TIFR-2018 (Mathematical Physics Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. If a 2×2 matrix M is given by

$$M=\begin{pmatrix} 1 & (1-i)/\sqrt{2} \\ (1+i)/\sqrt{2} & 0 \end{pmatrix}$$

Then det exp $M=$

(a) e^2 (b) e (c) $2i\sin\sqrt{2}$ (d) $\exp\left(-2\sqrt{2}\right)$

Ans. : (b)

Solution:

Remember $Det\left(e^M\right)=e^{Trace\,M}$

Trace M = 1 + 0 = 1

So $Det\left(e^M\right)=e^{Trace\,M}=e^1$

Hence (b) is the correct result.

Q2. Consider the two equations

$\frac{x^2}{3}+\frac{y^2}{2}=1$ $x^3-y=1$

How many simultaneous real solutions does this pair of equations have?

Ans. : 4

Solution:

$\frac{x^2}{3}+\frac{y^2}{2}=1..........(1),$ $x^3-y=1.........(2)$

From (2) $x^3-1=y,$ put the value of y in (1)

We get, $\frac{x^2}{3}+\frac{\left(x^3-1\right)^2}{2}=1\Rightarrow\frac{x^2}{3}+\frac{x^6+1-2x^3}{2}=1\Rightarrow 2x^2+3x^6+3-6x^3=6$

Rearranging, we get, $3x^6-6x^3+2x^2-3=0...............(3)$

(3) is a sixth order polynomial. It has six possible solutions

Positive Solutions:

$3x^6-6x^3+2x^2-3=0.$ There are three sign changes.

Thus maximum no. of positive roots $=3$

Negative solutions:

Replace x by $-x$, we get $3x^6-6x^3+2x^2-3=0.$

One sign charge is encountered.

Thus, equation (3) has maximum of 1 negative root. Combining we get that (1) and (2) have 4 real solutions.

SECTION B- (only for Int.-Ph.D. candidates)

Q3. If $y(x)$ satisfies the differential equation $y''-4y'+4y=0$ with boundary conditions

$$y(0)=1 \text{ and } y'(0)=0, \text{ then } y\left(-\frac{1}{2}\right)=$$

(a) $+\frac{2}{e}$ (b) $\frac{1}{2}\left(e+\frac{1}{e}\right)$ (c) $\frac{1}{e}$ (d) $\frac{e}{2}$

Ans. : (a)

Solution:

Auxiliary equation is $D^2-4D+4=0 \Rightarrow (D-2)^2=0 \Rightarrow D=2,2 \quad \Rightarrow y=(A+Bx)e^{2x}$

Now $y(0)=1=(A+B\cdot 0)e^{20}=A \quad \Rightarrow A=1$

So $y=(1+Bx)e^{2x} \Rightarrow y'=(1+Bx)2e^{2x}+Be^{2x}$

$y'(0)=0=(1+B\cdot 0)\cdot 2\cdot e^0+Be^0 \Rightarrow 0=2+0+B \Rightarrow B=-2$

Hence $y=(1-2x)e^{2x} \Rightarrow y\left(-\frac{1}{2}\right)=\left(1-2\times\left(-\frac{1}{2}\right)\right)e^{2\times\left(-\frac{1}{2}\right)}=2e^{-1}=\frac{2}{e}$

(a) is the correct answer.

Q4. Given the following xy data

x	1.0	2.0	3.0	4.0	5.0
y	0.002	0.601	0.948	1.21	1.42

Which of the following would be the best curve, with constant positive parameters a and b, to fit this data?

(a) $y=ax-b$ (b) $y=a+\exp bx$ (c) $y=a\log_{10}bx$ (d) $y=a-\exp(-bx)$

Ans. : (c)

Solution:

	$\Delta x=1$	$\Delta x=1$	$\Delta x=1$	$\Delta x=1$	
x	1.0	2.0	3.0	4.0	5.0
y	0.002	0.601	0.948	1.21	1.42

Δy	0.599	0.347	0.262	0.21
$\frac{\Delta y}{\Delta x}$	0.599	0.347	0.262	0.21

The increments in x, y have been evaluated between two consecutive values. The derivative has also been evaluated at successive readings.

Thus, the given function is such that

(1) For equal increments in x, increments in y are not equal. Hence cannot be approximated by a linear function

(2) Function is increasing

(3) Derivative is +ve but decreasing

Thus, our expected solution should be an increasing function of x with a positive but decreasing derivative. Let's analyze the given solutions for these requirements.

(a) $y = ax - b$, $\frac{dy}{dx} = a = \text{constant}$

But the derivative of the given function is variable. Ruled Out

(b) $y = a + e^{bx}$, $\frac{dy}{dx} = be^{bx}$ - function is increasing

But the derivative of the given function is decreasing. Ruled Out

(c) $y = a\log_{10} bx$, $\frac{dy}{dx} = \frac{a}{bx}$ $\rightarrow$ derivative is thus decreasing for +ve a and b

Function itself is increasing. The given function is also increasing with a decreasing derivative. Hence 'c' is possible.

(d) $y = a - e^{-bx}$, $\frac{dy}{dx} = be^{-bx}$

$\uparrow \uparrow$

function is decreasing derivative is decreasing

But given function is increasing with decreasing derivative

Hence d is also ruled out.

Thus, 'c' is the correct answer.

Q5. Evaluate the integral $\int_{-\infty}^{+\infty} dx \exp\left(-x^2\right)\cos\left(\sqrt{2}x\right)$

Ans. :

Solution: $I = \int_{-\infty}^{\infty} e^{-x^2} \frac{e^{i\sqrt{2}x} + e^{-i\sqrt{2}x}}{2} dx = \frac{1}{2}\int_{-\infty}^{\infty} e^{-x^2+i\sqrt{2}x} + \int_{-\infty}^{\infty} e^{-x^2-i\sqrt{2}x} dx = I_1 + I_2$

$$I_1 = \frac{1}{2}\int_{-\infty}^{\infty} e^{-\left(x^2 - i\sqrt{2}x + \left(\frac{i}{\sqrt{2}}\right)^2 - \left(\frac{i}{\sqrt{2}}\right)^2\right)} dx = \frac{1}{2}\int_{-\infty}^{\infty} e^{-\left(x-\frac{i}{\sqrt{2}}\right)^2} e^{\frac{i^2}{2}} dx.$$

Put $x - \frac{i}{\sqrt{2}} = t \Rightarrow dx = dt$, Limits remain unchanged

$$I_1 = \frac{1}{2} e^{-\frac{1}{2}} \int_{-\infty}^{\infty} e^{-\left(x - \frac{i}{\sqrt{2}}\right)^2} dx = \frac{1}{2} e^{-\frac{1}{2}} \int_{-\infty}^{\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2} e^{-\frac{1}{2}} \qquad \because \int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

$$I_2 = \frac{1}{2} \int_{-\infty}^{\infty} e^{-\left(x^2 + i\sqrt{2}x + \left(\frac{i}{\sqrt{2}}\right) - \left(\frac{i}{\sqrt{2}}\right)^2\right)} dx = \frac{1}{2} e^{-\frac{1}{2}} \int_{-\infty}^{\infty} e^{-\left(x + \frac{i}{\sqrt{2}}\right)^2} dx = \frac{1}{2} e^{-\frac{1}{2}} \int_{-\infty}^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2} e^{-\frac{1}{2}}$$

Thus $I = \frac{\sqrt{\pi}}{2} e^{-\frac{1}{2}} + \frac{\sqrt{\pi}}{2} e^{-\frac{1}{2}} = \sqrt{\pi}\, e^{-\frac{1}{2}}$

SECTION B-(Only for Ph.D. candidates)

Q6. The Fourier series which reproduces, in the interval $0 \le x < 1$, the function

$$f(x) = \sum_{n=-\infty}^{+\infty} \delta(x-n)$$

Where n is an integer, is

(a) $1 + 2\cos 2\pi x + 2\cos 4\pi x + 2\cos 6\pi x + \cdots + (to\ \infty)$

(b) $1 + \cos \pi x + \cos 2\pi x + \cos 3\pi x + \cdots (to\ \infty)$

(c) $\cos \pi x + \cos 2\pi x + \cos 3\pi x + \cdots (to\ \infty)$

(d) $(\cos \pi x + \sin \pi x) + \frac{1}{2}(\cos 2\pi x + \sin 2\pi x) + \frac{1}{3}(\cos 3\pi x + \sin 3\pi x) + \cdots (to\ \infty)$

Ans. : (b)

Solution:

Delta function is an even function, so $b_n = 0$.

Let the Fourier series be $f(x) = \frac{a_0}{2} + \sum_{n=1} a_n \cos \frac{n\pi x}{\ell}$, Here $\ell = 1$

(1) $f(x) = \frac{a_0}{2} + \sum_{n=1} a_n \cos n\pi x$

Integrating both sides w.r.t 'x' from 0 to 1

$$\int_0^1 f(x)\,dx = \frac{a_0}{2} \int_0^1 dx + \sum_{n=1} a_n \int_0^1 \cos n\pi x\, dx$$

The second term on right hand side is zero.

