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JEST Physics-2024

Solution-Mathematical Methods

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Section A (MCQ)

Correct answer: +1, wrong answer: −1/3.

Q12. Let $(G,\circ)$ be a discrete group of order 4 where the group operation '$\circ$' among the various elements of $G=\{e,a,b,c\}$ is given by the following multiplication table:

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Which of the following is correct?

(A) $(G,\circ)$ is non-cyclic and abelian.

(B) $(G,\circ)$ is cyclic and abelian.

(C) $(G,\circ)$ is cyclic and non-abelian.

(D) $(G,\circ)$ is non-cyclic and non-abelian.

Ans.: (A)

Q13. Consider the Fourier transform of a function $f(x)$ defined as

$$g(p)=\int_{-\infty}^{\infty} f(x)\exp(ipx)\,dx, \text{ where } f(x)=\frac{1}{\sqrt{|x|}}$$

Which of the following is the correct form of $g(p)$ for some constant?

(A) $g(p)=\frac{\beta}{|p|}$

(B) $g(p)=\frac{\beta}{p}$

(C) $g(p)=\frac{\beta}{p^2}$

(D) $g(p)=\frac{\beta}{\sqrt{|p|}}$

Ans.: (D)

Solution:

$$g(p)=\int_{-\infty}^{\infty}\frac{1}{\sqrt{|x|}}\cdot e^{ipx}dx \text{. Let } |x|=t^2 \Rightarrow \pm dx = 2t\,dt$$

$$\Rightarrow g(p)\propto\int_{-\infty}^{\infty}\frac{1}{t}e^{ipt^2}(-2tdt)\propto\int_{-\infty}^{\infty}e^{ipt^2}dt$$

$$\because \int_{-\infty}^{\infty}e^{-ax^2+bx}dx=\sqrt{\frac{\pi}{a}}e^{b^2/4a} \quad \Rightarrow a=ip,\ b=0$$

$$\Rightarrow g(p)\propto\sqrt{\frac{\pi}{ip}}e^0=\frac{\beta}{\sqrt{p}},\ \beta\rightarrow\text{constant}$$

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Section B (MCQ)

Correct answer: +3, wrong answer: −1.

Q3. The singular matrix $A=\begin{pmatrix}2&3&3\\3&6&3\\3&3&6\end{pmatrix}$ commutes with the matrix $B=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$

The eigenvalues of A are

(A) (0, 3, 13) (B) (0, 3, 11) (C) (0, 0, 12) (D) (0, 2, 5)

Ans.: (B)

Solution: Sum of eigenvalues of A i.e. $\sum\lambda_i=\text{Trace A}=2+6+6=14$

(A) $\sum\lambda_i=0+3+13=16$ (B) $\sum\lambda_i=0+3+11=14$

(C) $\sum\lambda_i=0+0+12=12$ (D) $\sum\lambda_i=0+2+5=7$

Q4. A polynomial $C_n(x)$ of degree n defined on the domain $x\in[-1,1]$ satisfies the differential equation $\left(1-x^2\right)\frac{d^2C_n}{dx^2}-x\frac{dC_n}{dx}+n^2C_n=0$

The polynomials satisfy the orthogonality relation

$\int_{-1}^{1}\sigma(x)C_n(x)C_m(x)dx=0$ for $n\neq m$. What is $\sigma(x)$?

(A) $\left(1-x^2\right)^{-1/2}$ (B) $\left(1-x^2\right)$

(C) 1 (D) $\exp\left(-x^2\right)$

Ans.: (A)

Solution: The given D.E. has form of Legendre D.E. so $c_n(x)$ is Legendre polynomial. Thus

$c_0(x)=1,\ c_1(x)=x,\ c_2(x)=\frac{1}{2}\left(3x^2-1\right),\ c_3=\frac{1}{2}\left(5x^3-3x\right)$

Let us check the given option for $n\neq m$

(A) $I=\int_{-1}^{1}\left(1-x^2\right)^{-1/2}c_0(x)c_1(x)dx=\int_{-1}^{1}\frac{x}{\sqrt{1-x^2}}dx$

Put $1-x^2=t^2\Rightarrow -2xdx=2tdt\Rightarrow xdx=-tdt\ \Rightarrow I=\int_0^0\left(-\frac{t}{t}\right)dt=0$

(B) $I=\int_{-1}^{1}\left(1-x^2\right)c_0(x)c_1(x)dx=\int_{-1}^{1}\left(1-x^2\right)xdx\neq 0$

(C) $I=\int_{-1}^{1}1\cdot c_0(x)c_1(x)\,dx=\int_{-1}^{1}x\,dx=0$ also check $I=\int_{-1}^{1}1\cdot c_0(x)c_2(x)\,dx=\int_{-1}^{1}\frac{1}{2}\left(3x^2-1\right)dx\neq 0$

(D) $I=\int_{-1}^{1}e^{-x^2}c_0(x)c_1(x)\,dx=\int_{-1}^{1}xe^{-x^2}dx=0$ also check

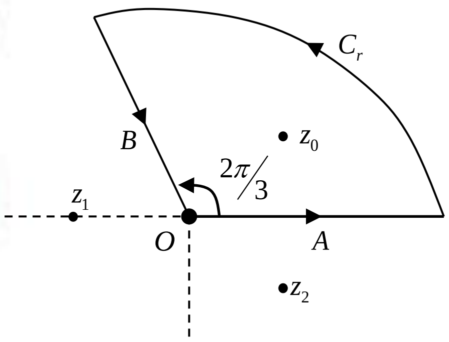
$$I=\int_{-1}^{1}e^{-x^2}c_0(x)c_2(x)\,dx=\int_{-1}^{1}e^{-x^2}\frac{1}{2}\left(3x^2-1\right)dx\neq 0$$

Q14. What is the value of $\int_0^\infty \frac{dx}{1+x^3}$?

(A) $\frac{2\pi}{3\sqrt{3}}$ (B) $\frac{\pi}{3\sqrt{3}}$ (C) $\frac{2\pi}{\sqrt{3}}$ (D) $\frac{\pi}{3}$

Ans.: (A)

Solution: $I=\int_0^\infty\frac{1}{1+x^3}dx$. Let $f(z)=\frac{1}{1+z^3}$



$\because\ z=re^{i\theta}$ along A, $\theta=0^\circ$ so $z=r\Rightarrow z^3=r^3$

along B: $z=re^{i2\pi/3}\Rightarrow z^3=r^3$

Consider the contour shown in figure

$$\oint_c\frac{1}{1+z^3}dz=\int_A\frac{1}{1+z^3}dz+\int_{C_r}\frac{1}{1+z^3}dz+\int_B\frac{1}{1+z^3}dz$$

$$\lim_{r\to\infty}\int_{C_r}\frac{1}{1+z^3}dz=0,\quad \int_A\frac{1}{1+z^3}dz=\int_0^\infty\frac{1}{1+r^3}dr=I,\qquad \because z=re^{i\theta},\theta=0^\circ$$

$$\int_B\frac{1}{1+z^3}dz=\int_\infty^0\frac{e^{i2\pi/3}dr}{1+r^3}=-e^{i2\pi/3}I\qquad \text{along B: } z=re^{i2\pi/3}\Rightarrow z^3=r^3$$

$$\Rightarrow\oint_c\frac{1}{1+z^3}dz=I-Ie^{i2\pi/3}=2\pi i\sum\text{Res}\,f(z)$$

$f(z)=\frac{1}{1+z^3}$ is non-analytic if $z^3+1=0\Rightarrow z=(-1)^{1/3}=e^{i\left(\frac{\pi+2k\pi}{3}\right)}$, $k=0,1,2$

$\Rightarrow z_0=e^{i\pi/3},z_1=e^{i\pi},z_3=e^{i5\pi/3}$. Thus $\text{Res}(z=z_0)=\lim_{z\to z_0}\frac{z-z_0}{1+z^3}=\left.\frac{1}{3z^2}\right|_{z=z_0}=\frac{1}{3z_0^2}=\frac{1}{3e^{i2\pi/3}}$

Thus $I\left(1-e^{i2\pi/3}\right)=2\pi i\left(\frac{1}{3e^{2\pi/3}}\right)\Rightarrow I\left(e^{-i\pi/3}-e^{i\pi/3}\right)=2\pi i\left(\frac{1}{3e^{i\pi}}\right)=-\frac{\pi}{3}2i$

$$\Rightarrow I=\frac{\pi}{3}\frac{2i}{e^{i\pi/3}-e^{-i\pi/3}}=\frac{\pi}{3}\cdot\frac{1}{\sin\pi/3}=\frac{\pi}{3}\cdot\frac{1}{\sqrt{3}/2}=\frac{2\pi}{3\sqrt{3}}$$

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q3. Consider the rotation matrix $R=\begin{pmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{pmatrix}$

Let ϕ be the angle of rotation. What is the value of $\sec^2\phi$?

Answer: 4

Solution:

$$R^TR=\begin{bmatrix} 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \end{bmatrix}\begin{bmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

$R\rightarrow$ orthogonal matrix

$$\det R=\begin{vmatrix} 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{vmatrix}=\frac{2}{3}\left(\frac{4}{9}+\frac{2}{9}\right)+\frac{1}{3}\left(\frac{4}{9}-\frac{1}{9}\right)+\frac{2}{3}\left(\frac{4}{5}+\frac{2}{9}\right)=\frac{4}{9}+\frac{1}{9}+\frac{4}{9}=1$$

$\Rightarrow$ It involves rotation

Eigenvalues of matrix R are $\lambda_1, \lambda_2, 1$

We will write a matrix $D=C^{-1}RC$ in new coordinate system where z axis is the axis of rotation

i.e. $D=\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ compare with $\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Thus $2\cos\phi+1=\sum\lambda_i=\text{Trace R}=\frac{2}{3}+\frac{2}{3}+\frac{2}{4}=\frac{6}{3}=2$

$\Rightarrow \cos\phi=\frac{1}{2}\Rightarrow \sec\phi=2\Rightarrow \sec^2\phi=4$

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Solution- Classical Mechanics

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Section A (MCQ)

Correct answer: +1, wrong answer: −1/3.

Q2. A classical system has the following action: $S=\int\left(\dot{q}^2+\alpha q\dot{q}+\beta q^2\dot{q}\right)dt$ where q is the generalized coordinate, and α and β are constants. Which of the following statements is true about the dynamics of the system?

(A) The dynamics depends only on α

(B) The dynamics is independent of α and β

(C) The dynamics depends only on β

(D) The dynamics depends on the ratio $\frac{\alpha}{\beta}$

Ans.: (B)

Solution: $\because L=\dot{q}^2+\alpha q\dot{q}+\beta q^2\dot{q}$

$$\Rightarrow \frac{\partial L}{\partial \dot{q}}=2\dot{q}+\alpha q+\beta q^2,\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=2\ddot{q}+\alpha\dot{q}+2\beta q\dot{q} \text{ and } \frac{\partial L}{\partial q}=\alpha\dot{q}+2\beta q\dot{q}$$

LEM $\boxed{\ddot{q}=0}$. It is independent of α and β.

