TIFR-2016 (Mathematical Physics Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. If x is a continuous variable which is uniformly distributed over the real line from $x=0$ to $x\to\infty$ according to the distribution

$$f(x)=\exp(-4x)$$

then the expectation value of $Cos4x$ is

(a) zero (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

Ans. : (b)

Solution:

$$\langle\cos 4x\rangle=\frac{\int_0^\infty \cos 4x\, e^{-4x}}{\int_0^\infty e^{-4x}\,dx}=\frac{I_1}{I_2}$$

$$I_1=\int_0^\infty \cos 4x\, e^{-4x}\,dx$$

$$=\int_0^\infty\left(\frac{e^{4ix}+e^{-4ix}}{2}\right)e^{-4x}\,dx=\frac{1}{2}\int_0^\infty e^{(4i-4)x}+\frac{1}{2}\int_0^\infty e^{-(4i+4)x}=\frac{1}{2}\int_0^\infty e^{-(4-4x)x}+\frac{1}{2}\int_0^\infty e^{-(4i+4)x}$$

$$I_1=\frac{1}{2}\left[\frac{e^{-(4-4i)x}}{-(4-4i)}\Bigg|_0^\infty\right]+\frac{1}{2}\left[\frac{e^{-(4i+4)x}}{-(4i+4)}\right]_0^\infty=\frac{1}{2}\left[\frac{1}{4i-4}\times\left[e^{-\infty}-e^{-0}\right]\right]+\frac{1}{2}\left[\frac{-1}{(4i+4)}\left[e^{-\infty}-e^{-0}\right]\right]$$

$$I_1=\frac{1}{2(4i-4)}[0-1]-\frac{1}{2(4i+4)}\times(-1)=-\frac{1}{8(i-1)}+\frac{1}{8(i+1)}=\frac{1}{8}\left(\frac{-(i+1)+i-1}{i^2-1}\right)$$

$$I_1=\frac{1}{8}\left(\frac{-\cancel{i}-1+\cancel{i}-1}{(-1-1)}\right)=\frac{-2}{8\times(-2)}=\frac{1}{8}$$

$$I_2=\int_0^\infty e^{-4x}dx=-\frac{1}{4}\left|e^{-4x}\right|_0=-\frac{1}{4}\left[e^{-\infty}-e^{-0}\right]=\frac{1}{4}$$

$$\Rightarrow\langle\cos 4x\rangle=\frac{\frac{1}{8}}{\frac{1}{4}}=\frac{1}{8}\times 4=\frac{1}{2}$$

The answer is therefore, (b).

Q2. If the eigenvalues of a symmetric 3×3 matrix A are $0,1,3$ and the corresponding eigenvectors can be written as

$$\begin{pmatrix}1\\1\\1\end{pmatrix},\quad \begin{pmatrix}1\\0\\-1\end{pmatrix},\quad \begin{pmatrix}1\\-2\\1\end{pmatrix}$$

respectively, then the matrix A^4 is

(a) $\begin{pmatrix}41 & -81 & 40\\ -81 & 0 & -81\\ 40 & -81 & 41\end{pmatrix}$

(b) $\begin{pmatrix}-82 & -81 & 79\\ -81 & 81 & -81\\ 79 & -81 & 83\end{pmatrix}$

(c) $\begin{pmatrix}14 & -27 & 13\\ -27 & 54 & -27\\ 13 & -27 & 14\end{pmatrix}$

(d) $\begin{pmatrix}14 & -13 & 27\\ -13 & 54 & -13\\ 27 & -13 & 14\end{pmatrix}$

Ans. : (c)

Solution. :

As eigenvalues are given.

So, the diagonal matrix is $D=\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 3\end{pmatrix} \Rightarrow D^4=\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 81\end{pmatrix}$

As eigen vectors are given, So, $P=\begin{pmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{pmatrix}$

Now, we can find P^{-1}; $|P|=1\begin{vmatrix}0 & -2\\ -1 & 1\end{vmatrix}-1\begin{vmatrix}1 & -2\\ 1 & 1\end{vmatrix}+1\begin{vmatrix}1 & 0\\ 1 & -1\end{vmatrix}=-2-3-1=-6$

Co-factor of first row elements are –2, –3, –1

Co-factor of second row elements are –2, 0, 2

Co-factor of third row elements are –2, 3, –1

So, $P^{-1}=\frac{-1}{6}\begin{vmatrix}-2 & -3 & -1\\ -2 & 0 & 2\\ -2 & 3 & -1\end{vmatrix}^T=\frac{-1}{6}\begin{bmatrix}-2 & -2 & -2\\ -3 & 0 & 3\\ -1 & 2 & -1\end{bmatrix}$

Now $A^4=PD^4P^{-1}=\frac{-1}{6}\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 81\end{bmatrix}\begin{bmatrix}-2 & -2 & -2\\ -3 & 0 & 3\\ -1 & 2 & -1\end{bmatrix}$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

$$=\frac{-1}{6}\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 0\\ -3 & 0 & 3\\ -81 & 162 & -81\end{bmatrix}=\frac{-1}{6}\begin{bmatrix}-3-81 & 162 & 3-81\\ 162 & -2\times 162 & -2\times(-81)\\ 3-81 & 162 & -3-81\end{bmatrix}$$

$$=\frac{-1}{6}\begin{bmatrix}-84 & 162 & -78\\ 162 & -324 & 162\\ -78 & 162 & -84\end{bmatrix}=\begin{bmatrix}14 & -27 & 13\\ -27 & 54 & -27\\ 13 & -27 & 14\end{bmatrix}$$

Which is same as 'c'.

Hence 'c' is the answer.

SECTION B- (only for Int.-Ph.D. candidates)

Q3. The integral $\int_0^{\infty}\frac{dx}{x}\left[\exp\left(-\frac{x}{\sqrt{3}}\right)-\exp\left(-\frac{x}{\sqrt{2}}\right)\right]$ evaluates to

(a) zero (b) 2.03×10^{-2} (c) 2.03×10^{-1} (d) 2.03

Ans. : (a)

Solution. :

$$I=\int_0^{\infty}\frac{1}{x}e^{-\frac{x}{\sqrt{3}}}dx-\int_0^{\infty}\frac{1}{x}e^{-\frac{x}{\sqrt{2}}}dx$$

Let $I_1=\int_0^{\infty}\frac{1}{x}e^{-\frac{x}{\sqrt{3}}}dx$

Put $x=\sqrt{3}t,\ dx=\sqrt{3}dt$; at $x=0,\quad t=0$ and at $x=\infty,\ t=\infty$

Thus $I_1=\int_0^{\infty}\frac{1}{\cancel{\sqrt{3}}t}e^{-\frac{\sqrt{3}t}{\sqrt{3}}}\cancel{\sqrt{3}}\ dt=\int_0^{\infty}t^{-1}e^{-t}dt=\Gamma(0)$

Similarly, we can show $I_2=\int_0^{\infty}\frac{1}{x}e^{-\frac{x}{\sqrt{2}}}dt=\int_0^{\infty}t^{-1}e^{-t}dt=\Gamma(0)$

Thus $I=I_1-I_2=\Gamma(0)-\Gamma(0)=0$

Thus, the answer is (a).

Q4. The function $y(x)$ satisfies the differential equation

$$x\frac{dy}{dx}=y(\ln y-\ln x+1)$$

with the initial condition $y(1)=3$. What will be the value of $y(3)$?

Ans. : 9

Solution:

$$x\frac{dy}{dx}-y\ln y=-y\ln x+y$$

Divide by xy we get

$$\frac{\cancel{x}}{\cancel{x}y}\frac{dy}{dx}-\frac{\cancel{y}}{x\cancel{y}}\ln y=-\frac{\cancel{y}}{x\cancel{y}}\ln x+\frac{\cancel{y}}{x\cancel{y}}$$

$$\frac{1}{y}\frac{dy}{dx}-\frac{1}{x}\ln y=-\frac{1}{x}\ln x+\frac{1}{x}$$

Put $\ln y=z\Rightarrow\frac{1}{y}\frac{dy}{dx}=\frac{dz}{dx}$

$$\Rightarrow\frac{dz}{dx}-\frac{z}{x}=-\frac{1}{x}\ln x+\frac{1}{x}\qquad\left[\frac{dz}{dx}+Pz=Q\right]$$

$$\text{I.F.}=e^{\int Pdx}=-\int_e\frac{1}{x}dx=e^{-\ln x}=e^{\ln x^{-1}}=\frac{1}{x}$$

$$\frac{1}{x}\frac{dz}{dx}-\frac{z}{x^2}=-\frac{1}{x^2}\ln x+\frac{1}{x^2}$$ Multiplying with I.F.

$$\frac{d\left(\frac{z}{x}\right)}{dx}=\frac{-1}{x^2}\ln x+\frac{1}{x^2}$$

Integrating we get ,

$$\frac{z}{x}=\int\frac{-1}{x^2}\ln x+\int\frac{1}{x^2}+c=-\left[\ln x\int\frac{1}{x^2}dx-\int\frac{d}{dx}(\ln x)\left(\int\frac{1}{x^2}dx\right)dx\right]-\frac{1}{x}+c$$

$$\frac{z}{x}=-\left[\ln x\times\frac{-1}{x}-\int\frac{1}{x}\times\frac{-1}{x}dx\right]-\frac{1}{x}+c=-\left[\frac{-\ln x}{x}+\int\frac{1}{x^2}dx\right]-\frac{1}{x}+c$$

$$\frac{z}{x}=-\left[\frac{-\ln x}{x}-\frac{1}{x}\right]-\frac{1}{x}+c=\frac{\ln x}{x}+\cancel{\frac{1}{x}}-\cancel{\frac{1}{x}}+c\quad\frac{z}{x}=\frac{\ln x}{x}+c$$

$$z=\ln x+cx\Rightarrow\ln y=\ln x+cx\Rightarrow y=e^{\ln x}e^{cx}\Rightarrow y=\frac{1}{x}e^{cx}$$

$$y(1)=3\Rightarrow 3=\frac{1}{1}\cdot e^{c\cdot 1}\Rightarrow c=\ln 3$$

$$y=\frac{1}{x}e^{(\ln 3)x}=\frac{1}{x}e^{x\ln 3}=\frac{1}{x}e^{\ln 3^x}\Rightarrow y=\frac{3^x}{x}$$

$$y(3)=\frac{3^3}{3}=\frac{27}{3}=9$$ Answer

$$y(1)=\frac{3^1}{1}=3$$ Initial condition is also verified

SECTION B-(Only for Ph.D. candidates)