Therefore, we have $\int_0^1 \sum_{n=-\infty}^{\infty} \delta(x-n)\,dx = \frac{a_0}{2}$

Only $n = 0$ will contribute on left hand side, other integrals will be zero.

$$\frac{a_0}{2} = \int_0^1 \delta(x-0)\,dx = 1 \Rightarrow a_0 = 2$$

Multiplying (1) by $\cos m\pi x$ and integrate with respect to x from $x = 0$ to 1

$$\int_0^1 f(x)\cos m\pi x\,dx = \frac{a_0}{2}\int_0^1 \cos m\pi x\,dx + \sum_{n=1}^{\infty} a_n \int_0^1 \cos n\pi x \cos m\pi x\,dx$$

The first term on right hand side is zero. Only $n = m$ term will be finite in the second term.

$$\int_0^1 \sum_{n=\infty}^{\infty} \delta(x-n)\cos m\pi x\,dx = 0 + a_m \int_0^1 \cos^2 m\pi x\,dx$$

Only n= 0=m=x, integral will be finite on left hand side

$$\int_0^1 \delta(x-0)\cos m\pi x\,dx = \frac{a_m}{2}\int_0^1 (1+\cos 2m\pi x)\,dx$$

$$\cos(m\pi 0) = \frac{a_m}{2}\left| x + \frac{\sin 2m\pi x}{2m\pi}\right|_0^1 \quad \Rightarrow 1 = \frac{a_m}{2}\times 1 \Rightarrow a_m = 2$$

Thus $f(x) = \frac{2}{2} + \sum 2\cos n\pi x$

$$f(x) = 1 + 2(\cos \pi x + \cos 2\pi x + \cos 3\pi x + ...)$$

Except for some constants the answer matches 'b'. So 'b' may be correct answer.

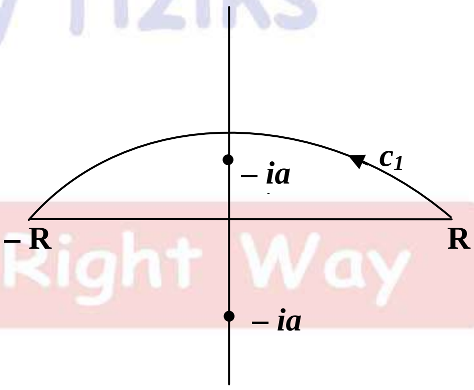
Q7. The value of the integral $\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2}$ is.

(a) $1/2a$ (b) $1/2\pi a$ (c) $\exp(-a)/a$ (d) $\pi a \exp(-a)$

Ans. : (c)

Solution:

Consider $\int_{-\infty}^{\infty} \frac{\cos ax}{x^2+a^2}dx$, $f(z) = \frac{e^{iaz}}{(z^2+a^2)}$



$$\left[\because \text{Real of } \frac{e^{iaz}}{z^2+a^2} = \frac{\cos az}{z^2+a^2}\right]$$

Poles are given by $z^2 + a^2 = 0 \Rightarrow z = \pm ia$

$z = -ia$ lies at side the center

Let's calculate the residue at $z = ia$

$$\underset{z=ia}{\text{Res}} = \underset{z\to ia}{Lt} \frac{(z-ia)e^{iz}}{(z-ia)(z+ia)} = \frac{e^{i(ia)}}{(ia+ia)} = \frac{1}{2ia}e^{i^2 a} = \frac{1}{2ia}e^{-a}$$

So, by Residue Theorem $\oint_C f(z)\,dz = 2\pi i \sum$ Residue

$$\int_{-R}^{R} f(z)\,dz + \int_{C_1} f(z)\,dz = 2\pi i \times \frac{1}{2ia} e^{-a}$$

$$\underset{R\to\infty}{Lt} \int_{-R}^{R} f(z)\,dz + \underset{R\to\infty}{Lt} \int_{C_1} f(z)\,dz = \frac{\pi}{a} e^{-a}$$. The second integral is just zero.

$z = x$ over real axis

$$\int_{-\infty}^{\infty} \frac{e^{ix}}{x^2+a^2} dx = \frac{\pi}{a} e^{-a} \Rightarrow \int_{-\infty}^{\infty} \frac{\cos ax}{x^2+a^2} dx = \text{Re} \int_{-\infty}^{\infty} \frac{e^{iax}}{x^2+a^2} dx = \text{Re}\left(\frac{\pi}{a} e^{-a}\right) = \frac{\pi}{a} e^{-a}$$

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\cos ax}{x^2+a^2} dx = \frac{1}{\pi} \cdot \frac{\pi}{a} e^{-a} = \frac{1}{a} e^{-a}$$

(c) is correct answer.

TIFR-2018 (EMT Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. The characteristic impedance of a co-axial cable is independent of the

(a) Dielectric medium between the core and the outer mesh

(b) Outer diameter

(c) Length of the cable

(d) Core diameter

Ans: (c)

Q2. Consider a dipole antenna with length ℓ, charge q and frequency ω. The power emitted by the antenna at a large distance r is P. Now suppose the length ℓ is increased to $\sqrt{2}\ell$, the charge is increased to $\sqrt{3}q$ and the frequency is increased to $\sqrt{5}\omega$. By what factor is the radiated power increased?

Ans. : 30

Solution. : $\langle P \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$ where $p_0 = q_0 d$

$$\frac{\langle P_2 \rangle}{\langle P_1 \rangle} = \frac{q_2^2 d_2^2 \omega_2^2}{q_1^2 d_1^2 \omega_1^2} \Rightarrow \frac{\langle P_2 \rangle}{\langle P_1 \rangle} = \frac{\left(\sqrt{3}q\right)^2 \left(\sqrt{2}l\right)^2 \left(\sqrt{5}\omega\right)^2}{q^2 l^2 \omega^2} = 30$$

Q3. Calculate the self-energy, in Joules, of a spherical conductor of radius $8.5cm$, which carries a charge $100\mu C$.

Ans. : 529.4

Solution. :

Capacitance of spherical conductor is $C = 4\pi\varepsilon_0 R$.

Self-energy $U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2R} = \left(9\times10^9\right)\frac{\left(100\times10^{-6}\right)^2}{2\times8.5\times10^{-2}}$

$$\Rightarrow U = \frac{90\times10^2}{17} = \frac{9000}{17} = 529.4 \text{ Joules}$$

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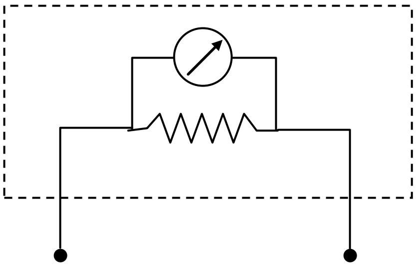
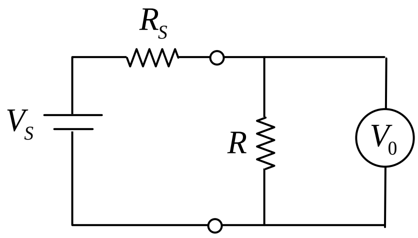
Q4. A realistic voltmeter can be modeled as an ideal voltmeter with an input resistor in parallel a shown below:



Such a realistic voltmeter, with input resistance $1k\Omega$, gives a reading of $100mV$ when connected to a voltage source with source resistance 50Ω. .What will a similar voltmeter, with input resistance $1M\Omega$, read in mV, when connected to the same voltage source?

Ans. : 105

Solution. : Given that $R_S = 50\Omega$, $R = 1k\Omega$ then



$V_0 = 100mV$.

Th $V_0 = \frac{R}{R+R_S} \times V_S \Rightarrow 100mV = \frac{1000\Omega}{1000\Omega + 50\Omega} \times V_S$ us

$\Rightarrow V_S = 100mV \times \frac{1050\Omega}{1000\Omega} = 105\, mV$.

When $R_S = 50\Omega$, $R = 1M\Omega$ then $V_0 = \frac{10^6\,\Omega}{10^6\,\Omega + 50\Omega} \times 105mV \approx 105mV$.

SECTION B- (only for Int.-Ph.D. candidates)

Q5. An atom of atomic number Z can be modeled as a point positive charge surrounded by a rigid uniformly negatively charged solid sphere of radius R. The electric Polarisability α of this system is defined as $\alpha = \frac{p_E}{E}$

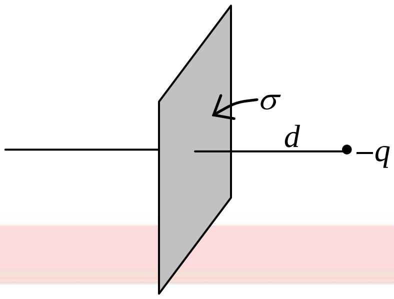
Where p_E is the dipole moment induced on application of electric field E which is small compared to the binding electric field inside the atom. It follows that $\alpha =$

(a) $\frac{4\pi\varepsilon_0}{R^3}$ (b) $\frac{8\pi\varepsilon_0}{R^3}$ (c) $4\pi\varepsilon_0 R^3$ (d) $8\pi\varepsilon_0 R^3$

Ans. : (c)

Solution. : $E = \frac{1}{4\pi\varepsilon_0}\frac{qd}{R^3} \Rightarrow p_E = qd = \left(4\pi\varepsilon_0 R^3\right)E \Rightarrow \alpha = 4\pi\varepsilon_0 R^3$

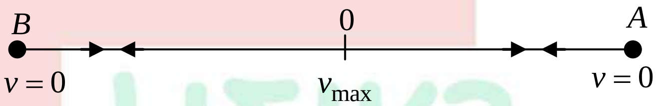
Q6. Consider an infinite plane with a uniform positive charge density σ as shown below. A negative point charge $-q$ with mass m is held at rest at a distance d from the sheet and released. It will then undergo oscillatory motion. What is the time period of this oscillation?