Q7. Two classical particles moving in three dimensions interact via the potential

$$V=K\left[\left(x_1^2+y_1^2\right)+\left(x_2^2+y_2^2\right)+\left(z_1-z_2\right)^2\right],$$

where K is a constant, and $\left(x_1,y_1,z_1\right)$ and $\left(x_2,y_2,z_2\right)$ are the Cartesian coordinates of the two particles. Let $\left(p_1^x,p_1^y,p_1^z\right)$ and $\left(p_2^x,p_2^y,p_2^z\right)$ be the components of the linear momenta of the two particles, and $\left(L_1^x,L_1^y,L_1^z\right)$ and $\left(L_2^x,L_2^y,L_2^z\right)$ the components of the corresponding angular momenta. Which of the following statements is true?

(A) L_1^z,L_2^z and $\left(p_1^z+p_2^z\right)$ are conserved

(B) L_1^z and L_2^z are not separately conserved but $L_1^z+L_2^z$ is conserved

(C) $\left(L_1^x+L_2^x\right),\left(L_1^y+L_2^y\right),\left(L_1^z+L_2^z\right)$ are conserved

(D) $\left(L_1^x+L_2^x\right)$ and $\left(L_1^y+L_2^y\right)$ are conserved

Ans.: (A)

Solution:

(i) $L_1^z=x_1p_{y1}-y_1p_{x1}$

$$H=\frac{p_{1x}^2+p_{1y}^2}{2m}+\frac{p_{2x}^2+p_{2y}^2}{2m}+k\left[x_1^2+y_1^2+x_2^2+y_2^2+z_1^2+z_2^2-2z_1z_2\right]$$

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$$\left[L_1^z, H\right] = x_1\left[p_{y1}, y_1^z\right]k + p_{y1}\left[x_1 + \frac{p_{x1}^2}{2m}\right] - y_1\left[p_{x1}, x_1^2\right]k - p_{x1}\left[y_1, \frac{p_{y1}^2}{2m}\right]$$

$$\Rightarrow \left[L_1^z, H\right] = -2x_1y_1k + \frac{p_{y1}p_{x1}}{m} + 2x_1y_1k - \frac{p_{x1}p_{y1}}{m}$$

$\Rightarrow \left[L_1^z, H\right] = 0$, Similarly $\left[L_2^z, H\right] = 0$

(ii) $\left[p_{1z} + p_{2z}, H\right] = k\left[p_{1z}, z_1^2\right] - 2kz_2\left[p_{1z}, z_1\right] + k\left[p_{2z}, z_2^2\right] - 2kz_1\left[p_{2z}, z_2\right]$

$\Rightarrow \left[p_{1z} + p_{2z}, H\right] = -2kz_1 + 2kz_2 - 2kz_2 + 2kz_1 \quad \Rightarrow \left[p_{1z} + p_{2z}, H\right] = 0$

So, L_1^z, L_2^z and $p_{1z} + p_{2z}$ are conserved quantities during the dynamics of the system.

Q8. A cylindrical rigid block has principal moments of inertia I about the symmetry axis and $2I$ about each of the perpendicular axes passing through the center of mass. At some instant, the components of angular momentum about the center of mass in the body-fixed principal axis frame is (l, l, l), with $l > 0$. What is the cosine of the angle between the angular momentum and the angular velocity?

(A) $\frac{2}{3}$ (B) $\frac{2}{\sqrt{6}}$ (C) $\frac{2\sqrt{2}}{3}$ (D) $\frac{5}{3\sqrt{3}}$

Ans.: (C)

Solution:

$\vec{L} = \overline{\overline{I}}\,\vec{\omega}$

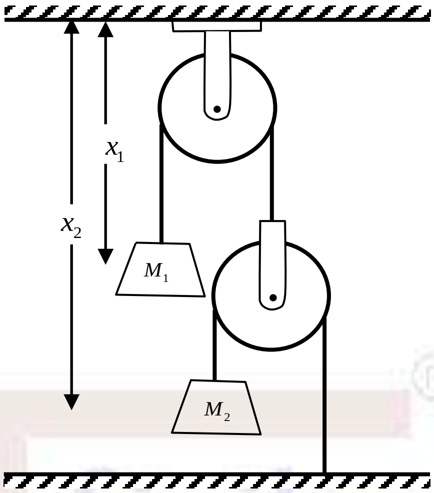
$$\begin{pmatrix} l \\ l \\ l \end{pmatrix} = \begin{pmatrix} 2I & 0 & 0 \\ 0 & 2I & 0 \\ 0 & 0 & I \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \Rightarrow \omega_x = \frac{l}{2I}, \omega_y = \frac{l}{2I}, \omega_z = \frac{l}{I}$$

$$\vec{L} = l\hat{x} + l\hat{y} + l\hat{z}, \quad \vec{\omega} = \frac{l}{2I}\hat{x} + \frac{l}{2I}\hat{y} + \frac{l}{I}\hat{z}$$

$$\cos\theta = \frac{\vec{L}\cdot\vec{\omega}}{L\omega} = \frac{l^2/2I + l^2/2I + l^2/I}{\sqrt{l^2 + l^2 + l^2}\sqrt{l^2/4I^2 + l^2/4I^2 + l^2/I^2}} = \frac{4l^2/2I}{l\sqrt{3}\sqrt{6l^2/4I^2}} = \frac{4l^2/2I}{l^2/2I\,\sqrt{18}} = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}}$$

$$\Rightarrow \cos\theta = \frac{2\sqrt{2}}{3}$$

Q9. Consider a mass-pulley system as shown in the figure. The heights of the blocks as measured from the ceiling are x_1 and x_2, as shown in the figure.



What is the constraint between x_1 and x_2?

(A) $x_2 + 2x_1 =$ constant (B) $x_2 - x_1 =$ constant

(C) $x_2 + x_1 =$ constant (D) They are unconstrained

Ans.: (A)

Solution:

$T_1 = 2T_2$

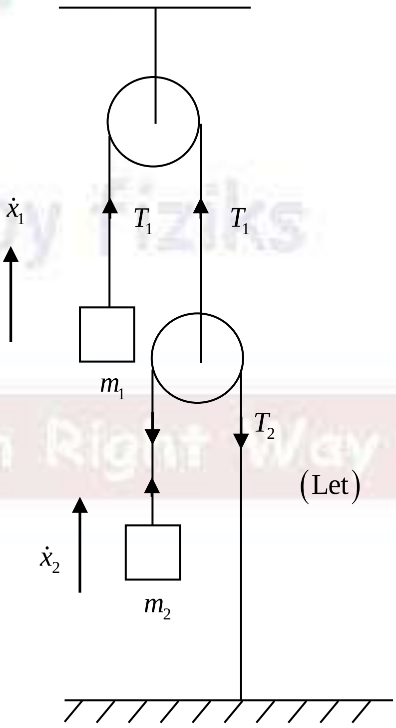
$T_1\dot{x}_1 + T_2\dot{x}_2 = 0$

$2T_2\dot{x}_1 + T_2\dot{x}_2 = 0$

$2\dot{x}_1 + \dot{x}_2 = 0$

$\frac{d}{dt}\left(2x_1 + x_2\right) = 0$

$2x_1 + x_2 =$ constant



Q10. Let q and p be the canonical phase space coordinates of a system, where q is the generalized coordinate and p is the generalized momentum. Let us make a transformation of the generalized coordinate as $Q = q^2$. Which of the following functions is canonically conjugate to Q?

(A) $\frac{p^2}{2q^2}$ (B) $\frac{p}{q}$ (C) p^2 (D) $\frac{p}{2q}$

Ans.: (D)

Solution: $\left[\frac{p}{2q}, Q\right] = \left[\frac{p}{2q}, q^2\right] = \frac{1}{2q}\left[p, q^2\right] = 1$ $\quad \because Q = q^2$

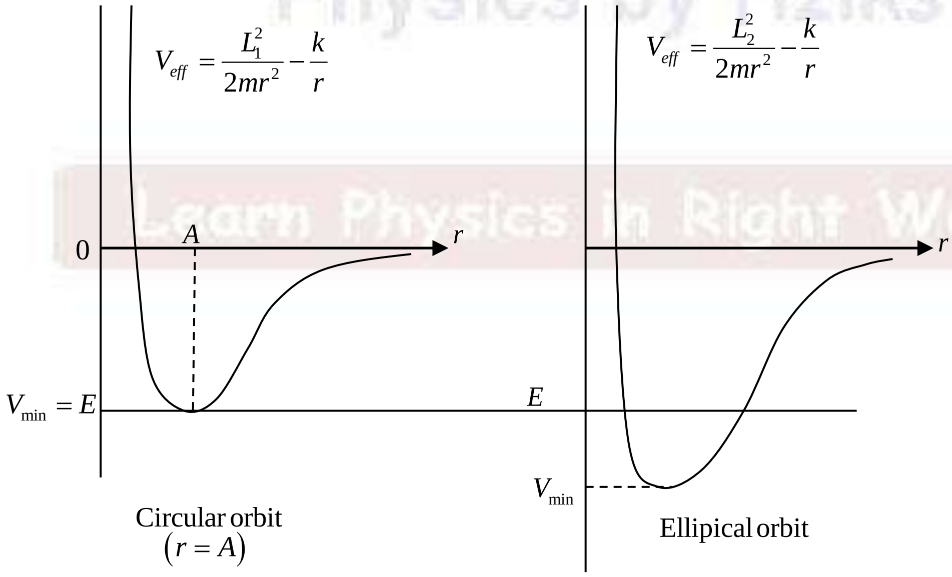
So, $\frac{p}{2q}$ is canonically conjugate to Q.

Q18. A satellite of mass 2000 kg is placed in an elliptic orbit around Earth with semi major axis A. Assume that the total energy of the orbiting satellite is E and the angular momentum is L. Through a series of manoeuvres, the elliptic orbit is changed to a circular orbit with radius A. For the orbit change described, which of the following is true?

(A) E does not change, but L changes.
(B) E changes, but L does not change.
(C) Both E and L change.
(D) Neither E nor L changes.

Ans.: (A)

Solution:



In both cases energy is equal to $E = -\frac{GMm}{2A}$

but angular momentum will be different as can be observed from the plots of effective potentials.

Section B (MCQ)

Correct answer: +3, wrong answer: −1.

Q5. Consider a particle of mass m and nonzero angular momentum ℓ subjected to a central force potential $V(r)=k\ln r$, where k is a positive constant. What is the radius R at which it can have a circular orbit? Will the circular orbit be stable or unstable?

(A) $R=\frac{\ell}{\sqrt{2km}}$ and stable

(B) $R=\frac{\ell}{\sqrt{km}}$ and unstable

(C) $R=\frac{\ell}{\sqrt{km}}$ and stable

(D) $R=\frac{\ell}{\sqrt{2km}}$ and unstable

Ans.: (C)

Solution:

$$V_{eff}=\frac{l^2}{2mr^2}+k\ln r$$

$$\frac{\partial V_{eff}}{\partial r}=0\Rightarrow-\frac{l^2}{mr_0^3}+\frac{k}{r_0}=0\Rightarrow r_0=\frac{l}{\sqrt{km}}$$

$$\frac{\partial^2 V_{eff}}{\partial r^2}=\frac{3l^2}{mr^4}-\frac{k}{r^2}$$

$$\left.\frac{\partial^2 V_{eff}}{\partial r^2}\right|_{r=r_0}=\frac{2mk^2}{l}>0 \text{ stable circular orbit.}$$

Q11. Two trains, each having proper length L_0 are moving towards each other with the same speed v but in opposite directions as measured by an observer in an inertial frame. What is the length of one of the trains as measured by an observer in the other train?