Q5. The value of the integral

$$\oint_C \frac{\sin z}{z^6} dz$$

where C is the circle of centre $z=0$ and radius $=1$

(a) $i\pi$ (b) $\frac{i\pi}{120}$ (c) $\frac{i\pi}{60}$ (d) $\frac{-i\pi}{6}$

Ans. : (c)

Solution. :

$$\frac{\sin z}{z^6} = \frac{1}{z^6}\left[z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^6}{6!} + ...\right] = \left[\frac{1}{z^5} - \frac{1}{z^3 3!} + \frac{1}{z\,5!} - \frac{z^6}{6!} + ...\right]$$

Pole is given by $z^5 = 0$. 0 is pole of fifth order.

The residue is just the coefficient of $\frac{1}{z} = \frac{1}{5!}$

Thus $\oint \frac{\sin z}{z^6} dz = 2\pi i \sum$ Residues $= \frac{\cancel{2}\pi i}{5\times 4\times 3\times \cancel{2}} = \frac{\pi i}{60}$

Q6. Given the infinite series

$$y(x) = 1 + 3x + 6x^2 + 10x^3 + ... + \frac{(n+1)(n+2)}{2}x^n + ...$$

find the value of $y(x)$ for $x = \frac{6}{7}$

Ans. : 343

Solution. :

This is just the expansion of

$$y(x) = (1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + ...$$

Thus, $y\left(\frac{6}{7}\right) = \left(1 - \frac{6}{7}\right)^{-3} = \left(\frac{1}{7}\right)^{-3} = 7^3 = 343$

TIFR-2016 (EMT Question and Solution)

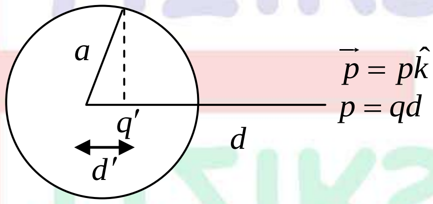
SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. A grounded conducting sphere of radius a is placed with its centre at the origin. A point dipole of dipole moment $\vec{p} = p\hat{k}$ is placed at a distance d along the x-axis, where $\hat{i},\hat{k}$ are the units vector along the x and z-axes respectively. This leads to the formation of an image dipole of strength $\vec{p}'$ at a distance d' from the centre along the x-axis. If $d' = a^2/d$, then $\vec{p}' =$

(a) $-\frac{a^4 p}{d^4}\hat{k}$ (b) $-\frac{a^3 p}{d^3}\hat{k}$ (c) $-\frac{a^2 p}{d^2}\hat{k}$ (d) $-\frac{ap}{d}\hat{k}$

Ans.: (b)

Solution:



The image charge due to conducting sphere will from at a distance $d' = \frac{a^2}{d}$ from the origin with charge $q' = -q\frac{a}{d}$

$$|\vec{p}'| = q' \times d' = \left(-q\frac{a}{d}\right)\left(\frac{a^2}{d}\right) = -q\frac{a^3}{d^2} = -a^3\frac{qd}{d^3} = -\frac{a^3}{d^3}|\vec{p}|$$

$\vec{p}' = -\frac{a^3}{d^3}p\hat{k}$ Option (b)

Q2. A long, solid dielectric cylinder of radius a is permanently polarized so that the polarization is everywhere radially outward, with a magnitude proportional to the distance from the axis of the cylinder, i.e., $\vec{P} = \frac{1}{2}P_0 r\hat{r}$. The bound charge density in the cylinder is given by

(a) $-P_0$ (b) P_0 (c) $-\frac{P_0}{2}$ (d) $\frac{P_0}{2}$

Ans.: (a)

Solution:

$$\vec{P}=\frac{1}{2}P_0 r\hat{r}$$

$$\rho_b=-\left(\vec{\nabla}\cdot\vec{P}\right)=-\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left(\frac{1}{2}P_0 r\right)\right\}$$ Use cylindrical co-ordinates

$$\rho_b=-\frac{1}{r}\frac{P_0}{2}\frac{\partial}{\partial r}\left(r^2\right)=-\frac{1}{r}\frac{P_0}{2}2r=-P_0$$

Option (a)

Q3. A circular loop of fine wire of radius R carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the loop. If the breaking tension of the wire is T_b, the wire will break when the magnetic field exceeds

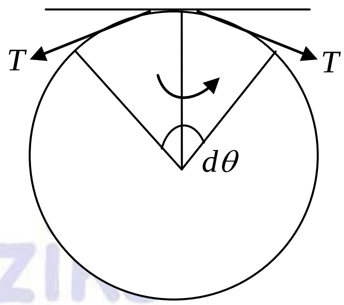
(a) $\frac{T_b}{IR}$ (b) $\frac{T_b}{2\pi IR}$ (c) $\frac{\mu_0 T_b}{2\pi IR}$ (d) $\frac{\mu_0 T_b}{4\pi IR}$

Ans.: (a)

Solution:

Outward magnetic force on an infinitesimal length dl of wire

$F=IBdl,\quad dl=Rd\theta$



$$T_L=T_B\sin\frac{d\theta}{2}=T_B\frac{d\theta}{2}$$

Total inward for force $=2T_L=T_B d\theta$

$F_{out}=F_{in}$

$IBRd\theta=T_B d\theta$

$B=\frac{T_B}{IR}$ Option (a)

Q4. In a glass fibre, light propagates by total internal reflection from the inner surface. A very short pulse of light enters a perfectly uniform glass fibre at $t=0$ and emerges from the other end of the fibre with negligible losses. If the refractive index of the glass used in the fibre is 1.5 and its length is exactly $1.0\ km$, the time t at which the output pulse will have completely exited the fibre will be

(a) $5.0\ \mu s$ (b) $7.5\ \mu s$ (c) $25\ ns$ (d) $750\ ns$

Ans.: (b)

Solution:

Time taken by the pulse

$$t=\frac{d}{v}=\frac{\mu l}{c/\mu}=\mu^2\frac{\ell}{c}=(1.5)^2\frac{(1000m)}{3\times10^8 m/s}=\frac{2.25}{3}\times10^{-5}s=\frac{22.5}{3}\quad \mu s=7.5\,\mu s$$

Q5. In an ionization experiment conducted in the laboratory, different singly- charged positive ions are produced and accelerated simultaneously using a uniform electric field along the x-axis. If we need to determine the masses of various ions produced, which of the following methods will NOT work

(a) Detect them at a fixed distance from the interaction point along x-axis and measure their time of arrival.

(b) Apply a uniform magnetic field along y-axis and measure the deviation.

(c) Apply a uniform electric field along y-axis and measure the deviation.

(d) Apply a uniform electric field along y-axis and a (variable) uniform magnetic field along z-axis simultaneously and note the zero deviation.

Ans.: (b)

Solution:

Take $\vec{F}=q\vec{E}+q\left(\vec{v}\times\vec{B}\right)$

Since $\vec{E}=E\hat{i}$

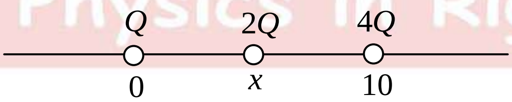
We need magnetic field to counter this to select a velocity field. Then

$$qE=qvB,\ v=\frac{E}{B}$$

Option (b)

SECTION B- (only for Int.-Ph.D. candidates)

Q6. Three positively charged particles lie on a straight line at positions $0, x$ and 10 as indicated in the figure below. Their charges are $Q, 2Q$, and $4Q$ cm respectively.

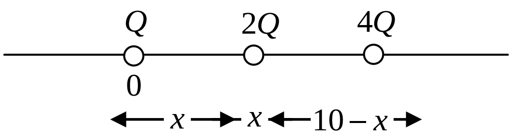


If the charges at $x=10$ are fixed and the charge at x is movable, the system will be in equilibrium when $x=$

(a) 8 (b) 2 (c) $\frac{20}{3}$ (d) $\frac{10}{3}$

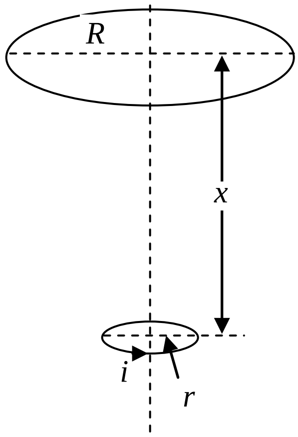
Ans.: (d)

Solution: For equilibrium force on $2Q$ will be zero



$$\frac{1}{4\pi\varepsilon_0}\frac{(2Q)Q}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{(2Q)(4Q)}{(10-x)^2}\Rightarrow\left(\frac{10-x}{x}\right)^2=4\Rightarrow x=\frac{10}{3}$$ Option (d)

Q7. Consider the following system. Two circular loops of wire are placed horizontally, having a common axis passing vertically through the centre of each coil (see figure). The lower loop has radius r and carries a current i as shown in the figure. The upper loop has a radius $R\,(R>>r)$ and is at a distance $x\,(x>>r)$ above it.