[You may assume that the point charge can move freely though the charged plane without disturbing the charge density.].

Ans. :

Solution. :



Force acting on the charged particle $F = qE = q\dfrac{\sigma}{\varepsilon_0}$

Acceleration of the particle $= \dfrac{F}{m} = \dfrac{q\sigma}{m\varepsilon_0}$

For the motion of charge particle

From $A \to 0$:

$$d = \frac{1}{2}at^2 = \frac{1}{2}\frac{q\sigma}{m\varepsilon_0}t^2 \quad t = \sqrt{\frac{2m\varepsilon_0 d}{q\sigma}}$$

So time taken by the particle to complete on cycle $(A \to O \to B \to O \to A)$ is

$$T = 4t = 4\sqrt{\frac{2m\varepsilon_0 d}{q\sigma}}$$

SECTION B-(Only for Ph.D. candidates)

Q7. The electrostatic charge density $\rho(r)$ corresponding to the potential

$$\phi(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left(1+\frac{\alpha r}{2}\right)\exp(-\alpha r) \qquad \text{is } \rho =$$

(a) $q\delta(r) - 2q\alpha^3\exp(-\alpha r)$

(b) $q\delta(r) - q\dfrac{\alpha^3}{4}\exp(-\alpha r)$

(c) $-q\delta(r) - 2q\alpha^3\exp(-\alpha r)$

(d) $q\delta(r) - q\dfrac{\alpha^3}{2}\exp(-\alpha r)$

Ans. : (d)

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Solution. :

$$\vec{E}=-\vec{\nabla}\phi=-\frac{\partial\phi}{\partial r}\hat{r}=-\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left[\frac{1}{r}\left(1+\frac{\alpha r}{2}\right)\exp(-\alpha r)\right]\hat{r}=-\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left[\left(\frac{1}{r}+\frac{\alpha}{2}\right)\exp(-\alpha r)\right]\hat{r}$$

$$\Rightarrow \vec{E}=-\frac{q}{4\pi\varepsilon_0}\left[\left(\frac{1}{r}+\frac{\alpha}{2}\right)(-\alpha)\exp(-\alpha r)+\exp(-\alpha r)\left(-\frac{1}{r^2}\right)\right]\hat{r}$$

$$\Rightarrow \vec{E}=\frac{q}{4\pi\varepsilon_0}\left[\exp(-\alpha r)\left(\frac{\alpha}{r}+\frac{\alpha^2}{2}\right)\hat{r}+\exp(-\alpha r)\frac{\hat{r}}{r^2}\right]$$

$$\Rightarrow \vec{\nabla}.\vec{E}=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\times\exp(-\alpha r)\left(\frac{\alpha}{r}+\frac{\alpha^2}{2}\right)\right\}+\vec{\nabla}.\left\{\exp(-\alpha r)\frac{\hat{r}}{r^2}\right\}\right]$$

$$\Rightarrow \vec{\nabla}.\vec{E}=\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left\{\exp(-\alpha r)\left(\alpha r+\frac{\alpha^2 r^2}{2}\right)\right\}+\exp(-\alpha r)\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right)+\frac{\hat{r}}{r^2}.\vec{\nabla}\{\exp(-\alpha r)\}\right]$$

$$\Rightarrow \vec{\nabla}.\vec{E}=\frac{q}{4\pi\varepsilon_0}\left[\frac{\exp(-\alpha r)}{r^2}\left\{(\alpha+\alpha^2 r)-\alpha\left(\alpha r+\frac{\alpha^2 r^2}{2}\right)\right\}+\exp(-\alpha r)\times 4\pi\delta(r)+\frac{\hat{r}}{r^2}.\{-\alpha\exp(-\alpha r)\hat{r}\}\right]$$

$$\Rightarrow \vec{\nabla}.\vec{E}=\frac{q}{4\pi\varepsilon_0}\exp(-\alpha r)\left[\left(\frac{\alpha}{r^2}+\frac{\alpha^2}{r}\right)-\left(\frac{\alpha^2}{r}+\frac{\alpha^3}{2}\right)+4\pi\delta(r)-\frac{\alpha}{r^2}\right]$$

$$\Rightarrow \vec{\nabla}.\vec{E}=\frac{q}{4\pi\varepsilon_0}\exp(-\alpha r)\left[-\frac{\alpha^3}{2}+4\pi\delta(r)\right]=\frac{1}{\varepsilon_0}\exp(-\alpha r)\left[q\delta(r)-\frac{q}{4\pi}\frac{\alpha^3}{2}\right]$$

$$\Rightarrow \rho=\varepsilon_0\left(\vec{\nabla}.\vec{E}\right)=\left[q\delta(r)-\frac{q}{4\pi}\frac{\alpha^3}{2}\right]\exp(-\alpha r)\;\Rightarrow \rho=q\delta(r)-\frac{q}{4\pi}\frac{\alpha^3}{2}\exp(-\alpha r)$$

Best option is (d)

Q8. A plane electromagnetic wave, which has an electric field

$$\vec{E}(\vec{x},t)=\left(P\hat{i}+Q\hat{j}\right)\exp i\omega\left(t-\frac{z}{c}\right)$$

is passing through vacuum. Here P, Q and ω are all constants, while c is the speed of light in vacuo. What is the average energy flux per unit time (in SI units) crossing a unit area placed normal to the direction of propagation of this wave, in terms of the above constants?

Solution.:

$$\left\langle\vec{S}\right\rangle=\frac{1}{2}c\varepsilon_0E^2\hat{k}=\frac{1}{2}c\varepsilon_0\left(P^2+Q^2\right)\hat{k}$$

TIFR-2018 (Quantum Mechanics Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. A particle is confined inside a one-dimensional box of length ℓ and left undisturbed for a long time. In the most general case, its wave-function MUST be

(a) The ground state of energy.

(b) Periodic, where ℓ equals an integer number of periods.

(c) Any one of the energy eigenfunctions.

(d) A linear superposition of the energy eigenfunctions.

Ans. :(d)

Solution:

A particle is confined inside a one dimensional box of length l. The energy eigenvalue and their corresponding eigenfunction are given.

$$\phi_n = \sqrt{\frac{2}{l}}\sin\frac{n\pi x}{l}\ ;\ E_n = \frac{n^2\pi^2\hbar^2}{2ml^2}$$

Let the particle remains undisturbed for a long time t, the wave function of the particle is given by $\quad |\psi\rangle = \sum_{i=i}^{\omega} Ci|\phi_i\rangle = |\phi_1\rangle + C_2|\phi_2\rangle + ...C_n|\phi_n\rangle$

Thus, the wave function of the particle must be super position of energy eigen function.

Q2. A particle is in the ground state of a cubical box of side ℓ. Suddenly one side of the box changes from ℓ to 4ℓ. If p is the probability of finding the particle in the ground state of the new box, what is $1000p$?

Ans. : 058

Solution: The wave function of particle in the ground state of cubical box of side ℓ is given by

$$\psi(x) = \sqrt{\frac{2}{\ell}}\sin\frac{x\pi}{\ell}.$$

The wave function of particle in the ground state of new cubical box of side 4ℓ is given by

$$\psi'(x) = \sqrt{\frac{2}{4\ell}}\sin\frac{\pi x}{4\ell}$$

The probability of finding the particle in the ground state of the new box, is

$$p = \left|\langle\psi|\psi'(x)\rangle\right|^2$$

let us evaluate the value of $\langle\psi|\psi'(x)\rangle$

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$$\langle\psi|\psi'(x)\rangle=\sqrt{\frac{2}{\ell}}\cdot\sqrt{\frac{2}{4\ell}}\int_0^{\ell}\sin\frac{\pi x}{\ell}\sin\frac{\pi x}{4\ell}dx=\frac{1}{\ell}\int_0^{\ell}\sin\frac{\pi x}{\ell}\sin\frac{\pi x}{4\ell}dx$$

$$\langle\psi|\psi'(x)\rangle=\frac{1}{2\ell}\int_0^{\ell}\left[\cos\frac{3\pi x}{4\ell}-\cos\frac{5\pi x}{4\ell}\right]dx=\frac{1}{2\ell}\left[\frac{4\ell}{3\pi}\left(\sin\frac{3\pi x}{4\ell}\right)-\frac{4\ell}{5\pi}\left(\sin\frac{5\pi x}{4\ell}\right)\right]_0^{\ell}$$

$$\langle\psi|\psi'(x)\rangle=\frac{1}{2\ell}\left[\frac{4\ell}{3\pi}\left(\sin\frac{3\pi}{4}\right)-\frac{4\ell}{5\pi}\left(\sin\frac{5\pi}{4}\right)\right]=\left[\frac{1}{3}\sin\left(\frac{\pi}{4}\right)+\frac{1}{5}\sin\left(\frac{\pi}{4}\right)\right]\left(\frac{2}{\pi}\right)$$

$$\langle\psi|\psi'(x)\rangle=\left[\frac{1}{3}+\frac{1}{5}\right]\left(\frac{1}{\sqrt{2}}\right)\left(\frac{2}{\pi}\right)=\frac{\sqrt{2}}{\pi}\left[\frac{5+3}{15}\right]=\frac{8\sqrt{2}}{\pi 15}$$

The probability of finding the particle in ground state of the new box is given by

$$p=\left|\langle\psi|\psi'(x)\rangle\right|^2=\left(\frac{8\sqrt{2}}{\pi 15}\right)^2$$

The value of $1000p$ is $1000p=1000\left(\frac{8\sqrt{2}}{\pi 15}\right)^2=1000\times\frac{64\times 2}{(3.14)^2\times 225}\Rightarrow 1000p=058$

Q3. The wave-function ψ of a particle in a one-dimensional harmonic oscillator potential is given by

$$\psi=\left(\frac{1}{\pi\ell^2}\right)^{1/4}\left(1+\frac{\sqrt{2}x}{\ell}\right)\exp\left(-\frac{x^2}{2\ell^2}\right)$$

Where $\ell=100\mu m$. Find the expectation value of the position x of this particle, in μm.