(A) $L_0\left(\frac{c^2-v^2}{c^2+v^2}\right)$

(B) $L_0\sqrt{\left(\frac{c^2-v^2}{c^2+v^2}\right)}$

(C) $L_0\sqrt{1-\frac{v^2}{4c^2}}$

(D) $L_0\sqrt{1-\frac{v^2}{c^2}}$

Ans.: (A)

Solution: $v_{1E}=v_1$, $v_{2E}=-v$. Thus $v_{12}=\frac{v-(-v)}{1-\frac{v(-v)}{c^2}}=\frac{2v}{1+\frac{v^2}{c^2}}$

$$L_{12}=L_0\sqrt{1-v_{12}^2/c^2}=L_0\sqrt{1-\frac{4v^2/c^2}{\left(1+\frac{v^2}{c^2}\right)^2}}\Rightarrow L_{12}=L_0\left(\frac{c^2-v^2}{c^2+v^2}\right)$$

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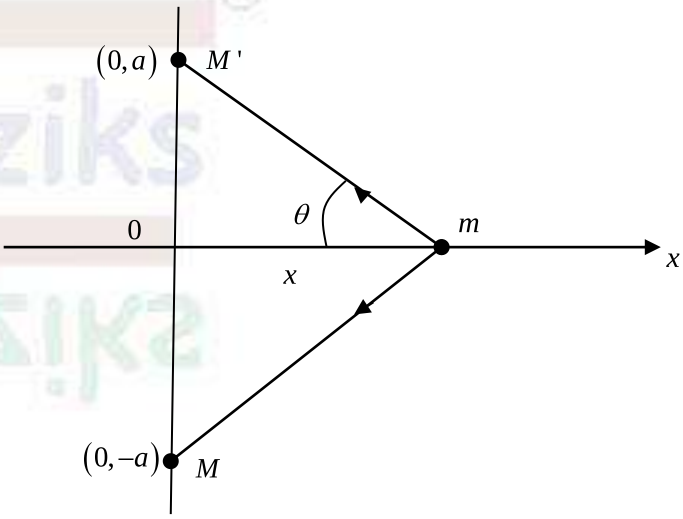
Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q6. A point mass m constrained to move along the X-axis is under the influence of gravitational attraction from two point particles each of mass M fixed at the points $(x=0, y=a)$ and $(x=0, y=-a)$. Find the time period of small oscillations of the mass m in units of $\pi\sqrt{\frac{a^3}{8GM}}$, where G is the universal gravitational constant.

Answer: 4

Solution:



Net force acting on mass m is

$$F = -2F_x = -2\frac{GMm}{(x^2+a^2)}\cos\theta$$

$$ma = -\frac{2GMm\,x}{(x^2+a^2)^{3/2}}$$

$$a = -\frac{2GM}{a^3}x \qquad [\because x \ll a]$$

$$\omega = \sqrt{\frac{2GM}{a^3}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{a^3}{2GM}} = 4\pi\sqrt{\frac{a^3}{8GM}}$$

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Solution- Electromagnetic Theory

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Section A (MCQ)

Correct answer: +1, wrong answer: −1/3.

Q1. Three equal charges $+q$ are placed at the corners of an equilateral triangle. A test charge constrained to move on the plane of the triangle is placed at the centre of the triangle. Which of the following statements about the test charge is true?

(A) Stability of the equilibrium depends on the sign of the test charge.

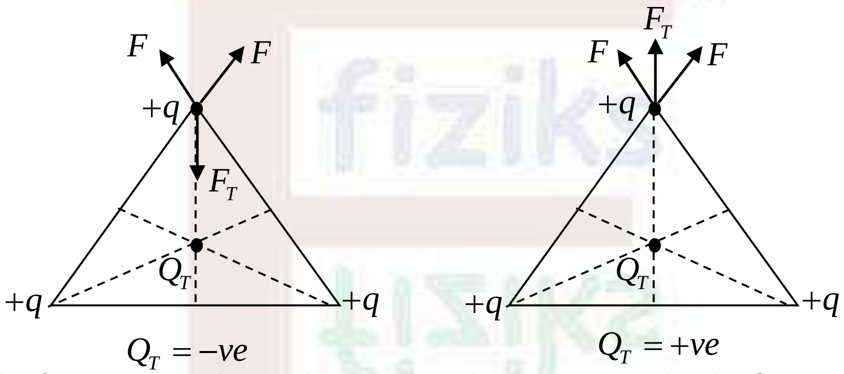
(B) It is in a stable equilibrium.

(C) It is not in an equilibrium.

(D) It is in an unstable equilibrium.

Ans.: (A)

Solution:



Let F be the force on $+q$ charge due to other charges and F_T be the force on $+q$ charge due to test charge Q_T.

The system will be in equilibrium if net force on any charge $+q$ is zero.

$\Rightarrow$ stability of the equilibrium depends on the sign of the test charge.

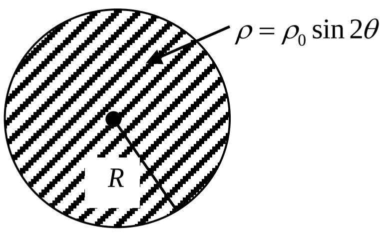
Q11. A solid sphere of radius R has a volume charge density $\rho = \rho_0 \sin 2\theta$. How does the leading term in the electrostatic potential depend on the distance r far away from the charged sphere?

(A) r (B) $\frac{1}{r}$

(C) $\frac{1}{r^2}$ (D) Does not depend on r

Ans.: (C)

Solution: According to multipole expansion



$$V(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{1}{r}\int_V \rho(r')d\tau' + \frac{1}{r^2}\int_V r'\cos\theta'\rho(r')d\tau' + \ldots\right]$$

$$\int_V \rho d\tau' = \int_0^R\int_0^\pi\int_0^{2\pi} \rho_0 \sin 2\theta' \times r'^2 \sin\theta dr' d\theta' d\phi' = \rho_0 \frac{R^3}{3} \times 2\pi \int_0^\pi 2\sin^2\theta' \cos\theta' d\theta' = 0$$

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$$\int_V r'\cos\theta\rho d\tau' = \int_0^R\int_0^\pi\int_0^{2\pi} r'\cos\theta\left(\rho_0\sin 2\theta'\right)r'^2\sin\theta' dr d\theta' d\phi'$$

$$\Rightarrow \int_V r'\cos\theta\rho d\tau' = \rho_0\times\frac{R^4}{4}\times 2\pi\int_0^\pi 2\sin^2\theta\cos^2\theta d\phi \neq 0. \quad \text{Thus } V\propto\frac{1}{r^2}$$

Q14. What is the power of a light source emitting photons of wavelength 600 nm at the rate of one photon per second? Planck's constant $h = 6.6\times10^{-34}$ Joule sec and speed of light $c = 3\times10^8$ m/sec.

(A) $3.3\times10^{-18}W$ (B) $3.3\times10^{-19}W$

(C) $6.0\times10^{-19}W$ (D) $6.0\times10^{-18}W$

Ans.: (B)

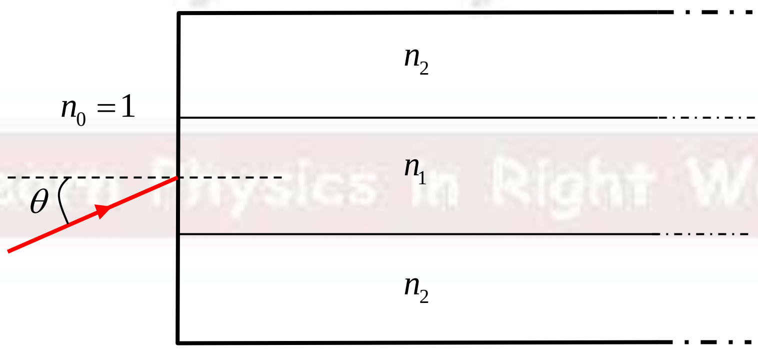
Solution: Given a leaser source $\lambda = 600\,\text{nm}$, $\frac{dN}{dt} = 1$ Photon/sec, $h = 6.6\times10^{-34}$ Js, $c = 3\times10^8$ m/s

$$P = \frac{dN}{dt}\frac{hc}{\lambda} = \frac{NE}{t} = \frac{U}{t}$$, i.e. energy emitted per sec.

$$P = \frac{1\times6.6\times10^{-34}\,Js\times3\times10^8\,ms^{-1}}{600\times10^{-9}\,m} = 3.3\times10^{-19}W$$

Q17. A step index optical fiber has refractive indices $n_1 = 1.474$ for core region and $n_2 = 1.470$ for the cladding region. A ray of light is incident from air into the core through the cross section of the fiber at an angle θ with the normal. What is the limiting value of θ below which the light ray will be totally internally reflected?

Refractive index of air is taken as 1.



(A) 4.222° (B) 58.194°

(C) 2.862° (D) 6.229°

Ans.: (D)

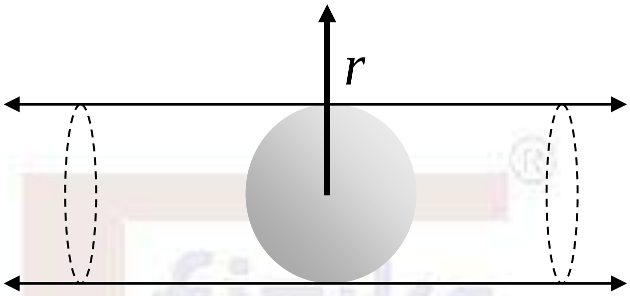
Solution:

$$\theta_a = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right) = \sin^{-1}\left(\sqrt{(1.474)^2 - (1.470)^2}\right) = \sin^{-1}\sqrt{(0.0118)} = \sin^{-1}(0.1085) \approx 6.229°$$

Section B (MCQ)

Correct answer: +3, wrong answer: −1.

Q12. An infinitely long cylinder of radius R has uniform volume charge density. A spherical region of radius R is carved out of it, as shown in the figure. At what value of r (the radial coordinate in a cylindrical system, with origin at the center of the sphere) is the electric field maximum?



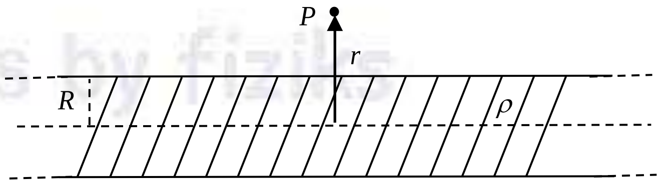
(A) $r = \frac{3}{2}R$ (B) $r = R$

(C) $r = \frac{2}{3}R$ (D) $r = \frac{4}{3}R$

Ans.: (D)

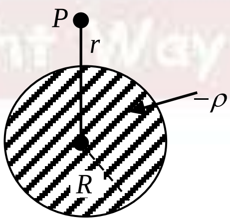
Solution:

Let charge density of cylinder is ρ and assume a sphere of charge density $-\rho$ is imbedded inside the cylinder.