If the lower loop is held fixed and the upper loop moves upwards with a uniform velocity $v=\frac{dx}{dt}$, then the induced e.m.f. and the direction of the induced current in this loop will be

(a) $3i\mu_0\pi r^2\frac{R^2v}{2x^4}$; anti-clockwise (b) $2i\mu_0\pi r^2\frac{R^2v}{2x^4}$; clockwise

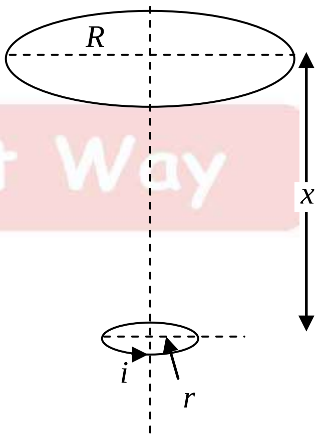
(c) $3i\mu_0\pi r^2\frac{R^2v}{2x^3}$; anti-clockwise (d) $2i\mu_0\pi r^2\frac{R^2v}{3x^3}$; clockwise

Ans.: (a)

Solution:

The magnetic field produced by smaller loop at the centre of larger loop is

$$B_1=\frac{\mu_0}{2}\frac{ir^2}{\left(x^2+r^2\right)^{3/2}}$$



Flux through bigger loop $=B_1\times\pi R^2$

$$\phi=\frac{\mu_0}{2}\frac{ir^2}{\left(x^2+r^2\right)^{3/2}}\times\pi R^2$$

Induced e.m.f $\varepsilon=-\frac{d\phi}{dl}=-\frac{\mu_0 i\pi R^2r^2}{2}\frac{d}{dt}\left(x^2+r^2\right)^{-3/2}$

$$\varepsilon=-\frac{\mu_0 i\pi}{2}R^2r^2\left(-\frac{3}{2}\right)\left(x^2+r^2\right)^{-5/2}\times 2x\frac{dx}{dt}$$

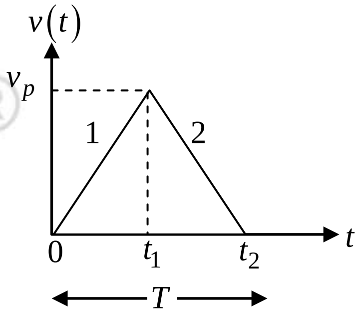
$$\Rightarrow\varepsilon=3\frac{\mu_0}{2}i\pi R^2r^2\times\frac{xv}{\left(x^2+r^2\right)^{5/2}}=3\frac{\mu_0 i\pi r^2R^2v}{2x^4}$$

Q8. Consider a sawtooth waveform which rises linearly from 0 Volt to 1 Volt in 10 ns and then decays linearly to $0V$ over a period of 100 ns. Find the r.m.s. voltage in units of milliVolt?

Ans.: 577

Solution:

Let us do it generally



Equation (1): $v_1(+) = \left(\frac{t}{t_1}\right)v_p$, $\quad \left\langle v_1^2 \right\rangle = \frac{1}{t_1}\int_0^{t_1}\left[v_1(t)\right]^2 dt$

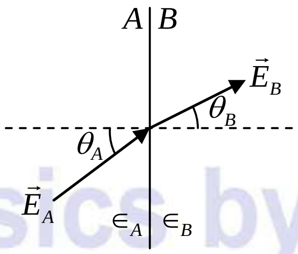
Equation (2):

$$v_2(+) = \left(\frac{t_2 - t}{t_2 - t_1}\right)v_p, \quad \left\langle v_2^2 \right\rangle = \frac{1}{(t_2 - t_1)}\int_{t_1}^{t_2}\left[v_2(t)\right]^2 dt$$

$$V_{rms} = \sqrt{\left\langle v_1^2 \right\rangle + \left(v_2^2 \right)} = \frac{v_p}{\sqrt{3}} = \frac{1}{\sqrt{3}} Volts = \frac{1000}{\sqrt{3}} mV = 577\, mV$$

SECTION B-(Only for Ph.D. candidates)

Q9. Two semi-infinite slabs A and B of dielectric constant $\in_A$ and $\in_B$ meet in a plane interface, as shown in the figure below.



If the electric field in slab A makes an angle θ_A with the normal to the boundary and the electric field in slab B makes an angle θ_B with the same normal (see figure), then

(a) $\cos\theta_A = \frac{\in_A}{\in_B}\cos\theta_B$ (b) $\sin\theta_A = \frac{\in_A}{\in_B}\sin\theta_B$

(c) $\tan\theta_A = \frac{\in_A}{\in_B}\tan\theta_B$ (d) $\sin\theta_A = \frac{\in_B}{\in_A}\sin\theta_B$

Ans. : (c)

Solution. :

$$\frac{\tan\theta_A}{\tan\theta_B} = \frac{E_A^{\parallel} / E_A^{\perp}}{E_B^{\parallel} / E_B^{\perp}} = \frac{E_B^{\perp}}{E_A^{\perp}} = \frac{\varepsilon_A}{\varepsilon_B} \qquad \because E_A^{\parallel} = E_B^{\parallel}, \quad \varepsilon_A E_A^{\perp} = \varepsilon_B E_B^{\perp}$$

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H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

TIFR-2016 (Quantum Mechanics Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. A one-dimensional harmonic oscillator of mass m and natural frequency ω is in the quantum state

$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$$

at time $t=0$, where $|n\rangle$ denotes the eigenstate with eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. It follows that the expectation value (x) of the position operator $\hat{x}$ is

(a) $x(0)\left[\cos\omega t + \frac{1}{\sqrt{3}}\sin\omega t\right]$

(b) $x(0)[\cos\omega t - \sin\omega t]$

(c) $x(0)\left[\cos\omega t - \frac{1}{2}\sin\omega t\right]$

(d) $x(0)\left[\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

Ans.: (c)

Solution:

We have $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$ (1)

The wave function at any later time t is

$$|\psi(t)\rangle = \frac{1}{\sqrt{3}}|0\rangle e^{\frac{-iE_0 t}{\hbar}} + \frac{i}{\sqrt{3}}|1\rangle e^{\frac{-iE_1 t}{\hbar}} + \frac{i}{\sqrt{3}}|2\rangle e^{\frac{-iE_2 t}{\hbar}} \quad (2)$$

The bra of eq.(2) is

$$\langle\psi| = \frac{1}{\sqrt{3}}\langle 0|e^{\frac{iE_0 t}{\hbar}} - \frac{i}{\sqrt{3}}\langle 1|e^{\frac{iE_1 t}{\hbar}} - \frac{i}{\sqrt{3}}\langle 2|e^{\frac{iE_2 t}{\hbar}} \quad (3)$$

The average of position for the given wave function at later time t is

$$\langle x\rangle = \langle\psi(t)|\hat{x}|\psi(t)\rangle = \frac{1}{3}\langle 0|\hat{x}|0\rangle + \frac{1}{3}\langle 1|\hat{x}|1\rangle + \frac{1}{3}\langle 2|\hat{x}|2\rangle$$

$$+\frac{i}{3}\langle 0|\hat{x}|1\rangle\left(e^{-\frac{i(E_1-E_0)}{\hbar}t} - e^{\frac{i(E_1-E_0)}{\hbar}t}\right) + \frac{1}{3}\langle 2|\hat{x}|1\rangle\left(e^{-\frac{i(E_2-E_1)}{\hbar}t} + e^{\frac{i(E_2-E_1)}{\hbar}t}\right)$$

$$+\frac{1}{3}\langle 2|\hat{x}|0\rangle\left(e^{-\frac{i(E_2-E_0)}{\hbar}t} - e^{\frac{i(E_2-E_0)}{\hbar}t}\right) \quad (4)$$

Using the general result for harmonic oscillator derived using dagger operator,

$$\langle x\rangle = \langle n|\hat{x}|n\rangle = 0; \langle 0|\hat{x}|2\rangle = 0;$$

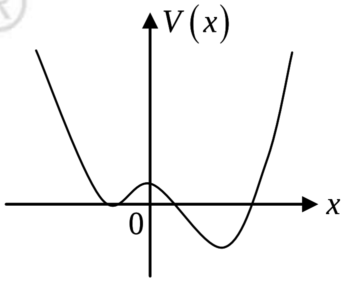
$$\langle 0|\hat{x}|1\rangle = \langle 0|\hat{x}|1\rangle = \sqrt{\frac{h}{2m\omega}};\langle 2|\hat{x}|1\rangle = \langle 1|\hat{x}|2\rangle = \sqrt{\frac{h}{m\omega}} \qquad (5)$$

Substituting eq. (5) in eq. (4), we obtain

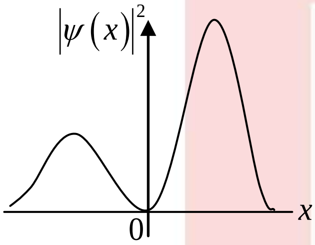
$$\langle x\rangle = \frac{2}{3}\sqrt{\frac{\hbar}{2m\omega}}\sin\omega t + \frac{2}{3}\sqrt{\frac{\hbar}{m\omega}}\cos\omega t = x(0)\left(\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right) \qquad (6)$$

where $\quad x(0) = \frac{2}{3}\sqrt{\frac{\hbar}{m\omega}} \qquad (7)$

Q2. A particle is confined inside a one- dimensional potential well $V(x)$, as shown on the right. One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is



(a)



