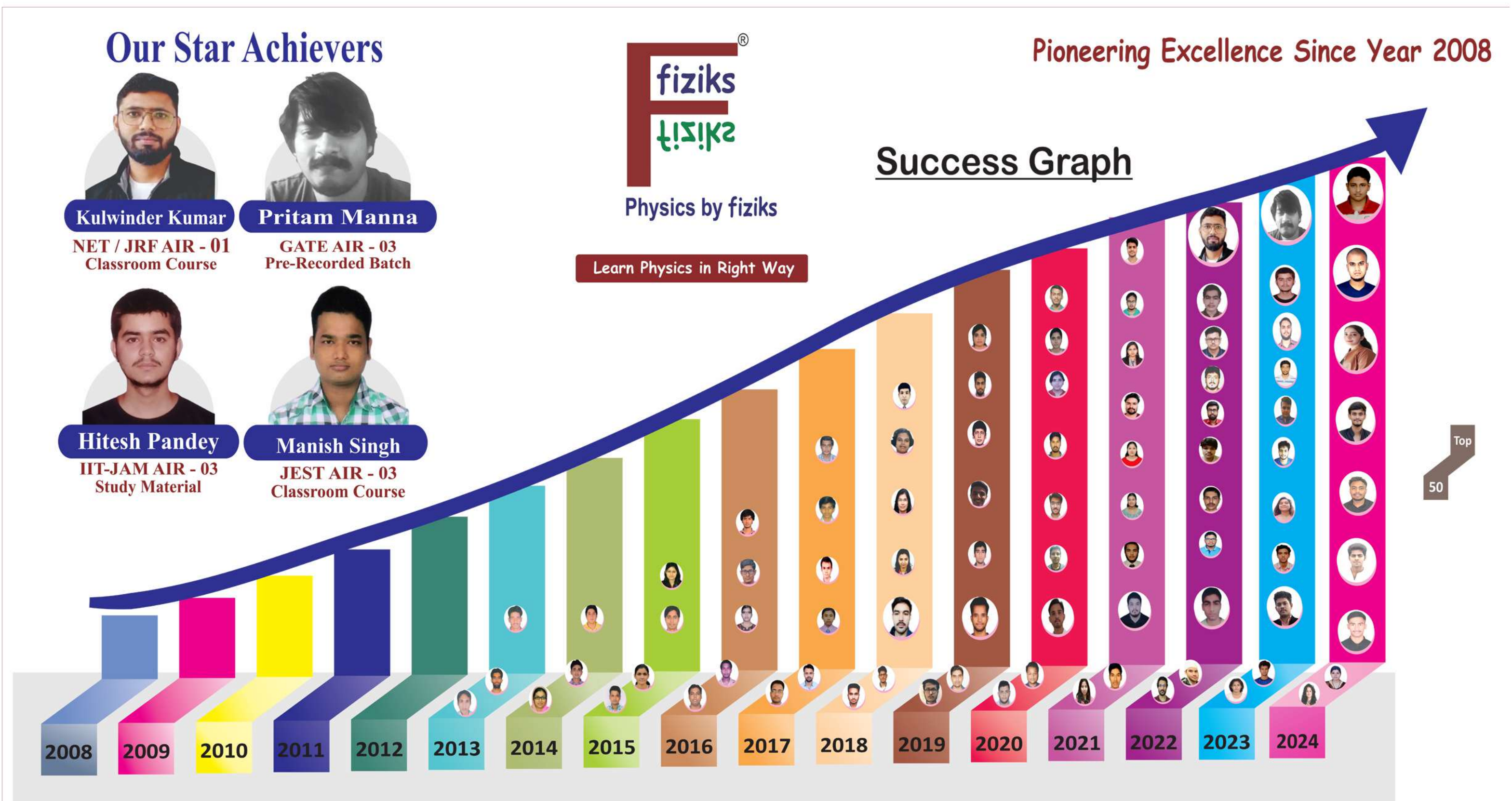

011-26865455, +9871145498

Head Office: House No. 40-D, Ground Floor, Jia Sarai Near IIT-Delhi, Hauz Khas, New Delhi-110016