Ans. : 71

Solution:

The wave function ψ of a particle in a one – dimensional oscillator potential is give by

$$\psi=\left(\frac{1}{\pi\ell^2}\right)^{1/4}\left(1+\frac{\sqrt{2}x}{\ell}\right)e^{-\frac{x^2}{2\ell^2}}$$

Let us determine the value of $\langle\psi|\psi\rangle$

$$\langle\psi|\psi\rangle=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\int_{-\infty}^{\infty}\left(1+\frac{\sqrt{2}x}{\ell}\right)^2 e^{-\frac{x^2}{\ell^2}}dx=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\int_{-\infty}^{\infty}\left(1+\frac{2}{\ell^2}x^2+\frac{2\sqrt{2}}{\ell}x\right)^2 e^{-\frac{x^2}{\ell^2}}dx$$

$$=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\left[\int_{-\infty}^{\infty}e^{-x^2/\ell^2}dx+\frac{2}{\ell^2}\int_{-\infty}^{\infty}x^2e^{-x^2/\ell^2}dx+\frac{2\sqrt{2}}{\ell}\int_{-\infty}^{\infty}xe^{-x^2/\ell^2}dx\right]$$

$$=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\left[\ell\sqrt{\pi}+\frac{2}{\ell^2}\frac{\sqrt{\pi}}{2}\ell^3+0\right]=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\left[\ell\sqrt{\pi}+\ell\sqrt{\pi}\right]=2\frac{\ell\sqrt{\pi}}{\ell\sqrt{\pi}}=2$$

where, we have used

$$\int_{-\infty}^{\infty} e^{-x^2/\ell^2}dx = \ell\sqrt{\pi}\ ;\ \int_{-\infty}^{\infty} x^2 e^{-x^2/\ell^2}dx = \frac{\ell^3\sqrt{\pi}}{2};\quad \int_{-\infty}^{\infty} xe^{-x^2/\ell^2}dx = 0$$

Now, we determine the value

$$\langle\psi|x|\psi\rangle = \left(\frac{1}{\pi\ell^2}\right)^{1/2}\int_{-\infty}^{\infty} x\left(1+\frac{\sqrt{2}x}{\ell}\right)^2 e^{-x^2/\ell^2}dx$$

$$=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\left[\int_{-\infty}^{\infty} xe^{-x^2/\ell^2}dx + \frac{2}{\ell^2}\int_{-\infty}^{\infty} x^3 e^{-x^2/\ell^2}dx + \frac{2\sqrt{2}}{\ell}\int_{-\infty}^{\infty} x^2 e^{-x^2/\ell^2}dx\right]$$

$$=\left(\frac{1}{\pi\ell^2}\right)^{1/2}\left[0+0+\frac{2\sqrt{2}}{\ell}\frac{\sqrt{\pi}\ell^3}{2}\right] = \sqrt{2}\ell$$

where, we have used

$$\int_{-\infty}^{\infty} xe^{-x^2/\ell^2}dx = 0\ ;\ \int_{-\infty}^{\infty} x^3 e^{-x^2/\ell^2}dx = 0;\ \int_{-\infty}^{\infty} x^2 e^{-x^2/\ell^2}dx = \frac{\sqrt{\pi}}{2}\ell^3$$

The expectation value of the position x of this particle is defined as

$$\langle x\rangle = \frac{\langle\psi|x|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{\sqrt{2}\ell}{2} = \frac{\sqrt{2}\ell}{2} = \frac{\sqrt{2}\times 100}{2} = 71\ \mu m$$

SECTION B- (only for Int.-Ph.D. candidates)

Q4. A particle in a one-dimensional harmonic oscillator potential is described by a wave-function $\psi(x,t)$. If the wave function changes to $\psi(\lambda x,t)$ then the expectation value of kinetic energy T and the potential energy V will change, respectively, to

(a) $\lambda^2 T$ and $\lambda^2 V$ (b) T/λ^2 and $\lambda^2 V$

(c) T/λ^2 and V/λ^2 (d) $\lambda^2 T$ and V/λ^2

Ans. : (d)

Solution: A particle in a one-dimensional harmonic oscillator potential is daubed by a wave function $\psi(x,t)$. The expectation value of kinetic energy T and potential energy V are defined as

$$\langle T\rangle = \langle\psi|T|\psi\rangle = \left\langle\psi\left|\frac{p^2}{2m}\psi\right.\right\rangle = \frac{1}{2m}\langle\psi|p^2\psi\rangle = \frac{1}{2m}\left\langle\psi\left|-\hbar^2\frac{\partial^2}{\partial x^2}\psi\right.\right\rangle = \frac{-\hbar^2}{2m}\left\langle\psi\left|\frac{\partial^2\psi}{\partial x^2}\right.\right\rangle$$

and $\langle V\rangle = \langle\psi|V|\psi\rangle = \left\langle\psi\left|\frac{1}{2}m\omega^2x^2\right|\psi\right\rangle = \frac{1}{2}m\omega^2\langle\psi|x^2|\psi\rangle = \frac{1}{2}m\omega^2\langle\psi(x)|x^2|\psi(x)\rangle$

If the wave function changes to $\psi(\lambda x,t)$, then the new kinetic energy is given by.

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$$\langle T'\rangle = \langle\psi(\lambda x,t)|T|\psi(\lambda x,t)\rangle = \left\langle\psi(\lambda x,t)\left|\frac{p^2}{2m}\psi(\lambda x,t)\right.\right\rangle = \frac{1}{2m}\langle\psi(\lambda x,t)|\,p^2\psi(\lambda x,t)\rangle$$

$$= \frac{-\hbar^2}{2m}\left\langle\psi(\lambda x,t)\left|\frac{\partial^2\psi}{\partial x^2}(\lambda x,t)\right.\right\rangle$$

we as define the new variable, $\lambda x = y \Rightarrow \frac{\partial}{\partial x} = \frac{\lambda\partial}{\partial y} \Rightarrow \frac{\partial^2}{\partial x^2} = \frac{\lambda^2\partial^2}{\partial y^2}$

$$T' = \frac{-\hbar^2}{2m}\left\langle\psi(y,t)\frac{\lambda^2\partial^2}{\partial y^2}\psi(y,t)\right\rangle = \lambda^2\left(\frac{-\hbar^2}{2m}\left\langle\psi(y,t)\left|\frac{\partial^2}{\partial y^2}\psi(y,t)\right.\right\rangle\right) = \lambda^2 T\,,$$

Here, we have defined $T = \frac{-\hbar^2}{2m}\left\langle\psi(y,t)\left|\frac{\partial^2}{\partial y}\psi(y,t)\right.\right\rangle$

The potential energy of the $V' = \langle\psi(\lambda x,t)|V|\psi(\lambda x,t)\rangle = \left\langle\psi(\lambda x,t)\left|\frac{1}{2}m\omega x^2\right|\psi(\lambda x,t)\right\rangle$

$$\Rightarrow V' = \frac{1}{2}m\omega^2\langle\psi(\lambda x,t)|x^2|\psi(\lambda x,t)\rangle$$

Let us define the new variable, $\lambda x = y$; or $x = \frac{y}{\lambda}$; substituting this in above equation we get

$$V' = \frac{1}{2}m\omega^2\langle\psi(y,t)|y^2|\psi(y,t)\rangle = \frac{1}{\lambda^2}\left[\frac{1}{2}m\omega^2\langle\psi(y,t)|y^2|\psi(y,t)\rangle\right] = \frac{1}{\lambda^2}V$$

Where we defined, $V = \frac{1}{2}m\omega^2\langle\psi(y,t)|y^2|\psi(y,t)\rangle$

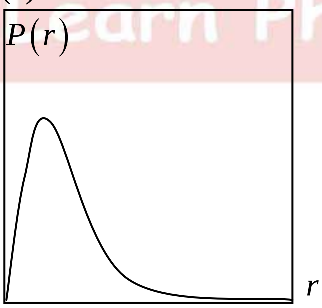
Thus, the expectation value of kinetic energy T' and potential energy V' for the new wave function are given by $T' = \lambda^2 T$; $V' = \frac{1}{\lambda^2}V$

Q5. An electron is in the $2s$ level of the hydrogen atom, with the radial wave-function