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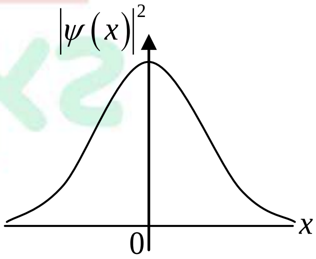
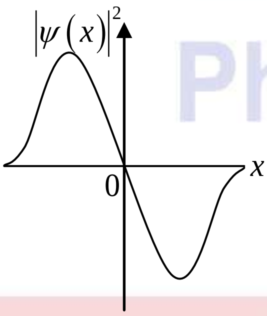
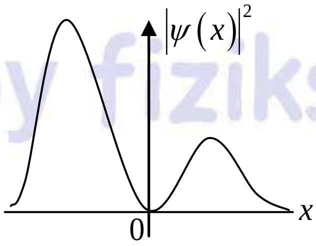


(c)



(d)



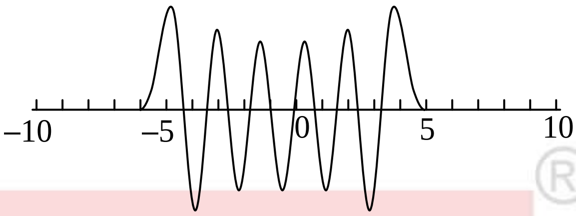
Ans.: (a)

Solution:

We solve this question by using elimination method. The potential has its peak at the origin and is located in the left region. The probability of finding the particle at origin must be zero and then it starts increasing in both regions. The particle would have highest probability of finding the particle in the Right region then left region as the curve is more step. These two characteristics are satisfied only in option A. Hence, the answer is (a).

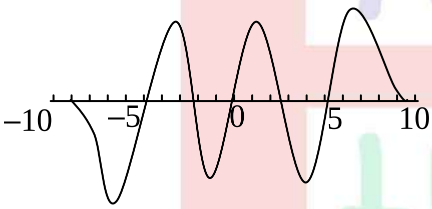
SECTION B- (only for Int.-Ph.D. candidates)

Q3. Two harmonic oscillators A and B are in excited eigenstates with the same excitation energy E, as measured from their respective ground state energies.The natural frequency of A is twice that of B.

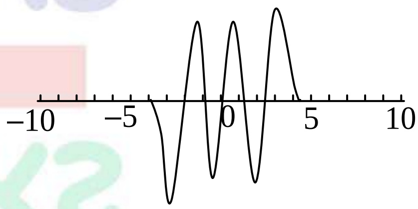


If the wavefunction of B is as sketched in the above picture, which of the following would best represent the wavefunction of A?

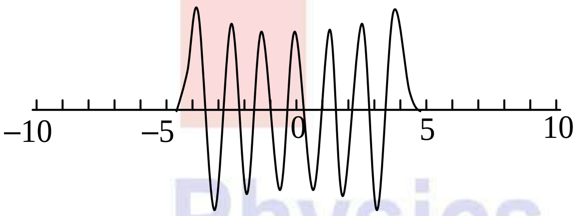
(a)



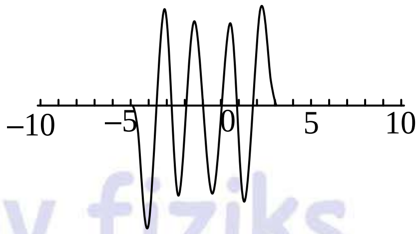
(b)



(c)



(d)



Ans.: (a)

Solution:

There are 10 nodes in the wave function of excited state A i.e., A has 10 nodes.

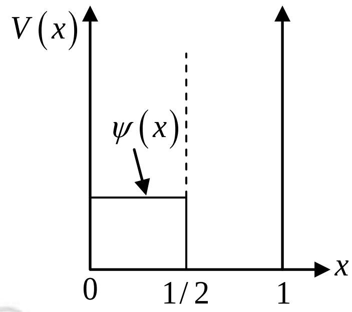
$\therefore\ E_A = 10\,h\omega$, 10 nodes :the given wave function is even.

$E_B = 5\,h\omega$ $n = 5$: the wave function for the excited state must be an odd function.

The correct form of wave function for the excited state is represented by the option (b).

Q4. A particle is confined in a one-dimensional box of unit length, i.e. $L=1$, i.e. in a potential

$$V(x)=\begin{cases}0 & if\ 0<x<1\\ \infty & \text{otherwise}\end{cases}$$



The energy eigenvalues of this particle are denoted $E_0, E_1, E_2, E_3, \ldots$

In a particular experiment, the wavefunction of this particle, at $t=0$, is given by

$$\psi(x)=\begin{cases}\sqrt{2} & if\ 0<x<\dfrac{1}{2}\\ 0 & \text{otherwise}\end{cases}$$

If, simultaneously, i.e. at $t=0$, a measurement of the energy of the particle is made, find $100p_3$, where p_3 is the probability that the measurement will yield the energy E_3

Ans.: 00

Solution:

The eigen function and energy for the give potential in the unit of $L=1$ is

$$\psi_n(x,0)=\sqrt{2}\sin n\pi x, E_n=\frac{n^2\pi^2\hbar^2}{2m}, n=1,2,3,4 \qquad (1)$$

And any arbitrary wave function can be expanded in this basis, that is,

$$\psi(r,t)=\sum_{n=1}^{\infty}a_n\psi_n(x)e^{-iE_nt/\hbar} \qquad (2)$$

The value of coefficient a_n are calculated from the initial given wave function. i.e ,

$$\psi(x,0)=\sum_{n=1}^{\infty}a_n\psi_n(x) \qquad (3)$$

or $$\int_0^a\psi(x,0)\psi_k(x)dx=\sum_{n=1}^{\infty}a_n\int_0^a\psi_n(x)\psi_k(x)dx=\sum_{n=1}^{\infty}a_n\delta_n \qquad (4)$$

that,

$$a_k=\int_0^a\psi(x,0)\psi_k(x)dx \qquad (5)$$

The energy E_3 is obtained for $k=4$, so the coefficient $k=4$, so the coefficient a_4 is

$$a_4=\int_0^{1/2}\sqrt{2}\cdot\sqrt{2}\sin 4\pi x=2\int_0^{1/2}\sin 4\pi x dx \qquad (6)$$

$$=-2\left[-\cos 4\pi x\right]_0^{1/2}=2\left[-1-\cos 2\pi\right]=0 \qquad (7)$$

The probability of $n=4$ state is

$$P_3|a_4|^2=0 \qquad (8)$$

So the $$100P_3=0 \qquad (9)$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

SECTION B-(Only for Ph.D. candidates)

Q5. A particle moving in one dimension is confined inside a rigid box located between $x=\frac{-a}{2}$ and $x=\frac{a}{2}$. If the particle is in its ground state

$$\psi_0(x)=\sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$$

the quantum mechanical probability of its having a momentum p is given by

(a) $\frac{8\hbar^4}{\left(\pi^2\hbar^2-p^2a^2\right)^2}\cos^2\frac{pa}{2\hbar}$

(b) $\frac{\pi^2\hbar^4}{\left(\pi^2\hbar^2-p^2a^2\right)^2}\sin^2\frac{pa}{2\hbar}$

(c) $\frac{2\hbar^4}{\left(\pi^2\hbar^2+p^2a^2\right)^2}\cos^2\frac{pa}{2\hbar}$

(d) $\frac{16\hbar^4}{\left(\pi^2\hbar^2-p^2a^2\right)^2}$

Ans.: (d)

Solution:

We have $\psi_0(x)=\sqrt{\frac{2}{a}}\cos\frac{\pi x}{a}$ (1)

The momentum wave function is given by

$$=\sqrt{\frac{2}{a}}\int_{-a/2}^{+a/2}e^{-i\frac{px}{\hbar}}\cos\frac{\pi x}{a}dx=\sqrt{\frac{2}{a}}\int_{-a/2}^{+a/2}e^{-i\frac{px}{\hbar}}\cos\frac{\pi x}{a}dx \quad (2)$$

Rearranging eq. $(s16.02)$ in terms of wave vector, we get

$$\psi(p)=\sqrt{\frac{2}{a}}\int_{-a/2}^{+a/2}e^{-ikx}\cos\frac{\pi x}{a}dx=\left[\frac{\sqrt{\frac{2}{a}}e^{ikx}a\left[\left(\pi\sin\frac{\pi x}{a}\right)-iak\cos\frac{\pi x}{a}\right]}{\left(\pi^2-a^2k^2\right)}\right]_{-a/2}^{+a/2}=\frac{2\sqrt{2}\pi\cos\frac{ak}{2}}{\sqrt{\frac{1}{a}}\left(\pi^2-a^2k^2\right)} \quad (3)$$

where we have used

$$p=\hbar k \quad (4)$$

The probability of the particle in momentum space is

$$\left|\psi(p)\right|^2=\frac{8\pi^2a}{\left(\pi^2-a^2k^2\right)^2}\cos^2\frac{ak}{2} \quad (5)$$

Again using eq. (4) in eq. (5), we obtain

$$\left|\psi(p)\right|^2=\frac{8\pi^2a}{\left(\pi^2-\frac{a^2p^2}{\hbar^2}\right)^2}\cos^2\frac{ap}{2h}=\frac{8\pi^2a\hbar^4}{\left(\pi^2\hbar^2-p^2a^2\right)^2}\cos^2\frac{pa}{2\hbar} \quad (6)$$

There is an additional term of $\left(\pi^2a\right)$ in the numerator.