Field outside long cylinder $\vec{E}_1 = \frac{\rho R^2}{2\in_0 r}\hat{r}$,

Field outside Sphere $\vec{E}_2 = -\frac{\rho R^3}{3\in_0 r^2}\hat{r}$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{\rho R^2}{2\in_0 r} - \frac{\rho R^3}{3\in_0 r^2}\right)\hat{r}$$

For maximum or minimum value of E :

$$\frac{dE}{dr} = 0 \Rightarrow \frac{\rho R^2}{\in_0}\left[-\frac{1}{2r^2} + \frac{2R}{3r^3}\right] = 0 \Rightarrow \frac{1}{2r^2} = \frac{2R}{3r^3} \Rightarrow r = \frac{4}{3}R$$

For maximum value of E : $\frac{d^2E}{dr^2} = -ve$

$$\frac{d^2E}{dr^2} = \frac{\rho R^2}{\in_0}\left[\frac{1}{r^3} - \frac{2R}{r^4}\right] = \frac{\rho R^2}{\in_0 r^3}\left[1 - \frac{2R}{r}\right] = \frac{\rho R^2}{\in_0}\frac{27}{64R^3}\left(1 - \frac{3}{2}\right) \Rightarrow \frac{d^2E}{dr^2} = -ve$$

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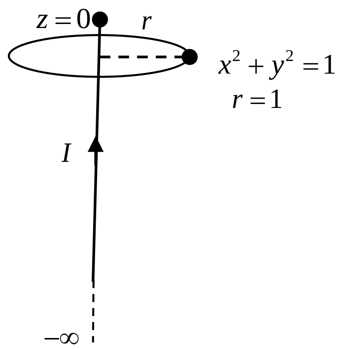
Section C (NAT) Correct answer: +3, wrong answer: 0.

Q2. A semi-infinite, thin wire extending from $-\infty$ to zero along the z-axis carries a constant current I in the positive z-direction. The wire is charge-neutral except at $z=0$, where the inflowing charge is accumulated. What is the absolute value of the line integral $\frac{4}{\mu_0 I}\oint \vec{B}\cdot d\vec{l}$ along the circle $x^2+y^2=1$? $\vec{B}$ is the magnetic field and μ_0 is the permeability in free space. Assume that the accumulated charge at $z=0$ is a point charge.

Answer: 2

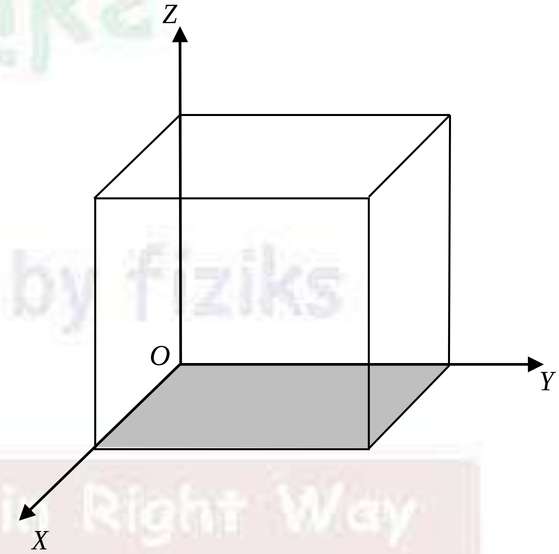
Solution: Magnetic field due to semi-infinite wire at one end is given by



$$\vec{B}=\frac{\mu_0 I}{4\pi r}\hat{\phi}$$

$$\oint_{\text{line}} \vec{B}\cdot d\vec{l}=\int_0^{2\pi}\left(\frac{\mu_0 I}{4\pi r}\hat{\phi}\right)\cdot\left(rd\phi\hat{\phi}\right)=\frac{\mu_0 I}{4\pi r}\times 2\pi r=\frac{\mu_0 I}{2}\Rightarrow\frac{4}{\mu_0 I}\oint_{line}\vec{B}\cdot d\vec{l}=2$$

Q8. A magnetic vector potential is given as $\vec{A}=6\hat{i}+yz^2\hat{j}+(3y+z)\hat{k}$. Find the corresponding outgoing magnetic flux through the five faces (excluding the shaded one) of a unit cube with one corner at the origin, as shown in the figure.



Answer: 0

Solution: $\because \oint_s \vec{B}\cdot d\vec{a}=0\Rightarrow\int_{s_1}\vec{B}\cdot d\vec{a}+\int_{\substack{\text{rest}\\ \text{surface}}}\vec{B}\cdot d\vec{a}=0\Rightarrow\int_{\substack{\text{rest}\\ \text{surface}}}\vec{B}\cdot d\vec{a}=-\int_{s_1}\vec{B}\cdot d\vec{a}=-\int_{s_1}\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{a}$

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 6 & yz^2 & 3y+z\end{vmatrix}=\hat{i}(3-2yz)-\hat{j}(0-0)+\hat{k}(0-0)=\hat{i}(3-2yz)=3\hat{i} \qquad \because z=0$$

$$\because d\vec{a}=-dxdy\hat{k}\Rightarrow\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{a}=0\ \Rightarrow\int_{\substack{\text{rest}\\ \text{surface}}}\vec{B}\cdot d\vec{a}=0$$

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Solution- Quantum Mechanics

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Section A (MCQ) -Correct answer: +1, wrong answer: −1/3.

Q3. A and B are 2×2 Hermitian matrices. $|a_1\rangle$ and $|a_2\rangle$ are two linearly independent eigenvectors of A. Consider the following statements:

1. If $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B, then $[A,B]=0$.

2. If $[A,B]=0$, then $|a_1\rangle$ and $|a_2\rangle$ are eigenvectors of B.

Mark the correct option.

(A) Both statements 1 and 2 are true.

(B) Statement 2 is true but statement 1 is false.

(C) Statement 1 is true but statement 2 is false.

(D) Both statements 1 and 2 are false.

Ans.: (C)

Solution: Statement 1:

Let $A|a_1\rangle=\lambda_1|a_1\rangle$ and $A|a_2\rangle=\lambda_2|a_2\rangle$ also $B|a_1\rangle=\beta_1|a_1\rangle$ and $B|a_2\rangle=\beta_2|a_2\rangle$

Now $[A,B]|a_1\rangle=AB|a_1\rangle-BA|a_1\rangle=\beta_1 A|a_1\rangle-\lambda_1 B|a_1\rangle=\beta_1\lambda_1|a_1\rangle-\lambda_1\beta_1|a_1\rangle=0$

$\Rightarrow [A,B]=0$. Thus statement 1 is correct.

Statement 2: If $[A,B]=0$, it does not necessarily imply that every eigenvector of A is automatically an eigenvector of B. Thus statement 2 is not generally true.

Q6. A particle moving in one dimension has the wave function

$$\psi(x)=\exp\left[-\alpha\left(x-\frac{ik_0}{\alpha}\right)^2\right]\sin^2(k_1x)$$

where α is real positive and k_0,k_1 are real. The expectation value of momentum is

(A) 0 (B) $2\hbar k_0$ (C) $\hbar k_0$ (D) $\hbar k_1$

Ans.: (B)

Solution: Given $\psi(x)=\exp\left[-\alpha\left(x-\frac{ik_0}{\alpha}\right)^2\right]\sin^2(k_1x)=\exp\left[-\alpha\left(x^2-\frac{k_0^2}{\alpha^2}-\frac{2ik_0}{\alpha}x\right)\right]\sin^2(k_1x)$

$$\Rightarrow \psi(x)=\sin^2(k_1x)\exp\left[-\alpha x^2+\frac{k_0^2}{\alpha}+2ik_0x\right]=\sin^2(k_1x)e^{\left(-\alpha x^2+\frac{k_0^2}{\alpha}\right)}\cdot e^{2ik_0x}$$

Expectation value of momentum is

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$$\langle p\rangle = \frac{\int_{-\infty}^{+\infty}\psi^*\left(-i\hbar\frac{\partial}{\partial x}\right)\psi dx}{\int_{-\infty}^{+\infty}\psi^*\psi dx} = \frac{(-i\hbar)(2ik_0)\int_{-\infty}^{+\infty}\sin^4(k_1x)^2 e^{\left(-\alpha x^2+\frac{k_0^2}{2}\right)}dx}{\int_{-\infty}^{+\infty}\sin^4(k_1x)e^{2\left(-\alpha x^2+\frac{k_0^2}{2}\right)}dx} = 2\hbar k_0$$

Q23. Two electrons have orbital angular momentum quantum numbers $l_1 = 3$ and $l_1 = 2$, respectively. Let $L^z = L_1^z + L_2^z$, where L_1^z and L_2^z are the z-components of the respective angular momentum operators. How many linearly independent states have L_z quantum number $m = 2$?

(A) 11 (B) 3

(C) 4 (D) 0

Ans.: (C)

Solution: $l_1 = 3, \quad m_1 = -3,-2,-1,0,+1,+2,+3$

$l_2 = 2, \quad m_2 = -2,-1,0,+1,+2$

Since $L^z = L_1^z + L_2^z$. Therefore, $m = m_1 + m_2$

The possible value of m_1 and m_2 for $m = 2$ are

$m_1 = 3, m_2 = m - m_1 = 2-3 = -1$; $\quad m_1 = 2, m_2 = 2-2 = 0$

$m_1 = 1, m_2 = 2-1 = 1$; $\quad m_1 = 0, m_2 = 2-0 = 2$

$m_1 = -1, m_2 = 2+1 = 3$, this is not allowed.

Each of these combinations represents a different linearly independent state. Hence, there are 4 linearly independent state with L^z quantum numbers $m = 2$. Thus, correct options is (c).

Q24. A quantum particle is subjected to the potential $V(x) = ax + bx^2$, where a and b are constants. What is the mean position of the particle in the first excited state?

(A) $\frac{a}{2b}$ (B) $-\frac{a}{2b}$ (C) $-\frac{a}{b}$ (D) $\frac{a}{b}$

Ans.: (B)

Solution: $V(x) = ax + bx^2 = b\left[x^2 + \frac{a}{b}x\right] = b\left[\left(x+\frac{a}{2b}\right)^2 - \left(\frac{a}{2b}\right)^2\right] = b\left(x+\frac{a}{2b}\right)^2 - \frac{a^2}{4b}$

The potential can be interpreted as a harmonic oscillator potential centered at $x_0 = -\frac{a}{2b}$

$\therefore \langle x\rangle = x_0 = -\frac{a}{2b}$

Section B (MCQ): Correct answer: +3, wrong answer: −1.

Q2. A two-level quantum system has the Hamiltonian $H = \hbar\omega_0 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

At $t = 0$, the system is in the state $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

What is the earliest time $t > 0$ at which a measurement of $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ will yield the value –1 with probability one?