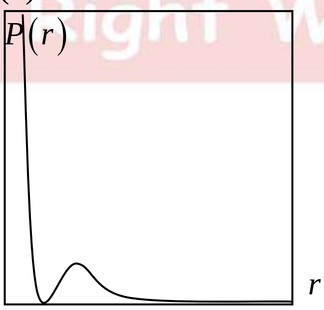
$$\psi(r) = \frac{1}{2\sqrt{2}a_0^{3/2}}\left(2 - \frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right).$$

The probability $P(r)$ of finding this electron between distances r to $r+dr$ from the centre is best represented by the sketch

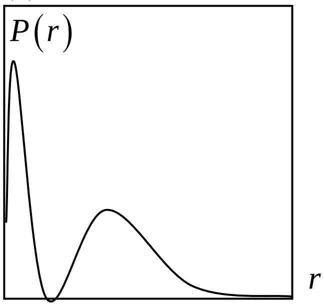
(a)



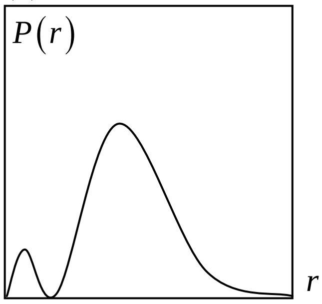
(b)



(c)



(d)



Ans. : (d)

Solution:

The probability density for the radial wave function for $2s$ orbital of hydrogen atom is correctly represented in the option (b).

Q6. Given a particle confined in a one-dimensional box between $x=-a$ and $x=+a$, a student attempts to find the ground state by assuming a wave-function

$$\psi(x)=\begin{cases} A\left(a^2-x^2\right)^{3/2} & \text{for } |x|\le a \\ 0 & \text{for } |x|>a \end{cases}$$

The ground state energy E_m is estimated by calculating the expectation value of energy with this trial wave-function. If E_0 is the true ground state energy, what is the ratio E_m/E_0?

Ans. : $\dfrac{21}{8\pi^2}$

Solution. :

The particle is confined in one-dimensional box between $x=-a$ and $x=+a$, the ground state. Energy of the particle is given by $E_0=\dfrac{\pi^2\hbar^2}{2m(2a)^2}=\dfrac{\pi^2\hbar^2}{8ma^2}=\dfrac{h^2}{32ma^2}$

The trial wave function of the particle is given by $\psi(x)=\begin{cases} A\left(a^2-x^2\right)^{3/2} & \text{for } |x|\le a \\ 0 & \text{for } |x|>a \end{cases}$

The normalization constable A is given by

$$\langle\psi(x)|\psi(x)\rangle=A^2\int_{-a}^{a}\left(a^2-x^2\right)^3 dx=1 \text{ or } A^2\int_{-a}^{a}\left(a^6-x^6-3a^2x^2\left(a^2-x^2\right)\right)dx=1$$

$$\text{or } A^2\int_{-a}^{a}\left(a^6-x^6-3a^4x^2+3a^2x^4\right)dx=1 \text{ or } 2A^2\int_{0}^{a}\left(a^6-x^6-3a^4x^2+3a^2x^4\right)dx=1$$

or $2A^2\left[a^6x-\frac{x^7}{7}-3a^4\frac{x^3}{3}+3a^2\frac{x^5}{5}\right]_0^a=1$ or $2A^2\left[a^7-\frac{a^7}{7}-3\frac{a^7}{3}+3\frac{a^7}{5}\right]=1$

or $2A^2\frac{16}{35}a^7=1\Rightarrow A=\sqrt{\frac{35}{32a^7}}$

The normalized trial wave function is defined as

$$\psi(x)=\begin{cases}\sqrt{\frac{35}{32a^7}}\left(a^2-x^2\right)^{3/2} & \text{for}\,|x|\le a\\ 0 & \text{for}\,|x|\le 0\end{cases}$$

The Hamiltonian for this system is defined

$$H=\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

The energy for this system is

$$E=\left\langle\psi\left|\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}\right.\right\rangle=-\frac{-\hbar^2}{2m}\left\langle\psi\left|\frac{\partial^2\psi}{\partial x^2}\right.\right\rangle$$

$$=\left(\frac{35}{32a^7}\right)\frac{-\hbar^2}{2m}\int_{-a}^{a}\left(a^2-x^2\right)^{3/2}\frac{\partial^2}{\partial x^2}\left(a^2-x^2\right)^{3/2}d$$

$$=\left(\frac{35}{32a^7}\right)\frac{\hbar^2}{2m}\int_{-a}^{a}\left(a^2-x^2\right)^{3/2}\left[3\left(a^2-x^2\right)^{1/2}-3x^2\left(a^2-x^2\right)^{-1/2}\right]d$$

$$=\left(\frac{35}{32a^7}\right)\frac{\hbar^2}{2m}\int_{-a}^{a}\left(3\left(a^2-x^2\right)^2-3x^2\left(a^2-x^2\right)\right)d$$

$$=\left(\frac{35}{32a^7}\right)\frac{3\hbar^2}{2m}\int_{-a}^{a}\left(a^4+x^4-2a^2x^2-a^2x^2+x^4\right)dx$$

$$=\left(\frac{35}{32a^7}\right)\frac{3\hbar^2.2}{2m}\int_{0}^{a}\left(a^4+2x^4-3a^2x^2\right)dx=\left(\frac{35}{32a^7}\right)\left(\frac{3\hbar^2.2}{2m}\right)\left(a^4x+\frac{2x^5}{5}-a^2x^3\right)_0^a$$

$$=\frac{3\hbar^2}{2m}\left(\frac{35}{32a^7}\right)2\left(a^5+\frac{2}{5}a^5-a^5\right)$$

$$=\frac{35}{32a^7}.\frac{3\hbar^2}{2m}.2.\frac{2a^5}{5}=\frac{21}{16}\frac{\hbar^2}{ma^2}$$

If E_0 is the true ground state energy, the ratio E_m/E_0 is

$$\frac{E_m}{E_0}=\frac{\left(21/16\hbar^2/ma^2\right)}{\left(\pi^2\hbar^2/2ma^2\right)}=\frac{21}{16}\times\frac{2}{\pi^2}=\frac{21}{8\pi^2}$$

SECTION B-(Only for Ph.D. candidates)

Q7. The state of a spin-1 particle is given by

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|1,\,-1\rangle + |1,\,0\rangle \exp\frac{i\pi}{3} + |1,\,1\rangle \exp\frac{2i\pi}{3}\right)$$

Where $|S,\,M_s\rangle$ denote the spin eigenstates with eigenvalues $h^2 S(S+1)$ and hM_s respectively. Find $\langle S_x\rangle$, i.e. the expectation value of the x component of the spin.

Ans. : $\sqrt{2}/3\hbar$

Solution. :

The state $|\psi\rangle$ of a spin -1 particle is given by

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|1,-1\rangle + |1,0\rangle \exp\left(\frac{i\pi}{3}\right) + |1,1\rangle \exp\left(\frac{i2\pi}{3}\right)\right)$$

The $\hat{S}_x$ operator in terms of ladder operator is defined as $\hat{S}_x = \left(\frac{S_+ + S_-}{2}\right)$

where as , S_+ and S_- are defined as.

$$S_+|s,m\rangle = \hbar\sqrt{(s-m)(s+m+1)}\,|s,m\rangle\,; \qquad S_-|s,m\rangle = \hbar\sqrt{(s+m)(s-m+1)}\,|s,m\rangle$$

Applying $\hat{S}_x$ on the operator, we get

$$\hat{S}_x|\psi\rangle = \frac{(S_+ + S_-)}{2}\left(\frac{1}{\sqrt{3}}\left(|1,-1\rangle + |1,0\rangle e^{i\pi/3} + |1,1\rangle e^{2i\pi/3}\right)\right)$$

$$= \frac{1}{2\sqrt{3}}\left(S_+\left(|1,-1\rangle + |1,0\rangle e^{i\pi/3} + |1,1\rangle e^{2i\pi/3}\right) + S_-\left(|1,-1\rangle + |1,0\rangle e^{i\pi/3} + |1,1\rangle e^{2i\pi/3}\right)\right)$$

$$= \frac{1}{2\sqrt{3}}\left(\left(\hbar\sqrt{(1+1)(1-1+1)}\,|1,0\rangle + \hbar\sqrt{(1+0)(1+0+1)}e^{i\pi/3}\,|1,1\rangle + \hbar\sqrt{(1-1)(1+1+1)}e^{2i\pi/3}\,|1,2\rangle\right)\right.$$

$$\left.+\left(\hbar\sqrt{(1-1)(1+1+1)}\,|1,-2\rangle + \hbar\sqrt{(1+0)(1-0+1)}\,|1-1\rangle e^{i\pi/3} + \hbar\sqrt{(1+1)(1-1+1)}e^{2i\pi/3}\,|1,0\rangle\right)\right)$$

$$S_x|\psi\rangle = \frac{1}{2\sqrt{3}}\left(\sqrt{2}\hbar|1,0\rangle + \sqrt{2}\hbar|1,1\rangle e^{i\pi/3} + \sqrt{2}\hbar\ e^{i\pi/3}|1,-1\rangle + \sqrt{2}\hbar e^{i2\pi/3}|1,0\rangle\right)$$

$$S_x|\psi\rangle = \frac{\sqrt{2}\hbar}{2\sqrt{3}}\left(e^{i\pi/3}|1,-1\rangle + \left(1+e^{2i\pi/3}\right)|1,0\rangle + e^{i\pi/3}|1,1\rangle\right)$$