Q6. Consider two spin-$\frac{1}{2}$ identical particles A and B, separated by a distance r, interacting through a potential

$$V(r)=\frac{V_0}{r}\vec{S}_A.\vec{S}_B$$

where V_0 is a positive constant and the spins are $\vec{S}_{A,B}=\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ in terms of the Pauli spin matrices. The expectation values of this potential in the spin- singlet and triplet states are

(a) Singlet: $-\frac{V_0}{3r}$, Triplet: $\frac{V_0}{r}$

(b) Singlet: $-\frac{3V_0}{r}$, Triplet: $\frac{V_0}{r}$

(c) Singlet: $\frac{3V_0}{r}$, Triplet: $-\frac{V_0}{r}$

(d) Singlet: $-\frac{V_0}{r}$, Triplet: $\frac{3V_0}{r}$

Ans.: (b)

Solution:

The addition of the spin of the two identical particle A and B is

$$\left(\vec{S}_A+\vec{S}_B\right)^2=\vec{S}_A^2+\vec{S}_B^2+2\vec{S}_A\cdot\vec{S}_B \qquad (1)$$

Taking the average of the eq. $(s16.01)$, we get

$$\left\langle\vec{S}_A\cdot\vec{S}_B\right\rangle=\frac{1}{2}\left[\left\langle\left(\vec{S}_A+\vec{S}_B\right)^2\right\rangle-\left\langle\vec{S}_A^2\right\rangle-\left\langle\vec{S}_B^2\right\rangle\right] \qquad (2)$$

But, we also have

$$\vec{S}_A+\vec{S}_B=\begin{cases}0\sin glet\\1triplet\end{cases};\left\langle\vec{S}_A^2\right\rangle=S_A\left(S_A+1\right)\hbar^2;\left\langle\vec{S}_B^2\right\rangle=S_B\left(S_B+1\right)\hbar^2; \qquad (3)$$

$$\left\langle\left(\vec{S}_A+\vec{S}_B\right)^2\right\rangle=\left(S_A+S_B\right)\left(S_A+S_B+1\right)\hbar^2$$

Substituting eq.$(s16.03)$ in eq. $(s16.02)$, we get

$$\left\langle\vec{S}_A\cdot\vec{S}_B\right\rangle_{\text{singlet}}=\frac{1}{2}\left[0-\frac{1}{2}\times\frac{3}{2}-\frac{1}{2}\times\frac{3}{2}\right]\hbar^2=\frac{1}{2}\left[-\frac{3}{4}-\frac{3}{4}\right]=-\frac{3}{4}\hbar^2 \qquad (4)$$

and also

$$\left\langle\vec{S}_A\cdot\vec{S}_B\right\rangle_{\text{Triplet}}=\frac{1}{2}\left[2-\frac{3}{4}+\frac{3}{4}\right]\hbar^2=\frac{1}{4}\hbar^2 \qquad (5)$$

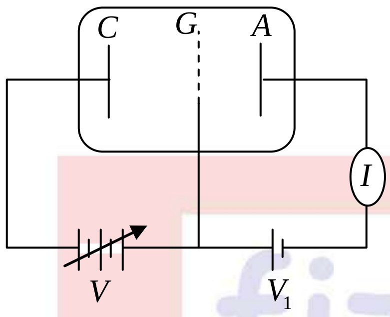
Thus, the expectation values of this potential in the spin singlet and triplet states are

$$F_n\text{ singlet}-\frac{3V_0}{r};F_n\text{ Triplet}\frac{V_0}{r} \qquad (6)$$

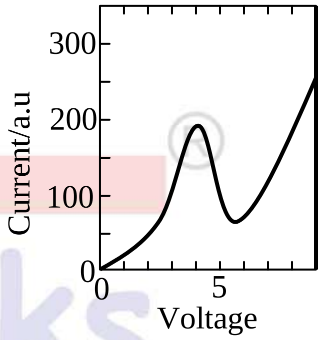
Here, we have neglected the term $\frac{\hbar}{2}$ as the spin is defined as $S=\sigma$ instead of $S=\sigma\frac{\hbar}{2}$.

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q7. In the experiment shown in figure (i) below, the emitted electrons from the cathode (C) are made to pass through the mercury vapor filled in the tube by accelerating them using a grid (G) at potential V positive w.r.t. the cathode. The electrons are collected by the anode (A).



(i)



(ii)

The variation of electron current (I) as a function of V is given in figure (ii)..The shape of this curve must be interpreted as due to

(a) ionization of mercury atoms.

(b) an emission line from mercury atoms.

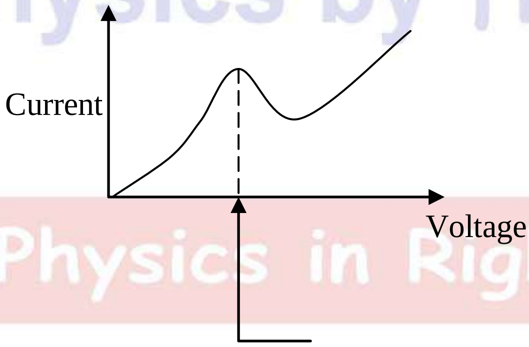
(c) attachment of electrons to mercury atoms.

(d) resonant backscattering of electrons to cathode from grid.

Ans.: (b)

Solution: The given apparatus is for the Frank Hertz experiment.

Frank Hertz experiment



The Peak corresponds to the first excitation of line of mercury atom.

Q8. A quantum mechanical plane rotator consists of two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the xy plane about their centre of mass. Suppose that the initial state of the rotor is given by

$$\psi(\phi, t=0) = A\cos^2\phi$$

where φ is the angle between one mass and the x axis, while A is a normalization constant. Find the expectation value of $3\hat{L}_z^2$ in this state, in units of $\hbar^2$

Ans.: 4

Solution:

We have

$$\psi(\psi, t=0) = A\cos^2\phi \tag{1}$$

The normalization constant is determined from the normalization condition

$$\langle\psi|\psi\rangle = A^2\int_0^{2\pi}\cos^4\phi = 1 \Rightarrow A^2\frac{3\pi}{4} = 1 \Rightarrow A = \sqrt{\frac{4}{3\pi}} \tag{2}$$

The normalized wave function is given by

$$\psi = \sqrt{\frac{4}{3\pi}}\cos^2\phi \tag{3}$$

The expectation value of $\langle L_z^2\rangle$ is

$$3\langle L_z^2\rangle = -3\left(\frac{4\pi}{3}\right)\int_0^{2\pi}\cos^2\phi\hbar^2\frac{\partial^2}{\partial\phi^2}\cos^2\phi\, d\psi, \hbar^2 = 1 \tag{4}$$

$$= -\frac{4}{\pi}\int_0^{2\pi}\cos^2\phi\frac{\partial}{\partial\phi}(-2\cos\phi\sin\phi)\, d\psi = -\frac{4}{\pi}\int_0^{2\pi}\cos^2\phi\frac{\partial}{\partial\phi}(-\sin 2\phi)\, d\psi$$

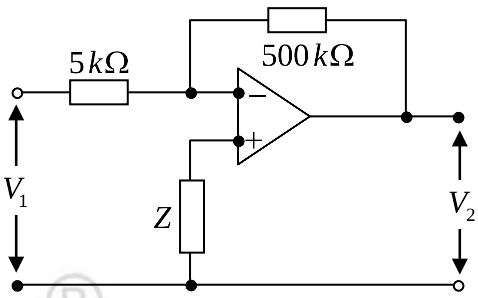
$$= +\frac{4}{\pi}\times 2\int_0^{2\pi}\cos^2\phi\cos 2\phi d\psi = \frac{8}{\pi}\int_0^{2\pi}\cos^2\phi\cos 2\phi d\psi = \frac{8}{\pi}.\frac{\pi}{2} = 4 \tag{5}$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

TIFR-2016 (Electronics Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. In the generalized operational amplifier circuit shown on the right, the op-amp has a very high input impedance $(Z > 50\ M\Omega)$ and an open gain of 1000 for the frequency range under consideration. Assuming that the op-amp draws negligible current, the voltage ratio $\frac{V_2}{V_1}$ is approximately



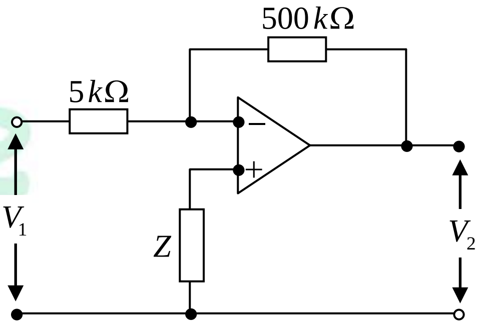
(a) -190 (b) -190 (c) -90 (d) 80

Ans.: (c)

Solution:

This is an inverting amplifier

$A = 1000$, $R_1 = 5k$, $R_f = 500k$



Feedback gain

$$A_f = \frac{V_2}{V_1} = -KA/(1+AB)$$

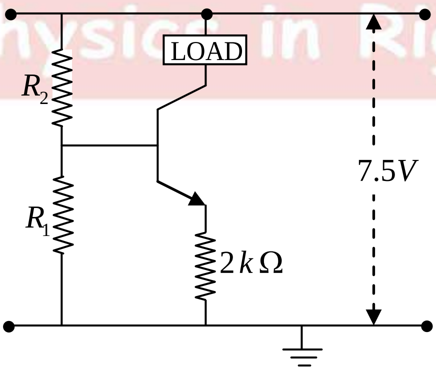
$$B = \frac{R_1}{R_1 + R_f} = \frac{5}{5+500}, \quad K = \frac{R_f}{R_1 + R_f} = \frac{500}{5+500}$$

$$A_f = -1000\left(\frac{500}{505}\right)\frac{1}{\left[1+\frac{1000\times 5}{505}\right]} = -90.82 \sim -90$$

Option (c)

Q2. In the transistor circuit shown on the right, assume that the voltage drop between the base and the emitter is $0.5\ V$.

What will be the ratio of the resistances $\frac{R_2}{R_1}$ in order to make this circuit function as a source of constant current, $I = 1mA$?