(A) $\frac{\pi}{\omega_0}$ (B) $\frac{2\pi}{\omega_0}$ (C) $\frac{\pi}{2\omega_0}$ (D) Never

Ans.: (C)

Solution: The eigenvalues and eigenvectors of $H = \hbar\omega_0 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

are $\lambda_1 = +\hbar\omega_0$, $\lambda_2 = -\hbar\omega_0$ and $|\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

The state $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ can be written as $|\psi(0)\rangle = c_1|\phi_1\rangle + c_2|\phi_2\rangle$

where $c_1 = \langle\phi_1|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}$ and $c_2 = \langle\phi_2|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}$

The state at time t is $|\psi(t)\rangle = e^{iHt/\hbar}|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[e^{-i\omega_0 t}|\phi_1\rangle + e^{i\omega_0 t}|\phi_2\rangle\right] = \frac{1}{2}\begin{bmatrix} e^{-i\omega_0 t} + e^{+i\omega_0 t} \\ e^{-i\omega_0 t} - e^{-i\omega_0 t} \end{bmatrix}$

The eigenvalues and eigenvectors of σ_z are ± 1 and $|\chi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\chi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The state $|\psi(t)\rangle$ can be written as $|\psi(t)\rangle = d_1|\chi_1\rangle + d_2|\chi_2\rangle$

where $d_1 = \langle\chi_1|\psi(t)\rangle = e^{-i\omega_0 t} + e^{+i\omega_0 t}$ and $d_2 = \langle\chi_2|\psi(t)\rangle = e^{-i\omega_0 t} - e^{+i\omega_0 t}$

when σ_z operates on $|\psi(t)\rangle$, the outcome will be either +1 or –1.

The probability of getting value –1 is

$$P(-1) = |d_2|^2 = \left|e^{-i\omega_0 t} - e^{i\omega_0 t}\right|^2 = \left(1 + 1 - 2\cos(\omega_0 t)\right) = 2\sin^2\left(\frac{\omega_0 t}{2}\right)$$

where, $P(-1) = 1$ $\therefore 2\sin^2\left(\frac{\omega_0 t}{2}\right) = 1 \Rightarrow \sin^2\left(\frac{\omega_0 t}{2}\right) = \frac{1}{2} \Rightarrow \frac{\omega_0 t}{2} = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{2\omega_0}$

Thus correct option is (c).

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Q6. Consider a particle of mass m moving in a three-dimensional delta-function potential well $V(\vec{r}) = -\alpha\delta^3(\vec{r})$, where $\alpha > 0$. Which of the following is an allowed expression for the energy of a bound state for some dimensionless proportionality constant $\beta > 0$?

(A) $-\dfrac{\beta\hbar^6}{\alpha^2 m^3}$ (B) $\dfrac{\beta\hbar^6}{\alpha^2 m^3}$ (C) $-\dfrac{\beta\alpha^2 m}{\hbar^2}$ (D) $\dfrac{\beta\alpha^2 m}{\hbar^2}$

Ans.: (A)

Solution: Since bound state energies are always negative. Therefore option (B) and (D) are not correct whereas option (A) and (C) have different dimensions therefore correct option can be obtained by checking dimensions.

Dimension of $\delta^3(r)$ or $D(\delta^3(r)) = [L^{-3}]$. Now $D(V(\vec{r})) = [M^1L^2T^{-2}]$

$$\Rightarrow D(\alpha) = \frac{[M^1L^2T^{-2}]}{[L^{-3}]} = [M^1L^5T^{-2}]$$

In option (A): $D(E) = D\left(-\dfrac{\beta\hbar^6}{\alpha^2 m^3}\right) = \dfrac{[M^1L^2T^{-1}]^6}{[M^2L^{10}T^{-4}][M^3]} = \dfrac{[M^6L^{12}T^{-6}]}{[M^5L^{10}T^{-4}]} = [M^1L^2T^{-2}]$

Thus option (A) is correct answer.

Q7. A particle with energy $E > 0$ is incident from the right $(x > 0)$ on a one-dimensional potential composed of a delta-function barrier at $x = 0$ and a hard wall at $x = -a$:

$$V(x) = \begin{cases} \alpha\delta(x), & x > -a \\ \infty, & x \le -a \end{cases}$$

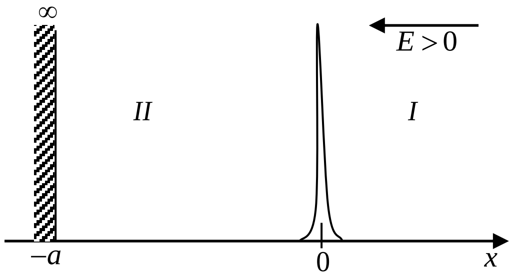
where $\alpha > 0$ and $a > 0$. Let us define $k^2 = \dfrac{2mE}{\hbar^2}$ and the dimensionless quantities: $\xi = ka$ and $\beta = \dfrac{\hbar^2}{2m\alpha a}$. For some energy E the particle reflects from the barrier without any phase shift. Which of the following transcendental equations determines this energy? [Note that in the presence of the delta function barrier, the derivative of the wave function has a discontinuity at $x = 0 : \psi'(0^+) - \psi'(0^-) = \dfrac{\psi(0)}{\beta a}$]

(A) $\tan\xi = \beta\xi$ (B) $\tan\xi = -\beta\xi$

(C) $\tanh\xi = \beta\xi$ (D) $\tanh\xi = -\beta\xi$

Ans.: (B)

Solution: Schrodinger equation in region II is



$$\frac{d^2\psi_{II}}{dx^2}+\frac{2mE}{\hbar^2}\psi_{II}=0 \Rightarrow \psi_{II}=Ae^{ikx}+Be^{-ikx}$$

The ψ_{II} can also be written as

$$\psi_{II}=A\sin(kx)+B\cos(kx)$$

The Schrodinger equation in region I is

$$\frac{d^2\psi_I}{dx^2}+\frac{2mE}{\hbar^2}\psi_I=0 \Rightarrow \psi_I=C\cos(kx)$$

Apply boundary condition

At $x=-a, \psi_{II}=0 \Rightarrow A\sin(-ka)+B\cos(-ka)=0$

$$\Rightarrow -A\sin(ka)+B\cos(ka)=0 \Rightarrow \tan(ka)=\frac{B}{A}$$

Also, at $x=0$, $\psi_{II}=\psi_I \Rightarrow B=C$

Also given, at $x=0$; $\psi'(0^+)-\psi'(0^-)=\frac{\psi(0)}{\beta a}$

$$\Rightarrow Ck\sin(0)-Ak\cos(0)+Bk\sin(0)=\frac{B}{\beta a} \Rightarrow -Ak=\frac{B}{\beta a}$$

$$\Rightarrow \frac{B}{A}=-\beta ka \Rightarrow \tan(ka)=-\beta ka \qquad (\text{Since } \varepsilon=ka)$$

$\Rightarrow \tan(\xi)=-\beta\xi$. Thus correct option is (B)

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Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q4. A quantum harmonic oscillator of mass m and angular frequency ω is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|287\rangle + |288\rangle)$ where $|n\rangle$ denotes the n^{th} normalized energy eigenstate of the harmonic oscillator. Let $L_0 = \sqrt{\frac{\hbar}{m\omega}}$ denote the oscillator size and $\langle \hat{x} \rangle$ denote the expectation value of the position operator in the state $|\psi\rangle$. What is the value of $\frac{\langle \hat{x} \rangle}{L_0}$?

You may use the form of the position operator in terms of the raising and lowering operators: $\hat{x} = \frac{L_0}{\sqrt{2}}(a + a^\dagger)$.

Answer: 12

Solution:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|287\rangle + |288\rangle)$$

Now $\langle x \rangle = \langle \psi | x | \psi \rangle \; = \frac{1}{2}\left[\langle 287|x|287\rangle + \langle 288|x|288\rangle + \langle 287|x|288\rangle + \langle 288|x|287\rangle\right]$

Since $\langle 287|x|287\rangle = 0$ and $\langle 288|x|288\rangle = 0$

whereas

$$\langle 287|x|288\rangle = \frac{L_0}{\sqrt{2}}\left[\langle 287|a|288\rangle + \langle 287|a^+|288\rangle\right] = \frac{L_0}{\sqrt{2}}\left[\sqrt{288}\langle 287|287\rangle + \sqrt{289}\langle 287|289\rangle\right]$$

$$\Rightarrow \langle 287|x|288\rangle = \frac{L_0}{\sqrt{2}}\left[\sqrt{288}\right]$$

Similarly

$$\langle 288|x|287\rangle = \frac{L_0}{\sqrt{2}}\left[\langle 288|a|287\rangle + \langle 288|a^+|287\rangle\right] = \frac{L_0}{\sqrt{2}}\left[\sqrt{287}\langle 288|286\rangle + \sqrt{288}\langle 288|288\rangle\right]$$

$$\Rightarrow \langle 288|x|287\rangle = \frac{L_0}{\sqrt{2}}\sqrt{288}$$

Now, $\langle x \rangle = \frac{1}{2}\left[0 + 0 + \frac{L_0}{\sqrt{2}}\sqrt{288} + \frac{L_0}{\sqrt{2}}\sqrt{288}\right] = L_0 \times 12$ and $\frac{\langle x \rangle}{L_0} = 12$

Thus correct answer is 12.

Q5. A quantum mechanical particle of mass m is conned in a one dimensional infinite potential well whose walls are located at $x=0$ and $x=1$. The wave function of the particle inside the well is $\psi(x)=\mathcal{N}\left[x\ln x+(1-x)\ln(1-x)\right]$ for some normalization constant $\mathcal{N}$. An experimentalist measures the position of the particle on an ensemble of a large number of identical systems in the same state. The mean of the outcomes is found to be $\frac{1}{n}$, where n is an integer. What is n?

Answer: 2

Q7. The radial part of the electronic ground state wave function of the Hydrogen atom is

$$R_{10}(r)=\frac{1}{\sqrt{\pi a_0^3}}\exp\left(-\frac{r}{a_0}\right),$$

where a_0 is the Bohr radius. If $\langle r\rangle$ and r_{mp} denote the expectation value and the maximum probable value of the radial coordinate, respectively, compute $\frac{8\langle r\rangle}{3r_{mp}}$.

Answer: 4

Solution:

Given $R_{10}(r)=\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$

This is the ground state of H-atom i.e. $n=1$

Now $\langle r\rangle=\frac{1}{2}\left[3n^2-l(l+1)\right]a_0=\frac{3a_0}{2}$ where $r_{mp}=a_0$

$$\therefore\ \frac{8\langle r\rangle}{3r_{mp}}=\frac{8}{3}\times\frac{3a_0/2}{a_0}=4$$

Thus correct answer is 4.

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Solution- Thermodynamics and Statistical Mechanics

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Section A (MCQ)

Correct answer: +1, wrong answer: −1/3.

Q4. An ideal gas initially at pressure P_i undergoes the following sequence of processes:

1. A reversible adiabatic expansion that doubles its volume.
2. A reversible isothermal compression that restores its original volume.
3. A reversible isothermal expansion that doubles its volume.
4. A reversible adiabatic compression that restores its original volume.

If the final pressure of the gas is P_f , which of the following is true?