The expectation value of $\hat{S}_x$ is given by

$$\langle S_x\rangle = \frac{\sqrt{2}\hbar}{2.3}\left(e^{i\pi/3}\langle 1,-1|1,-1\rangle + \left(1+e^{2i\pi/3}\right)e^{-i\pi/3}\langle 1,0|1,0\rangle + e^{-2i\pi/3}e^{i\pi/3}\langle 1,1|1,1\rangle\right)$$

$$\Rightarrow \langle S_x\rangle = \frac{\sqrt{2}\hbar}{6}\left[e^{i\pi/3} + e^{-i\pi/3} + e^{i\pi/3} + e^{-i\pi/3}\right]$$

$$\Rightarrow \langle S_x\rangle = \frac{\sqrt{2}\hbar}{6}.2\left[e^{i\pi/3} + e^{-i\pi/3}\right]$$

$$\Rightarrow \langle S_x\rangle = \frac{2\sqrt{2}\hbar}{3}.\frac{\left[e^{i\pi/3} + e^{-i\pi/3}\right]}{2} = \frac{2\sqrt{2}\hbar}{3}\cos\frac{\pi}{3} = \frac{\sqrt{2}}{3}\hbar$$

Thus, the expectation value of x component of the spin S_x is $\sqrt{2}/3\hbar$.

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Q8. A particle of mass m moves in a two-dimensional space (x, y) under the influence of a Hamiltonian.

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{4}m\omega^2\left(5x^2 + 5y^2 + 6xy\right)$$

Find the ground state energy of this particle in a quantum-mechanical treatment.

Ans. : $1.6\hbar\omega$

Solution. :

The Hamiltonian for a particle of man moving in two dimensional space (x, y) is given by

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + \frac{1}{4}m\omega^2\left(5x^2 + 5y^2 + 6xy.\right)$$

Let us define the new variable $x = \frac{X-Y}{\sqrt{2}},\ y = \frac{X+Y}{\sqrt{2}}$

Substituting the value of co-ordinate x and y in the potential term, we have

$$V(X,Y) = \frac{1}{4}m\omega^2\left[5\left(\frac{X-Y}{\sqrt{2}}\right)^2 + 5\left(\frac{X+Y}{\sqrt{2}}\right)^2 + 6\frac{\left(X^2-Y^2\right)}{2}\right]$$

$$= \frac{1}{4}m\omega^2\left(\frac{5}{2}\left(X^2+Y^2-2XY\right) + \frac{5}{2}\left(X^2+Y^2+2XY\right) + 3\left(X^2-Y^2\right)\right)$$

$$= \frac{1}{4}m\omega^2\left(5X^2+5Y^2+3X^2-3Y^2\right) = \frac{1}{4}m\omega^2\left(8X^2\right) + \frac{1}{4}m\omega^2\left(2Y^2\right)$$

$$= \frac{1}{2}m\omega^2\left(4X^2\right) + \frac{1}{2}m\omega^2Y^2 = \frac{1}{2}m(2\omega)^2X^2 + \frac{1}{2}m\omega^2Y^2$$

$$= \frac{1}{2}m\omega_X^2Y^2 + \frac{1}{2}m\omega_Y^2Y^2$$

where, we have defined $\omega_X = 2\omega;\ \omega_Y = \omega..$

The Hamiltonian operator modifies to

$$H = \frac{p_X^2}{2m} + \frac{p_Y^2}{2m} + \frac{1}{2}m\omega_X^2X^2 + \frac{1}{2}m\omega_Y^2Y^2$$

The energy corresponding to this Hamiltonian is given by

$$E_{nxny} = \left(n_X + \frac{1}{2}\right)\hbar\omega_X + \left(n_Y + \frac{1}{2}\right)\hbar\omega_Y$$

The ground state energy of this particle is given by

$$E_{00} = \frac{1}{2}\hbar\omega_X + \frac{1}{2}\hbar\omega_Y = \frac{1}{2}\hbar(2\omega) + \frac{1}{2}\hbar\omega = \frac{3}{2}\hbar\omega = 1.6\hbar\omega.$$

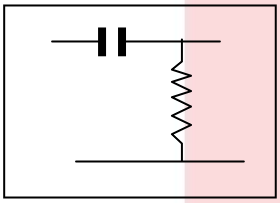
TIFR-2018 (Electronics Question and Solution)

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

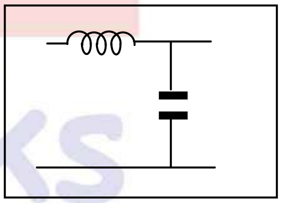
Q1. The figure below shows an unknown circuit, with an input and output voltage signal.

From the form of the input and output signals, one can infer that the circuit is likely to be

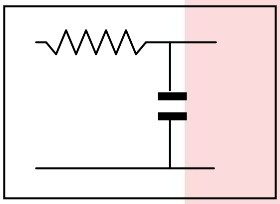
(a)



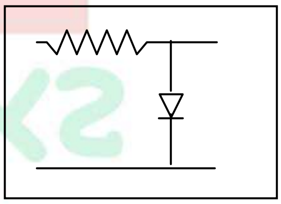
(b)



(c)



(d)



Ans: (a)

Solution. :

Its passive differentiator.

Q2. In Boolean terms, $(A+B)(A+C)$ is equal to

(a) ABC (b) $A+BC$

(c) $A(B+C)$ (d) $(A+B+C)(A+B)$

Ans: (b)

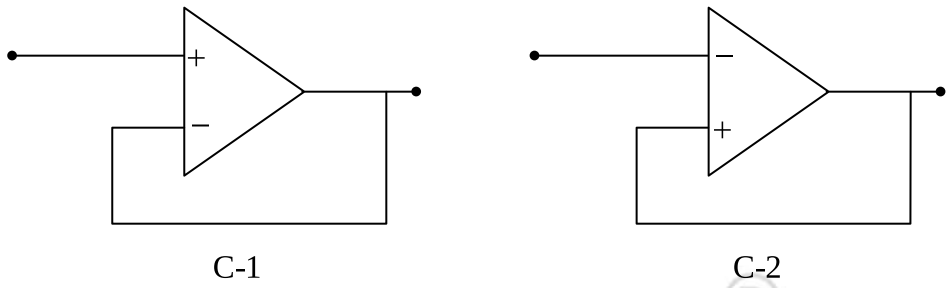
Solution. :

$Y=(A+B)(A+C)=AA+AC+AB+BC \Rightarrow Y=A+AC+BC$

$\Rightarrow Y=A(1+C)+BC \Rightarrow Y=A+BC$

SECTION B- (only for Int.-Ph.D. candidates)

Q3. Consider the following circuits C-1 and C-2.



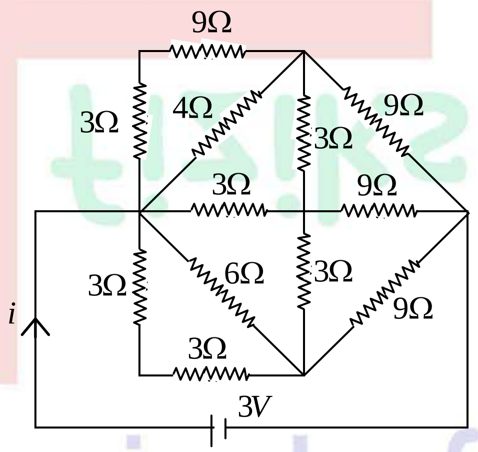
You can apply the golden rules of an ideal op-amp to

(a) Both C-1 and C-2 (b) Neither C-1 nor C

(c) Only C-1 (d) Only C-2

Ans. : (c)