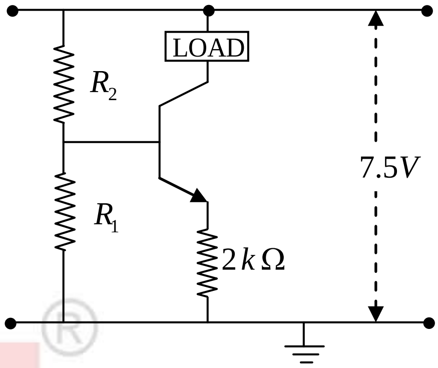


(a) 4.5 (b) 3.0 (c) 2.5 (d) 2.0

Ans.: (d)

Solution:

This is a constant current circuit



$V_B = 7.5\dfrac{R_1}{R_1+R_2}$

$V_{BE} = 0.5V$ (Given)

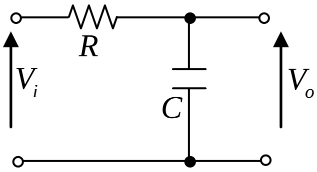
$V_B = I(\text{load})R_E + V_{BE}$

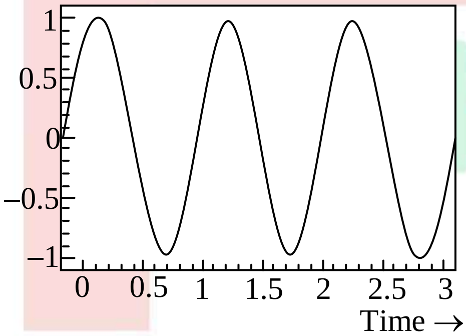
$\therefore \dfrac{7.5\,R_1}{R_1+R_2} = \left(10^{-3}A\right)\left(2\times10^{3}\Omega\right)+0.5 = 2.5$

$\dfrac{R_1}{R_1+R_2} = \dfrac{2.5}{7.5} = \dfrac{1}{3} \Rightarrow 3R_1 = R_1 + R_2$

$2R_1 = R_2 \Rightarrow \dfrac{R_2}{R_1} = 2.0$

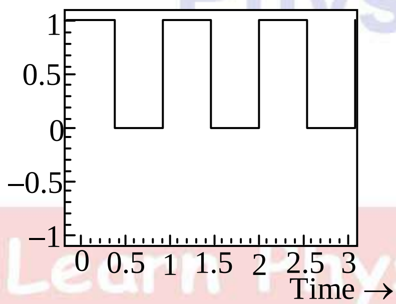
Q3. For the circuit depicted on the right, the input voltage V_i is a simple sinusoid as shown below, where the time period is much smaller compared to the time constant of this circuit.



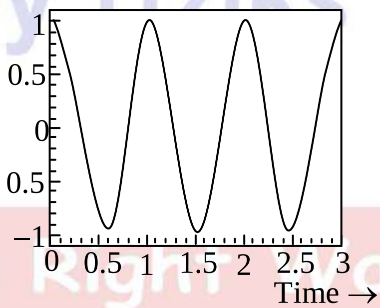


The voltage V across C is best represented by

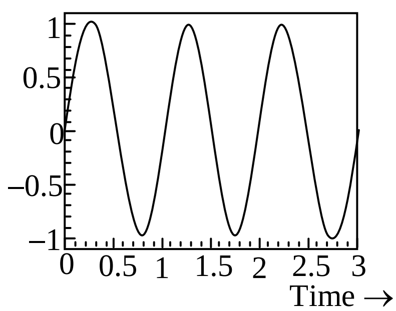
(a)



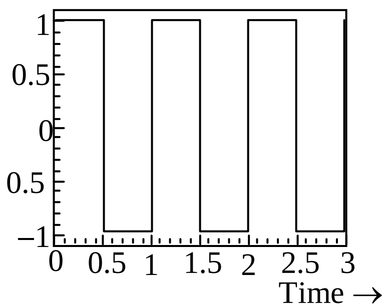
(b)



(c)



(d)



Ans.: (b)

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H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
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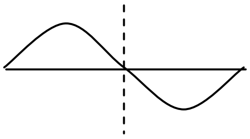
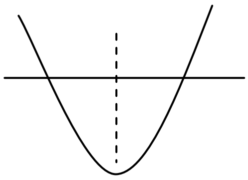
Solution:

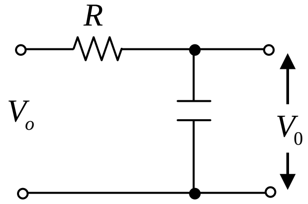
This is a circuit of integrator

Vi : Sine wave



V_0 : Cosine wave





SECTION B- (only for Int.-Ph.D. candidates)

Q4. A student in the laboratory is provided with a bunch of standard resistors as well as the following instruments

— Voltmeter accurate to $0.1\,V$ — Thermometer accurate to $0.1\,C$

— Ammeter accurate to $0.01\,A$ — Stop watch accurate to $0.05\,s$

— Constant current source (ideal) — Constant voltage source (ideal)

Using this equipment (and nothing else), the student is expected to measure the resistance R of one of the given resistors. The least accurate result would be obtained by

(a) measuring the Joule heating.

(b) passing a constant current and measuring the voltage across it.

(c) measuring the current on application of a constant voltage across it.

(d) the Wheatstone bridge method.

Ans. : (a)

Solution.:

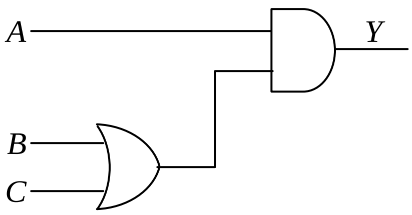
Due to Joule heating power will be lost due to radiation in the resistor.

Q5. In a digital circuit for three input signals (A, B and C) the final output (Y) should be such that for inputs

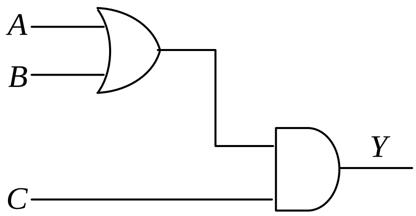
A	B	C
0	0	0
0	0	1
0	1	0

the output (Y) should be low and for all other cases it should be high. Which of the following digital circuits will give such output?

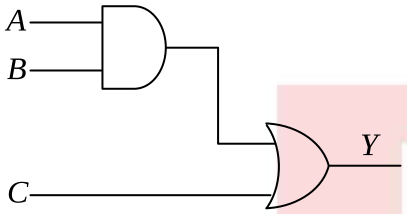
(a)



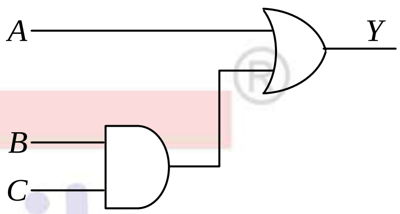
(b)



(c)



(d)



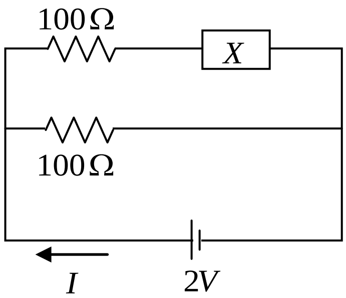
Ans. : (a)

Solution.: $Y = A.(B+C)$

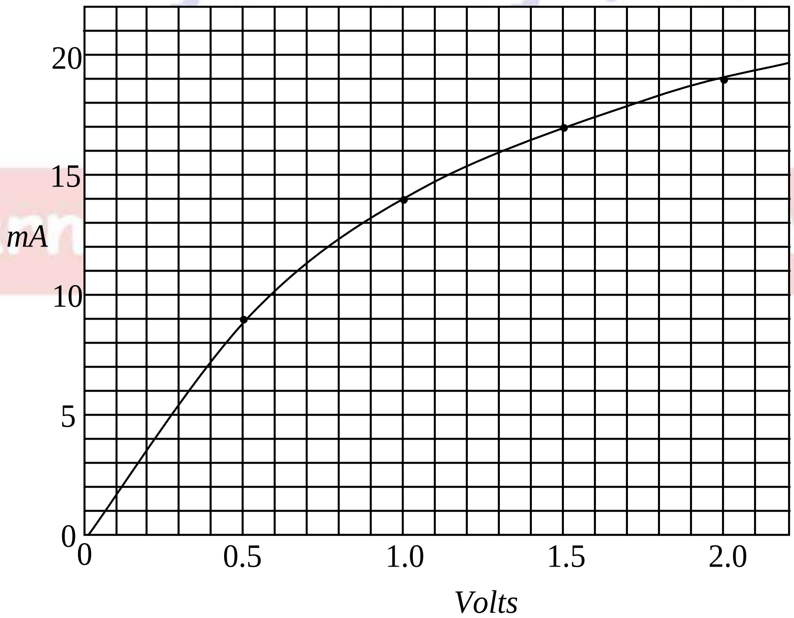
A	B	C	$Y = A.(B+C)$
0	0	0	0
0	0	1	0
0	1	0	0

SECTION B-(Only for Ph.D. candidates)

Q6. The circuit shown below contains an unknown device X



The current-voltage characteristics of the device X were determined and shown in the plot given below.

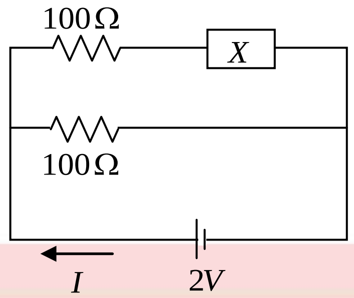


Determine the current I (in mA) flowing through the device X.