(A) $P_f = P_i$

(B) $P_f > P_i$

(C) $P_f < P_i$

(D) The relation between P_f and P_i depends on the initial conditions

Ans.: (A)

Solution:

Following four processes are involved

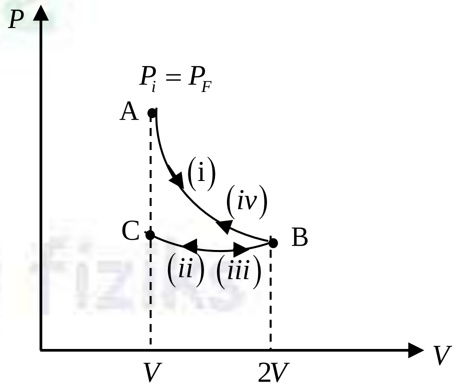
(i) Reversible adiabatic expansion, $V \to 2V$

(ii) Reversible Isothermal compression, $2V \to V$

(iii) Reversible Isothermal expansion, $V \to 2V$

(iv) Reversible Adiabatic compression, $2V \to V$

These four processes are presented on PV-diagram



Note that on same PV diagram

$$\left(\frac{dP}{dV}\right)_{\text{adiabatic}} = \gamma\left(\frac{dP}{dV}\right)_{\text{isothermal}}$$

i.e. adiabatic curve is more steeper than isothermal.

In first step, Reversible adiabatic expression, $V \to 2V$ and pressure changes from $P_i = P_A$ to P_B.

In second step, Reversible Isothermal compression, $2V \to V$, system moves from B to C with relatively lower slope.

In third step, Reversible Isothermal expansion, system retrace path CB and reaches back at B with pressure P_B.

In fourth step, it traces BA by following Reversible Adiabatic compression from $2V \to V$.

$\therefore$ Final pressure $P_F = P_A = P_i$.

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Q5. A quantum oscillator with energy levels

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega, \qquad n = 0,1,2...$$

is in equilibrium at a low enough temperature T so that the occupation of all states with $n \geq 2$ is negligible. What is the mean energy of the oscillator as a function of the inverse temperature $\beta = \left(\frac{1}{k_B T}\right)$?

(A) $\hbar\omega\left[\frac{1}{2}+\frac{1}{1-\exp(\beta\hbar\omega)}\right]$ (B) $\hbar\omega\left[\frac{1}{2}+\frac{1}{1+\exp(\beta\hbar\omega)}\right]$

(C) $\hbar\omega\left[1+\exp(-\beta\hbar\omega)\right]$ (D) $\hbar\omega\left[1-\exp(-\beta\hbar\omega)\right]$

Ans.: (B)

Solution:

The given $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$, $n = 0,1,2$

occupation for $n \geq 2$ is negligible, $\therefore$ consider $n = 0,1$. $E_0 = \frac{\hbar\omega}{2}, E_1 = \frac{3}{2}\hbar\omega$

Partition function is $Q = e^{-\beta E_0} + e^{-\beta E_1} = e^{-\frac{\beta\hbar\omega}{2}} + e^{-\frac{3\beta\hbar\omega}{2}} \Rightarrow Q = e^{-\frac{\beta\hbar\omega}{2}}\left[1+e^{-\beta\hbar\omega}\right]$

$$\ln Q = \ln\left(e^{-\frac{\beta\hbar\omega}{2}}\right) + \ln\left(1+e^{-\beta\hbar\omega}\right) = -\frac{\beta\hbar\omega}{2} + \ln\left(1+e^{-\beta\hbar\omega}\right)$$

$\therefore$ Average energy $\langle E\rangle = -\frac{\partial}{\partial\beta}\ln Q = -\frac{\partial}{\partial\beta}\left[-\frac{\beta\hbar\omega}{2} + \ln\left(1+e^{-\beta\hbar\omega}\right)\right]$

$$\Rightarrow \langle E\rangle = \frac{\hbar\omega}{2} + \frac{1}{1+e^{-\beta\hbar\omega}}\left[-(-\hbar\omega)e^{-\beta\hbar\omega}\right] = \hbar\omega\left[\frac{1}{2}+\frac{1}{e^{\beta\hbar\omega}+1}\right]$$

Q19. Which of the following functions is not a valid thermodynamic function of internal energy U in terms of entropy S, volume V, and number of particles N? Here U_0, α, β, A, B and C are constants.

(A) $\frac{BS^2V^2}{N^3}$ (B) $\left(\frac{AV^2}{N}\right)\exp\left(\frac{\beta VN}{S^2}\right)$

(C) $U_0\exp\left(\frac{\alpha V^2 N}{S^2}\right)$ (D) $\frac{CN^2}{\sqrt{SV}}$

Ans.: (B)

Solution:

$$U = U(S,V,N) \Rightarrow dU = \left(\frac{\partial U}{\partial S}\right)dS + \left(\frac{\partial U}{\partial V}\right)dV + \left(\frac{\partial U}{\partial N}\right)dN$$

$$\because\ dU = TdS - PdV + \mu dN$$

$$\Rightarrow T = \left(\frac{\partial U}{\partial S}\right)_{N,V}, P = -\left(\frac{\partial U}{\partial V}\right)_{S,N}, \mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$$

Thus for $U = U(S,V,N)$ to be a perfect differential

$$-\left(\frac{\partial P}{\partial S}\right)_{N,V} = \left(\frac{\partial T}{\partial V}\right)_{S,N},\ \left(\frac{\partial \mu}{\partial S}\right)_{N,V} = \left(\frac{\partial T}{\partial N}\right)_{S,V},\ \left(\frac{\partial \mu}{\partial V}\right)_{S,N} = -\left(\frac{\partial P}{\partial N}\right)_{S,V}$$

Let us check (B): $\left(\frac{AV^2}{N}\right)\exp\left(\frac{\beta VN}{S^2}\right)$

$$T = \left(\frac{\partial U}{\partial S}\right)_{N,V} = \left(\frac{AV^2}{N}\right)\exp\left(\frac{\beta VN}{S^2}\right)\left(\frac{-2\beta VN}{S^3}\right) = \left(\frac{-2\beta AV^3}{S^3}\right)\exp\left(\frac{\beta VN}{S^2}\right)$$

$$-P = \left(\frac{\partial U}{\partial V}\right)_{S,N} = \left(\frac{2AV}{N}\right)\exp\left(\frac{\beta VN}{S^2}\right) + \left(\frac{A\beta V^2}{S^2}\right)\exp\left(\frac{\beta VN}{S^2}\right)$$

$$\Rightarrow -P = \left(\frac{\partial U}{\partial V}\right)_{S,N} = \exp\left(\frac{\beta VN}{S^2}\right)\left[\frac{2AV}{N} + \frac{A\beta V^2}{S^2}\right]$$

$$\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \left(-\frac{AV^2}{N^2}\right)\exp\left(\frac{\beta VN}{S^2}\right) + \left(\frac{\beta AV^3}{NS^2}\right)\exp\left(\frac{\beta VN}{S^2}\right)$$

$$\Rightarrow \mu = \left(\frac{\partial U}{\partial N}\right)_{S,V} = \exp\left(\frac{\beta VN}{S^2}\right)\left[-\frac{AV^2}{N^2} + \frac{\beta AV^3}{NS^2}\right]$$

$$-\left(\frac{\partial P}{\partial S}\right)_{N,V} = \exp\left(\frac{\beta VN}{S^2}\right)\left(\frac{-2\beta VN}{S^3}\right)\left[\frac{2AV}{N} + \frac{A\beta V^2}{S^2}\right] + \exp\left(\frac{\beta VN}{S^2}\right)\left[0 - \frac{2A\beta V^2}{S^3}\right]$$

$$\Rightarrow -\left(\frac{\partial P}{\partial S}\right)_{N,V} = \exp\left(\frac{\beta VN}{S^2}\right)\left[\left(\frac{-2\beta VN}{S^3}\right)\left\{\frac{2AV}{N} + \frac{A\beta V^2}{S^2}\right\} - \frac{2A\beta V^2}{S^3}\right]$$

$$\Rightarrow -\left(\frac{\partial P}{\partial S}\right)_{N,V} = \exp\left(\frac{\beta VN}{S^2}\right)\left[\frac{-6A\beta V^2}{S^3} \frac{-2A\beta^2 V^3 N}{S^5}\right]$$

$$T = \left(\frac{\partial U}{\partial S}\right)_{N,V} = \left(\frac{-2\beta AV^3}{S^3}\right)\exp\left(\frac{\beta VN}{S^2}\right)$$

$$\left(\frac{\partial T}{\partial V}\right)_{S,N} = \left(\frac{-6\beta AV^4}{S^3}\right)\exp\left(\frac{\beta VN}{S^2}\right) + \left(\frac{-2\beta AV^3}{S^3}\right)\exp\left(\frac{\beta VN}{S^2}\right)\left(\frac{\beta N}{S^2}\right)$$

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$$\Rightarrow\left(\frac{\partial T}{\partial V}\right)_{S,N}=\exp\left(\frac{\beta VN}{S^2}\right)\left[-\frac{6\beta AV^4}{S^3}-\frac{2\beta^2 NAV^3}{S^5}\right]$$

Thus $-\left(\frac{\partial P}{\partial S}\right)_{N,V}\neq\left(\frac{\partial T}{\partial V}\right)_{S,N}$

Thus U is not a perfect differential. Hence $\left(\frac{AV^2}{N}\right)\exp\left(\frac{\beta VN}{S^2}\right)$ is not a valid thermodynamic function.

Q21. Given that the latent heat of liquefaction is 80 Cal/g, what is the change in entropy when 10 g of ice at 0°C is converted into water at the same temperature?

(A) $2.0\,\text{Cal}\,\text{K}^{-1}$ (B) $3.42\,\text{Cal}\,\text{K}^{-1}$

(C) $2.93\,\text{Cal}\,\text{K}^{-1}$ (D) $4.5\,\text{Cal}\,\text{K}^{-1}$

Ans.: (C)

Solution:

Process: Ice $\rightarrow$ Water @ 0°C = 273K

$L=80$ cal/g, $m=10\,\text{g}$

$$\Delta S=\frac{\Delta Q}{T}=\frac{mL}{T}=\frac{10\times 80}{273}=2.93\,\text{Cal}\,\text{k}^{-1}$$

Q22. Consider a system of N noninteracting spin-$\frac{1}{2}$ atoms subjected to a magnetic field with the Hamiltonian given by $H=-g\mu_B B\sum_{i=1}^{N}S_i^z$,

where g is the dimensionless Lande factor, μ_B is the Bohr magneton, B is the strength of the magnetic field, and S_i^z is the z-component of the spin of the i^{th} atom (S_i^z takes values $\pm\frac{1}{2}$). The system is in equilibrium at temperature T. What is the probability that the z-component of the spins corresponding to two given atoms have the same value? Take $\beta=\frac{1}{k_BT}$, where k_B is the Boltzmann constant.