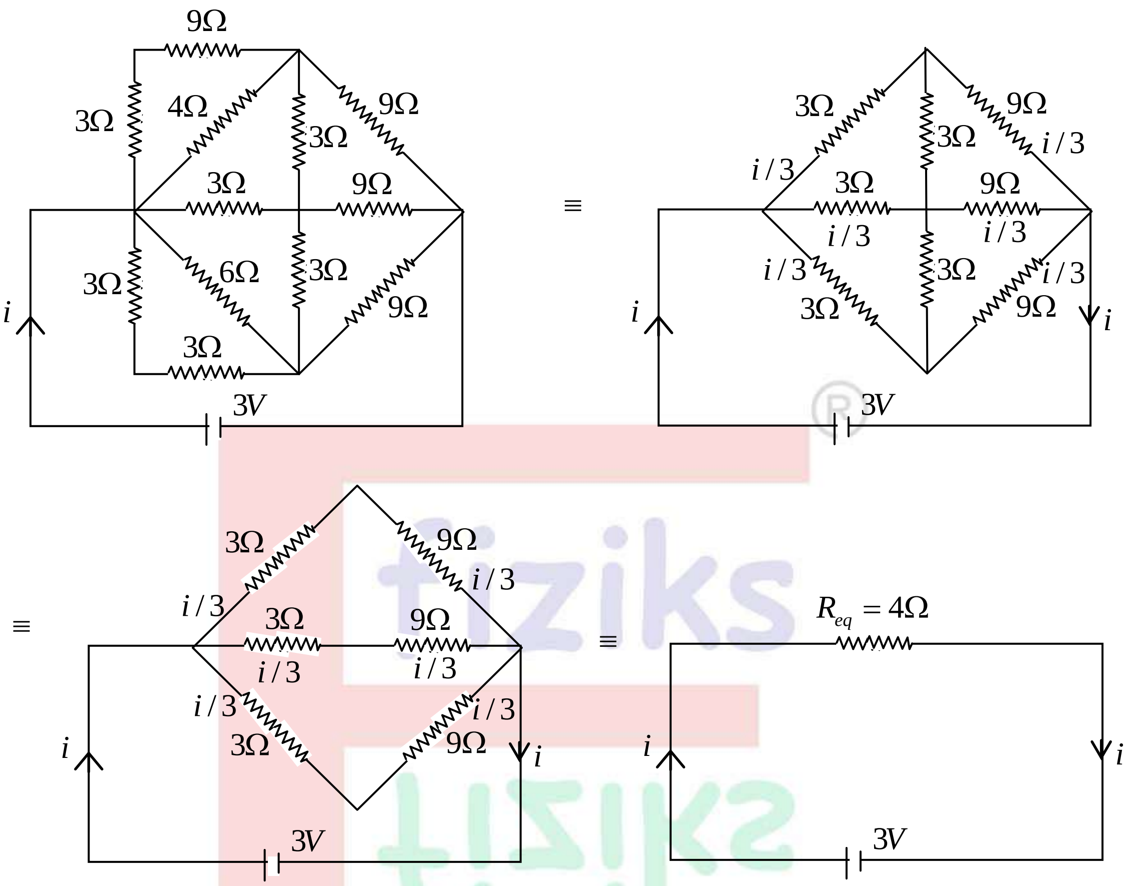
Q4. The current i flowing through the following circuit is



(a) $1.0A$ (b) $0.75A$ (c) $0.6A$ (d) $0.5A$

Ans. : (c)

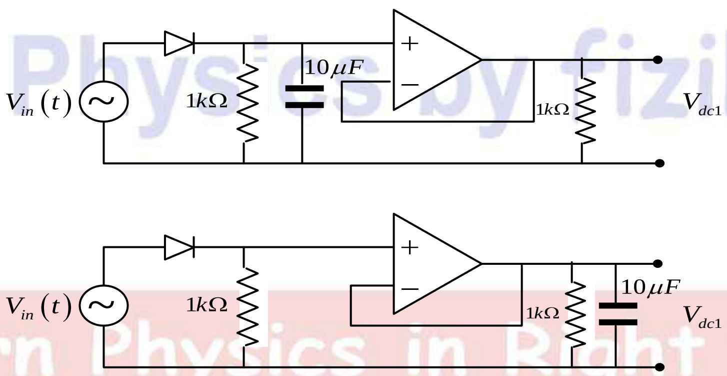
Solution. :



Thus current $i = \frac{3V}{4\Omega} = 0.75A$

SECTION B-(Only for Ph.D. candidates)

Q5. A signal $V_{in}(t) = 5\sin(100\pi t)$ is sent to both the circuits sketched below.



If the DC output voltage of the top circuit has a value V_{dc1} and the bottom circuit has a value V_{dc2}, then which of the following statements about the relative value of V_{dc1} and V_{dc2} is correct?

(a) It will depend on the slew rate of the op-amp.

(b) $V_{dc1} = V_{dc2}$

(c) $V_{dc1} < V_{dc2}$

(d) $V_{dc1} > V_{dc2}$

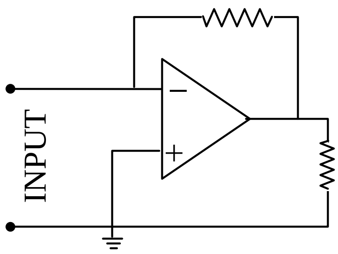
Ans. : (d)

Solution. :

In both case output will be a half wave rectified output. In first case output (V_{dc1}) will be without capacitor filter but in second case output (V_{dc2}) will be with capacitor filter.

So $V_{dc1} > V_{dc2}$

Q6. Consider the circuit shown on the right, which involves an op-amp and two resistors, with an input voltage marked INPUT.



Which of the following circuit components, when connected across the input terminals, is most likely to create a problem in the normal operation of the circuit?

(a) A voltage source with very high Thevenin resistance.

(b) A voltage source with a very low Thevenin resistance.

(c) A current source with a very high Norton resistance.

(d) A current source with a very low Norton resistance.

Ans. : (b)

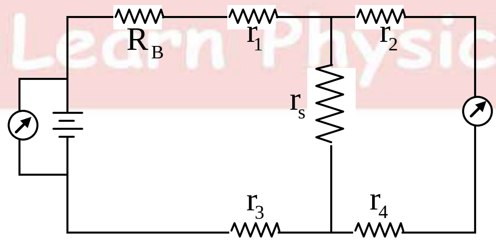
Solution. :

In inverting mode input resistance is equal to resistance applied at inverting input. So if we want high input resistance applied resistance must be very high. So there will be problem in the normal operation of the circuit if voltage source have a very low The venin resistance.

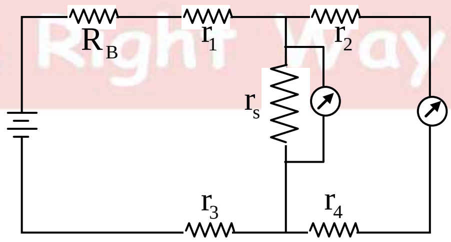
Q7. Which one of the following circuits, constructed only with resistors and voltmeters, will allow you to obtain the correct value of resistance r_s using the voltmeter readings? Note that the value of R_B is known while r_1, r_2, r_3, r_4 and r_s are all unknown.

[Assume that the voltmeters and resistors are ideal.]

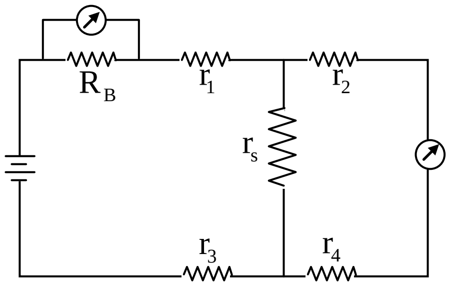
(a)



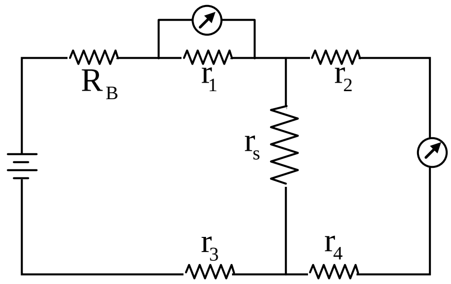
(b)



(c)



(d)



Ans: (c)

TIFR-2018 (Atomic- Molecular Physics Questions and Solution)

SECTION B-(Only for Ph.D. candidates)

Q1. The electron of a free hydrogen atom is initially in a state with quantum numbers $n=3$ and $\ell=2$. It then makes an electric dipole transition to a lower energy state. Which one of the given states could it be in after the transition?

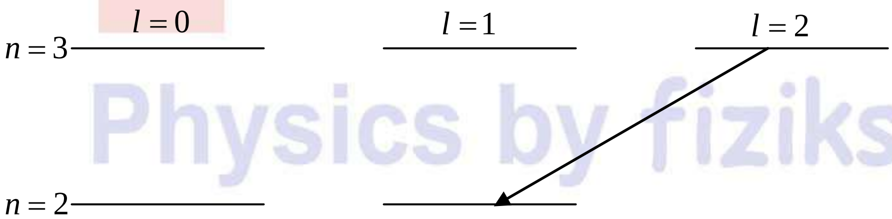
(a) $n=3,\ \ell=0$ (b) $n=2,\ \ell=1$ (c) $n=3,\ \ell=1$ (d) $n=2,\ \ell=2$

Ans.: (b)

Solution:

$\Delta n=$ any integer including zero

$\Delta l=\pm 1$



For the transition to a lower state, correct option is $n=2,\ l=1$

TIFR-2018 (Solid State Physics Questions and Solution)

SECTION B-(Only for Ph.D. candidates)

Q1. Consider a monatomic solid lattice at a low temperature $T \ll T_D$, where T_D is the characteristic Debye temperature of the solid $\left(T_D=\hbar\omega_m/k_B\right.$ where ω_m is the maximum possible frequency of the lattice vibrations). The heat capacity of the solid is proportional to

(a) T/T_D (b) T_D/T (c) $\left(T/T_D\right)^3$ (d) $\left(T_D/T\right)^2$

Ans: (c)

Solution:

Specific heat of monoatomic solid in three dimensions for $T \ll T_D$, is

$$C_V = \frac{12\pi^4}{5} N_A k \left(\frac{T}{T_D}\right)^3 \quad \Rightarrow \quad C_V \propto \left(\frac{T}{T_D}\right)^3$$

Thus, the correct answer is option (c)

<u>TIFR-2018 (Nuclear Physics Questions and Solution)</u>

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1. An electron enters a linear accelerator with a speed $v = 10ms^{-1}$. A vertical section of the accelerator tube is shown in the figure, where the lengths of the successive sections are designed such that the electron takes the same time $\tau = 20ms$ to traverse each section.