Ans.: 9.43

Solution:

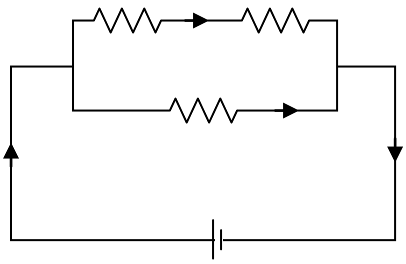
Static resistance of X at $2V = \frac{2}{18} \times 10^3 \Omega = \frac{2000}{18} \Omega = 111.1\ \Omega$



$\text{Re}q = (100 + 111.1) \,||\, (100) = (211.1) \,||\, (100) = (211.1) \,||\, (100) = 68.2\,\Omega$

I (Input) $= \frac{2}{68.2} = 29.32mA$

I (device) = I (Input) $\frac{100}{100+211} = 2932 \times \frac{100}{311} = 9.43mA$



100Ω $\quad 111.1 = 211.1$

$I \quad 29.32mA \quad 100\Omega \quad I_2$

TIFR-2016 (Atomic- Molecular Physics Questions and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. Two homonuclear diatomic molecules produce different rotational spectra, even though the atoms are known to have identical chemical properties. This leads to the conclusion that the atoms must be

(a) isotopes, i.e. with the same atomic number.

(b) isobars, i.e. with the same atomic weight.

(c) isotones, i.e. with the same neutron number.

(d) isomers, i.e. with the same atomic number and weight.

Ans.: (a)

Solution:

The rotational constant $B \propto \frac{1}{\mu}$

$e.g.$ HCL and DCL

SECTION B-(Only for Ph.D. candidates)

Q2. A continuous monochromatic ($\lambda = 600\ nm$) laser beam is chopped into $0.1\ ns$ pulses using some sort of shutter. Find the resultant line-width $\Delta\lambda$ of the beam in units of $10^{-3}\ nm$.

Ans.: 2

Solution:

$\Delta E\,\Delta t \sim \hbar \quad \Rightarrow h\,\Delta\nu\,\Delta t \sim \hbar$

$\Delta\nu \sim \dfrac{1}{2\pi\tau}, \qquad \Delta t = \tau = 0.1ns$

$\because \nu = \dfrac{c}{\lambda} \ \Rightarrow \Delta\nu = -\dfrac{c}{\lambda^2}\Delta\lambda$

$$\Rightarrow |\Delta\lambda| = \frac{\lambda^2}{c}|\Delta\nu| = \frac{\lambda^2}{2\pi\tau c} = \frac{\left(600\times10^{-9}\right)^2}{2\times3.14\times0.1\times10^{-9}\times3\times10^{8}} = \frac{36\times10^{-14}}{18.84\times10^{-2}} = 2\times10^{-12}$$

$\Rightarrow \Delta\lambda = 2\,pm = 2\times10^{-3}\ nm$

<u>TIFR-2016 (Solid State Physics Questions and Solution)</u>

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. In a simple cubic lattice of lattice constant $0.287\ nm$, the number of atoms per mm^2 along the (111) plane is

(a) 2.11×10^{13} (b) 1.73×10^{13} (c) 1.29×10^{13} (d) 1.21×10^{13}

Ans:

Solution:

The number of atoms per mm^2 along the (111) plane: $n = \dfrac{n_{eff}}{A}$

Where, n_{eff} is the effective number of atoms in the plane (111),it is triangular plane and for simple cubic this is

$$n_{eff} = \frac{1}{6}n_c + \frac{1}{2}n_f + n_i = \frac{1}{6}\times3 + \frac{1}{2}\times0 + 0 = \frac{1}{2}.$$

A is the area of the (111), and it is

$$A = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2 = \frac{\sqrt{3}}{2}a^2$$

Thus,

$$n=\frac{n_{eff}}{A}=\frac{1/2}{\frac{\sqrt{3}}{2}a^2}=\frac{1}{\sqrt{3}a^2}=\frac{1}{\sqrt{3}\times\left(0.287\times10^{-9}m\right)^2}=\frac{1}{\sqrt{3}\times\left(0.287\times10^{-6}mm\right)^2}$$

$$n=\frac{1}{\sqrt{3}\times\left(0.287\times10^{-6}mm\right)^2}=\frac{1}{0.497\times10^{-12}mm^2}=2.01\times10^{12}mm^2$$

Answers do not match.

SECTION B- (only for Int.-Ph.D. candidates)

Q2. The lattice constant of a material is of the order of a μm, and its bond energies are of the order of an eV. The bulk modulus of such a material, in Pascals, is of the order of

(a) 10^{-1} (b) 10^{-3} (c) 10^{-6} (d) 10^{9}

Ans: (a)

Solution:

Bulk modulus of crystal depends on the lattice constant R_0 and bond energy U_e as

$$\beta\propto\frac{U_e}{R_e^3}$$

$$\beta\propto\frac{eV}{\left(10^{-6}\right)^3}=\frac{1.6\times10^{-19}}{10^{-18}}\cong10^{-1}$$

Thus correct answer is option (a)

SECTION B-(Only for Ph.D. candidates)

Q3. The dispersion relation for electrons in the conduction band of a n-type semiconductor has the form $E(k)=ak^2+b$ where a and b are constants. It was observed that the cyclotron resonance frequency of such electrons is $\omega_0=1.8\times10^{11}\ rad\ S^{-1}$, when placed in a magnetic field $B=0.1\,Wm^{-2}$. It follows that the constant a must be about

(a) 10^{-36} (b) 10^{-28} (c) 10^{-32} (d) 10^{-38}

Ans: (d)

Solution:

The cyclotron resonance frequency of electron of charge (e) and effective mass m^* in the magnetic field of B is written as

$$\omega_c=\frac{eB}{m^*}$$

Effective mass of the electron is $m^*=\dfrac{\hbar^2}{\frac{d^2E(k)}{dk^2}}$

dispersion relation for electrons is $E(k) = ak^2 + b$

Differentiate $E(k)$ twice with respect to k : $\frac{dE}{dk} = 2ak$; $\quad \frac{d^2E}{dk^2} = 2a$

$$\text{Thus } m^* = \frac{\hbar^2}{\frac{d^2E(k)}{dk^2}} = \frac{\hbar^2}{2a}$$

The cyclotron frequency becomes

$$\omega_c = \frac{eB}{m^*} = \frac{eB}{\frac{\hbar^2}{2a}} = \frac{2a}{\hbar^2} eB \Rightarrow a = \frac{\omega_c \hbar^2}{2eB}$$

$$a = \frac{1.8\times10^{11}\times(1.05\times10^{-34})}{2\times1.6\times10^{-19}\times0.1} = 6.25\times10^{-38} eV - m^2$$

Thus the correct option is (d).

TIFR-2016 (Nuclear Physics Questions and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. In a fixed target experiment, a proton of total energy $200\ GeV$ is bombarded on a proton at rest and produces a nucleus ${}_A^Z N$ and its anti-nucleus ${}_A^Z \bar{N}$

$$p + p \to {}_A^Z N + {}_A^Z \bar{N} + p + p$$

The heaviest nucleus ${}_A^Z N$ that can be created has atomic mass number $A =$

(a) 15 (b) 9 (c) 5 (d) 4

Ans.: (b)

Solution:

Length of momentum 4-vector after the collision

$$\left(\frac{E_0^2}{c^2} - p^2\right)_f = \frac{(mc^2 + mc^2 + Amc^2 + Amc^2)^2}{c^2} - 0$$

Here maximum value of A will be produced when reaction products are at rest. So $p = 0$ is chosen.

$$\left(\frac{E_0^2}{c^2} - p^2\right)_f = 4(A+1)^2 m^2c^2$$

Length of momentum 4-vector after collision

$$\left(\frac{E_0^2}{c^2} - p^2\right)_i = \frac{(E_{01} + E_{02})^2}{c^2} - p_1^2 \qquad \text{Here } p_2 = 0,\ E_{02} = mc^2$$

$$\Rightarrow\left(\frac{E_0^2}{c^2}-p^2\right)_i=\frac{E_{01}^2}{c^2}+\frac{E_{02}^2}{c^2}+2\frac{E_{01}E_{02}}{c^2}-p_1^2=m^2c^2+\frac{m^2c^4}{c^2}+2\frac{E_{01}mc^2}{c^2}$$

$$\left(\frac{E_0^2}{c^2}-p^2\right)_i=\left(\frac{E_0^2}{c^2}-p^2\right)_f\quad\Rightarrow 4(A+1)^2m^2c^2=2m^2c^2+2E_{01}m\Rightarrow 2(A+1)^2=1+\frac{E_{01}}{mc^2}$$

For proton, $m=1\,GeV/c^2$; $\quad 2(A+1)^2=1+\dfrac{200\,GeV}{1GeV}\Rightarrow(A+1)^2=100.5$

$\Rightarrow A+1\approx 10\quad\Rightarrow A=9$

Q2. Consider a process in which atoms of Actinium-$226\left({}^{226}_{89}Ac\right)$ get converted to atoms of Radium-$226\left({}^{226}_{88}Ra\right)$ and the yield of energy is $0.64\ MeV$ per atom. This occurs through

(a) Both $p\to n+e^++\nu_e$ and $p+e^-\to n+\nu_e$

(b) Both $p\to n+e^++\nu_e$ and $n\to p+e^-+\bar{\nu}_e$

(c) only $p\to n+e^++\nu_e$

(d) only $p+e^-\to n+\nu_e$

Ans.: (d)

Solution:

For β^+ decay Q- value should be greater than $1.02\,MeV$. So option $(a),(b)$ and (c) are wrong.