(A) $\dfrac{\exp(-\beta g\mu_B B)}{2+\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}$ (B) $\dfrac{\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}{2+\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}$

(C) $\dfrac{\exp(\beta g\mu_B B)}{2+\exp(-\beta g\mu_B B)+\exp(\beta g\mu_B B)}$ (D) $\dfrac{1}{4}$

Ans.: (B)

Solution:

The possible arrangements of spins for atoms are

S_{z1}	S_{z2}	S_z	$H=-g\mu_B B\sum_{i=1}^{2} S_i^z$
$+\frac{1}{2}$	$+\frac{1}{2}$	1	$-g\mu_B B$
$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
$-\frac{1}{2}$	$-\frac{1}{2}$	-1	$g\mu_B B$

Therefore, partition function is $Q=2+e^{-\beta g\mu_B B}+e^{\beta g\mu_B B}$ ...(i)

Now, there are two arrangements (microstates) where two atoms have same spin. Corresponding probabilities are

$$P\left(+\frac{1}{2},+\frac{1}{2}\right)=\frac{e^{\beta g\mu_B B}}{Q} \quad \text{and} \quad P\left(-\frac{1}{2},-\frac{1}{2}\right)=\frac{e^{-\beta g\mu_B B}}{Q}$$

$\therefore$ Net probability of two atoms having same spin is $P=\dfrac{e^{\beta g\mu_B B}+e^{-\beta g\mu_B B}}{2+e^{-\beta g\mu_B B}+e^{\beta g\mu_B B}}$

Q25. The density of states of a system of N particles at energy E is

$$g(E,N)=\begin{cases}\dfrac{1}{(\hbar\omega)^N}\dfrac{E^{N-1}}{(N-1)!} & \text{for } E\geq 0\\ 0 & \text{for } E<0\end{cases}$$

where $\hbar$ is the Planck's constant and ω is a natural frequency of the system. Taking $\mathrm{k_B}$ to be the Boltzmann constant, compute the temperature of the system at energy E.

(A) $\dfrac{E}{Nk_B}$

(B) $\dfrac{1}{k_B}\left(\dfrac{E}{N}+\dfrac{1}{2}\hbar\omega\right)$

(C) $\dfrac{1}{k_B}\left(\dfrac{E}{N}+\hbar\omega\right)$

(D) $\dfrac{1}{k_B}\sqrt{\left(\dfrac{E}{N}\right)^2+(\hbar\omega)^2}$

Ans.: (A)

Solution: Given, $g(E,N)=\begin{cases}\dfrac{1}{(\hbar\omega)^N}\dfrac{E^{N-1}}{(N-1)!}, & E\geq 0\\ 0, & E<0\end{cases}$

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$$S = k_B \ln \Omega = k_B \ln\left[\frac{1}{(\hbar\omega)^N}\frac{E^{N-1}}{(N-1)!}\right] \Rightarrow S = k_B \ln\left(CE^{N-1}\right)$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right) = \frac{k_B}{CE^{N-1}} C(N-1)\frac{E^{N-1}}{E} = \frac{k_B}{E}(N-1)$$

$$\Rightarrow T = \frac{E}{(N-1)k_B} \approx \frac{E}{Nk_B}$$, where N is very large.

Section B (MCQ)

Correct answer: +3, wrong answer: −1.

Q1. The energy spectrum for a system of spinless noninteracting fermions consists of $(N+1)$ nondegenerate energy levels $0, \varepsilon, 2\varepsilon, ..., N\varepsilon (\varepsilon > 0)$. Let $x = \exp\left(-\frac{\varepsilon}{k_B T}\right)$, where k_B is the Boltzmann constant and T is the temperature. For N identical fermions in thermal equilibrium at temperature T, what is the average occupancy of the highest energy level?

(A) $\frac{x - x^{N+1}}{1 + x^{N+1}}$ (B) $\frac{x - x^{N+1}}{1 - x^{N+1}}$

(C) $\frac{x}{1 - x^N}$ (D) $\frac{x^N}{1 + x^N}$

Ans.: (B)

Q8. The speed distribution of the molecules of an ideal gas in equilibrium at inverse temperature $\beta\left(= \frac{1}{k_B T}\right)$ is found to obey the Maxwell distribution:

$$P(v) = Cv^2 \exp\left(-\frac{1}{2}\beta m v^2\right)$$

where m is the mass of a molecule and C is a normalization constant. Compute $\left(\langle v^4 \rangle\right)^{1/4}$.

(A) $\sqrt{\frac{11k_B T}{\pi m}}$ (B) $\sqrt{\frac{4k_B T}{m}}$ (C) $\sqrt{\frac{3k_B T}{m}}$ (D) $\sqrt{\frac{\sqrt{15}k_B T}{m}}$

Ans.: (D)

Solution:

$$\langle v^4 \rangle = \int_{-\infty}^{\infty} v^4 P(v) \Big/ \int_{-\infty}^{\infty} P(v) = 2\int_0^{\infty} v^4 P(v) \Big/ 2\int_0^{\infty} P(v) = \int_0^{\infty} v^6 e^{-\frac{\beta m v^2}{2}} dv \Big/ \int_0^{\infty} v^2 e^{-\frac{\beta m v^2}{2}} dv \quad ...(i)$$

$\because \int_0^\infty x^{m'} e^{-\alpha x^n} dx = \frac{1}{n} \frac{\sqrt{\left(\frac{m'+1}{n}\right)}}{\alpha^{\frac{m'+1}{n}}}$, here $\alpha = \frac{\beta m}{2}$

$$\langle v^4 \rangle = \frac{\frac{1}{2}\sqrt{\left(\frac{7}{2}\right)}}{\left(\frac{\beta m}{2}\right)^{7/2}} \Bigg/ \frac{1}{2}\frac{\sqrt{\left(\frac{3}{2}\right)}}{\left(\frac{\beta m}{2}\right)^{3/2}} = \frac{\frac{5}{2}\times\frac{3}{2}\sqrt{\pi}}{\frac{1}{2}\sqrt{\pi}} \times \left(\frac{2}{\beta m}\right)^2 = 15\left(\frac{kT}{m}\right)^2$$

$$\langle v^4 \rangle^{1/4} = \left[15\left(\frac{kT}{m}\right)^2\right]^{1/4} = \sqrt{\sqrt{15}\left(\frac{kT}{m}\right)}$$

Q10. The ratio of the molar specific heats of an ideal gas is $\gamma = \frac{c_p}{c_v} = \frac{3}{2}$. It undergoes a reversible isothermal expansion in which its volume doubles. Next, it undergoes a reversible isochoric process such that the change in entropy of the second process is equal to the change in entropy of the first process. What is the ratio of the final temperature to the initial temperature?

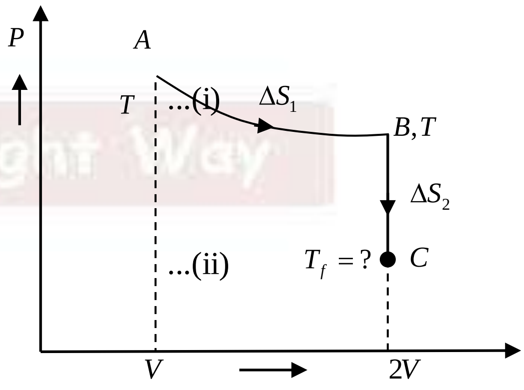
(A) 2 (B) $\sqrt{2}$ (C) 3 (D) $\frac{3}{2}$

Ans.: (B)

Solution: An ideal gas goes through following 2 processes

(i) Reversible process AB Isothermal expression, $V \to 2V$, ΔS_1

(ii) Reversible Isochoric process, ΔS_2, such that $\Delta S_2 = \Delta S_1$



$$\Delta S_1 = \frac{\Delta Q}{T} = \frac{nRT\ln\left(\frac{2V}{V}\right)}{T} \Rightarrow \Delta S_1 = nR\ln 2 = R\ln 2$$

Let $n = 1$ mole

$$\Delta S_2 = \int_B^C dS = \int n\frac{C_V dT}{T} = nC_V \ln\left(\frac{T_f}{T}\right) = C_V \ln\left(\frac{T_f}{T}\right)$$

Now, $\frac{C_P}{C_V} = \frac{3}{2} \quad \Rightarrow C_P = \frac{3}{2}C_V$

$$C_P - C_V = R \Rightarrow \frac{3}{2}C_V - C_V = R \Rightarrow C_V = 2R, C_P = 3R$$

$$\because \Delta S_1 = \Delta S_2 \quad \Rightarrow R\ln 2 = 2R\ln\left(\frac{T_f}{T}\right) = R\ln\left(\frac{T_f}{T_1}\right)^2 \quad \Rightarrow \frac{T_f}{T_1} = \sqrt{2}$$

Section C (NAT)

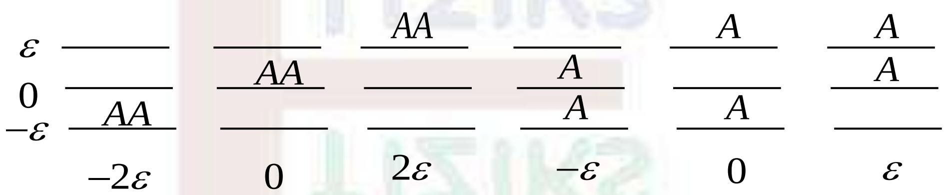
Correct answer: +3, wrong answer: 0.

Q1. A system of two noninteracting identical bosons is in thermal equilibrium at temperature T. The particles can be in one of three states with nondegenerate energy eigenvalues $-\varepsilon, 0$ and ε. The temperature T is such that $\exp\left(-\frac{\varepsilon}{k_B T}\right)=\frac{1}{2}$, where k_B is the Boltzmann's constant. The average energy of the system is found to be $\langle E\rangle=-\frac{n}{35}\varepsilon$, where n is an integer. What is the value of n?

Answer: 36

Solution: $N=2$ Bosons, non-interacting & identical.

Number of accessible microstates are



Energy of Microstate	Degeneracy
-2ε	1
2ε	1
$-\varepsilon$	1
ε	1
0	2

Therefore, partition function is $Q=2+e^{-\beta\varepsilon}+e^{\beta\varepsilon}+e^{-2\beta\varepsilon}+e^{2\beta\varepsilon}$ ...(i)

The average energy

$$\langle E\rangle=-\frac{\partial}{\partial\beta}\ln Q=\frac{-1}{Q}\left(-\varepsilon e^{-\beta\varepsilon}+\varepsilon e^{\beta\varepsilon}-2\varepsilon e^{-2\beta\varepsilon}+2\varepsilon e^{2\beta\varepsilon}\right) \quad ...(ii)$$

Therefore (ii) can be written as

$$\langle E\rangle=\frac{-1\left[-\varepsilon\times\frac{1}{2}+\varepsilon\times 2-2\varepsilon\times\left(\frac{1}{2}\right)^2+2\varepsilon(2)^2\right]}{2+\frac{1}{2}+\frac{2}{1}+\frac{1}{4}+\frac{4}{1}}$$

Given $e^{-\beta\varepsilon}=e^{-\frac{\varepsilon}{kT}}=\frac{1}{2}$

$$\Rightarrow\langle E\rangle=-\frac{36}{35}\varepsilon \qquad \therefore \boxed{n=36}$$

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Section A (MCQ)

Correct answer: +1, wrong answer: −1/3.

Q16. Where does the Fermi level of an n-type semiconductor lie?

(A) Near the valence band maximum.

(B) Near the conduction band minimum.

(C) At the middle of the energy gap.

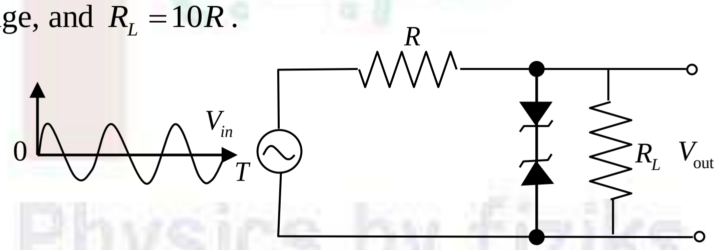
(D) Inside the valence band.