If the momentum of the electron increases by 2% every time it crosses the narrow gap between two sections, what is the length (in km) of the collider which will be required to accelerate it to $100kms^{-1}$?

Ans.: 100

Solution:

Number of gap	Velocity
1	$v_0\left(1+\frac{2}{100}\right)$
2	$v_0\left(1+\frac{2}{100}\right)^2$
3	$v_0\left(1+\frac{2}{100}\right)^3$
$\vdots$	$\vdots$
n	$v_0\left(1+\frac{2}{100}\right)^n$

$$\upsilon_0\left(1+\frac{2}{100}\right)^n = 10^5\, m/s \Rightarrow 10\left(1+\frac{2}{100}\right)^n = 10^5 \Rightarrow (1.02)^n = 10^4$$

$$\Rightarrow n = \frac{\log 10^4}{\log(1.02)} = 465.1 \quad \Rightarrow n = 465$$

So, the length of the collider

$$L = (10\times 0.02) + \left[10\left(1+\frac{2}{100}\right)\times 0.02\right] + \left[10\left(1+\frac{2}{100}\right)^2 \times 0.02\right] + \dots\dots\dots + \left[10\left(1+\frac{2}{100}\right)^{n-1} \times 0.02\right]$$

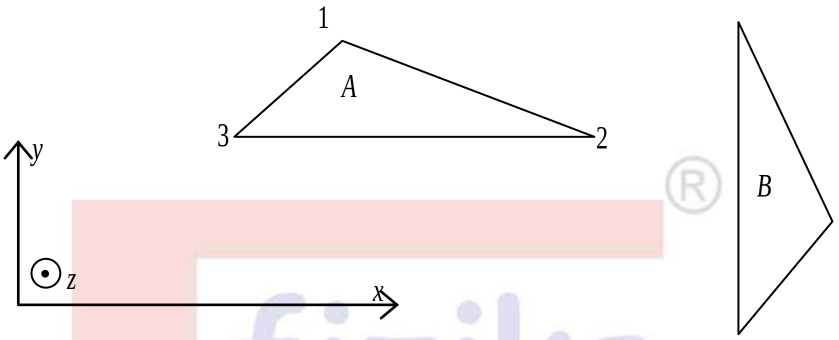
$$L = 10\times 0.02\times\left[1 + 1.02 + (1.02)^2 + \dots .(1.02)^{n-1}\right] = 0.2\times\frac{(1.02)^n - 1}{1.02 - 1}$$

$$L = 0.2\times\frac{10^4 - 1}{0.02} = 10\times 9999 = 99990 = 99.99\,km \approx 100\ km$$

General Physics

SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1.



Refer to the figure above. If the z-axis points out of the plane of the paper towards you, the triangle marked 'A' can be transformed (and suitably re-positioned) to the triangle marked 'B' by

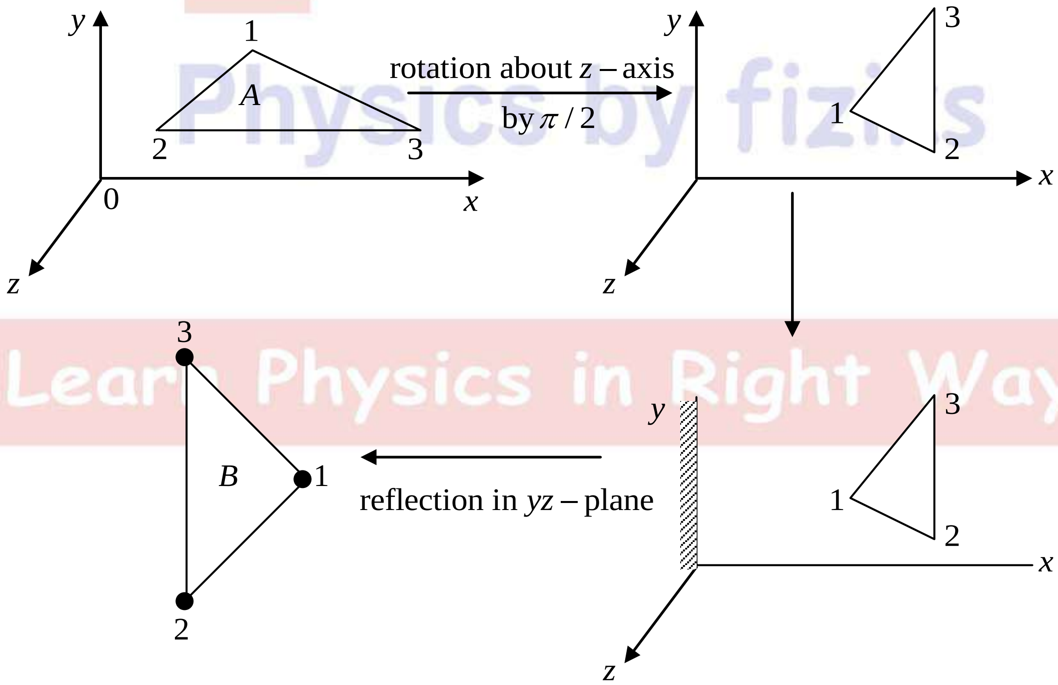
(a) Rotation about z - direction by $\pi/2$, then reflection in the yz -plane

(b) Reflection in the xz - plane, then rotation by $-\pi/2$ about z-direction

(c) Reflection in the yz - plane, then rotation by $\pi/2$ about z-direction

(d) Rotation about x - direction by $\pi/2$, then rotation by $-\pi/2$ in the yz -plane

Ans.: (a)



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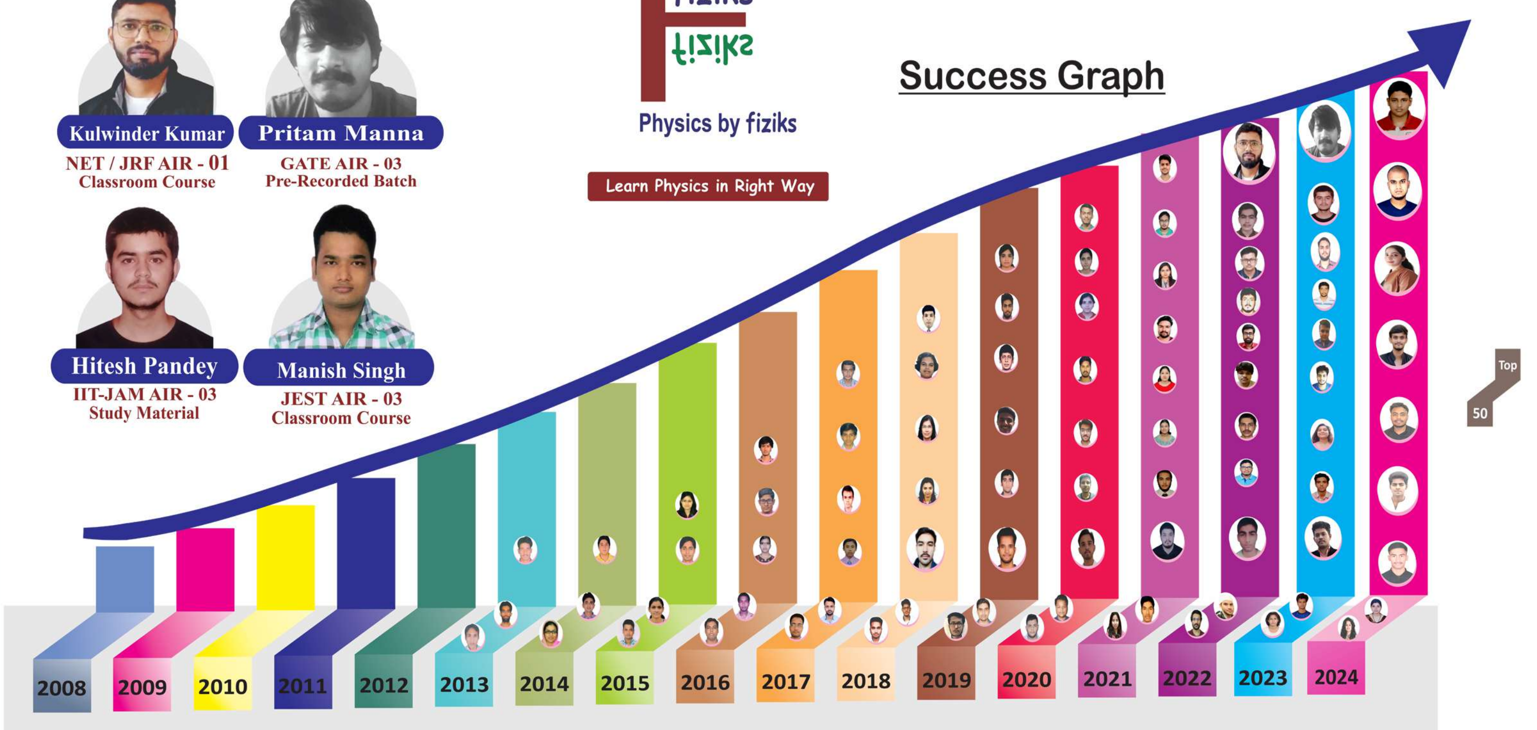

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