SECTION B-(Only for Ph.D. candidates)

Q3. The Weizsacker semi-empirical mass formula for an odd nucleus with Z protons and A nucleons may be written as: $\quad M(Z,A)=\alpha_1A+\alpha_2A^{\frac{2}{3}}+\alpha_3Z+\alpha_4Z^2$

where the a_i are constants independent of Z,A. For a given A, if Z_A is the number of protons of the most stable isobar, the total energy released when an unstable nuclide undergoes a single β^- decay to (Z_A,A) is

(a) α_3 (b) α_4 (c) $\alpha_4-\alpha_3$ (d) $\alpha_1+\alpha_2$

Ans.: (b)

Solution:

For most stable nuclei $(Z=Z_A)$; $\left(\dfrac{\partial M}{\partial Z}\right)_A=\alpha_3+2\alpha_4Z_A=0 \qquad (1)$

${}^A_ZX\to{}^A_{Z_A}Y+\beta^-+\bar{\nu}$; $\quad$ where $Z_A=Z+1$ or $(Z-Z_A)=-1 \qquad (2)$

$Q=M_X-M_Y$; $\quad$ Here M_X and M_Y are in terms of energy.

$$Q = \alpha_1 A + \alpha_2 A^{2/3} + \alpha_3 Z + \alpha_4 Z^2 - \alpha_1 A - \alpha_2 A^{2/3} - \alpha_3 Z_A - \alpha_4 Z_A^2$$

$$Q = \alpha_3 (Z - Z_A) + \alpha_4 (Z + Z_A)(Z - Z_A) = -\alpha_3 - \alpha_4 (Z_A - 1 + Z_A)$$

$$Q = -\alpha_3 - 2\alpha_4 Z_A + \alpha_4 = -(\alpha_3 + 2\alpha_4 Z_A) + \alpha_4 \qquad \Rightarrow Q = 0 + \alpha_4 = \alpha_4$$

Q4. Consider the hyperon decay $(1)\, A \to n + \pi^0$ it followed by $(2)\, \pi^0 \to \gamma\gamma$. If the isospin component, baryon number and strangeness quantum numbers are denoted by I_z, B and S respectively, then which of the following statements is completely correct?

(a) In (1) I_Z is not conserved, B is conserved, S is not conserved;
In (2) I_Z is conserved, B is conserved, S is conserved.

(b) In (1) I_Z is conserved, B is not conserved, S is not conserved; In (2) I_Z is conserved, B is conserved, S is conserved.

(c) In (1) I_Z is not conserved, B is conserved, S is not conserved; In (2) I_Z is not conserved, B is conserved, S is conserved.

(d) In (1) I_Z is not conserved, B is conserved, S is conserved; In (2) I_Z is conserved, B is conserved, S is conserved.

Ans.: (a)

Solution:

(1)	$\Lambda^\circ \to$	n	$+\ \pi^\circ$	
B	1	1	0	Conserved
S	-1	0	0	Not Conserved
I_z	0	$-\frac{1}{2}$	0	Not Conserved

(2)	$\pi^\circ \to$	γ	$+\ \gamma$	
B	0	0	0	Conserved
S	0	0	0	Conserved
I_z	0	0	0	Conserved

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q42. In a Rutherford scattering experiment, the number N of particles scattered in a direction θ, i.e. $\frac{dN}{d\theta}$, as a function of the scattering angle θ (in the laboratory frame) varies as

(a) $\csc^4\frac{\theta}{2}$ (b) $\csc^2\frac{\theta}{2}\cot\frac{\theta}{2}$ (c) $\csc^2\frac{\theta}{2}\tan^2\frac{\theta}{2}$ (d) $\sec^4\frac{\theta}{2}$

Ans: (b)

Solution:

Rutherford scattering:

$$\frac{dN}{d(\cos\theta)}\propto\frac{1}{(1-\cos\theta)^2}\qquad\Rightarrow\frac{dN}{\sin\theta d\theta}\propto\frac{1}{(1-\cos\theta)^2}$$

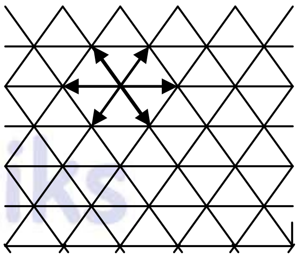
$$\frac{dN}{d\theta}\propto\frac{\sin\theta}{(1-\cos\theta)^2}\propto\frac{2\sin\theta/2\cos\theta/2}{\left[1-\left(1-2\sin^2\frac{\theta}{2}\right)\right]^2}$$

$$\Rightarrow\frac{dN}{d\theta}\propto\frac{\sin\theta/2\cos\theta/2}{\sin^4\theta/2}\propto\frac{1}{\sin^2\theta/2}\left(\frac{\cos\theta/2}{\sin\theta/2}\right)\propto\text{cosec}^2\frac{\theta}{2}\cot\frac{\theta}{2}$$

General Physics

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q3. In a triangular lattice a particle moves from a lattice point to any of its 6 neighbouring points with equal probability, as shown in the figure on the right.



The probability that the particle is back at its starting point after 3 moves is

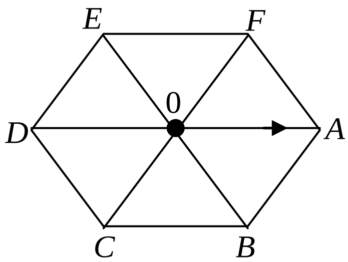
(a) $\frac{5}{18}$ (b) $\frac{1}{6}$ (c) $\frac{1}{18}$ (d) $\frac{1}{36}$

Ans: (c)

Solution:

The particle can take 1^{st} move in any six directions with equal probability of $\frac{1}{6}$.

Suppose it moves to A, now to be back at starting point 0 in two steps, it can either move to B or F.



Thus probability that it will move to B or $F = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

Next, suppose it moves to B, now to be back at O, he has got only option, i.e, probability must be 1.

Hence, total probability $= \frac{1}{6} \cdot \frac{1}{3} \cdot 1 = \frac{1}{18}$. (Option (c))

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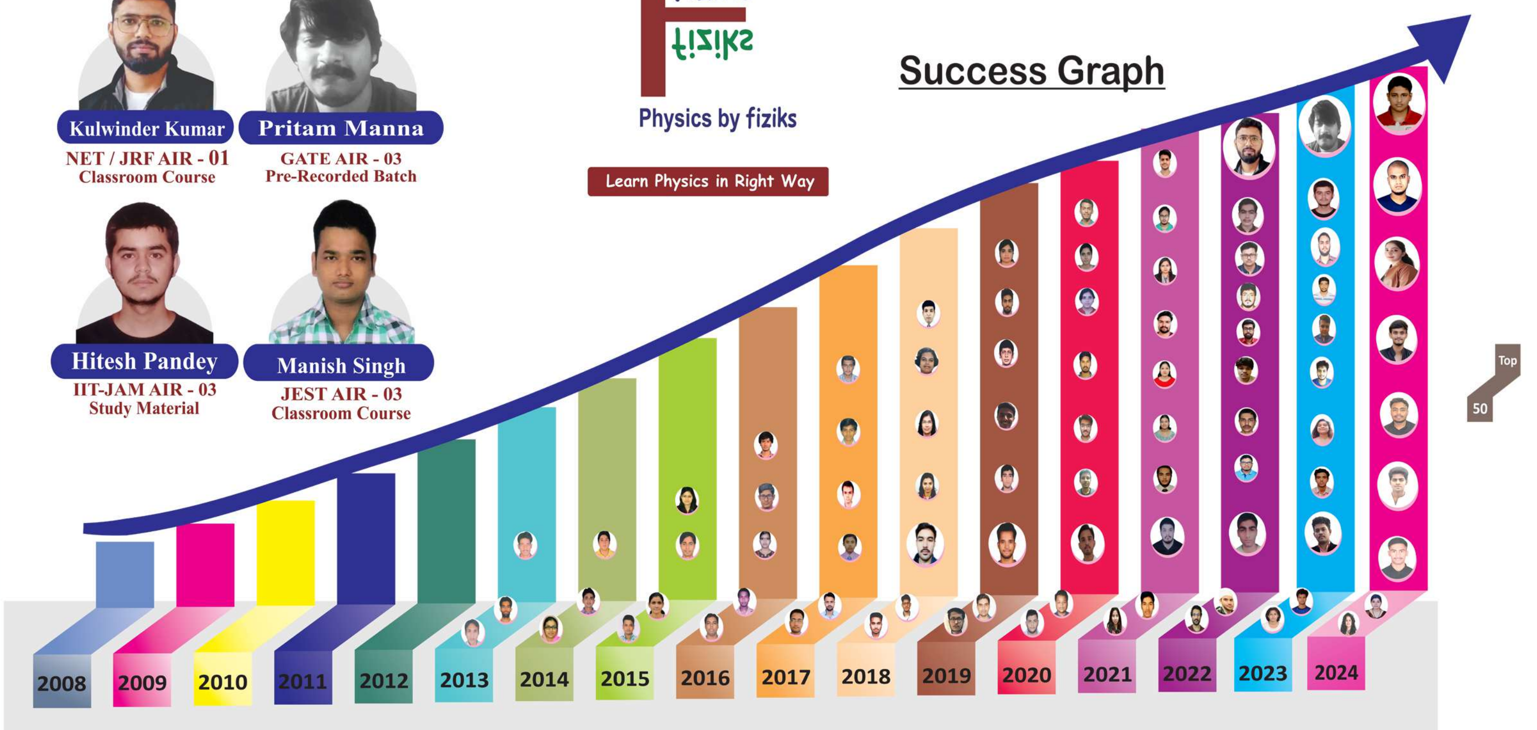

011-26865455, +9871145498

Head Office: House No. 40-D, Ground Floor, Jia Sarai
Near IIT-Delhi, Hauz Khas, New Delhi-110016