Ans.: (B)

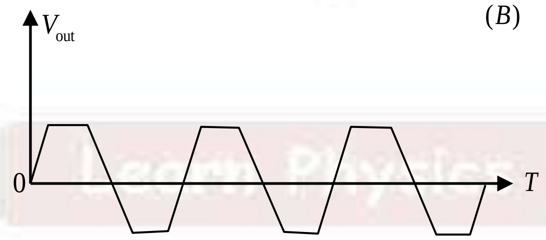
Solution:

Fermi-level lies near the conduction band Minimum.

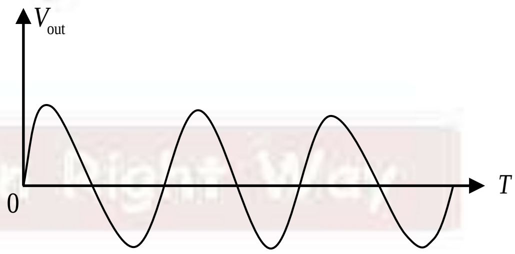
Q20. What is the output waveform of the circuit for the given input signal? Assume that the zener diodes are identical, amplitude of the input voltage V_{in} is twice the zener breakdown voltage, and $R_L = 10R$.



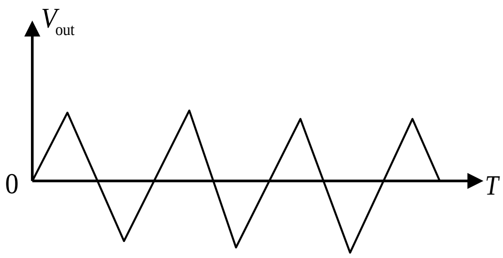
(A)



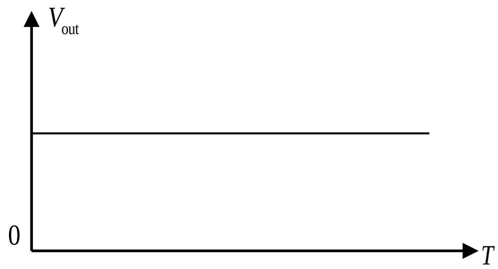
(B)



(C)



(D)



Ans.: (A)

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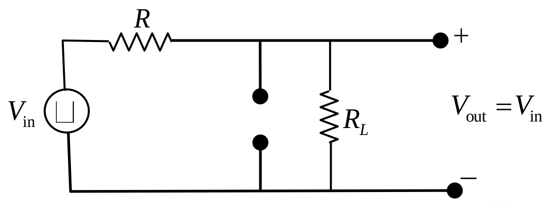
Solution:

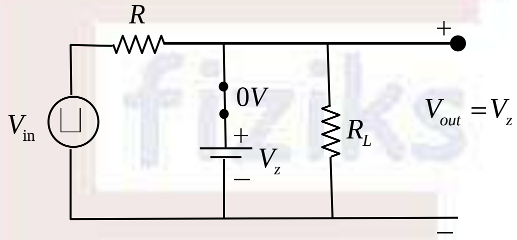
+ve half cycle: $D_1 \to F.B,\ D_2 \to R.B$

Let $V_z \to$ Breakdown, $V_r = 0$

$V_{\text{in}} < V_z,\ D_1 \to F.B\ D_2 \to$ R.F. (OFF) [Zener not in break down region]



$V_{\text{in}} > V_z, D_1 \to F.B\ D_2 \to$ R.F. (ON) [Zener in break down region]



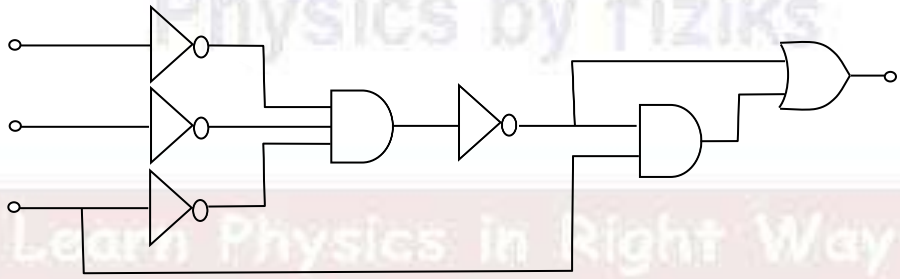
ve half cycle: $D_1 \to R.B.,\ D_2 \to F.B.$

same logic can be applied with negative polarity.

Section B (MCQ)

Correct answer: +3, wrong answer: −1.

Q13. What is the output of the following logic circuit?



(A) $X = A$ AND B AND C

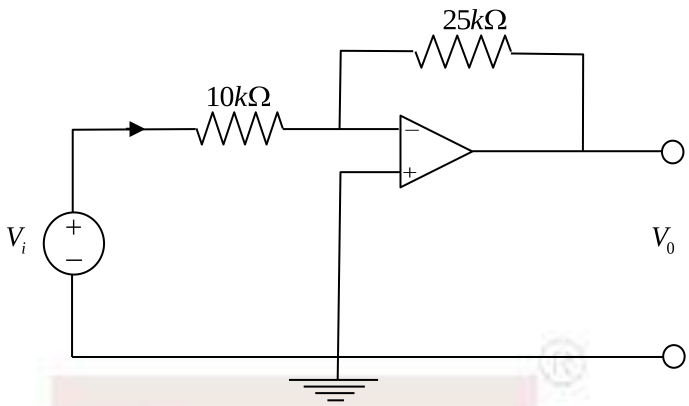
(B) $X = (A \text{OR} C)$ AND $(B \text{OR} C)$

(C) $X = (A \text{OR} C)$ AND $(B \text{OR} C)$ AND C

(D) $X = (\bar{A}$ OR $\bar{B}$ OR $\bar{C})$ AND C

This question is withdrawn since the labels are not shown in the figure. ALL CANDIDATES WILL BE AWARDED 3 MARKS.

Q15. What is the output voltage V_0 and current I in the $10k\Omega$ resistance of the following circuit? $V_i = 0.5V$



(A) $V_0 = -1.25V, I = 20\mu A$ (B) $V_0 = -0.4V, I = 50\mu A$

(C) $V_0 = -0.4V, I = 20\mu A$ (D) $V_0 = -1.25V, I = 50\mu A$

Ans.: (D)

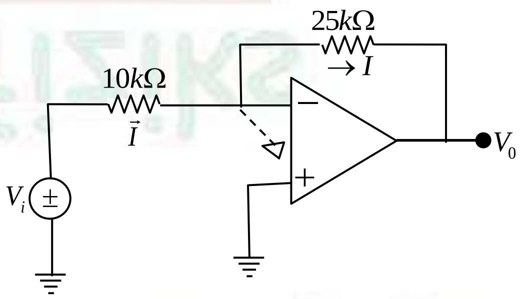
Solution:

$I = \frac{V_i - 0}{10k} = \frac{0.5}{10k} = 0.05\,\text{mA}$

or $I = 50\mu A$

$V_0 = -I \times 25k = -0.05 \times 25$

$\Rightarrow V_0 = -1.25$ volts



Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q9. In an intrinsic semiconductor at 300 K, the number density of electrons is $n_e = 2.5\times10^{20}\,\text{m}^{-3}$. If the mobility of electrons is $\mu_e = 0.4\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$ and the mobility of holes is $\mu_h = 0.2\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$, find the conductivity in units of mho/m. Charge of a proton $e = 1.6\times10^{-19}$ Coulomb.

Answer: 24

Solution:

Given $n_i = 2.5\times10^{20}\,\text{m}^{-3}$, $\mu_e = 0.4\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$, $\mu_h = 0.2\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$, $e = 1.6\times10^{-19}C$

conductivity in units of mho/m is $\sigma_i = n_i e(\mu_e + \mu_h) = 2.5\times10^{20}\times1.6\times10^{-19}(0.4+0.2)$

$\Rightarrow \sigma_i = 2.5\times0.6\times1.6\times10 = 1.5\times16 = 24$

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Solution- Waves and Optics

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Section A (MCQ)

Correct answer: +1, wrong answer: −1/3.

Q15. What is the right sequence of optical components to convert unpolarized light into circularly polarized light?

(A) Light source $\to$ linear polarizer $\to$ half wave plate

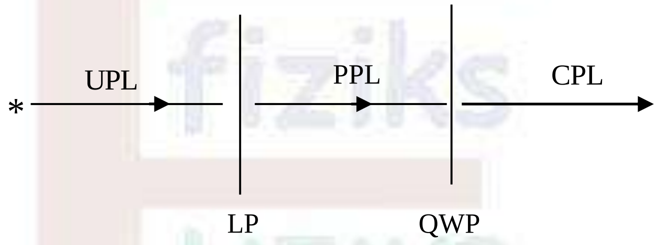
(B) Light source $\to$ quarter wave plate $\to$ half wave plate

(C) Light source $\to$ linear polarizer $\to$ quarter wave plate

(D) Light source $\to$ half wave plate $\to$ quarter wave plate

Ans.: (C)

Solution:



Section B (MCQ)

Correct answer: +3, wrong answer: −1.

Q9. A classical particle undergoing simple harmonic motion is confined to the region $(-a, a)$ on the X-axis. If a snapshot of the particle is taken at a random instant of time, what is the probability that it would be found in the region $\left(\frac{a}{2}, a\right)$?

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Ans.: (A)

Solution:

If $x = A\sin\theta$, then $\int_{\pi/6}^{5\pi/6} d\theta = \omega t \Rightarrow \frac{4\pi}{6} = \frac{2\pi}{T}t \Rightarrow \frac{t}{T} = \frac{1}{3} \Rightarrow P = \frac{t}{T} = \frac{1}{3}$

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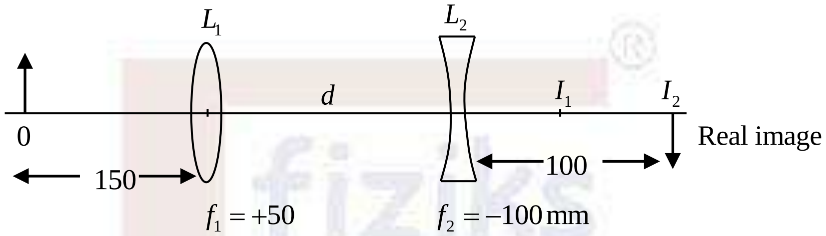
Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q10. An object of height 10 mm is located 150 mm to the left of a thin lens of focal length +50 mm. A second thin lens of focal length –100 mm is to be placed to the right of the first lens such that the real image of the object is located 100 mm to the right of the second lens. What should be the separation in mm between the two lenses?

Answer: 25

Solution:



First lens

$u_1 = -150\,\text{mm},\ f_1 = +50\,\text{mm}$

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{50} - \frac{1}{150} = \frac{2}{150} = \frac{1}{75} \Rightarrow v_1 = 75\,\text{mm}$$

Image I_1 will be on the right side of concave lens because it will act as virtual object for L_2 and its real image will be observed on the same side of lens L_2.

$u_2 = 75 - d$, $v_2 = 100$, $f_2 = -100$

$$\frac{1}{100} - \frac{1}{75-d} = \frac{1}{-100} \Rightarrow d = 25\,\text{mm}$$